5

Structural Design of Foundations

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CONTENTS

5.1 Introduction

Foundation substructures are structural members used to support walls and columns to transmit and distribute their loads to the ground. If these loads are to be properly transmitted, the substructure must be designed to prevent excessive settlement or rotation and to minimize differential settlement. In addition, it should be designed in such a way that the load bearing capacity of the soil is not exceeded and adequate safety against sliding and overturning is assured.

Cumulative floor loads of a building, a bridge, or a retaining wall are supported by the foundation substructure in direct contact with soil. The soil underneath the substructure becomes compressed and deformed during its interaction with the substructure. This deformation is the settlement that may be permanent due to dead loads or may be elastic due to transition live loads. The amount of settlement depends on many factors, such as the type of soil, the load intensity, the ground water conditions, and the depth of substructure below the ground level.

If the soil bearing capacity is different under different isolated substructures or footings of the same building a differential settlement will occur. Due to uneven settlement of supports the structural system becomes over stressed, particularly at column beam joints.

Excessive settlement may also cause additional bending and torsional moments in excess of the resisting capacity of the members, which could lead to excessive cracking and failures. If the total building undergoes even settlement, little or no overstressing occurs.

Therefore, it is preferred to have the structural foundation system designed to provide even or little settlement that causes little or no additional stresses on the superstructure. The layout of the structural supports varies widely depending upon the site conditions. The selection of the type of foundation is governed by the site-specific conditions and the optimal construction cost. In designing a foundation, it is advisable to consider different types of alternative substructures and arrive at an economically feasible solution. In the following sections, the design of a number of commonly used reinforced concrete foundation system types is presented. The reader is advised that, in keeping with the structural design practices in the United States, the English standard measurement units are adopted in the design procedures outlined in this chapter. However, the conversion facility in Table 5.1 is presented for the convenience of readers who are accustomed to the SI units.

5.2 Types of Foundations

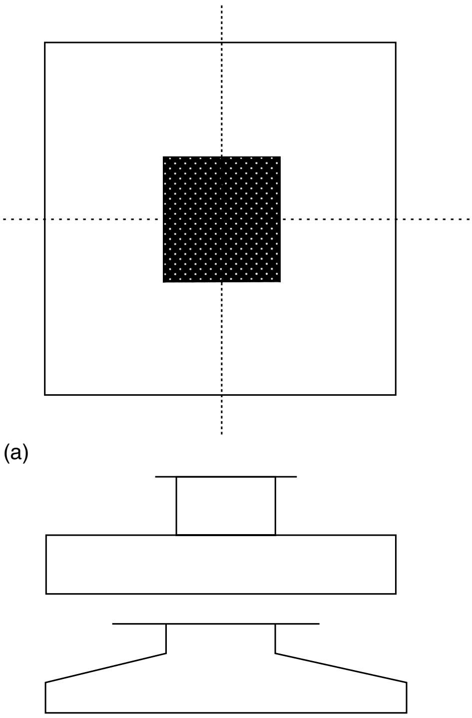
Most of the structural foundations may be classified into one of the following types:

1. *Isolated spread footings:* These footings are used to carry individual columns. These may be square, rectangular, or occasionally circular in plan. The footings may be of uniform thickness, stepped, or even have sloped top (Figure 5.1) and reinforced in both directions. They are one of most economical types of foundation, when columns are spaced at a relatively long distance.
2. *Wall footings:* They are used to support partitions and structural masonry walls that carry loads from floors and beams. As shown in Figure 5.2, they have a limited width and continuous slab strip along the length of the wall. The critical section for bending is located at the face of the wall. The main reinforcement is placed perpendicular to the wall direction. Wall footings may have uniform thickness, be stepped, or have a sloped top.

TABLE 5.1
Unit Conversion Table

From English	To SI	Multiply by	Quantity	From SI	To English	Multiply by
lbs/ft^3	N/m^3	157.1	Force/unit-volume	N/m^3	lbs/ft^3	0.0064
kips/ft^3	kN/m^3	157.1		kN/m^3	kips/ft^3	0.0064
lb-in.	N mm	112.98	Moment; or energy	N mm	lb-in.	0.0089
kip-in.	kN mm	112.98		kN mm	kip-in.	0.0089
lb-ft	N m	1.356		N m	lb-ft	0.7375
kip-ft	kN m	1.356		kN m	kip-ft	0.7375
ft-lb	Joule	1.356		Joule	ft-lb	0.7375
ft-kip	kJ	1.356		kJ	ft-kip	0.7375
sec/ft	sec/m	3.2808	Damping	sec/m	sec/ft	0.3048
Blows/ft	Blows/m	3.2808	Blow count	Blows/m	Blows/ft	0.3048

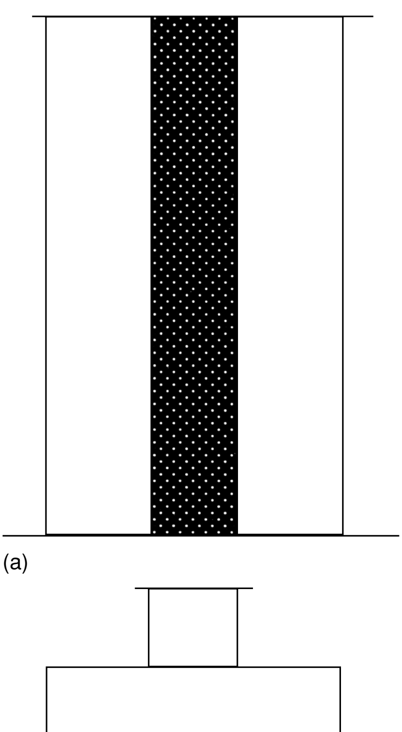
Source: Courtesy of the New York Department of Transportation.



(a)

(b)

FIGURE 5.1
Isolated spread footing: (a) plan; (b) elevation.



(a)

(b)

FIGURE 5.2
Wall footing: (a) plan; (b) elevation.

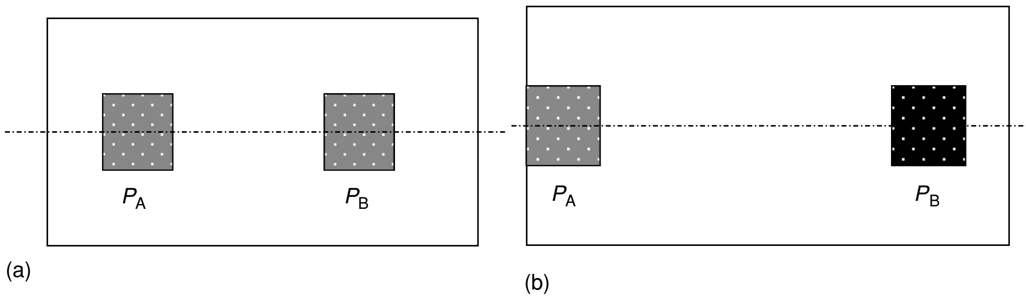


FIGURE 5.3
Combined rectangular footings: (a) equal column loads $P_A = P_B$; (b) unequal column loads $P_B > P_A$.

3. *Combined footings:* This type is used to support two or more column loads. They may be continuous with a rectangular or trapezoidal plan. The combined footing becomes necessary in situations where a wall column has to be placed on a property line that may be common in urban areas. Under such conditions, an isolated footing may not be suitable since it would have to be eccentrically loaded. It is more economical to combine the exterior column footings with an interior column footing as shown in Figure 5.3. The combined footings are more economical to construct in the case of closely spaced columns.
4. *Cantilever footings:* They are basically the same as combined footings except that they are isolated footings joined by a strap beam that transfers the effect of the bending moment produced by the eccentric column load at the exterior column (possibly located along the property line) to the adjacent interior column footing that lies at a considerable distance from it. Figure 5.4 shows an example of such a cantilever footing.
5. *Mat, raft, or continuous footing:* This is a large continuous footing supporting all of the columns and walls of a structure as shown in Figure 5.5. A mat or a raft footing is used when the soil conditions are poor and a pile foundation is not economical. In this case, the superstructure is considered to be theoretically floating on a mat or raft. This type of structure is basically an inverted floor system.

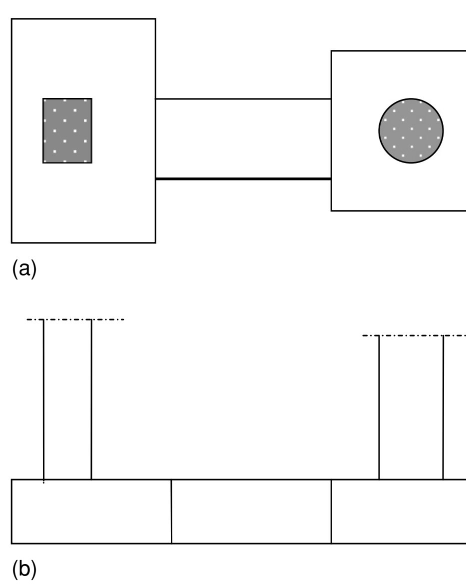


FIGURE 5.4
Strap or cantilever footing: (a) plan; (b) elevation.

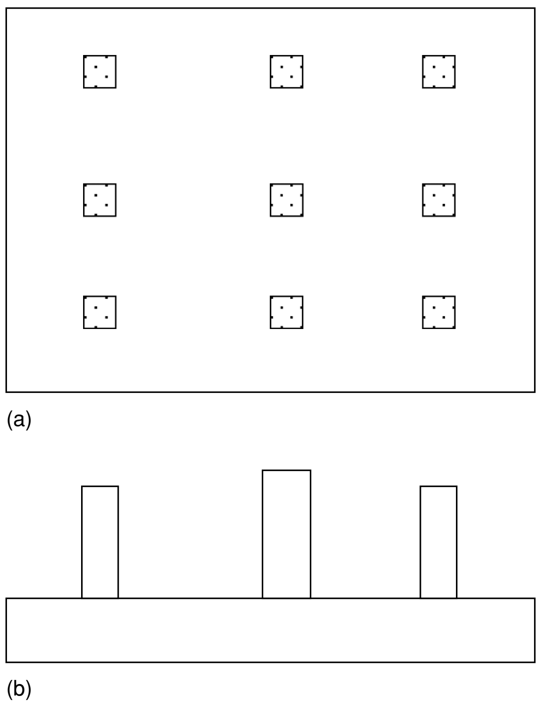


FIGURE 5.5
Raft or mat foundation: (a) plan; (b) elevation.

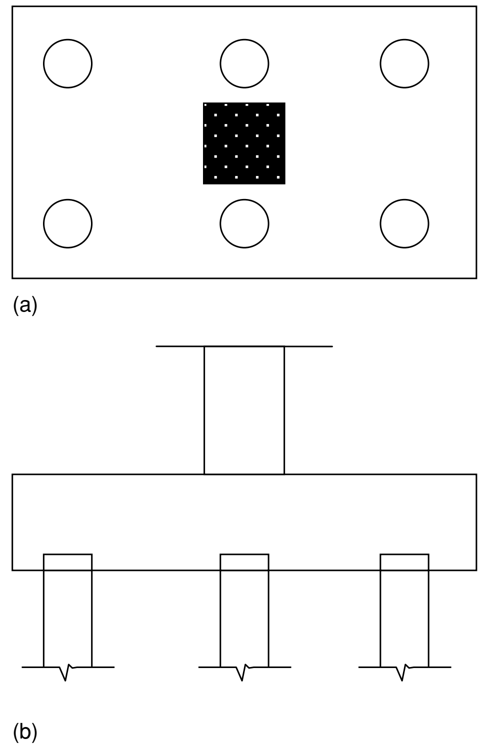


FIGURE 5.6
Pile foundation: (a) plan; (b) elevation.

6. *Pile foundations:* This type of foundation becomes essential when the supporting soil consists of poor layers of material to an extended depth such that an individual or mat foundation is not feasible. Figure 5.6 shows an example of such a footing.

5.3 Soil Pressure Distribution under Footings

The soil pressure distribution under a footing is a function of relative rigidity of the foundation, type, and stiffness of the soil. A concrete footing on cohesionless (sandy) soil will exhibit a pressure distribution similar to the one shown in Figure 5.7(a). The sand near the edges of the rigid footing tends to displace outward laterally when the footing is loaded whereas the rigid footing tends to spread pressure uniformly. On the other hand, the pressure distribution under a rigid footing in cohesive (clay) soil is similar to that shown in Figure 5.7(b). When the footing is loaded, the clay under the footing deflects in a bowl-shaped depression, relieving the pressure under the middle of the footing. However, for design purposes it is customary to assume that the soil pressures are linearly distributed, such that the resultant vertical soil force is collinear with the resultant applied force as shown in Figure 5.7(c).

To simplify the foundation design, footings are assumed to be rigid and the supporting soil layers elastic. Hence, the soil pressure under a footing is determined assuming linearly elastic action in compression. It is also assumed that there is no tensile strength

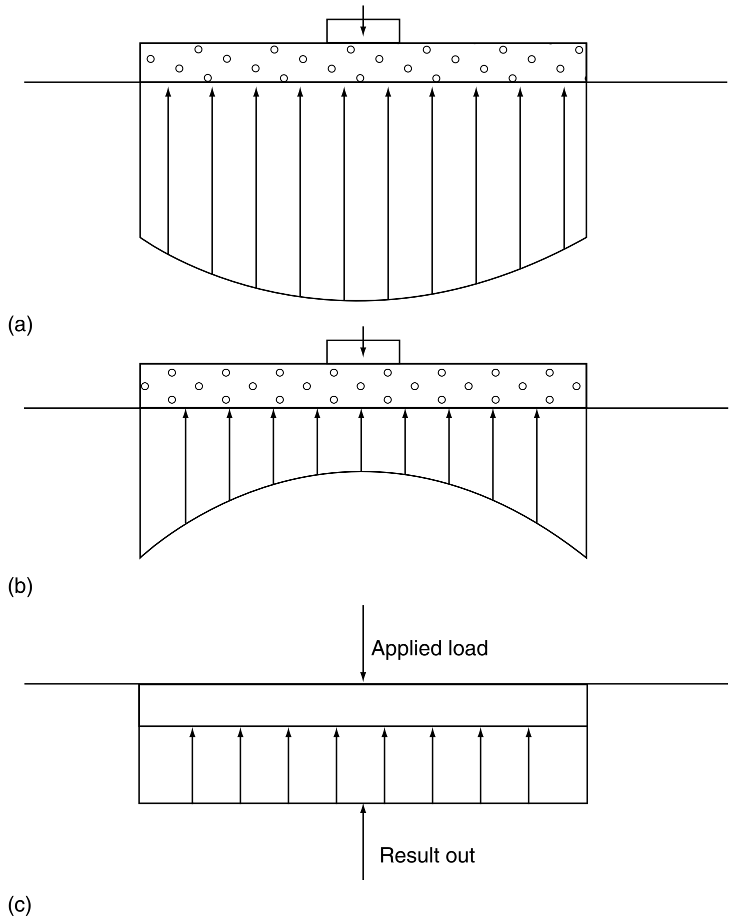


FIGURE 5.7
Pressure distribution under regular footings in different soil types: (a) pressure distribution in sandy soil; (b) pressure distribution in clayey soil; (c) simplified pressure distribution.

across the contact area between the footing and the soil. If a column footing is loaded with axial load P at or near the center of the footing, as shown in Figure 5.8, the contact pressure q under the footing is simply P/A. On the other hand, if the column is loaded with an axial load P and a moment of M, the stress under the footing is

$$q = P/A \pm MY/I \tag{5.1}$$

where q is the soil pressure under the footing at any point, P is the applied load, A is the area of footing $= BD$ (B is the width of footing and D is the length of footing), M is the moment, Y is the distance from centroidal axis to point where the stress is computed, and I is the second moment of area of the footing ($I = BD^3/12$).

If e is the eccentricity of the load relative to the centroidal axis of the area A, the moment M can be expressed as Pe. The maximum eccentricity e for which Equation (5.1) applies is the one that produces $q = 0$ at some point. However, the larger eccentricities will cause a part of the footing to lift of the soil. Generally, it is not preferred to have the footing lifted since it may produce an uneconomical solution. In cases where a larger moment is involved, it is advisable to limit the eccentricity to cause the stress $q = 0$ condition at

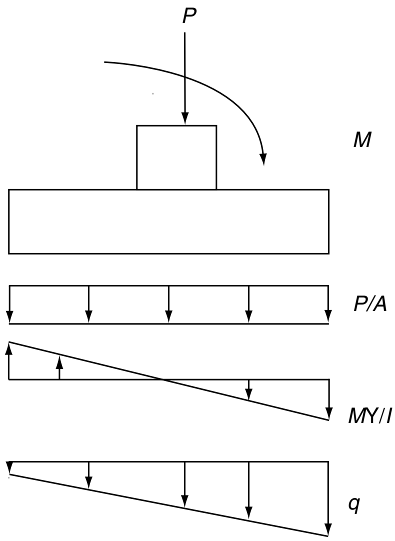


FIGURE 5.8
Pressure distribution under eccentric footings.

the edge of the footing. This will occur when the eccentricity e falls within the middle third of the footing or at a limit $B/6$ or $D/6$ from the centroidal point of footing. This is referred to as the *kern distance*. Therefore, the load applied within the kern distance will produce compression under the entire footing.

5.4 Determination of the Size of Footing

The footings are normally proportioned to sustain the applied factored loads and induced reactions that include axial loads, moments, and shear forces that must be resisted at the base of the footing or pile cap, in accordance with appropriate design requirements of the applicable codes. The base area of the footing or the number and the arrangement of piles are established after the permissible soil pressure or the permissible pile capacity has been determined by the principles of soil mechanics as discussed in Chapters 3, 4, and 6, on the basis of unfactored (service) loads such as dead, live, wind, and earthquake, whatever the combination that governs the specific design. In the case of footings on piles, the computation of moments and shear could be based on the assumption that the reaction from any pile is concentrated at the pile center.

5.4.1 Shear Strength of Footings

The strength of footing in the vicinity of the columns, concentrated loads, or reactions is governed by the more severe of two conditions: (a) wide beam action with each critical section that extends in a plane across the entire width needed to be investigated; and (b) footing subjected to two-way action where failure may occur by ''punching'' along a truncated cone around concentrated loads or reactions. The critical section for punching shear has a perimeter b_0 around the supported member with the shear strength computed in accordance with applicable provision of codes such as ACI 11.12.2. Tributary areas and corresponding critical sections for both wide-beam and two-way actions for isolated footing are shown in Figure 5.9.

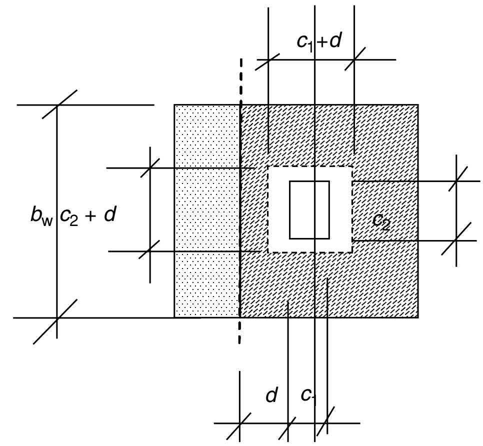


FIGURE 5.9
Tributary area and critical section for shear.

For footing design with no shear reinforcement, the shear strength of concrete V_c (i.e., $V_n = V_c$) is considered as the smallest of the following for two way action.

$$V_c = (2 + 4/\beta_c) b_0 d (f'_c)^{0.5} \quad \text{(ACI formula 11 – 36)} \tag{5.2}$$

$$V_c = (2 + \alpha_s/(b_0/d)) b_0 d (f'_c)^{0.5} \quad \text{(ACI formula 11 – 37)} \tag{5.3}$$

$$V_c = 4 b_0 d (f'_c)^{0.5} \quad \text{(ACI formula 11 – 38)} \tag{5.4}$$

where b_0 is the perimeter of critical section taken at $d/2$ from the loaded area

$$b_0 = 2(c_1 + c_2) + 4d \tag{5.5}$$

d is the effective depth of the footing, β_c is the ratio of the long side to the short side of the loaded area, and $\alpha_s = 40$ for interior columns, 30 for edge columns, and 20 for corner columns.

In the application of above ACI Equation 11-37, an ''interior column'' is applicable when the perimeter is four-sided, an ''edge column'' is applicable when the perimeter is three-sided, and finally a ''corner column'' is applicable when the perimeter is two-sided.

Design Example 5.1
Design for base area of footing (Figure 5.10).

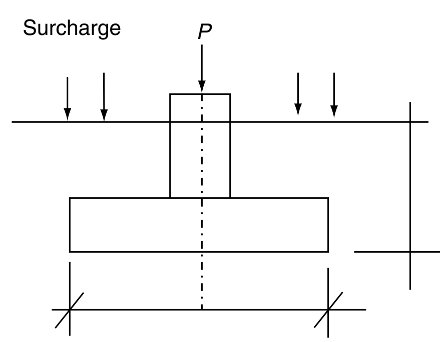


FIGURE 5.10
Illustration for Example 5.1.

Problem Statement
Determine the base area A_f required for a square footing of a three-storey building interior column with the following loading conditions:

Service dead load $=$ 400 kips
Service live load $=$ 280 kips
Service surcharge (fill) $=$ 200 psf
Permissible soil pressure $=$ 4.5 ksf
Column dimensions $= 24 \times 15$ in.

Solution
The base area of the footing is determined using service (unfactored) loads with the net permissible soil pressure.

1. *Determination of base area:*
Let us assume that the bottom of the footing is 4 ft below the ground level:

Average weight of soil $=$ 125.00 pcf
Total weight of surcharge $= (0.125 \times 4) + 0.2 = 0.70$ ksf
Permissible soil pressure $=$ 4.50 ksf
Net permissible soil pressure $= 4.5 - 0.7 = 3.80$ ksf

Given

Service DL $=$ 400.00 kips
Service LL $=$ 280.00 kips

Required base area of footing:

$$A_f = L^2 = \frac{400 + 280}{3.80} = 178.95 \text{ ft}^2$$
$$L = 13.38 \text{ ft}$$

Use a $13'\!-\!6'' \times 13'\!-\!6''$ square footing, $A_f = 182.25 \text{ ft}^2$.

2. *Factored loads and soil reaction:*
To proportion the footing for strength (depth and area of steel rebar) factored loads are used:

Safety factor for DL $=$ 1.40
Safety factor for LL $=$ 1.70

$$P_u = 1.4(400) + 1.7(280) = 1036 \text{ kips}$$
$$q_s = 5.68 \text{ ksf}$$

Example 5.2
For the design conditions of Example 5.1, determine the overall thickness of footing and the required steel reinforcement given that $f'_c = 3{,}000$ psi and $f_y = 60{,}000$ psi (Figure 5.11).

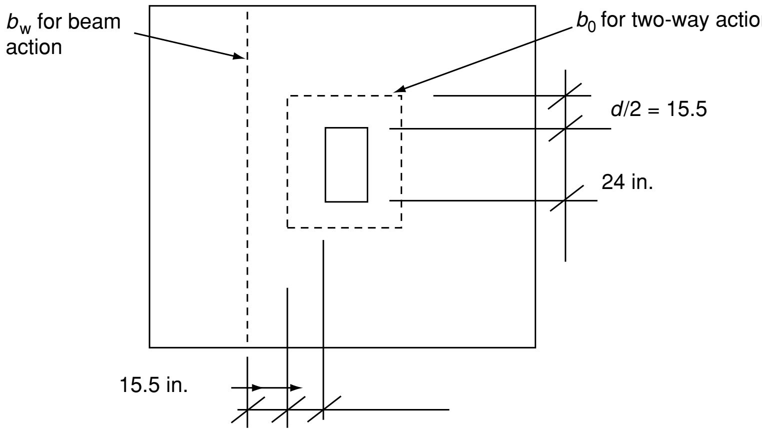


FIGURE 5.11
Illustration for Example 5.2.

Safety factor for DL = 1.40
Safety factor for LL = 1.70

$$P_u = 1.4(400) + 1.7(280) = 1036 \text{ kips}$$
$$q_s = 5.68 \text{ ksf}$$

Solution

Determine depth of shear based on the shear strength without any shear reinforcement. Depth required for shear usually controls the footing thickness. Both wide-beam action and two-way action for strength computation need to be investigated to determine the controlling shear criteria for depth.

Assume overall thickness, h = 36.00 in.
Clear cover to the rebar = 4.00 in.
Assumed rebar diameter = 1.00 in.
Effective diameter, d = 31.50 in.

(a) *Wide-beam action*

$V_u = q_s \times$ tributary area
b_w = 162.00 in.
Tributary area $= 13.5(13.5/2 - 15/24 - 31/12)$
$= 47.81 \text{ ft}^2$

$$V_u = 5.68 \times 47.81 \qquad f_y = 60,000 \text{ psi}$$
$$V_u = 271.79 \text{ kips} \qquad f'_c = 3000 \text{ psi}$$
$$\phi V_n = \phi(2 b_w d (f'_c)^{0.5}) \quad \text{where } \phi = 0.85$$
$$= 0.85(2 \times 162 \times 31 \times 3000^{0.5})$$
$$= 467.61 \text{kips} > V_u \quad \text{OK}$$

(b) *Two-way action*

$V_u = q_s \times$ tributary area

Tributary area $= (13.5 \times 13.5) - (24 + 31)(15 + 31)/144] = 164.68\ \text{ft}^2$

$$V_u = 5.68 \times 164.68 = 936.13\ \text{kips}$$

First determine the minimum of all three conditions as given below:

(i) $2 + 4/\beta_c = 2 + 4/1.6, \quad \beta_c = 24/15 = 1.60$
$= 4.50$

(ii) $2 + \alpha_s d/b_0 = 2 + 40 \times 31/202, \quad b_0 = 2(24 + 31) + 2(15 + 31) = 202$ in.
$= 8.14$

(iii) 4 (control)

$$fV_c = f4b_0d(f_c')0.5 = 0.85 \times 4 \times 202 \times 31 \times 3000 \times 0.5 = 1166.145143\ \text{kips}$$

Hence, the assumed total depth of 36 inch is OK.

(c) *Determination of reinforcement*

$q_s = 5.68$ kips
$b = 13.5$ ft
$d = 31$ in.

(i) Critical section for moment is at the face of column:

$$M_u = 5.68 \times 13.5 \times (13.5/2 - 15/24)^2/2$$
$$= 1439.488\ \text{ft-kips}$$

(ii) Compute the required area of reinforcement A_s as follows:
Compute

$$Q = \frac{12Mu}{\phi f_c' bd^2}$$

where $\phi = 0.9$ and $f_c' = 3$ ksi

$$Q = \frac{12 \times 1439.5}{0.9 \times 3.6 \times 13.5 \times 12 \times 31^2}, \quad f_y = 60\ \text{ksi}$$

$$Q = q(1 - 0.59q) = 0.0411, \text{ when } q = 0.0423, Q = 0.0412$$

$$A_s = qbdf_c'/f_y = 10.62\ \text{in.}^2$$

Check for $\rho_{min} = 0.0018 < 0.002115$ (provided), OK
Use 14 #8 bars each way, $A_s = 11.06$ in.^{2}
Note that less steel is required in the perpendicular direction, but for ease of bar placement use the same number of bars in the other direction.

(iii) Check for development of reinforcement:

The critical section for development of reinforcement is the same as for the moment (at the face of the column). However, the reinforcing bar should resist

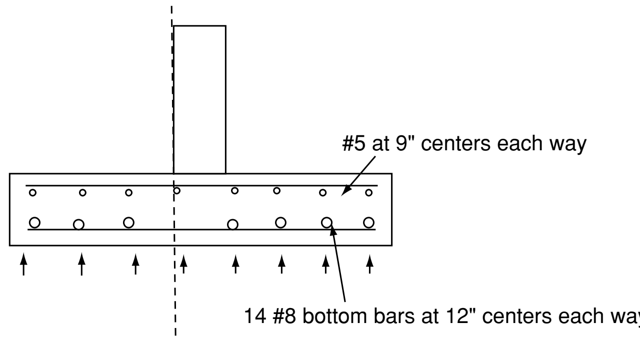


FIGURE 5.12
Determination of reinforcement for Example 5.2.

the moment at a foot distance from the edge of the footing. Hence, it is good practice to have rebars bent up at the end so that it provides a mechanical means of locking the bar in place.

The basic development length

$$l_d = 0.04A_b f_y/(f_c')^{0.5} = 0.04 \times 0.79 \times 60/(3)^{0.5} = 34.62 \text{ in.}$$

Clear spacing of bars $= (13.5 \times 12 - 2 \times 3 - 1.0)/(14 - 1) = 11.92$ in. $> 3D_b$, OK
Hence multiplier, ACI 12.2.3.1-3 $= 1$
Since cover is not less than $2.5D_b$ a reduction factor of 0.8 may be used. Hence, $l_d = 27.69$ in.
In any case l_d should not be less than 75% of the basic development length $=$ 25.96 in.
Hence, provide a development length $=$ 28 in.
But in reality the bar has a hook at the end. Hence it is satisfactory.

(iv) Temperature reinforcement (Figure 5.12):
It is good practice to provide a top layer of minimum distribution reinforcement to avoid cracking due to any rise in temperature caused by heat of hydration of cement or premature shrinkage of concrete.
It is advised to provide at least the minimum area of steel required in both directions.

$$A_s \min = 0.11A_g/2f_y = 0.11 \times 12 \times 36/(2 \times 60) \text{ (AASHTO LRFD provision)}$$
$$= 0.40 \text{ in.}^2/\text{ft}$$

Area of #5 $= 0.31$ in.^{2}

Provide #5 at 9 in. centers, $A_s = 0.41$ in.^{2}/ft

Use #5 bars at 9 in. centers in both directions.

5.5 Strip or Wall Footings

A wall footing generally has cantilevers out on both sides of the wall as shown in Figure 5.13. The soil pressure causes the cantilever to bend upward and, as a result, reinforcement is required at the bottom of the footing, as shown in Figure 5.13.

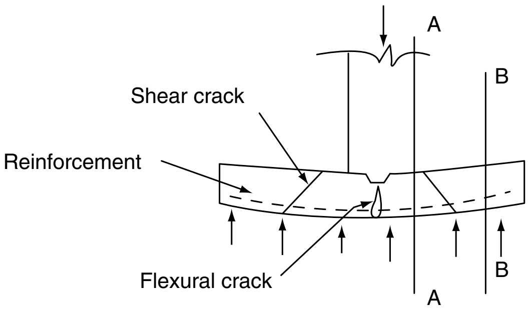


FIGURE 5.13
Structural action in a wall footing.

The critical sections for design for flexure and anchorage are at the face of the wall (section A–A in Figure 5.13). One-way shear is critical at the section at a distance d from the face of the wall (section B–B in Figure 5.13).

Example 5.3

A 8-in. thick wall is a part of a vertical load carrying member of an eight-storey condominium and hence carries seven floors and the roof. The wall carries a service (unfactored) dead load of 1.5 kips per foot per floor including the roof and a service live load of 1.25 kips per foot per floor. The allowable soil net bearing pressure is 5.0 ksf at the level of the base of the footing, which is 5 ft below the ground surface. The floor-to-floor height is 10 ft including the roof. Design the wall footing assuming $f'_c = 3{,}000$ psi and $f_y = 60{,}000$ psi.

Solution

(1) Estimate the total service load. Consider 1-ft width of the wall
Dead load from self weight of the wall $W_{d1} = (8 \times 10 + 5\text{: height}) \times (8/12\text{: thickness wall}) \times (0.15 \text{ kips/ft})$

$$W_{d1} = 8.50 \text{ kips/ft}$$

Dead load from floors

$$\begin{aligned} W_{d2} &= 8 \times 1.5 = 12.00 \text{ kips/ft} \\ \text{Total DL} &= W_{d1} + W_{d2} = 8.5 + 12.0 \\ &= 20.50 \text{ kips/ft} \\ \text{Liveload} &= 8 \times 1.25 \\ &= 10.00 \text{ kips/ft} \end{aligned}$$

Note that the net bearing pressure at the footing level is given, and hence the self-weight of the footing does not need to be considered.

(2) Compute the width of the wall

$$\text{Width required} = \frac{20.5 \text{ kips} + 10.0 \text{ kips}}{5 \text{ ksf}} = 6.1 \text{ ft}$$

Try a footing 6 ft 4 in. wide; $w = 6.33$

$$\text{Factored net pressure } q_u = \frac{1.4 \times 20.5 + 1.7 \times 10.0}{6.33} = 7.22 \text{ ksf}$$

In the design of the concrete and reinforcement, we will use $q_u = 7.22$ ksf.

(3) Check for shear
Shear usually governs the thickness of footing. Only one-way shear is significant for a wall footing. We need to check it at a distance d away from the face of the wall (section B–B in Figure 5.13).
Now let us assume a thickness of footing $=$ 16 in.

$$\begin{aligned} d &= 16 - 3 \text{ (cover)} - 0.5 \text{ (bar diameter)} \\ &= 12.5 \text{ in.} \end{aligned}$$

Clear cover (since it is in contact with soil) $=$ 3 in.

$$\begin{aligned} \phi &= 0.85 \\ f'_c &= 3000 \text{ psi} \\ f_y &= 60,000 \text{ psi} \\ V_u &= 7.22 \text{ ksf} \times (21.50/12 \times 1) \text{ ft}^2 = 12.9 \text{ kips/ft} \end{aligned}$$

$$\begin{aligned} \phi V_c &= \phi(2f_c'^{0.5}\, b_w d) = 0.85 \times 2 \times 3{,}000^{0.5} \times 12 \times 12.5/1{,}000 \\ &= 13.97 \text{ kips/ft} \end{aligned}$$

Since $\phi V_c > V_u$ the footing depth is satisfactory.

(4) Design of reinforcement
The critical section for moment is at the face of the wall section A–A in Figure 5.13. The tributary area for moment is shown shaded in Figure 5.14.

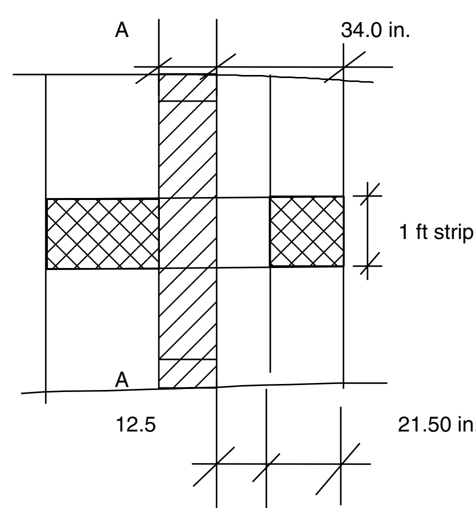


FIGURE 5.14
Plan view of footing (Example 5.3).

$$M_u = \frac{7.22(43.02/12)^2 \times 1}{2} = 29.01 \text{ ft-kips/ft}$$
$$M_u = \phi M_n = \phi A_s f_y jd$$

Let us assume $j = 0.9$, $jd = 11.25$

$$A_s = \frac{12,000\ M_u}{\phi f_y jd}, \quad \text{where } \phi = 0.9$$
$$A_s = 0.57 \text{ in.}^2/\text{ft}$$

From ACI sections 10.5.3 and 7.12.2
Minimum $A_s = 0.0018bh = 0.0018 \times 12 \times 16 = 0.35 \text{ in.}^2/\text{ft}$
Spacing of #5 bars at 6-in. centers, $A_s = 0.62 \text{ in.}^2/\text{ft}$; provide #5 bars at 6-in. centers.
Maximum spacing allowed in the ACI section 7.6.5 = $3h$ or 18 in.
Now compute

$$a = Asfy/0.85f'_c b$$
$$= \frac{0.62 \times 60{,}000}{0.85 \times 3{,}000 \times 12} = 1.22 \text{ in.}$$

$$\phi M_n = \frac{0.9 \times 0.62 \times 60{,}000(12.5 - 1.22/20)}{12000.00} = 33.18 \text{ ft-kips} > M_u$$

The design is satisfactory (Figure 5.15).

(5) Check the development length

Basic development length for #5 bars in 3,000 psi concrete = l_{db}
ACI code provision: furnish the following criterion:

$$l_{db} = 0.04 A_b f_y / f_c'^{0.5} = 0.04 \times 0.31 \times 60{,}000/3{,}000^{0.5}$$
$$= 14 \text{ in.}$$

ACI 12.2.3. (a) No transverse steel (stirrups): does not apply

(b) and (c) Do not apply if flexural steel is in the bottom layer
(d) Cover = 3 in. and clear spacing = 5.325 in. $>3d_b$ and therefore 12.3.3.1 (d) applies $\times 1.0$
ACI 12.2.3.4. Applies with a factor of 0.8

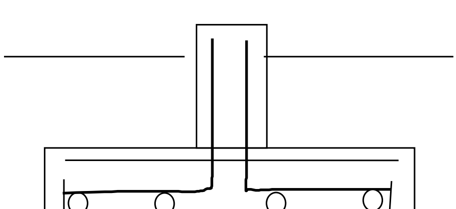


FIGURE 5.15
Configuration of reinforcement layout (Example 5.3).

ACI 12.2.4. Bottom bar, $\times 1.0$; normal weight concrete, $\times 1.0$; and standard deformed bar, $\times 1.0$

$$l_{db} = 14 \times 1.0 \times 0.8 = 10.87 \text{ in.}$$

ACI 12.2.3.6. $l_{db} => 0.03 d_b f_y / f_c'^{0.5} = 0.03 \times 5/8 \times 60{,}000/3{,}000^{0.5} = 21$ in.

The length of the bar from the maximum stress point at the face of the wall is 34 − 3 = 31 in., which is >21 in. and hence is satisfactory.

(6) Temperature and shrinkage ACI 7.12.2

$$\begin{aligned} A_s &= 0.0018bh = 0.0018 \times 12 \times 16 \\ &= 0.35 \text{ in.}^2 \end{aligned}$$

At least two thirds of this should be placed as top reinforcement in the transverse direction as the concrete exposed to the dry weather (low humidity and high temperature) until covered.

Provide #5 at 12-in. centers; $A_s = 0.31$ in.^{2}/ft and is thus satisfactory

$$\begin{aligned} A_s &= 0.0018bh = 0.0018 \times 76 \times 12 \\ &= 1.64 \text{ in.}^2/\text{ft} \end{aligned}$$

This reinforcement should be divided between top and bottom layers in the longitudinal direction (Figure 5.16).

Provide 6 #4 at 14-in. centers both top and bottom

$$\begin{aligned} A_s &= 6 \times 2 \times 0.2 = 2.40 \text{ in.}^2/\text{ft} \\ &> 1.64 \text{ in.}^2/\text{ft and hence is satisfactory} \end{aligned}$$

Example 5.4

You have been engaged as an engineer to design a foundation for a three-storey office building. It is required that the footings are designed for equal settlement under live loading. The footings are subjected to dead and live loads given below. However, statistics show that the usual load is about 50% for all footings. Determine the area of footing required for a balanced footing design. It is given that the allowable net soil bearing pressure is 5 ksf (Table 5.2–Table 5.4).

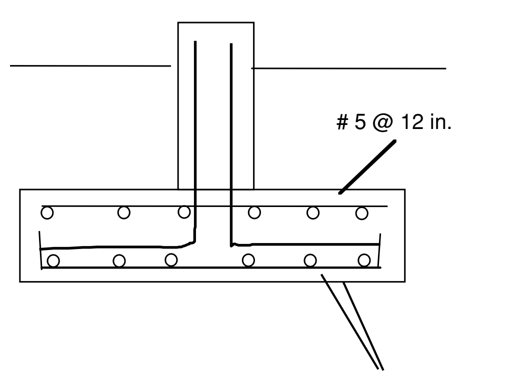


FIGURE 5.16
Details of reinforcement (Example 5.3).

Solution

TABLE 5.2

Details of Loads (Example 5.4)

Footing number	1	2	3	4	5	6
Dead load (kips)	130	170	150	190	140	200
Live load (kips)	160	210	200	180	210	250

TABLE 5.3

Computation of Load Ratios (Example 5.4)

Footing number	1	2	3	4	5	6
Ratio	1.231	1.235	1.333	0.947	1.500	1.250

TABLE 5.4

Computation of Factored Loads (Example 5.4)

Footing number	1	2	3	4	5	6
Usual load (DL + 0.5LL) (kips)	210	275	250	280	245	325

(a) Determine the footing that has the largest ratio of live load to dead load. Note that this ratio is 1.5 for footing #5.
(b) Calculate the usual load for all footings.
(c) Determine the area of footing that has the highest ratio of LL to DL (footing #5)

$$\text{Area of footing \# 5} = (\text{DL} + \text{LL})/(\text{allow soil pressure})$$

$$= \frac{140 + 210}{5} = 70 \text{ ft}^2$$

$$\text{Usual soil pressure under footing \#5} = (\text{usual load})/(\text{area of footing})$$

$$= \frac{245}{70} = 3.5 \text{ ksf}$$

(e) Compute the area required for each footing by dividing its usual load by the usual soil pressure footing #5. For example, for footing #1,

$$\text{Required area} = 210/3.5 = 60 \text{ ft}^2$$

For other footings, the computations are shown below:

TABLE 5.5

Computation of Areas (Example 5.4)

Footing number	1	2	3	4	5	6
Usual load (DL $\pm$ 0.5LL) (kips)	210	275	250	280	245	325
Required area (ft^2)	60	78.57	71.43	80.00	70.00	92.86

TABLE 5.6

Computation of Soil Pressure (Example 5.4)

Footing number	1	2	3	4	5	6
Soil pressure (ksf)	4.83	4.84	4.90	4.63	5.00	4.85

(f) For verification, compute the soil pressure under each footing for the given loads.

Note that the soil pressure under footing #5 is 5 ksf, whereas under other footings it is less than 5 ksf.

5.6 Combined Footings

Combined footings are necessary to support two or more columns on one footing as shown in Figure 5.17. When an exterior column is relatively close to a property line (in an urban area) and a special spread footing cannot be used, a combined footing can be used to support the perimeter column and an interior column together.

The size and the shape of the footing are chosen such that the centroid of the footing coincides with the resultant of the column loads. By changing the length of the footing, the centroid can be adjusted to coincide with the resultant loads. The deflected shape and the reinforcement details are shown for a typical combined footing in Figure 5.17(b) and an example is given below to further illustrate the design procedure of such combined footings.

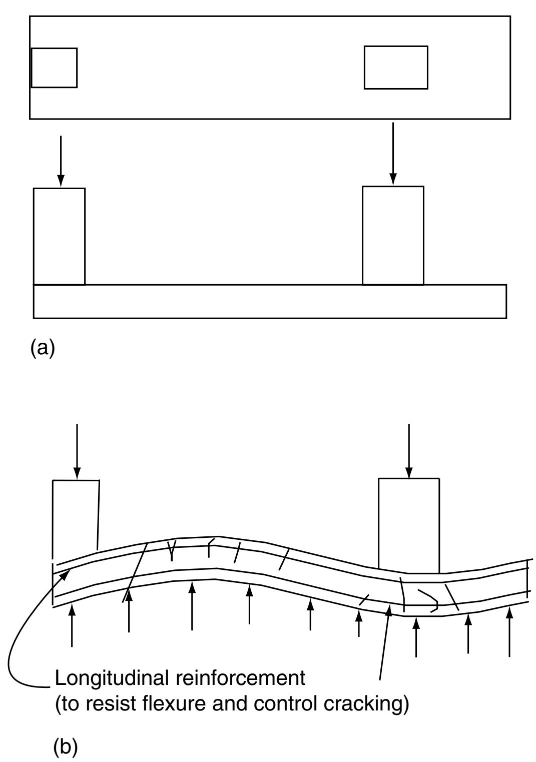


FIGURE 5.17
Typical combined footing: (a) under unloaded conditions; (b) under loaded conditions.

Example 5.5

In a three-storey building, an exterior column having a section of 24 in. × 18 in. carries a service dead load of 70 kips and service live load of 50 kips at each floor. At the same time the nearby 24 square interior column carries a service dead load of 100 kips and a live load of 80 kips at each floor. The architects have hired you as an engineer to design the footing for these columns. The specific site condition dictates that a combined footing be chosen as an economical solution. Both columns carry three floors above them and are located 18 ft apart. The geotechnical engineer has advised that the soil bearing pressure at about 4 ft below the ground is 5 ksf. The ground floor, which is going to be slab on grade with 6 in. concrete, supports a service live load of 120 psf. The soil below this floor is well compacted. The available concrete strength $f'_c = 3000$ psi and steel strength $f_y = 60{,}000$ psi. Design an economical footing.

Solution

Step 1. Determine the size and the factored soil pressure (Figure 5.18)

Allowable soil pressure = 5.00 ksf

Depth of soil above the base = 4.00 ft

Assume the unit weight of soil = 120.00 pcf

Allowable net soil pressure = 5 − 0.5 × 0.15 − 3.5 × 0.12 − 0.120 ksf = 4.33 ksf

$$\text{Area required for the footing} = \frac{3\times(70+50)+3\times(100+80)}{4.33}\,\text{ft}^2 = 208.09\ \text{ft}^2$$

The resultant of the column load located at X from the center of external column

$$X = \frac{18\times 3(100+80)}{3(70+50)+3(100+80)} = 10.80\ \text{ft}$$
$$= 129.60\ \text{in.}$$

Distance from the external face of the exterior column = 129.6 + 9 in. = 139 in. = 11.55 ft
Width of the footing = 208.09/(2 × 11.55) = 9 ft
Factored external column load = 3 × (1.4 × 70 + 1.6 × 50) kips = 534.00 kips
Factored internal column load = 3 × (1.4 × 100 + 1.6 × 80) kips = 804.00 kips
Total = 1338.00 kips

Now we can compute the net factored soil pressure which is required to design the footing.

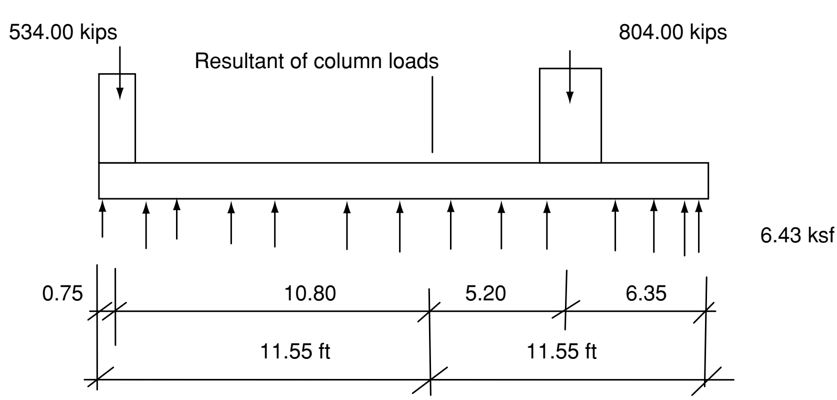


FIGURE 5.18
Factored load and factored net soil pressure (Example 5.5).

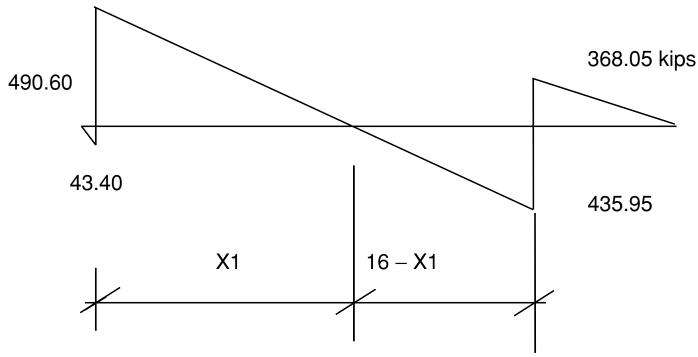


FIGURE 5.19
Shear force diagram (Example 5.5) (forces in kips).

$$q_{nu} = \frac{1338.0}{11.55 \times 9 \times 2.0} = 6.43 \text{ ksf}$$

Step 2. Draw the shear force and bending moment diagram (Figure 5.19)
If shear is zero at $X1$

$$X1 = \frac{16 \times 490.6}{490.6 + 435.95} = 8.47 \text{ ft}$$

Step 3. Determine the thickness of footing (Figure 5.20)

In this case, the footing acts as a wide (9 ft) heavy duty beam. It is better to determine the thickness based on the moment and check it for shear. We can start with minimum reinforcement of $200/f_y$ as per ACI Section 10.5.1.

$$\frac{200}{f_y} = \rho = 0.0033$$

$$f'_c = 3{,}000 \text{ psi}$$
$$f_y = 60{,}000 \text{ psi}$$
$$\phi = 0.9$$
$$\frac{M_n}{bd^2 f'_c} = \frac{\rho f_y}{f'_c}[1 - 0.59\rho f_y/f'_c]$$

and

$$M_n = \frac{M_u}{\phi} = \frac{2063.30}{0.90} = 2292.51 \text{ ft-kips}$$
$$\frac{\rho f_y}{f'_c}\left[1 - 0.59\rho f_y/f'_c\right] = \frac{0.0033 \times 60{,}000}{3000}(1 - 0.59 \times 0.0033 \times 6{,}000/3{,}000) = 0.064$$

Therefore,

$$\frac{M_n}{bd^2 f'_c} = 0.064044 = \frac{12{,}000 \times 2292.51}{3{,}000 \times 9 \times 12 \times d^2}$$

$$d^2 = 1325.77 \text{ and hence } d = 36.41 \text{ in.}$$

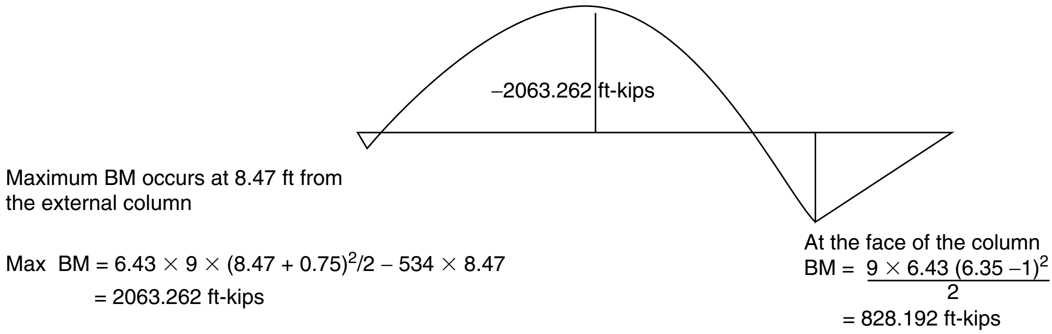


FIGURE 5.20
Bending moment diagram (Example 5.5).

Now we will choose the total depth $h = 40$ in. and the area of steel will be more than minimum steel required.

$h = 40.00$ in.
$d = 40 - 4$ (cover to reinforcement of 3 in. and 1 in. for reinforcement)
$= 36.00$ in.

Step 4. Check two-way shear at the interior column

The critical perimeter is a square with sides 24 in. + 36 in. = 60.00 in.

Therefore,

$$b_0 = 4 \times 60 = 240.00 \text{ in.}$$

The shear, V_u, is the column load corrected for (minus) the force due to the soil pressure on the area enclosed by the above perimeter, see Figure 5.21.

$$V_u = 804 - 6.43 \times (60/12)^2$$
$$= 643.25 \text{ kips}$$
$$\phi = 0.85$$

ϕV_c is the smallest of the following:

(a) $\phi V_c = \phi(2 + 4/\beta) b_0 d f_c'^{0.5} = 0.85(2 + 4)240 \times 36 \times (3000)^{0.5} = 2413.5$ kips $(\beta = 1)$

(b) $\phi V_c = \phi(2 + d\alpha/b_0) b_0 d f_c'^{0.5} = 0.85(2 + 40 \times 36/240)240 \times 36 \times (3000)^{0.5}$ where $\alpha = 40$ $\phi V_c = 3217.98$ kips

(c) $\phi V_c = \phi 4 b_0 \mathrm{d} f_c'^{0.5} = 0.85 \times 4 \times 240 \times 36 \times (3000)^{0.5} = 1608.99$ kips

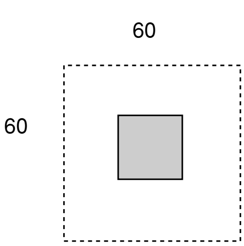


FIGURE 5.21
Shear perimeter (Example 5.5 Interior Column).

Since V_u is less than the smallest value of 1609 kips of all three above conditions, the depth of the footing is adequate to support the interior square column load.

Step 5. Check two-way shear at the exterior column

The critical perimeter is a rectangle with sides of $24 + 36 = 60.00$ in. and a width of $18 + 36/2 = 36.00$ in. The shear, V_u, is the column load minus the force due to the soil pressure on the area enclosed by the above perimeter

$$\begin{aligned}
b_0 &= 60 + 2 \times 36 = 132.00 \text{ in.}\\
b_1 &= 36.00 \text{ in.}\\
b_2 &= 60.00 \text{ in.}\\
V_u &= 534.0 - 6.43 \times (60 \times 36/144)\\
&= 437.55 \text{ kips}\\
\phi &= 0.85
\end{aligned}$$

The shear perimeter around the column is three-sided, as shown in Figure 5.22. The distance from line B–C to the centroid of the shear perimeter is given by X_2

$$\begin{aligned}
X_2 &= 2(b_1 d^2/2)/(2b_1 d + b_2 d)\\
&= 2\left(\frac{36 \times 36^2}{2}\right)/(2 \times 36 \times 36 + 60 \times 36)\\
&= 9.82 \text{ in.}
\end{aligned}$$

The force due to the soil pressure on the area enclosed by the perimeter is

$$= 6.43 \times (60 \times 36/144) = 96.45 \text{ kips}$$

Then, summing up the moment about the centroid of the shear perimeter gives

$$\begin{aligned}
M_u &= 534 \times (36 - 9.82 - 9) - 96.45 \times (18 - 9.82)\\
&= 8385.16 \text{ in.-kips}
\end{aligned}$$

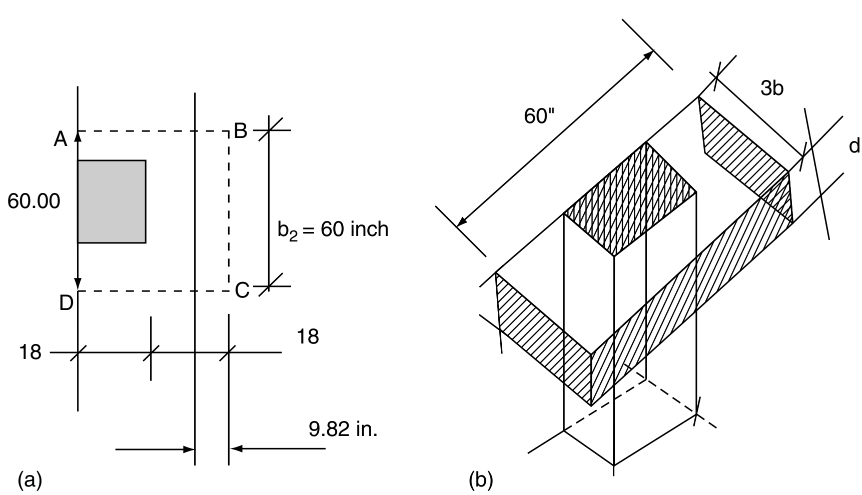


FIGURE 5.22
Illustration of the shear perimeter (Example 5.5 (a) Exterior column; (b) interior column).

This moment must be transferred to the footing through shear stress and flexure. The moment of inertia of the shear perimeter j_c is

$$j_c = 2[[36 \times 36^3/12] + [36 \times 36^3/12] + [36 \times 36][18 - 9.82^2]] + [(60 \times 36) \times 9.82^2]$$
$$j_c = 646590.5 + 208293.984 = 854884.5 \text{ in.}^4$$

The fraction of moment transferred by flexure is

$$\gamma_f = \frac{1}{1 + 2(b_1/b_2)^{0.5}/3} = \frac{1}{1 + 2(36/60)^{0.5}/3} = 0.659$$

The fraction transferred by shear $= (1 - \gamma_f) = (1 - 0.659) = 0.341$

The shear stress due to the direct shear and due to moment transfer will be additive at points A and D in Figure 5.22, giving the largest shear stresses on the critical shear perimeter

$$V_u = \frac{V_u}{b_0 d} + \frac{(1 - \gamma_f)M_u C}{j_c}$$

$$= \frac{437.55}{132 \times 36} + \frac{0.341 \times 8385.16 \times (36 - 9.82)}{854884.5}$$
$$= 0.092078 + 0.087564$$
$$= 0.180 \text{ kips}$$

Now we will compute ϕV_c from the following condition using ACI equations 11-36 to 11-38, which is the smallest of the following:

(a) $\phi V_c = \phi(2 + 4/\beta)f_c'^{0.5} = 0.85(2 + 4/(24/18)) \times (3000)^{0.5} = 0.2328$ ksi
(b) $\phi V_c = \phi(2 + d\alpha/b_0)f_c'^{0.5} = 0.85(2 + 30 \times 36/132)(3000)^{0.5}$ where $\alpha = 30\ \phi V_c$
$= 0.474$ ksi
(c) $\phi V_c = 4f_c'^{0.5} = 0.85 \times 4 \times (3000)^{0.5} = 0.186$ ksi

Since V_u is less than the smallest value of 0.186 ksi of all the above three conditions, the depth of the footing is adequate to support the interior square column load.

Step 6. Check one-way shear

The shear force diagram shows that the maximum shear is near the exterior column and, hence, one-way shear is critical at a distance d from the face of the exterior column:

$$V_u = 490.60 - (36 + 9)/12 \times (6.43 \times 9) = 490.60 - 217.01 = 273.59 \text{ kips}$$

$$\phi V_c = 0.85 \times 2(3000)0.5 \times (9 \times 12) \times 36/1000 = 362.02 \text{ kips}$$

Since V_u is less than ϕV_c, the depth is adequate to support the required shear condition.

Step 7. Design the flexural reinforcement

(a) The mid-span (negative moment):
$b = 108$ in.
$d = 36$ in.
$f_c' = 3$ ksi
$f_y = 60$ ksi
$M_u = 2063.3$ ft-kips

Per ACI handbook,

$$\frac{12M_u}{0.9f_c'bd^2} = \frac{12 \times 2063.3}{0.9 \times 3 \times 108 \times 36^2} = Q = 0.0655$$
$$= q(1 - 0.59q) = 0.0671 \quad \text{for } q = 0.0700$$
$$A_s = bdqf_c'/f_y = 13.61 \text{ in.}^2$$

Provide #8 bars at 6-in. spacing at the top. Total area $A_s = 14.137\,\text{in.}^2$

(b) At the face of interior column (positive moment):

$b = 108$ in.
$d = 36$ in.
$f_c' = 3$ ksi
$f_y = 60$ ksi
$M_u = 828.192$ ft-kips

Per ACI handbook,

$$\frac{12M_u}{0.9f_c'bd^2} = \frac{12 \times 2063.3}{0.9 \times 3^{0.5} \times 108 \times 36^2} = Q = 0.0455$$
$$= q(1 - 0.59q) = 0.0457 \quad \text{for } q = 0.0470$$
$$A_s = dqf_c'/f_y = 9.14 \text{ in.}^2$$

Provide #8 bars at 6-in. spacing at the bottom. Total area $A_s = 14.137\,\text{in.}^2$, which is satisfactory.

Check for minimum area of steel required $= (200/f_y)bd = 12.96\,\text{in.}^2$

Since the provided area of steel is greater than the minimum steel required, the flexural reinforcement provided is adequate.

Step 8. Check the development length

Basic development length l_{db} for #8 bars in 3000 psi concrete (ACI code provision) is given by

$$l_{db} = 0.04Ab/f_c^{0.5} = 0.04 \times 0.79 \times 60{,}000/3000^{0.5}$$
$$= 35 \text{ in.}$$

ACI 12.2.3.1:
(a) No transverse steel (stirrups): does not apply
(b) Does not apply
(c) Does not apply if flexural steel is in the bottom layer
(d) Cover $= 3$ in. and clear spacing $= 5.0$ in $> 3d_b$ and therefore 12.3.3.1 (d) applies with a factor of 1.0

ACI 12.2.3.4 Applies with a factor of 0.8
ACI 12.2.4 Bottom bar, $\times 1.0$; normal wt concrete, $\times 1.0$; and standard deformed bar, $\times 1.0$
ACI 12.2.4.1 Top bar with a factor of 1.3

$$l_{db} = 35 \times 1.0 \times 0.8 \times 1.3 = 36 \text{ in. (top bar)}$$

ACI 12.2.3.6 $l_{db} = > 0.03 d_b f_y / f_c' 0.5 = 0.03 \times 1 \times 60{,}000/3000^{0.5} = 33$ in.

The length of the bar from the maximum stress point at the face of the columns is $67 - 3 = 64$ in., which is >36 in. and hence is satisfactory.

Step 9. Temperature and shrinkage reinforcement (ACI 7.12.2)

$$A_s = 0.0018bh = 0.0018 \times 12 \times 40$$
$$= 0.86 \text{ in.}^2/\text{ft}$$

At least two thirds of this should be placed as top reinforcement in the transverse direction as the concrete exposed to the dry and hot weather until covered by earth (backfill).

Provide #7 at 12 in. on centers; $A_s = 0.6 \text{ in.}^2/\text{ft}$

$$A_s = 0.0018bh = 0.0018 \times 12 \times 40 = 0.86 \text{ in.}^2/\text{ft}$$

This reinforcement should be divided between the top and the bottom and should provide two thirds of it at the top since it will be exposed to temperature and half of it at the bottom layers in the longitudinal direction or provide 6 #7 at 12-in. centers at the top at the interior column and at the bottom at the exterior column.

$$A_s = 2 \times 0.61 = 1.22 \text{ in.}^2/\text{ft}$$
$$> 0.86 \text{ in.}^2/\text{ft}, \quad \text{satisfactory}$$

Step 10. Design of the transverse ''beam''

The transverse strips under each column will be assumed to transmit the load evenly from the longitudinal beam strips into the column strip. The width of the column strip will be assumed to extend $d/2$ on either side of the interior column and one side of the exterior column (Figure 5.23).

(a) The maximum factored load for the interior column = 804 kips
This load is carried by a 9-ft beam and, hence,

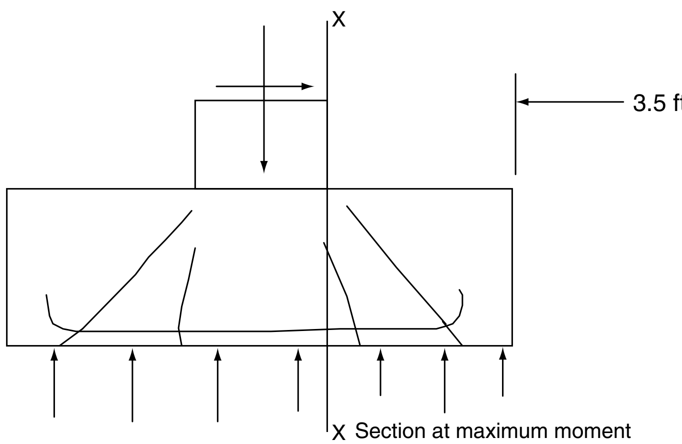


FIGURE 5.23
Cross section of beam (Example 5.5).

$$\text{load per ft} = 804/9 = 89.33 \text{ kips}$$
$$\text{Maximum } M_u = 89.33 \times 3.5^2/2$$
$$= 547.15 \text{ ft-kips}$$
$$b = 60 \text{ in.}$$
$$d = 36 \text{ in.}$$
$$f'_c = 3 \text{ ksi}$$
$$f_y = 60 \text{ ksi}$$
$$M_u = 547.15 \text{ ft-kips}$$

Per ACI handbook,

$$\frac{12M_u}{0.9f'_c bd^2} = \frac{12 \times 547.15}{0.9 \times 3^{0.5} \times 60 \times 36^2} = Q = 0.0542$$

$$= q(1 - 0.59q) = 0.0544 \quad \text{for } q = 0.0563$$

$$A_s = bdqf'_c/f_y = 6.08 \text{ in.}^2$$

Provide #8 bars at 6-in. spacing at the bottom. This amounts to a total of ten bars

$$\text{Total } A_s = 7.85 \text{ in.}^2, \text{ Satisfactory}$$

(b) The maximum factored load for the interior column = 534 kips
This load is carried by a 9-ft beam and, hence, load per ft = 534/9

$$= 59.33 \text{ kip}$$
$$\text{Maximum } M_u = 59.33 \times 3.5^2/2$$
$$= 363.42 \text{ ft-kips}$$
$$b = 42 \text{ in.}$$
$$d = 36 \text{ in.}$$
$$f'_c = 3 \text{ ksi}$$
$$f_y = 60 \text{ ksi}$$
$$M_u = 363.42 \text{ ft-kips}$$

Per ACI handbook,

$$\frac{12M_u}{0.9f'_c bd^2} = \frac{12 \times 363.42}{0.9 \times 3^{0.5} \times 42 \times 36^2} = Q = 0.0514$$
$$= q(1 - 0.59q) = 0.0523 \quad \text{for } q = 0.0540$$
$$A_s = bdqf'_c/f_y = 4.08 \text{ in.}^2$$

Provide #8 bars at 6-in. spacing at the bottom. This amounts to a total of seven bars

$$A_s = 5.50 \text{ in.}^2 \text{ satisfactory}$$

Step 11. Details of reinforcement (Figure 5.24)

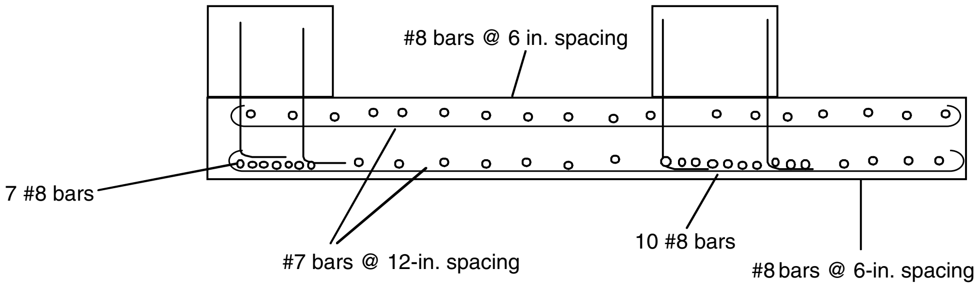


FIGURE 5.24
Details of reinforcement (Example 5.5).

5.7 Pile Foundations

A structure is founded on piles if the soil immediately below its base does not have adequate bearing capacity, or if the foundation cost estimate indicates that a pile foundation may be more economical and safer than any other type of foundation. In this discussion, we will consider only piles that are commonly available and driven into the ground by a mechanical driving devise known as a pile driver. Please note that the general principles are also applicable to other types of pile foundations, with minor modifications. Piles may be divided into three categories based on the method of transferring the load into the ground (Figure 5.25–Figure 5.28):

1. *Friction piles in coarse-grained very permeable soils:* These piles transfer most of their loads to the soil through skin friction. The process of driving such piles close to each other (in groups) greatly reduces the porosity and compressibility of the soil within and around the group.
2. *Friction piles in very fine-grained soils of low permeability:* These piles also transfer their loads to the soils through skin friction. However, they do not compact the soil during driving as in case 1. Foundations supported by piles of this type are commonly known as floating pile foundations.
3. *Point bearing piles:* These piles transfer their loads into a firm stratum or a soil layer. Depending on the geographical location, these piles have to be driven to a considerable depth below the base of the footing.

In practice, piles are used to transfer their loads into the ground using a combination of the above mechanisms.

5.7.1 Analysis of Pile Groups

The function of a pile cap, a relatively rigid body, is to distribute the loads to each pile in a group of piles. The loads could be vertical or horizontal loads or moments from the superstructure. The horizontal forces at the base are generally resisted by battered or raked piles. The batter can be as steep as 1 on 1, but it is economical to limit the batter to

1.0 horizontal to 2.5 vertical (approximately 22° of an inclination to vertical). The horizontal forces may be carried by vertical piles in the form of shear and moments. The shear capacity of piles is limited by the material property of the pile. However, it is advisable to resist by the horizontal component of the axial load in a battered pile.

When a footing consisting of N number of piles is subjected to a vertical load of P, moments of M_x and M_y, and a horizontal force of H, the following equation can be used to determine the force attributed to each pile. After determining the force in each pile, the horizontal resistance force may be provided by battering or raking the piles to develop adequate horizontal resistance:

$$\text{Load in a pile} = \frac{P}{N} \pm \frac{M_x d_x}{\sum d_x^2} \pm \frac{M_y d_y}{\sum d_y^2} \tag{5.6}$$

where P is the total vertical load in the pile cap, M_x is the moment at the pile cap about the x-axis, M_y is the moment of the pile cap about the y-axis, d_x is the x-directional distance of the pile from the center of the pile group, and d_y is the y-directional distance of the same pile from the center of the pile group.

The above principle is illustrated by the following example in an actual design situation.

Example 5.6

You have been engaged as the engineer to design the footing of a pier foundation for a major bridge. The bridge engineer has determined that the foundation needs to be designed for a factored load of 3650 kips, a transverse factored moment of 7050 ft-kips, and a longitudinal moment of 2400 ft-kips. The bridge pier is 8 ft (longitudinal direction) × 10 ft (transverse direction). The bridge engineer has proposed to use 18-in. square PC piles. The geotechnical engineer has recommended limiting the pile capacity to 325 kip (factored load). The group has to resist a lateral force of 125 kips in the transverse direction and 75 kips in the longitudinal direction. The bridge engineer has estimated that 17 to 18 piles would be adequate. The shear capacity of the 18-in. square pile is limited to 10 kips.

$$f_c' = 4{,}600 \text{ psi}$$

$$f_y = 60{,}000 \text{ psi}$$

Solution

This is a bridge foundation design example and hence AASHTO provisions apply:

1. Determine the number of piles and the spacing required to resist the given loading condition. It is given that the bridge engineer presumes that 18 piles would be required. The spacing between piles is more than three times the pile diameter $= 3 \times 18 = 54$ in. Provide piles at spacing of 60 in. (5 ft) in both directions.

2. Determine the size of the pile cap or the footing.
   Careful study of the situation indicates that the pile cap should provide higher resistance in the transverse direction. An edge distance of 2 ft should be sufficient. If the pile group is arranged with five piles in the transverse direction and four piles in the longitudinal direction,

   Length of the pile cap $= 4 \times 5\,\text{ft} + 2 \times 2\,\text{ft} = 24\,\text{ft}$

Width of the pile cap $= 3 \times 5\,\text{ft} + 2 \times 2 = 19\,\text{ft}$
$h = 5.00$ ft
$d = 3.75$ ft
$L = 24.00$ ft
$B = 19.00$ ft

3. Analysis of pile group.

   Analysis of the pile group can be carried out in a tabular form as given below:
   Pile load analysis — Transverse direction (Table 5.7)
   Pile load analysis — Longitudinal direction (Table 5.8)
   Combined loading effect: Load per pile (kips) (Table 5.9)

$$\text{Axial load due to moment } M_y = \frac{7050}{1000} \times 10 = 70.5 \text{ kips}$$

4. Consider one-way shear action:
   Critical section for shear $=$ d from the face of the piles

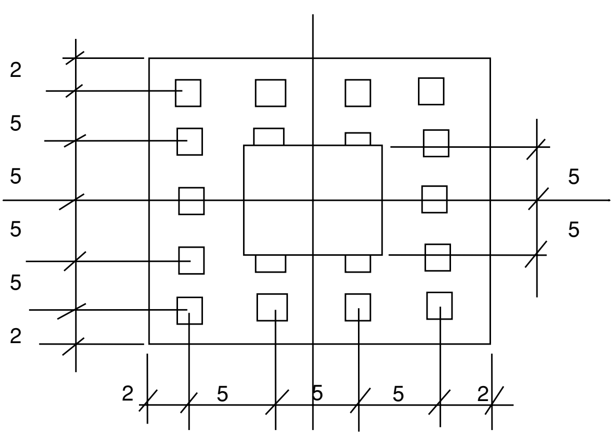


FIGURE 5.25
Arrangement of piles and the pile cap.

TABLE 5.7
Computations for Transverse Directional Analysis

N	X	d^2	$N \times d^2$	P_u kips 3650.00	M_y ft-kips 7050.00	$P_u \pm M_y$ kips	M_x ft-kips 2400.00
4	10	100	400	202.78	70.50	273.28	
4	5	25	100	202.78	35.25	238.03	
2	0	0	0	202.78	0.00	202.78	
4	−5	25	100	202.78	−35.25	167.53	
4	−10	100	400	202.78	−70.50	132.28	
Total 18			1000				

$b = 228$ in. (19.0 ft)

Shear loads $= 1093.11$ kips

Shear area $= 10{,}260$ in.^{2} (228×45)

Shearstress $= 106.54$ psi $< 2f_c'^{0.5} = 135.65$ psi, Satisfactory

5. Consider two-way shear action:
   Critical section for shear $= d/2$ from the face of a single pile.

   $b_0 = 99$ in.
   Shear loads $= 302.67$ kips

   Shear area $= 4455$ in.^{2} (45×99)

   Shearstress $= 67.938$ psi $< 2f_c'^{0.5} = 135.65$ psi, Satisfactory

6. Flexural behavior — Determination of reinforcements in the longitudinal direction:

   Minimum cover to reinforcement $= 3$ in.

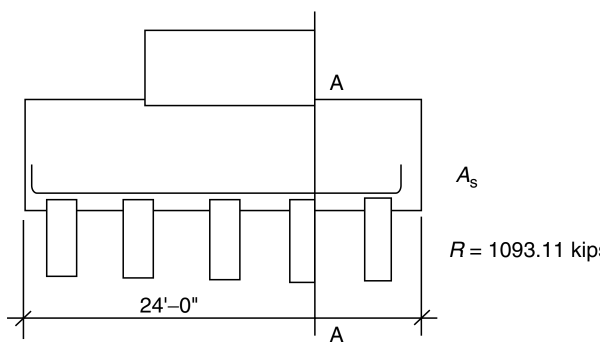


FIGURE 5.26
Pile cap layout — longitudinal direction.

TABLE 5.8

Computations for Longitudinal Directional Analysis

N	X	d^2	N $\times$ d^2	M_x ft-kips
5	7.5	56.25	281.25	29.39
4	2.5	6.25	25	9.80
4	−2.5	6.25	25	−9.80
5	−7.5	56.25	281.25	−29.39
	Total		612.5	

TABLE 5.9

Computations of Combined Analysis

	Transverse					
Longitudinal	T1	T2	T3	T4	T5	
L1	161.67	196.92	232.17	267.42	302.67	1160.85
L2	142.07	177.32		247.82	283.07	
L3	122.48	157.73		228.23	263.48	
L4	102.89	138.14	173.39	208.64	243.89	

Notes: Horizontal force $=$ 8.10 kips; high load on a pile $=$ 302.67 kips. $\sum = 1093.11$ kips

$R = 1093.11$ kips
Max. moment at A $= 1093.11 \times 5$ kip-ft
$M_u = 5514.75$ kip-ft
$f'_c = 4.6$ ksi
$b = 216$ in.
$d = 45.00$ in.
$Q = 12M_u/\phi f'_c bd^2 = q(1 - 0.59q)$
$f_y = 60{,}000$ psi
$Q = 0.04348 < q_{min} = (200/f_y) \times x$
$A_s = qbd/x$, where $x = f_y/f'_c = 13.04$
$= 32.40$ in.^{2}
Number of #10 bars $= 26$ bars
Spacing $= 8.15$ in. in the longitudinal direction
Check for minimum reinforcement (AASHTO Section 8.17.1):

$\phi M_n > 1.2M_{cr}$
$M_{cr} = 7.5f_c'^{0.5} \times l_g/y_t$

where

$l_g = bh^3/12 = 3{,}888{,}000$ in.^{4}
$y_t = 30$ in.
$M_{cr} = 5493.69$ kip-ft
$1.2M_{cr} = 6592.42$ kip-ft
$\phi M_n = \phi A_s f_y jd$ kip-ft
$A_s = 27$ #10 bars $= 34.29$ in.^{2}
$T = A_s f_y = 2057.4$ kips
$C = 0.85\beta baf'_c$

where

$\beta = 0.85 - (4{,}600 - 4{,}000)/1{,}000 \times 0.05 = 0.82$
$C = 692.539a$
$a = T/C = 2.97$ in., $j_d = 43.5146$ in.
$\phi M_n = 6714.52$ ft-kips $> 1.2M_{cr}$, Satisfactory

Provide 28 #10 bars at the bottom and provide #7 bars at the top at 8-in. spacing.

7. Determination of reinforcement in transverse direction:

Min. cover $= 3$ in.
$R = 1160.83$ kips

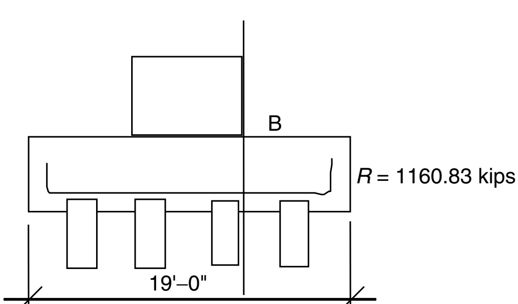


FIGURE 5.27
Pile cap layout — transverse direction.

Max. moment at B $= 1120.3 \times 3.75$ kip-ft
$f_c' = 4{,}600$ psi
$M_u = 4201.1$ kip-ft
$f_y = 60{,}000$ psi
$f_c' = 4.6$ ksi $\quad f_y = 60$ ksi
$b = 288$ in.
$d = 43.75$ in.
$Q = 12M_u/\phi f_c' bd^2 = q(1 - 0.59q)$
$f_c' = 4{,}600$ psi
$Q = 0.0221 \Rightarrow q = 0.027$
$q = 0.043478 < = q_{\min} = \left(\frac{200}{f_y}\right)x$ where $x = \frac{f_y}{f_c'}$
$A_s = qbd/x$, where $x = f_y/f_c' = 13.04 = 42$ in.^{2}
Number of #10 bars $= 33.071$ bars
Spacing $= 8.5$ in.

Check for minimum reinforcement AASHTO Section 8.17.1:
$\phi M_n \geqslant 1.2M_{cr}$
$M_{cr} = 7.5f_c'0.5 \times l_g/y_t$
where $l_g = bh^3/12 = 518{,}4000$ in.^{4}
$Y_t = 30$ in.
$M_{cr} = 7324.9$ kip-ft
$1.2M_{cr} = 8789.9$ kip-ft
$\phi M_n = \phi A_s f_y j_d$ kip-ft
$A_s = 38$ #10 bars $= 47.752$ in.^{2}
$T = A_s f_y = 2865.1$ kips
$C = 0.85\beta baf_c'$, where $\beta = 0.85 - (4{,}600 - 4{,}000)/1{,}000 \times 0.05 = 0.82$
$C = 923.39a$
$a = T/C = 3.10$ in.
$jd = 42.19858$ in.
$\phi M_n = 9067.8$ kip-ft $> 1.2M_{cr}$, Satisfactory
Provide #10 bars at 7.5-in. spacing at the bottom.

8. Lateral force resistance of the pile cap:

The lateral force could be due to a centrifugal force, wind force, or even due to earthquake motions. In this example, it is a combination of forces per section 3 of AASHTO specifications. The lateral forces were computed per AASHTO and found that the piles need to resist the following forces:

$F_x = 75$ kips
$F_y = 125$ kips
$R = (F_x^2 + F_y^2)^{0.5} = 145.77$ kips
Lateral force per pile $= 145.77/18.00 = 8.10$ kips

Since the lateral force of 8.1 kips $<$ 10 kips per pile, no pile requires any battering or raking.

9. Shrinkage and temperature reinforcement:

   Average R_H = 75%
   Assumed shrinkage = 150 microstrains
   Correction for $R_H = 1.4 - 0.01 \times R_H$
   S_H = 97.5 microstrains
   $E_c = 4E + 0.6$ psi
   Concrete stress = $E_c \times S_H$ psi
   = 376.93 psi
   Depth of shrinkage effect = 5.00 in. from the surface
   The shrinkage induced force per ft = 22.616 kips

   This force has to be resisted by steel reinforcement. Otherwise the concrete will develop cracking.
   Required steel to prevent cracking $A_s = 22.616/0.85f_y$, A_s = 0.443445 in.^{2}/ft

10. Temperature effect:
    Temperature rise during the initial stage of concrete curing does more damage to concrete than at latter stages

    Temperature rise could be = 25°C
    Temperature strains = αt and $\alpha = 6.5 \times 10^{-6}$
    = 162.5 microstrains
    Concrete stress = $162.5 \times E_c/3$
    = 209.4 psi
    Assuming the depth of the temperature rising effect to be 6 in.
    Temperature-induced force = 15.077 kips/ft
    The required steel area = $15.077/0.85f_y$
    = 0.2956 in.^{2}/ft
    Total area = 0.7391 in.^{2}/ft
    Spacing of #7 bars = 9.9043 in.
    Provide #7 bars at 9-in. spacing at the top and the vertical face.
    Some transportation agencies recognize shrinkage and temperature-related cracking of RC members and require that the minimum reinforcement is provided. For example, Florida Department of Transportation requires the following:

    Two-way cage reinforcement must be provided on all faces of pier footings

    (1) 5 bars at 12-in. centers as minimum
    (2) When the minimum dimension exceeds 3.28 ft and volume–surface area ratio is greater than 12 in.

$$V/A = 20.39 > 12 \text{ in.}$$

The pile cap meets the mass concrete requirements

$$\Sigma A_b \Rightarrow S(2d_c + d_b)/100$$

where A_b = minimum area of bar (mm^2) = 285
S = spacing of bar (mm) = 300
d_c = concrete cover measured to the center of the bar (mm) = 85.73
d_b = diameter of the reinforcing bar = 19.05

$$2d_c + d_b = 190.5 \text{ mm}$$

But $(2d_c + d_b)$ need not be greater than 75 mm

$$\sum A_b = 0.75S = 225$$

Therefore, provide #6 bars at 12-in. centers.

11. Reinforcement development length:

    In this design, the reinforcement must be effective just outside of the piles within the pile caps. This is made possible by providing mechanically anchored bent-up bars (through a 90° bend). This is the most economical way of providing sufficient development. Otherwise the footing needs to be extended and may become uneconomical.

12. Reinforcement details

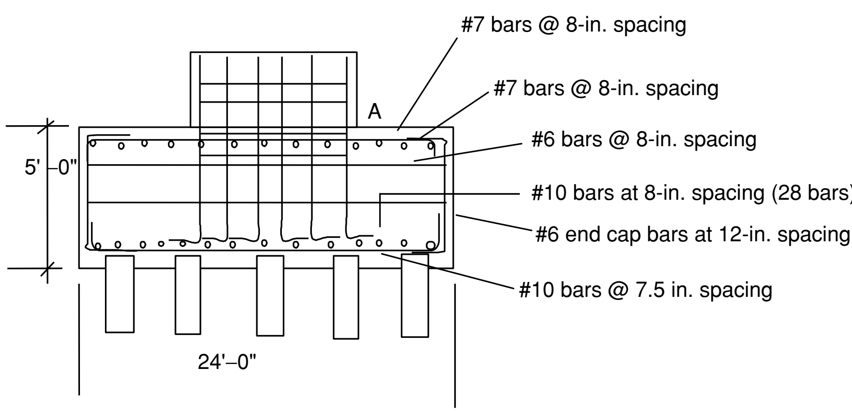


FIGURE 5.28
Typical pile cap details.

5.8 Design of Grade Beams

Example 5.7
One of your clients approaches you to design a foundation for a wood-framed (construction) building. The geotechnical engineer has advised you to use a grade beam supported by wooden timber piles. Twelve-inch diameter timber piles driven to a depth of 35 ft could carry a working load of 35 kips per pile. The grade beam has to carry the wall load of 2.5 kips per foot of dead load and 1.3 kips per foot of live load. The structural engineer advised you that the timber piles need to be staggered at least 1 ft 6-in. centers apart. If the building length is 85 ft, determine the pile spacing along the length of the building and design the grade beam given the following: $f_c' = 3{,}000$ psi, $f_y = 60{,}000$ psi. The frost depth is 2 ft 4 in (Figure 5.29 and Figure 5.30).

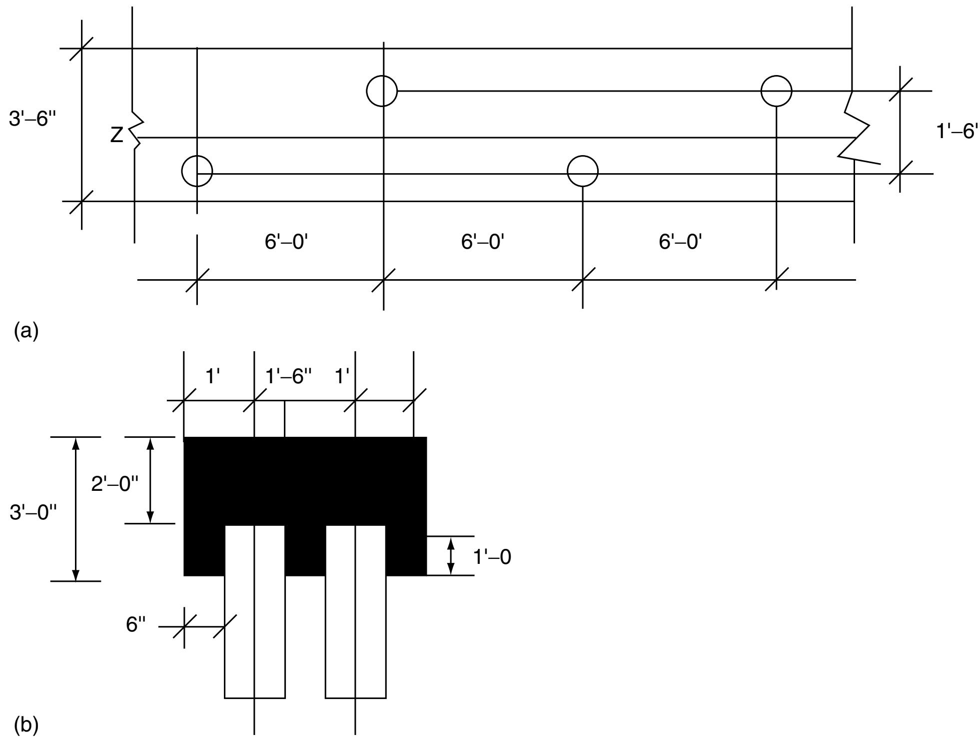


FIGURE 5.29
Illustration for Example 5.7: (a) front elevation; (b) side elevation.

Answer: Grade beam design
Data: Grade beam woodframe wall

$$\left.\begin{aligned} DL &= 2.5\ \text{kips/ft} \\ LL &= 1.3\ \text{kips/ft} \end{aligned}\right\} = 3.8\ \text{kips}$$

Grade beam supported by timber piles driven to 35 ft
Pile capacity = 35 kips
Ultimate load = 35 × 2 = 70 kips
Beam width = 3.5 ft
Beam depth >2 ft to 4 in. = 3.0 ft
Self weight of beam = 1.575 kips/ft

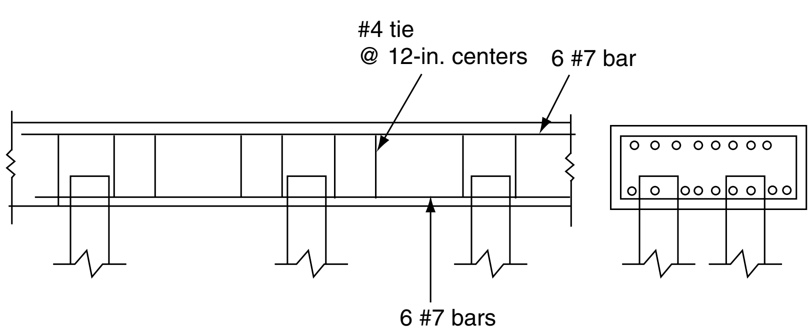


FIGURE 5.30
Reinforcement details for grade beam (Example 5.7).

When piles are spaced at 6–0
Load/pile = (1.575 + 3.8)6 = 32.25 kips < 35 kips
Thus, the design is adequate
W_d = 2.5 and 1.575 = 4.075 kips/ft
W_L = 1.3 kips/ft

$$W_u = 1.4 \times 4.075 \text{ and } 1.7 \times 1.3$$
$$= 7.92 \text{ kips/ft}$$

$$M_u = \frac{W_u l^2}{9} = \frac{7.92 \times 6^2}{9} = 31.7 \text{ kips/ft}$$
$$D = 36 \text{ in.} - 3 \text{ in.} - 0.5 \text{ in.} = 32.5 \text{ in.}$$
$$Q = \frac{12M_u}{0.9f'_c bd^2} = \frac{12 \times 31.7}{0.9 \times 3 \times 42 \times 32.5^2} = 0.0032 = q\ (1 - 0.59q) \Rightarrow q = 0.0033$$
$$A_s = bdq\frac{f'_c}{f_y} = \frac{42 \times 32.5 \times 0.0033 \times 3}{60} = 0.3 \text{ m}^2$$
$$\text{Minimum reinforcement} = \rho = \frac{200}{f_y} = \frac{200}{60,000}$$
$$A_s = \frac{200}{60,000} \times 42 \times 32.5 = 4.55 \text{ m}^2$$

A_s required < A_s min
Provide 6 #7 bars at the top and at the bottom

$$A_s = 6 \times 061 = 3.66 \text{ m}^2$$

A_s provided > A_s required × 1.33, OK.

Shear check:

$$\text{Shear force} = 0.55 \times 7.92 \times 6$$
$$Q = 26.14 \text{ kips}$$
$$\text{Shear stress} = \frac{Q}{bd} = \frac{26.14 \times 10^3}{42 \times 32.5}$$
$$= 19 \text{ psi}$$
$$< \sqrt{f'_c} = 54.8 \text{ psi}$$

provide #4 tie at 12 in. centers.

5.9 Structural Design of Drilled Shafts

The construction of high rise and heavier buildings in cities, where the subsurface conditions consist of relatively thick layers of soft to medium bearing strata overlying deep bedrock, led to the development of drilled shaft foundations. Therefore, the function of a drilled shaft (similar to pile foundations) is to enable structural loads to be taken down through deep layers of weak soil on to a hard stratum called for a very conservative value for bearing pressure for the hard strata around 8 to 10 kips per square foot.

However, the rapid advancement in the construction technology followed by the development of theories for design and analytical techniques, the use of computers, and full-scale testing led to the production of a better understanding of drilled shaft behavior. There are marked differences between the behavior of driven piles and drilled shaft. The drilled shaft is also known as caisson, drilled caisson, or drilled piers.

Drilled shafts have proved to be reliable foundations for transferring heavy loads from superstructure to be the suitable bearing strata beneath the surface of the ground. Economic advantages of a drilled shaft are often realized due to the fact that a very large drilled shaft can be installed to replace groups of driven piles, which in turn obviates the need for a pile cap. The drilled shaft is very often constructed to carry both vertical and horizontal loads.

5.9.1 Behavior of Drilled Shafts under Lateral Loads

Figure 5.31 shows views of two types of foundations used for column support in two buildings. Figure 5.31(a) shows two shaft foundations and Figure 5.31(b) shows a single-shaft support. The two-shaft system resists the wind moment by added tension and compression (a ''push–pull'' couple) in the shaft, although some bending is required to resist the wind shear, while the single-shaft foundation resists both the moment and shear produced by the wind load through bending.

5.9.2 Methodology for Design of Drilled Shafts

Drilled shafts are more often used to transfer both vertical and lateral loads. The design of a drilled shaft for lateral loading requires step-by-step procedures to be followed:

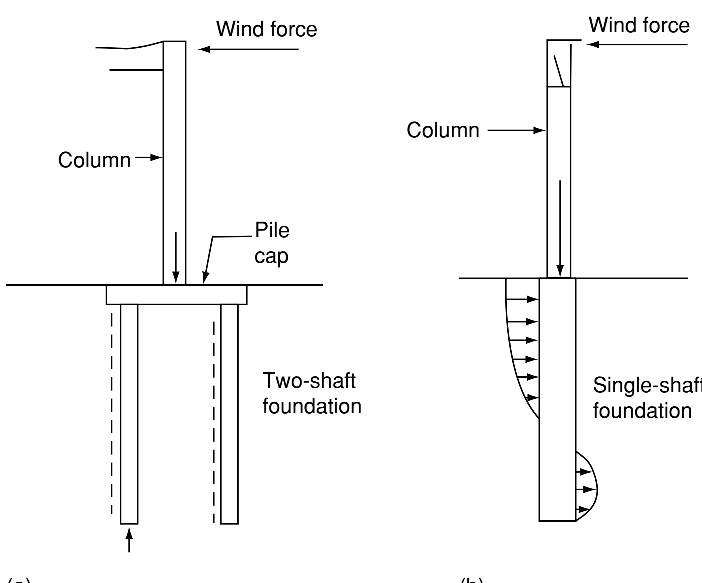


FIGURE 5.31
Elevation view of: (a) two-shaft foundation; (b) single-shaft foundation. (From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.)

1. Determine the depth of the drilled shaft to carry the computer-generated vertical load without undergoing excessive moment.
2. Determine the size (diameter) and mechanical properties of the concrete to resist the bending moment, shear force, and axial load that will be imposed on the drilled shaft by lateral loads in combination with axial loads.
3. Determine the deformation or stiffness of the drilled shaft in lateral translation and rotations to ensure that lateral deformation falls within acceptable limits.

There are three methods that can be used to analyze laterally loaded drill shafts. Brom's method can be used to estimate ultimate strength–state resistance. The other two methods include the ''characteristic load method'' and the ''*P–Y* methods,'' which can deal better with the nonlinear aspects of the problem. In the following section Brom's method is presented.

5.9.2.1 Brom's Method of Design

Brom's method is a straightforward hand-calculation method for lateral load analysis of a single drilled shaft or pile. The method calculates the ultimate soil resistance to lateral load as well as the maximum moment induced in the pile. Brom's method can be used to evaluate fixed or free head condition in either purely cohesive or purely cohesionless soil profiles. The method is not conducive to lateral load analyses in mixed cohesive and cohesionless soil profiles. For long fixed head piles in sands, the method can also over-predict lateral load capacities (Long, 1996). Therefore, for mixed profiles and for long fixed head shaft in sands, the COM624P program should be used. A step-by-step procedure developed by the New York State Department of Transportation (1977) on the application of Brom's method is provided below:

Step 1. Determine the general soil type (i.e., cohesive or cohesionless) within the critical depth below the ground surface (about 4 or 5 shaft diameters).

Step 2. Determine the coefficient of horizontal subgrade reaction, K_h, within the critical depth for cohesive or cohesionless soils

TABLE 5.10

Values of Coefficients of n_1 and n_2 for Cohesive Soils

Unconfined compression strength, q_u (kPa)	n_1
<8	0.32
48–191	0.36
>191	0.40
Pile material	n_2
Steel	1.00
Concrete	1.15
Timber	1.30

Source: From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd edn, American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.

(a) Cohesive soils:

$$K_h = \frac{n_1 n_2 80 q_u}{b} \tag{5.7}$$

where q_u is the unconfined compressive strength (kPa), b is the width or diameter of the shaft (m), and n_1 and n_2 are the empirical coefficients taken from Table 5.10

(b) Cohesionless soils:

Choose K_h from the Table 5.11. (The values of K_h given in Table 5.11 were determined by Terzaghi.)

Step 3. Adjust K_h for loading and soil conditions

(a) Cyclic loading (or earthquake loading) in cohesionless soil:
1. $K_h = \frac{1}{2} K_h$ from Step 2 for medium to dense soil.
2. $K_h = \frac{1}{4} K_h$ from Step 2 for loose soil.

(b) Static loads resulting in soil creep (cohesive soils)

1. Soft and very soft normally consolidated clays
   $K_h = (1/3 \text{ to } 1/6)K_h$ from Step 2
2. Stiff to very stiff clays
   $K_h = (1/4 \text{ to } 1/2)K_h$ from Step 2

Step 4. Determine the pile parameters

(a) Modulus of elasticity, E (MPa)

(b) Moment of inertia, I (m^4)

(c) Section modulus, S (m^3), about an axis perpendicular to the load plane

(d) Yield stress of pile material, f_y (MPa), for steel or ultimate compression strength, f_c (MPa), for concrete

(e) Embedded pile length, D (m)

(f) Diameter or width, b (m)

(g) Eccentricity of applied load e_c for free-headed piles — i.e., vertical distance between ground surface and lateral load (m)

(h) Dimensionless shape factor C_s (for steel piles only):

TABLE 5.11

Values of K_h in Cohesionless Soils

	K_h (kN/m^3)	
Soil Density	**Above Groundwater**	**Below Groundwater**
Loose	1,900	1,086
Medium	8,143	5,429
Dense	17,644	10,857

Source: From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.

1. Use 1.3 for pile with circular section
2. Use 1.1 for H-section pile when the applied lateral load is in the direction of the pile's maximum resisting moment (normal to the pile flanges)
3. Use 1.5 for H-section pile when the applied lateral load is in the direction of the pile's minimum resisting moment (parallel to the pile flanges)

(i) M_y the resisting moment of the pile

1. $M_y = C_s f_y S$ (kN m) (for steel piles)
2. $M_y = f_c S$ (kN m) (for concrete piles)

Step 5. Determine β_h for cohesive soils or η for cohesionless soils

(a) $\beta_h = \sqrt[4]{K_h b/(4EI)}$ for cohesive soil, or

(b) $\eta = \sqrt[5]{K_h/EI}$ for cohesionless soil

Step 6. Determine the dimensionless length factor

(a) $\beta_h D$ for cohesive soil, or

(b) ηD for cohesionless soil

Step 7. Determine if the pile is long or short

(a) Cohesive soil:

1. $\beta_h D > 2.25$ (long pile)
2. $\beta_h D < 2.25$ (short pile)

Note: It is suggested that for $\beta_h D$ values between 2.0 and 2.5, both long and short pile criteria should be considered in Step 9, and then the smaller value should be used.

(b) Cohesionless soil:

1. $\eta D > 4.0$ (long pile)
2. $\eta D < 2.0$ (short pile)
3. $2.0 < \eta D < 4.0$ (intermediate pile)

Step 8. Determine other soil parameters over the embedded length of pile

(a) The Rankine passive pressure coefficient for cohesionless soil, K_p

$K_p = \tan^2(45 + \phi/2)$ where ϕ is the angle of internal friction

(b) The average effective weight of soil, y (kN/m^3)

(c) The cohesion, c_u (kPa)

$c_u = \frac{1}{2}$ the unconfined compressive strength, q_u

Step 9. Determine the ultimate lateral load for a single pile, Q_u

(a) Short free or fixed-headed pile in cohesive soil
Use D/b (and e_c/b for the free-headed case), enter Figure 5.32, select the corresponding value of $Q_u/c_u b^2$, and solve for Q_u (kN)

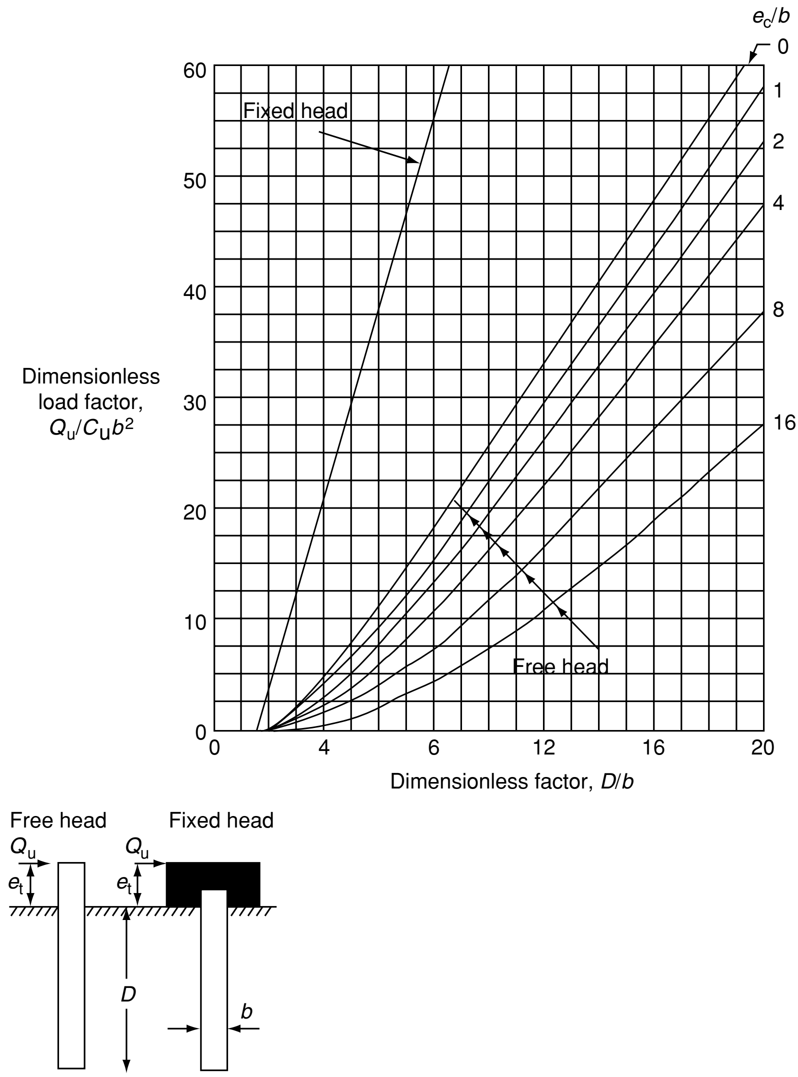


FIGURE 5.32
Ultimate lateral load capacity of short piles in cohesive soils. (From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.)

(b) Long free or fixed-headed pile in cohesive soil
Using M_y/c_ub^3 (and e_c/b for the free-headed case), enter Figure 5.33, select the corresponding value of Q_u/c_ub^2, and solve for Q_u (kN)

(c) Short free or fixed-headed pile in cohesionless soil
Use D/b (and e_c/D for the free-headed case), enter Figure 5.34, select the corresponding value of $Q_u/K_pb^3\gamma$, and solve for Q_u (kN)

(d) Long free or fixed-headed pile in cohesionless soil
Using $M_y/b^4\gamma K_p$ (and e_c/b for the free-headed case), enter Figure 5.35, select the corresponding value of $Q_u/K_pb^3\gamma$, and solve for Q_u (kN)

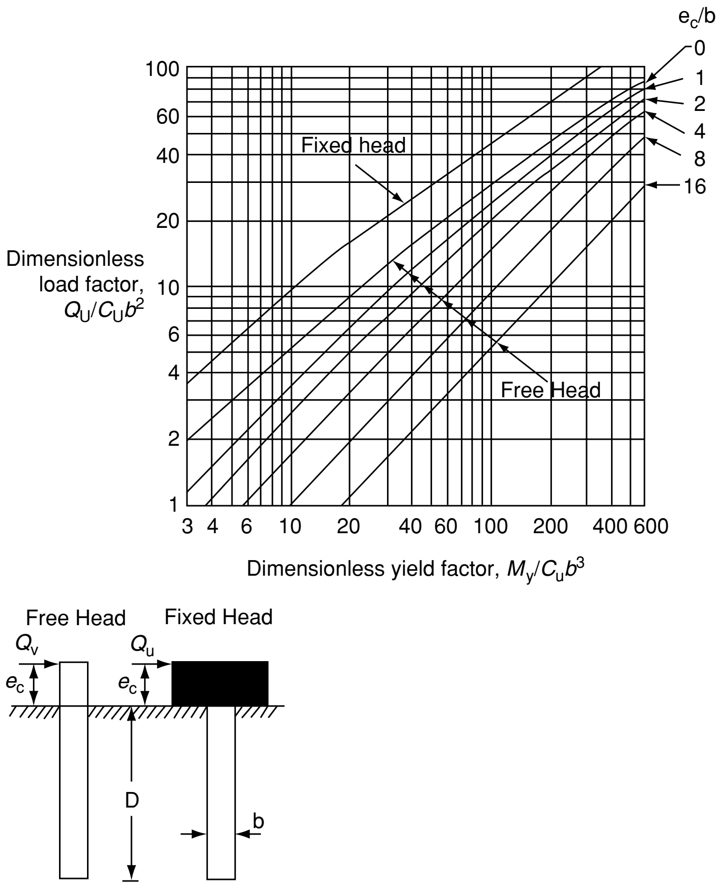


FIGURE 5.33
Ultimate lateral load capacity of long piles in cohesive soils. (From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.)

(e) Intermediate free or fixed-headed pile in cohesionless soil
Calculate Q_u for both short pile (Step 9c) and long pile (Step 9d) and use the smaller value.

Step 10: Calculate the maximum allowable working load for a single pile Q_m. Calculate Q_m, from the ultimate load Q_u determined in step 9 as shown in Figure 5.36.

$$Q_m = \frac{Q_u}{2.5} \text{ (kN)}$$

Step 11. Calculate the working load for a single pile, Q_a to (kN)
Calculate Q_a corresponding to a given design deflection at the ground surface y (m) or the deflection corresponding to a given design load (Figure 5.36). If Q_a and y are not given, substitute the value of Q_m (kN) from Step 10 for Q_a in the following cases and solve for Y_m (m):

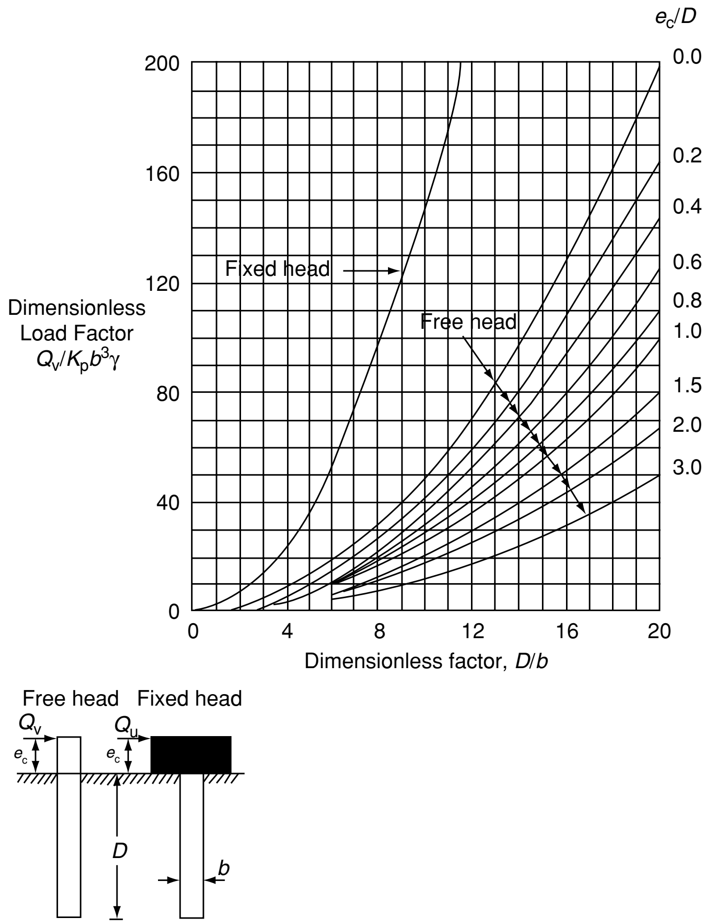


FIGURE 5.34
Ultimate lateral load capacity of short piles in cohesionless soils. (From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.)

(a) Free or fixed-headed pile in cohesive soil
Using $\beta_h D$ (and e/D for the free-headed case), enter Figure 5.37, select the corresponding value of $yK_h bD/Q_a$, and solve for Q_a (kN) or y (m)

(b) Free or fixed-headed pile in cohesionless soil
Using nD (and e/D for the free-headed case), enter Figure 5.38, select the corresponding value of $y(EI)^{3/5}K_h{}^{2/5}/Q_a D$, and solve for Q_a (kN) or y (m)

Step 12. Compare Q_a to Q_m

If $Q_a > Q_{m1}$ use Q_m and calculate y_m (Step 11)

If $Q_a < Q_m$ use Q_a and y

If Q_a and y are not given, use Q_m and y_m

Step 13. Reduce the allowable load from Step 12 for pile group effects and the method of pile installation

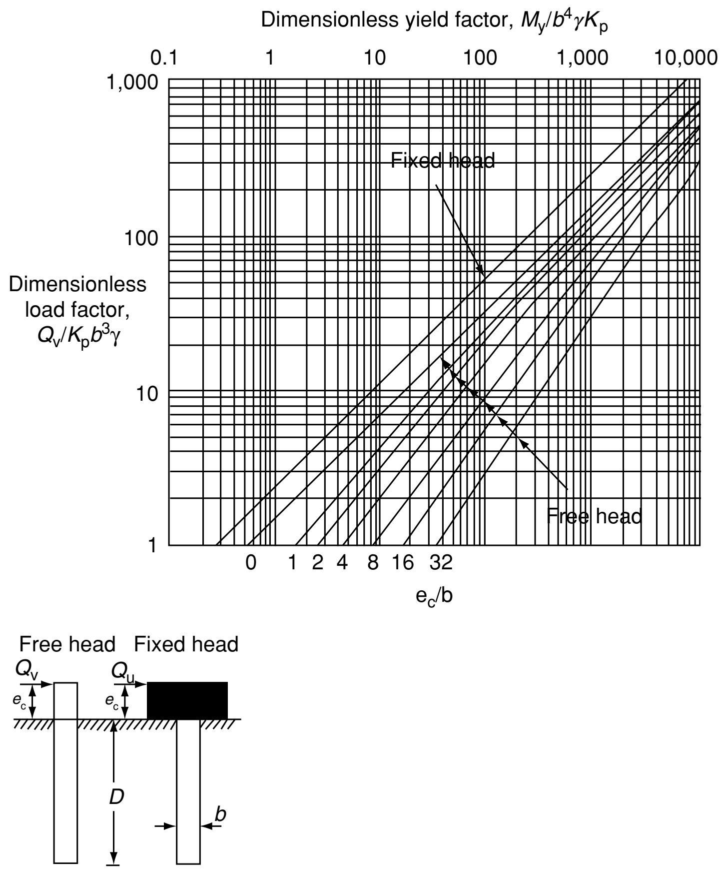


FIGURE 5.35
Ultimate lateral load capacity of long piles in cohesionless soils. (From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.)

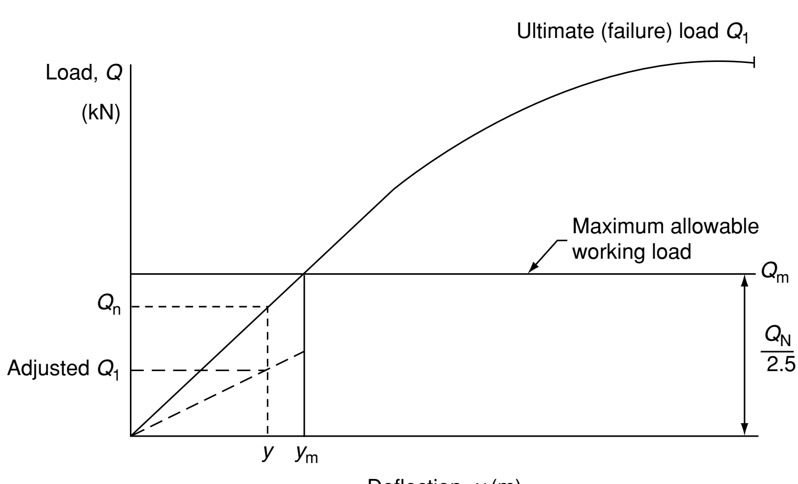


FIGURE 5.36
Load deflection relationship used in determination of Brom's maximum working load. (From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.)

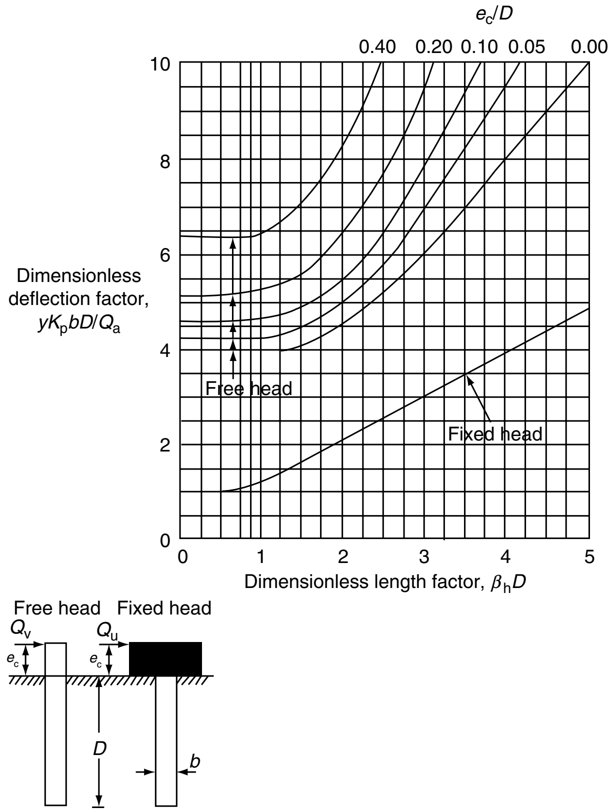


FIGURE 5.37
Lateral deflection at ground surface of piles in cohesive soils. (From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.)

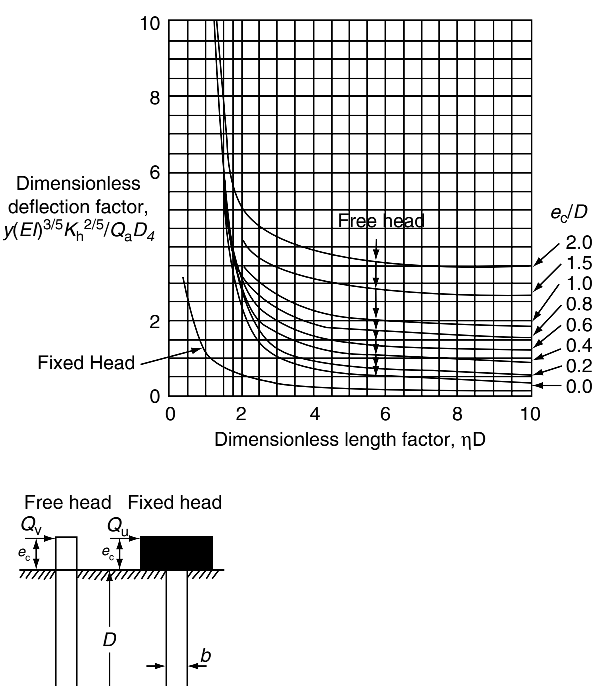


FIGURE 5.38
Lateral deflection at ground surface of piles in cohesionless soils. (From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.)

TABLE 5.12

Group Reduction Factors

Z	**Reduction Factor**
8b	1.0
6b	0.8
4b	0.65
3b	0.5

Source: From *LRFD Bridge Design Specifications, Customary U.S. Units*, 2nd ed., American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.

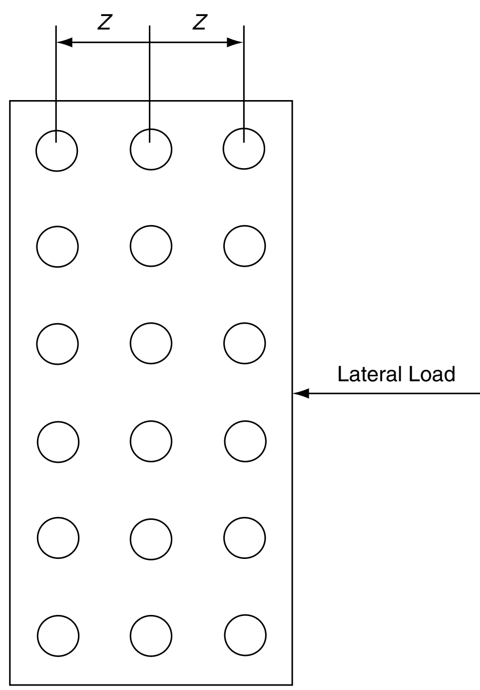


FIGURE 5.39
Guide for Table 5.12.

(a) Group reduction factor determined by the center-to-center pile spacing, z, in the direction of load (Table 5.12 and Figure 5.39)

(b) Method of installation reduction factor
   1. For driven piles use no reduction
   2. For jetted piles use 0.75 of the value from Step 13a

Step 14. Determine pile group lateral capacity
The total lateral load capacity of the pile group equals the adjusted allowable load per pile from Step 13b times the number of piles. The deflection of the pile group is the value selected in Step 12. It should be noted that no provision has been made to include the lateral resistance offered by the soil surrounding an embedded pile cap.

Example 5.8
Drill shaft design
You have been engaged as a foundation engineering consultant to design a drilled shaft for a building. Geotechnical engineers have recommended a drilled shaft or a group of piles. The value engineering analysis has indicated the drill shaft will be the most cost-effective solution. The structural engineer analyzing the building has given the following loading data that need to be transferred to the ground:

Working DL = 520 kips
LL = 314 kips
Working DL moment = 2,550 kip-ft
LL moment = 1,120 kip-ft
Working horizontal load = 195 kip

The attached borehole data (Figure 5.40) were given by the geotechnical engineer. You are required to design a single reinforced concrete drill shaft with concrete $f'_c = 4$ ksi and $f_y = 60$ ksi.

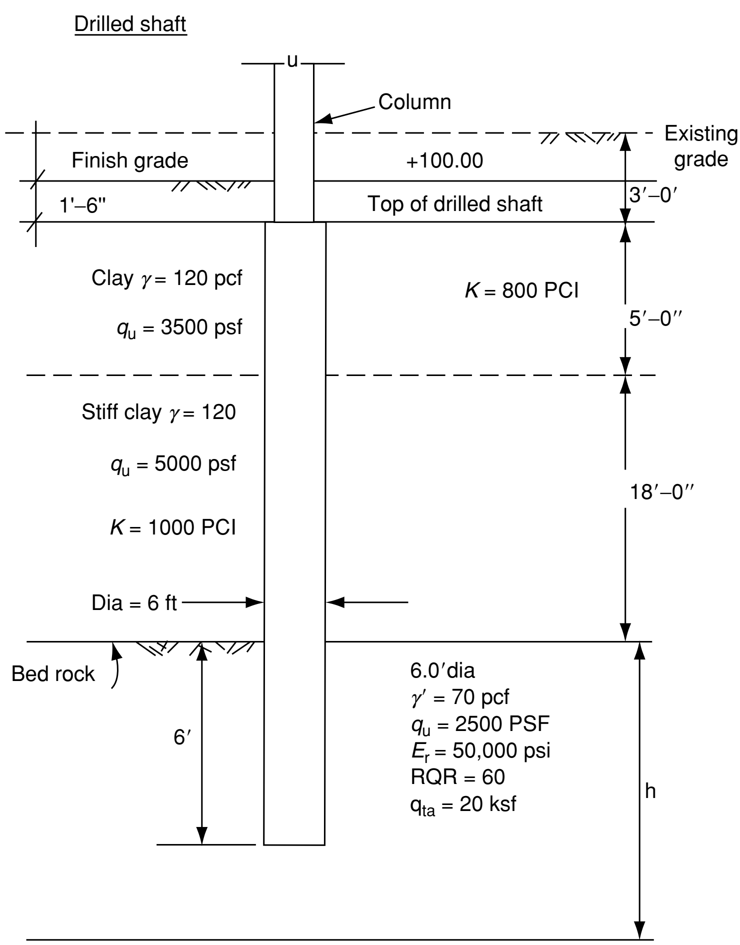


FIGURE 5.40
Soil profile for Example 5.8.

Drill shaft
Given:

DL = 520 kips
DLM = 2,550 kip-ft
LL = 314 kips
LLM = 1,120 kip-ft
V = 834 kips
M = 3,670 kips
H = 195 kips
Vertical load = 834 kips

$$\begin{aligned} \text{DLM} &= 2550.00 \\ \text{LLM} &= 1120.00 \\ \hline \text{Total M} &= 3670.00 \text{ kip-ft} \end{aligned}$$

Let us assume a 6-ft diameter drilled shaft:
$Q_u = Q_s + Q_T - W$ using a safety factor of

$$Q_a = \frac{1}{2.5}[Q_s + Q_T] - \frac{W}{2.5}$$
$$= q_{sa}A_s + q_{ta}A_T - \frac{W}{2.5}$$

Neglect resistance from the top layer of 5 ft. We will consider the second layer and assume the rock layer is the cohesive layer to determine the length of drilled shaft:

$$A_s = \pi D = 6\pi = 18.85 \text{ ft}^2/\text{ft}$$
$$A_{T2} = \frac{\pi D^2}{4} = \frac{5^2 \cdot 6^2 \pi}{4} = 9\pi = 28.27 \text{ ft}^2$$
$$A_{s2} = \pi_2 = 5.5\pi = 17.28 \text{ ft}^2/\text{ft}$$

Skin friction from stiff clay (second layer)

$$q_{sa} = 2.0 \text{ ksf}(\Leftarrow 5/2.5)$$
$$Q_s = 18.85 \times 2 \times 18 = 678 \text{ kips}$$

End bearing from bed shocket
Dia = 6.0 dia
Q_T = 20 × 28.27 = 565 kips

$$Q = 678 + 565 - 23 \times \left(\frac{\pi 6^2}{4}\right)\left(\frac{0.15}{2.5}\right)$$
$$= 1243 - 39.0$$
$$= 1204 \text{ kips} > V = 834 \text{ kips}$$

Therefore, take drill shaft at least 1 diameter depth into the rock, say 6 ft.
Now check for lateral loads

$$H = 195 \text{ ksf},\ M = 3{,}670 \text{ kips-ft} \quad e = \frac{M}{H} + 5 = \frac{3670}{195} + 5 = 23.82 \text{ ft}$$

From the top of stiff clay

Solution
Following the step-by-step procedure:

Step 1. Soil type within ($4 \times D =$) 24 ft depth
$=$ cohesive stiff clay
Step 2. Computation of coefficient of horizontal subgrade reaction, K_h, with the critical depth

$$q_u = \text{average value}\left(\frac{5 \times 3.500 + 18 \times 5000}{23}\right)$$
$$= 4674 \text{ psf}$$
$$= 224 \text{ kPa}$$

Concrete drilled shaft from Table 5.10

$$n_1 = 0.4;\ n_2 = 1.15$$
$$K_h = \frac{n.n_2 80 q_u}{b}$$

where $q_u = 224$ kPa

$$b = \frac{6}{3.281} = 1.8287 \text{ m}$$
$$K_h = \frac{0.4 \times 1.15 \times 80 \times 224^3}{1.8287}$$
$$= 4507.7 \text{ kN/m}$$
$$= 28.85 \text{ kips/ft}^3$$

Step 3. Adjust K_h for loading and soil conditions for stiff clays

$$K_h = \tfrac{1}{2} K_h (\text{from step 2})$$
$$= \tfrac{1}{2} \times 4507.7$$
$$= 2253.8 \text{ kN/m}^3$$
$$= 14.5 \text{ kips/ft}^3$$

Step 4. Determine shaft parameters (Figure 5.41)
(a) Modulus of elasticity

$$E_c = 57{,}000\sqrt{f_c^1} = 57{,}000\sqrt{4000}$$
$$= 3604996.5 \text{ psi}$$
$$E_c = 24{,}856.5 \text{ MPa} \qquad n = \frac{E_s}{E_c} = 8.0$$
$$E_s = 199{,}955 \text{ MPa}$$

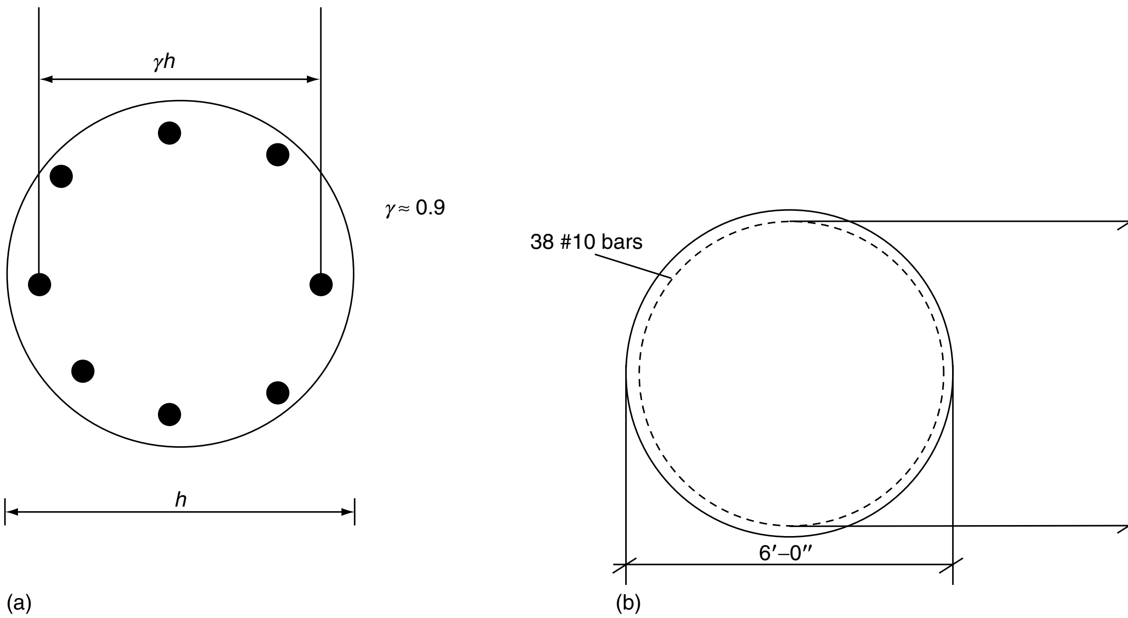


FIGURE 5.41
Shaft section with reinforcement layout: (a) schematic; (b) actual design.

(b) Moment of inertia

$$I_g = \frac{\pi D4}{64} = \frac{\pi \times 1.8287^4}{64} = 0.549\ \mathrm{m}^4$$
$$A_s = 48.2b\ \mathrm{in}^2$$
$$= 0.03113\ \mathrm{m}^2$$
$$I_{se} = 0.125 A_{sc}(\gamma D)^2$$

$$= 0.125 \times 0.03113 \times 1.6^2$$
$$= 0.010\ \mathrm{m}^4$$
$$I_e = I_g + (n-1)I_{se}$$
$$= 0.549 + (g-1)\ (0.010)\ \mathrm{m}^4$$
$$I_e = 0.619\ \mathrm{m}^4$$

(c) Section modulus $= \dfrac{I}{D/2} = \dfrac{0.619}{0.9144}$

$$S = 0.677\ \mathrm{m}^3$$
$$= 677 \times 10^{-3}\ \mathrm{m}^3$$

(d) Yield stress of drilled shaft

$$\begin{aligned}\text{Concrete} &= 4{,}000 \text{ psi}\\ &= 27.58 \text{ MPa}\end{aligned}$$

(e) Embedded shaft length
$D = 23$ ft $= 7.02$ m

(f) Diameter $= b = 1.8287$ m
(g) Eccentricity of applied load e_c
$e_c = 23.82$ ft $= 7.265$ m

(h) Resisting moment of pile M_y

$$\begin{aligned}M_y &= f_c' \cdot s\\ &= 27.58 \times 677 \times 10^{-3}\\ &= 18.671b \text{ kN m}\\ &= 13{,}584 \text{ kip} - \text{ft} \gg M_u\end{aligned}$$

Step 5. Determine βh for cohesive soils

$$\begin{aligned}\beta h &= \sqrt[4]{khb/4EI}\\ &= \left[\frac{2253.8 \times 1.8287}{4 \times (24.9 \times 10^6)(0.6122 \text{ m}^4)}\right]^4\\ &= \left[\frac{4121.5}{60.975 \times 10^6}\right]^{1/4}\\ &= 0.091 \text{ m}^{-1}\end{aligned}$$

Step 6. Determine the dimensionless length factor

$$\begin{aligned}\beta hD &= 0.091 \times 7.265\\ &= 0.66\end{aligned}$$

Step 7. Determine if the shaft is long or short (cohesive soil)
$\beta hD > 2.25$ (long)
$\beta hD < 2.25$ (short)
Since $\beta hD = 0.66$, it is a short drilled shaft
Step 8. Determine the soil parameters
Rankine passive pressure coefficient cohesionless soil
Since the soil that is of concrete, this design is a cohesive one

$$C_u = 1/2 q_u = \frac{224}{2} = 112 \text{ kPa}$$

Or $C_u = 2.35$ ksf

Step 9. Determine the dimensionless factor D/b

$$D/b = \frac{5+18}{6} = 3.833 \simeq 4.0$$

This is a fixed head (the building column is fixed at base). From Figure 5.32 (for cohesive soil),

$Q_u/c_u b^2 = 20.0$ (dimensionless b factor)

$$\begin{aligned} Q_u &= b^2 c_u (20) \\ &= 6^2 \times 2.35 \times 20 \\ &= 1692 \text{ kips} \quad \text{(horizontal force)} \end{aligned}$$

Step 10. Maximum allowable working load $= 1692/2.5 = 676.8$ kips which is > 195 kips

Step 11. Calculate the deflection y

Dimensionless factor $\beta_h D = 0.66$

From Figure 5.37 (lateral deflection at ground surface for cohesive soil),

Dimensionless factor $Y \times K_h (bd/Q_a) = 1.05$ (fixed head)

Replace $Q_a \cdot Q_m$ by the applied load

$$\begin{aligned} Y &= \frac{1.05 Q_m}{K_h b D} = \frac{1.05 \times 195.0}{14.5 \times 6 \times 29} \\ &= 0.081 \text{ ft} \\ &\approx 0.97 \text{ in.} \quad \text{OK} \end{aligned}$$

Shaft Design (Figure 5.42 and Figure 5.43; Table 5.13)

$$\begin{aligned} f'_c &= 4.0 \text{ ksi} \\ f_y &= 60 \\ A_s &= 38\#10 \text{ bars} \\ &= 48.26 \text{ m}^2 \\ A_g &= \frac{\pi B^2}{4} = \frac{\pi \times 72^2}{4} = 4071.5 \\ \rho_g &= 48.26/4071.5 = 0.012 = 1.2\% \\ \gamma &= \frac{(72 - 2(3+0.625) - 1.25)}{72} = 0.88 \end{aligned}$$

$$\begin{aligned} \text{Min } P &= 0.1 f'_c A_c \\ &= 0.1 \times 4 \times 4071.5 \\ &= 1628.6 \text{ kips} \gg 1261.8 \text{ kips} \\ P_u &= 1.4 \text{ DL} + 1.7 \text{ LL} \\ &= 1.4 \times 520 + 1.7 \times 314 \\ &= 1261.8 \text{ kips} \\ M_u &= 1.4 \times 2550 + 1.7 \times 1120 \\ &= 5474 \text{ kip-ft} \end{aligned}$$

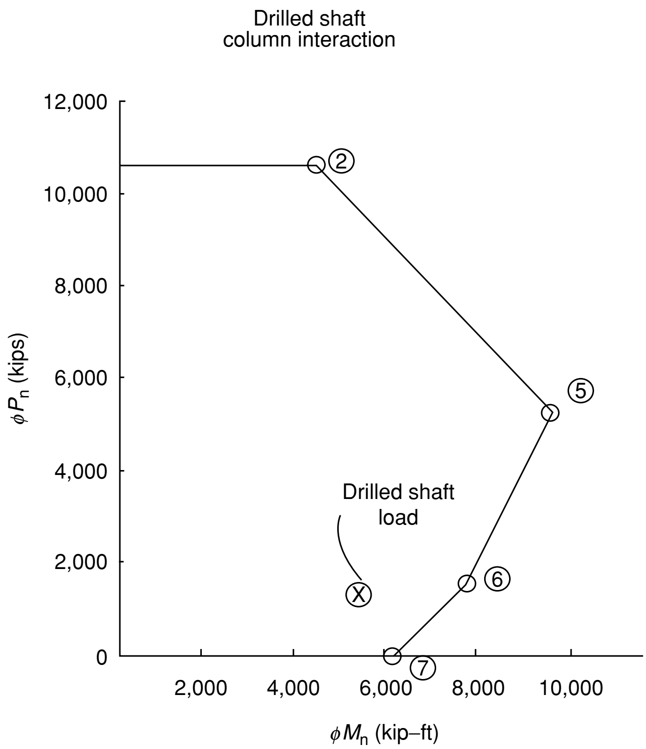


FIGURE 5.42
Drilled shaft column interaction.

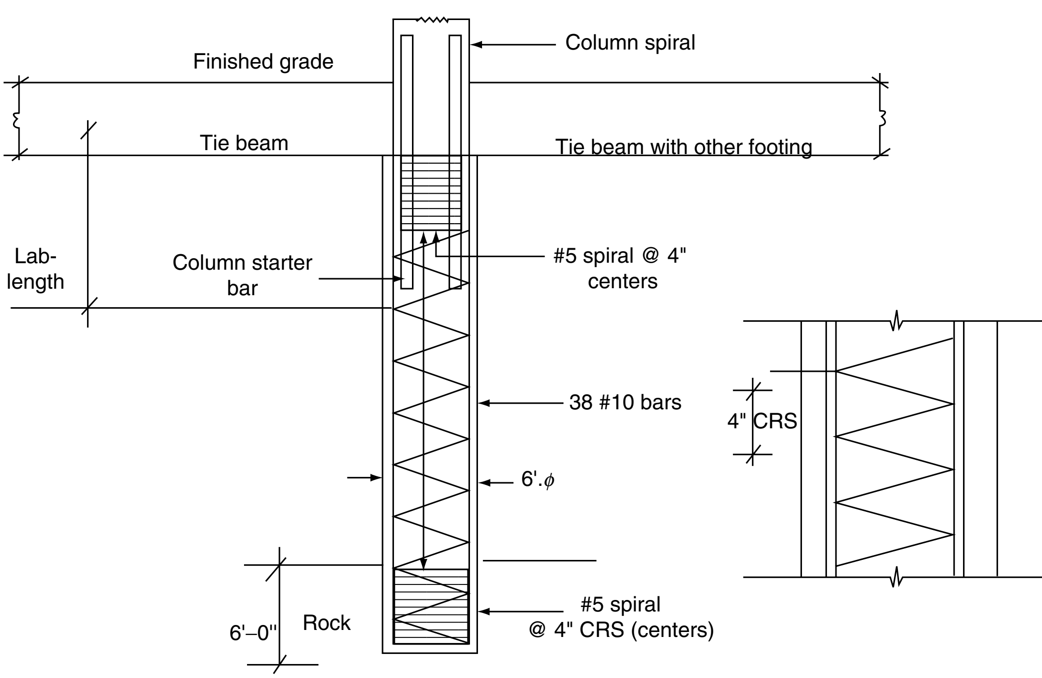


FIGURE 5.43
Rebar details of drilled shaft.

TABLE 5.13

Strength of Reinforced Column Sections from ACI Column Chart

ρ_g	p_f	$QP_n/4g$ (ksi)	ϕP_n (kips)	QM_n/Agh (ksi)	QM_n (k.ft)
0.012	2	2.60	10586	0.181	4422
0.012	5	1.318	5366	0.392	9576
0.012	6	0.400	1628.4	0.317	7743
0.012	7	0	0	0.254	6205

Source: LRFD Bridge Design Specifications, Customary U.S. Units, 2nd edn, American Association of State Highway and Transportation Officials, Washington, DC, 1998 (with 1999 interim revisions). With permission.

Example 5.10: Additional footing design example (rigid footing)

Machine Foundation Problem

As a foundation engineer you have been asked to design a machine foundation footing for a bakery mixer. The mixer loads are given below. The rear legs are subjected to additional shock load of 16 kips/ft^2. All four legs are identical with 100 in.^{2} area. Given that $f_c = 4$ ksi and $f_y = 60$ ksi, design the footing. Maximum allowable bearing pressure = 2.5 ksf (Figure 5.44–Figure 5.47).

Data from the manufacturer or mixer:

Net weight/leg DL = 10 kips/ft^2

Floor load, FL = 10 kips/ft^2

Shock load, SL = 16 kips/ft^2

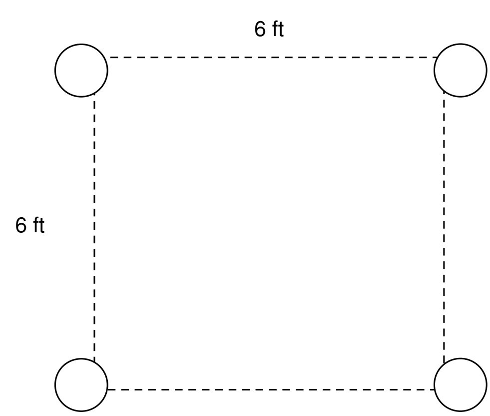


FIGURE 5.44
Illustration for Example 5.10 (plan).

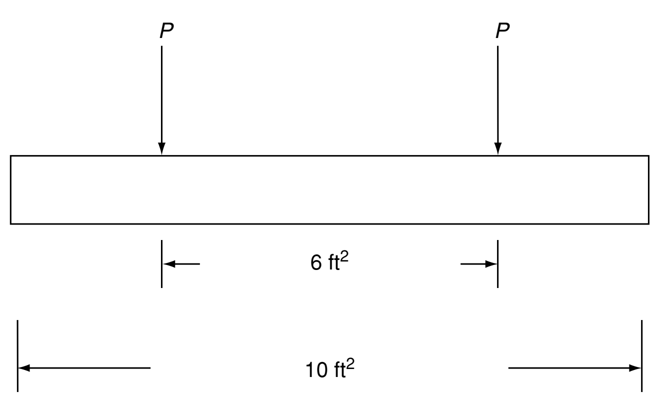


FIGURE 5.45
Illustration for Example 5.10 (elevation).

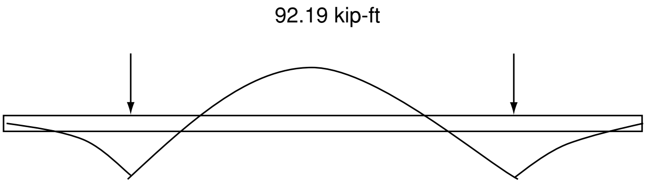


FIGURE 5.46
Bending moment diagram for Example 5.10.

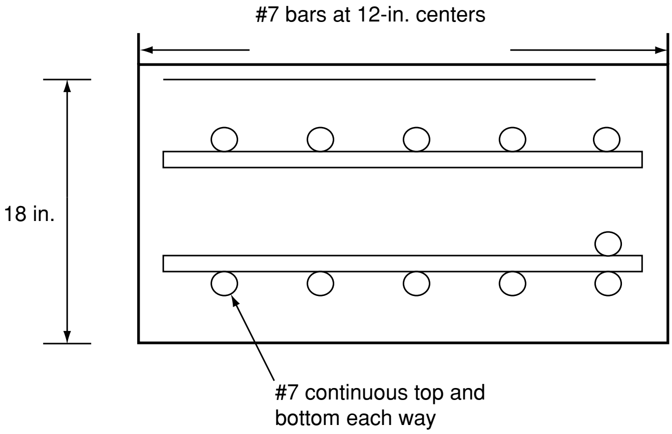


FIGURE 5.47
Reinforcement details (Example 5.10).

Load on rear legs $= 36$ kips/ft^2
Area of a rear leg $= 100$ in.^{2}
Area of a leg $= 0.69$ ft^2
Design load $= 1.6(\text{FL} + \text{SL} + \text{DL})$ for rear legs. Also note that a load factor of 1.6 is used

$P = 40.00$ kips

By considering the case with rear leg loading, we will design the footing for this load case:

Total load/rear legs $= 2P = 80$ kips
Width of the footing $= 4.00$ ft (4 ft strip) for worse condition
Length of the footing $= 10.00$ ft
Depth $= 1.50$ ft
$d = 13.50$ in.
Area $= 40.00$ ft^2, assume 3-in. cover
Pressure under the footing $= 2.225$ ksf < 2.5 ksf. OK

References

1. American Concrete Institute: ACI-318 Building Code Requirements for Reinforced Concrete and Commentary.
2. ACI 209 publication on shrinkage, Creep and Thermal Movements.
3. Notes on ACI 318-95 Building Code Requirements for Structural Concrete with Design Applica-

tion by Portland Cement Association.
4. AASHTO LRFD Bridge Design Specifications.
5. Florida Department of Transportation: Bridge Design Guidelines.
6. FHWA, 1998, *Design and Construction of Driven Pile Foundations*, Workshop Manual, vol. I, Publication Number FHWA HI-97-013, Revised, November.