Fundamentals of Electric Circuit

أسس الدارات الكهربائية

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CHAPTER 4 الفصل الرابع
AC – CIRCUITS
دارات التيار المتناوب

1) SINUSOIDS AND PHASORS الاشارات الجيبية والمطاور
2) SINUSOIDAL STEADY-STATE ANALYSIS دراسة الحالة المستقرة بالتيار المتناوب
3) AC POWER ANALYSIS دراسة الاستطاعة بالتيار المتناوب
4) FREQUENCY RESPONSE الاستجابة الترددية
5) Series Resonance الطنين التسلسلي
6) Parallel Resonance الطنين التفرعي
7) MAGNETICALLY COUPLED CIRCUITS الدارات المترابطة مغناطيسياً
8) Passive Filters المرشحات غير الفعالة

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الفصل الرابع: دارات التيار المتناوب
SINUSOIDS AND PHASORS
الإشارات الجيبية والمطاور

مقدمة: INTRODUCTION

A sinusoid : is a signal that has the form of the sine or cosine function

sinusoidal current is usually referred to as alternating current (ac).

ac circuits : Circuits driven by sinusoidal current or voltage sources.

SINUSOIDS

$$v(t) = V_m \sin \omega t$$

V_m = the *amplitude* of the sinusoid

ω = the *angular frequency* in radians/s

ωt = the *argument* of the sinusoid

$$T = \frac{2\pi}{\omega}$$

v(t) repeats itself every T seconds $v(t + T) = v(t)$

A periodic function :f (t) = f (t + nT), for all t and for all integers n.

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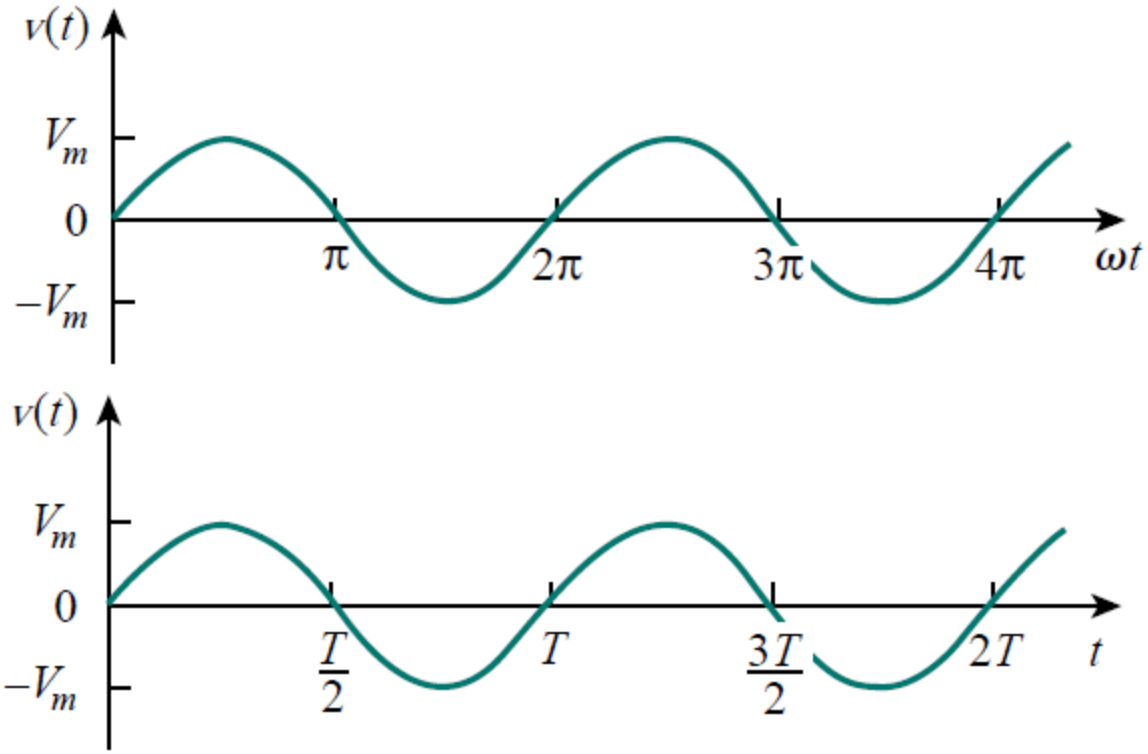
مقدمة: INTRODUCTION



$$f = \frac{1}{T} \qquad \omega = 2\pi f$$

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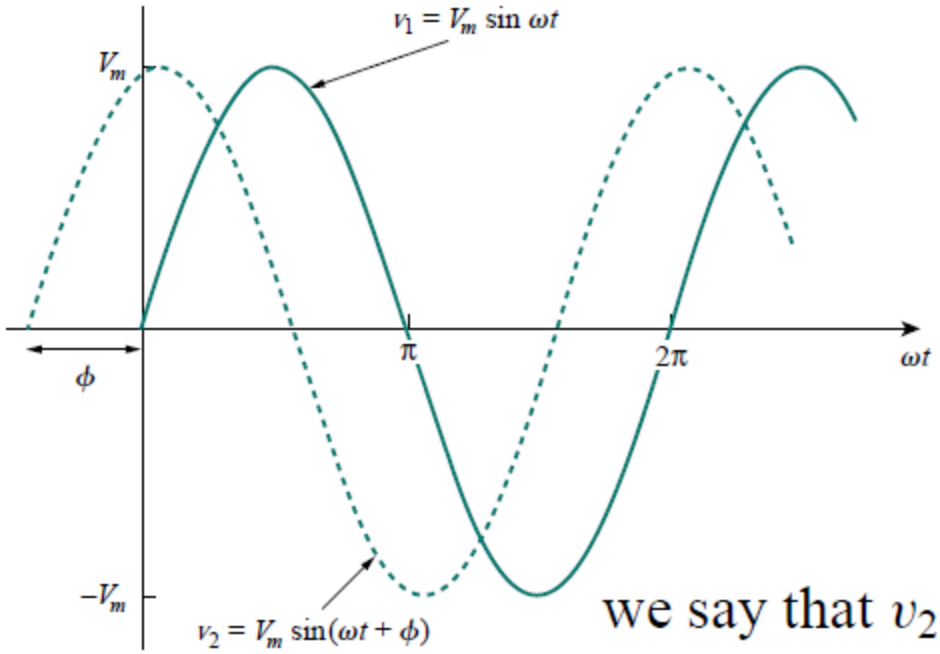
مقدمة: INTRODUCTION

$$v(t) = V_m \sin(\omega t + \phi)$$

الشكل العام لإشارة جيبية مع فرق بالطور Φ



$$v_1(t) = V_m \sin \omega t$$

$$v_2(t) = V_m \sin(\omega t + \phi)$$

الجهد V2 يتقدم بالطور Φ على الجهد V1
أو الجهد V1 يتأخر بالطور Φ عن V2

we say that v_2 *leads* v_1 by ϕ or that v_1 *lags* v_2 by ϕ

If $\phi \neq 0$ v_1 and v_2 are *out of phase*
If $\phi = 0$ v_1 and v_2 are *in phase*

A sinusoid can be expressed in either sine or cosine form

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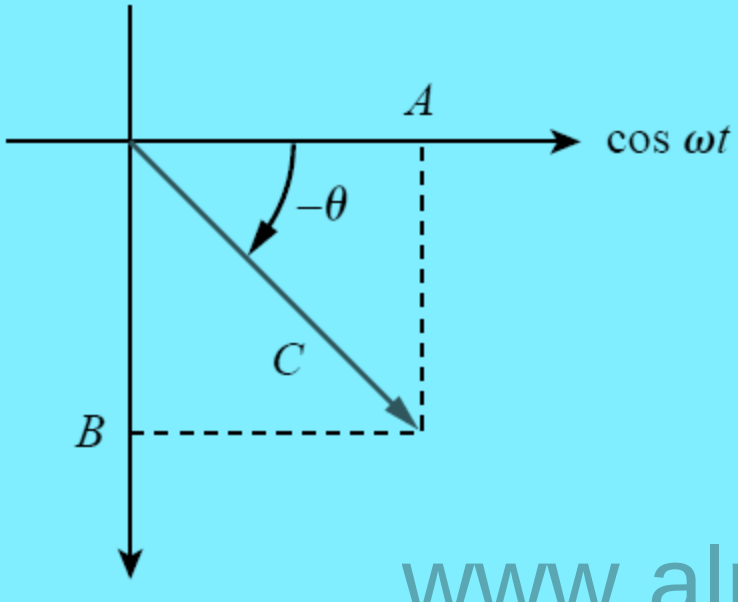
مقدمة: INTRODUCTION

$$A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$$

$$C = \sqrt{A^2 + B^2}, \qquad \theta = \tan^{-1}\frac{B}{A}$$



$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$

$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$

$$\sin(\omega t \pm 90^\circ) = \pm\cos\omega t$$

$$\cos(\omega t \pm 90^\circ) = \mp\sin\omega t$$

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PHASORS: المطاور

A phasor is a complex number that represents the amplitude and phase of a sinusoid:
هو عدد عقدي يمثل مطال وطور الإشارة الجيبية

$$v(t) = V_m \cos(\omega t + \phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)})$$

$$v(t) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t})$$

$$v(t) = \mathrm{Re}(\mathbf{V} e^{j\omega t})$$

$$\mathbf{V} = V_m e^{j\phi} = V_m \angle \phi$$

$\mathbf{V}$ is thus the *phasor representation* of the sinusoid $v(t)$

the term $e^{j\omega t}$ is implicitly present.: التردد موجود ضمنياً

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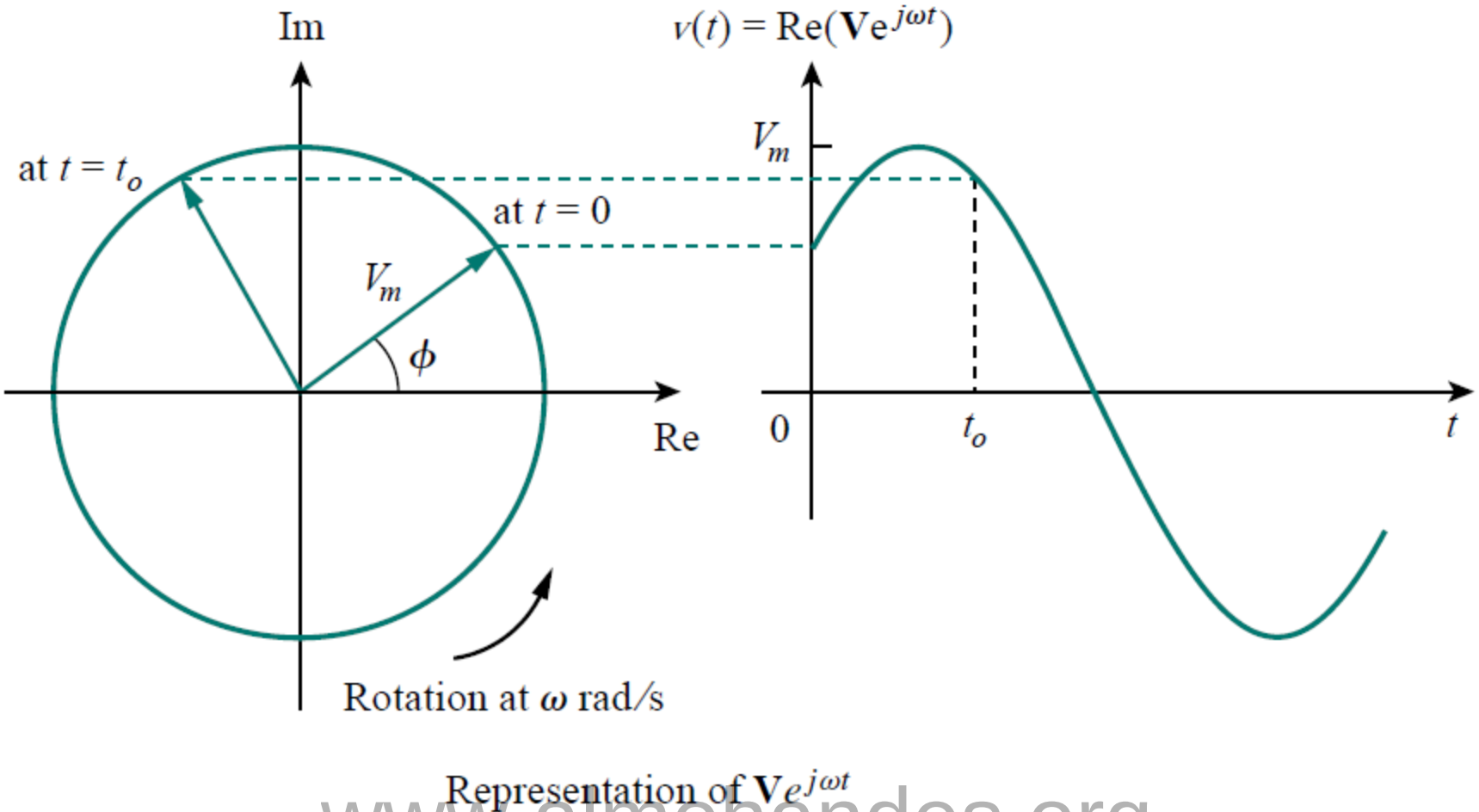
المطاور: PHASORS



Representation of $\mathbf{V}e^{j\omega t}$

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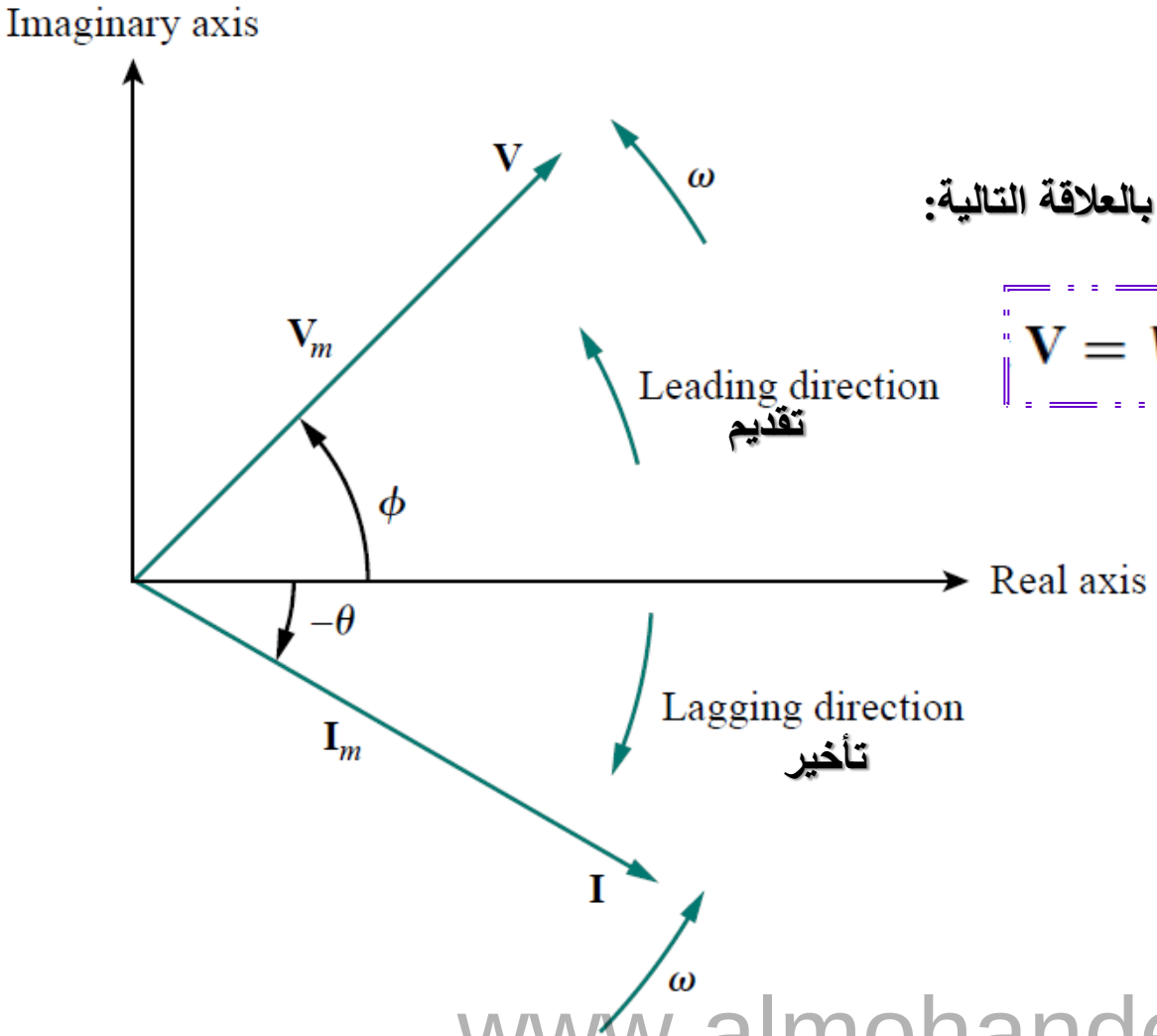
المطاور: PHASORS



التمثيل البياني لمطاوري الجهد والتيار المبينين بالعلاقة التالية:

$$\mathbf{V} = V_m \angle \phi \text{ and } \mathbf{I} = I_m \angle -\theta$$

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المطاور: PHASORS

$$v(t) = V_m \cos(\omega t + \phi) \iff \mathbf{V} = V_m \angle \phi$$

(Time-domain representation) ⟺ (Phasor-domain representation)

Sinusoid-phasor transformation.

Time-domain representation	Phasor-domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

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PHASORS: المطاور

مميزات استخدام المطاور:

$$\frac{dv}{dt} \iff j\omega\mathbf{V}$$

(Time domain) (Phasor domain)

$$\int v\,dt \iff \frac{\mathbf{V}}{j\omega}$$

(Time domain) (Phasor domain)

الفرق بين : $v(t)$ and $\mathbf{V}$

$v(t)$: تمثيل لحظي أو تابع للزمن وحقيقي

$\mathbf{V}$: تابع للتردد ومستقل عن الزمن وعقدي

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PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS: علاقات المطاور لعناصر الدارات الكهربائية

1- المقاومة:

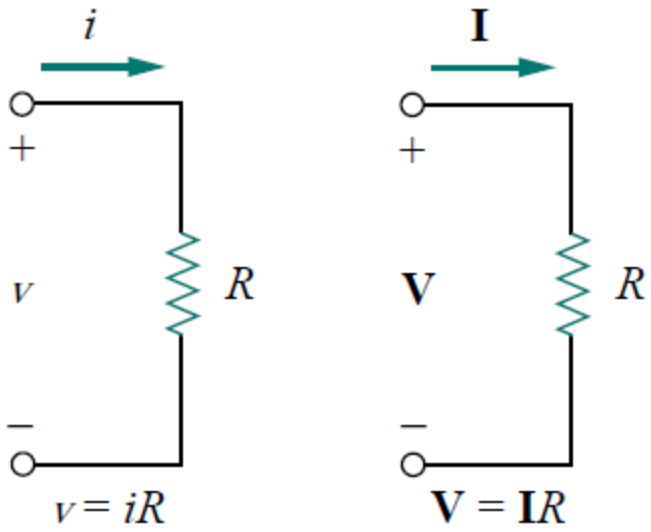
$$i = I_m \cos(\omega t + \phi)$$

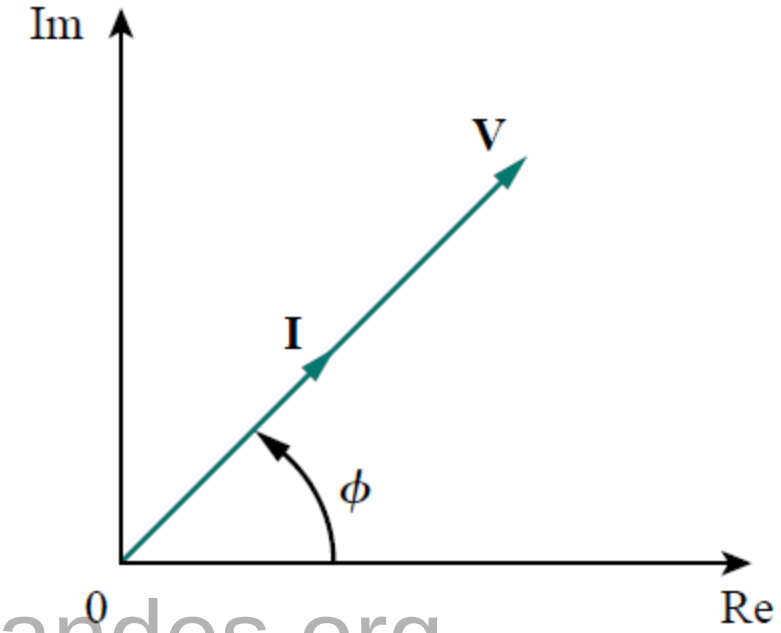
$$v = iR = RI_m \cos(\omega t + \phi)$$

$$\mathbf{V} = RI_m \angle \phi$$

$$\mathbf{I} = I_m \angle \phi$$

$$\mathbf{V} = R\mathbf{I}$$





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1- الملف :

$$i = I_m \cos(\omega t + \phi)$$

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$$

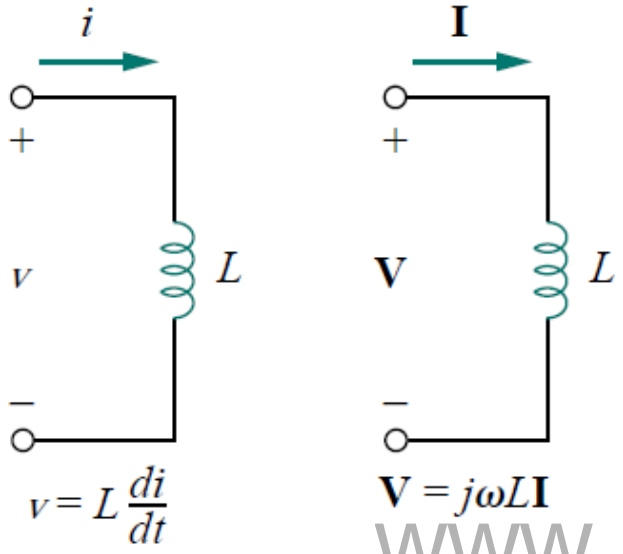
$$-\sin A = \cos(A + 90^\circ)$$

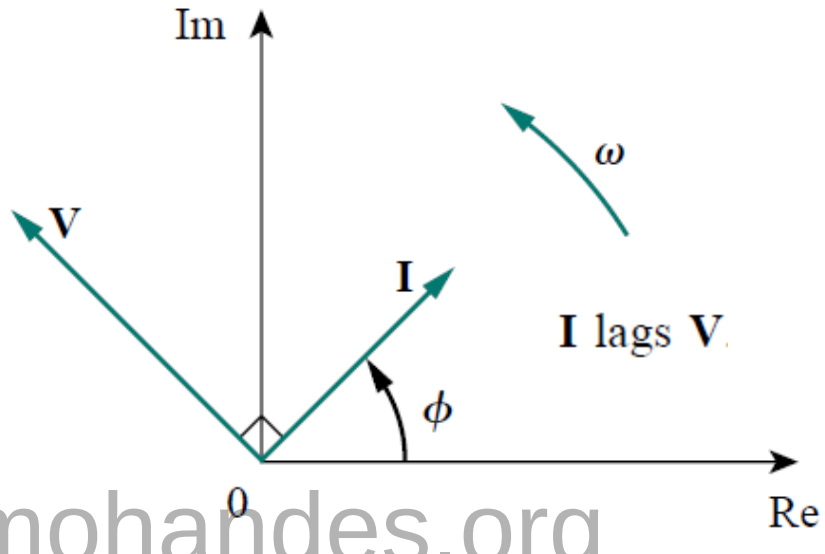
$$v = \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

$$\mathbf{V} = \omega L I_m e^{j(\phi+90^\circ)} = \omega L I_m e^{j\phi} e^{j90^\circ} = \omega L I_m \angle\phi\, e^{j90^\circ}$$

$$\mathbf{I} = I_m \angle\phi$$

$$\mathbf{V} = j\omega L\mathbf{I}$$





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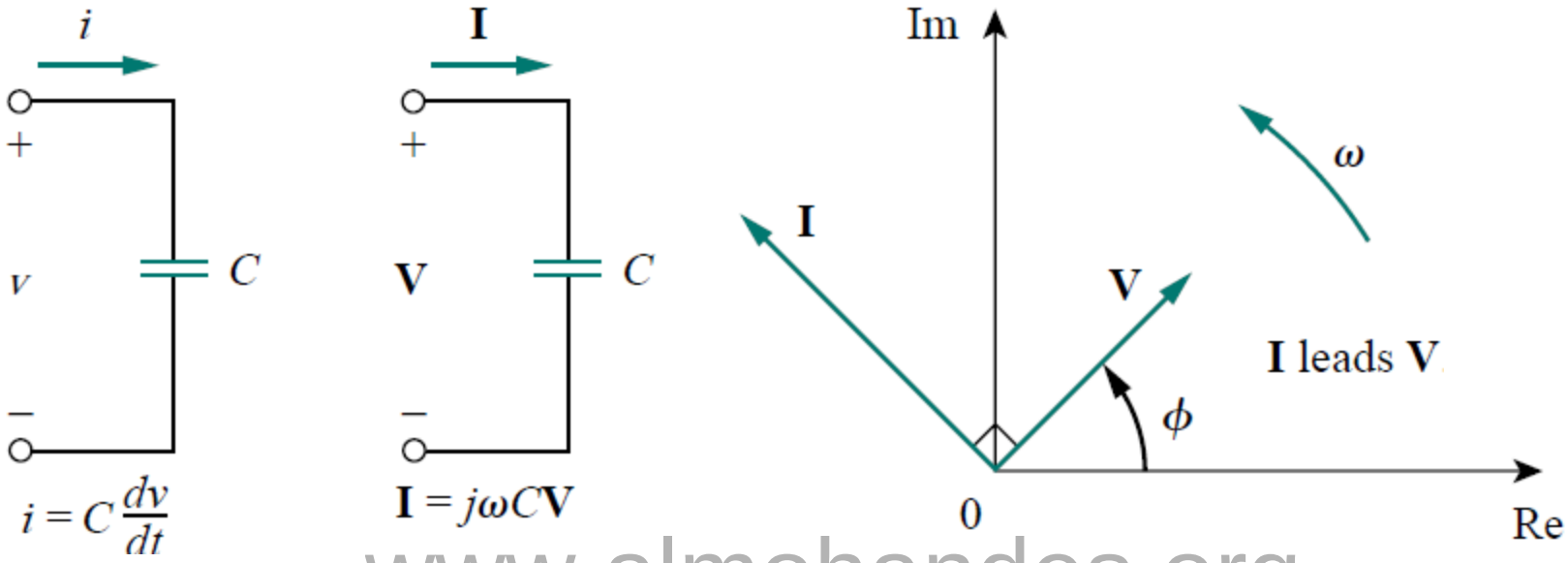
PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS:علاقات المطاور لعناصر الدارات الكهربائية

1- المكثف :

$$i = C\frac{dv}{dt} \qquad v = V_m \cos(\omega t + \phi)$$

$$\mathbf{I} = j\omega C\mathbf{V} \quad \Longrightarrow \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C} \qquad \mathbf{I} = I_m \angle \phi$$



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PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS:علاقات المطاور لعناصر الدارات الكهربائية

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

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IMPEDANCE AND ADMITTANCE: الممانعة والسماحية

The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms (Ω).

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \qquad \text{or} \qquad \mathbf{V} = \mathbf{ZI}$$

$$\mathbf{Z} = R + jX = |\mathbf{Z}|\angle\theta$$

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R}$$

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta$$

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

The admittance Y is the reciprocal of impedance, measured in siemens (S).

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}} \qquad \mathbf{Y} = G + jB$$

$G = \text{Re}\,\mathbf{Y}$ is called the *conductance*

$B = \text{Im}\,\mathbf{Y}$ is called the *susceptance*.

$$G + jB = \frac{1}{R + jX}\cdot\frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

$$G = \frac{R}{R^2 + X^2}, \qquad B = -\frac{X}{R^2 + X^2}$$

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Basic circuit laws (Ohm's and Kirchhoff's القوانين الأساسية:

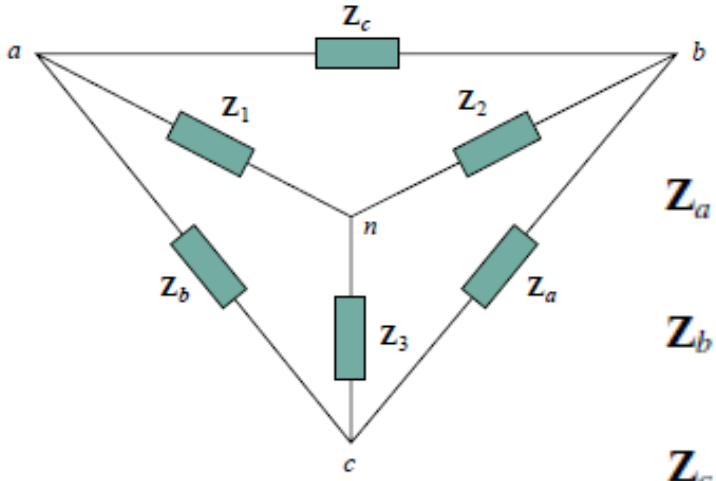
عملياً يمكن تطبيق جميع القوانين والعلاقات التي وجدناها سابقاً بحالة التيار المستمر على دارة تيار متناوب باستخدام المطاور

$$\mathbf{V} = \mathbf{ZI}$$

$$\sum \mathbf{I}_k = 0 \quad \text{(KCL)}$$

$$\sum \mathbf{V}_k = 0 \quad \text{(KVL)}$$

كذلك يمكن استخدام علاقات مجزئ الجهد والتيار وعلاقات التحويل من نجمي إلى مثلثي وبالعكس



Y-Δ

$$\mathbf{Z}_a = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_1}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_2}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_3\mathbf{Z}_1}{\mathbf{Z}_3}$$

Δ-Y

$$\mathbf{Z}_1 = \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_2 = \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

$$\mathbf{Z}_3 = \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_a + \mathbf{Z}_b + \mathbf{Z}_c}$$

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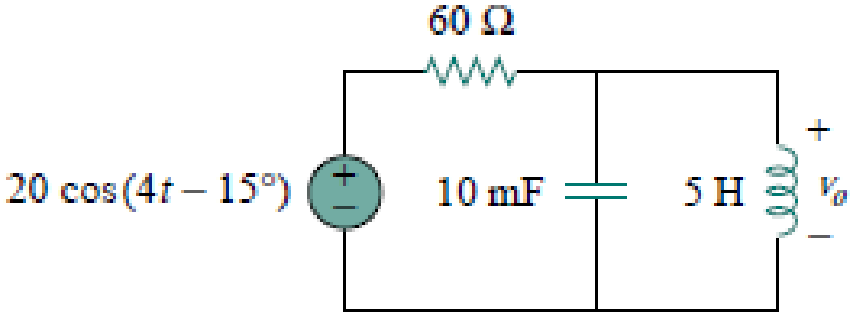
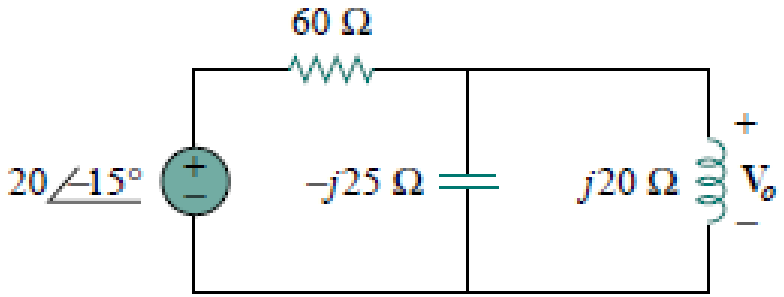
EXAMPLE

احسب $v_o(t)$ للدارة المبينة بالشكل التالي:

لإجراء الحل نقوم بتحويل الدارة من لمجال الزمني إلى المجال الترددي (phasor)





$$v_s = 20\cos(4t - 15^\circ) \quad \Longrightarrow \quad \mathbf{V}_s = 20\angle -15^\circ \text{ V}, \qquad \omega = 4$$

$$10 \text{ mF} \quad \Longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}}$$

$$= -j25\ \Omega$$

$$5 \text{ H} \quad \Longrightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

$$\mathbf{Z}_1 = 60\ \Omega \qquad \mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\angle -15^\circ)$$

$$= (0.8575\angle 30.96^\circ)(20\angle -15^\circ) = 17.15\angle 15.96^\circ \text{ V}$$

$$v_o(t) = 17.15\cos(4t + 15.96^\circ)\text{V}$$

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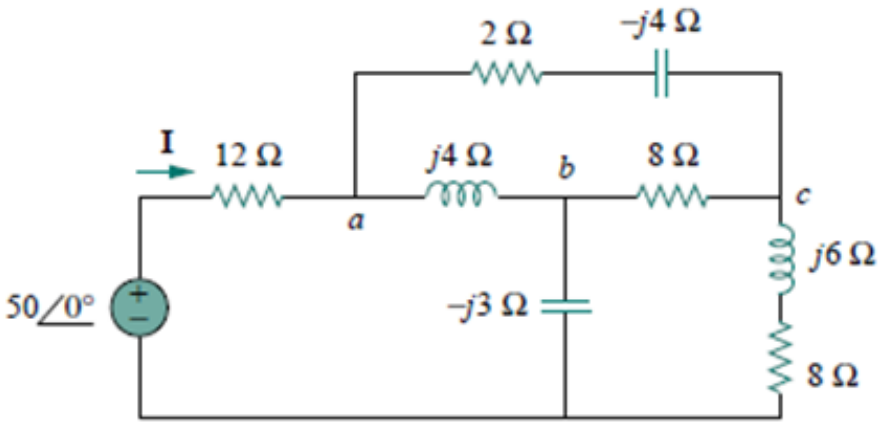
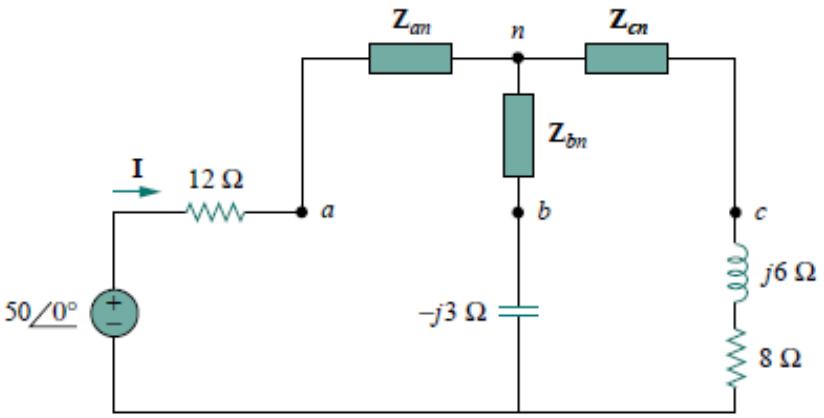
EXAMPLE

احسب التيار I للدارة المبينة بالشكل التالي:





$$\mathbf{Z}_{an} = \frac{j4(2-j4)}{j4+2-j4+8} = \frac{4(4+j2)}{10} = (1.6+j0.8)\ \Omega$$

$$\mathbf{Z}_{bn} = \frac{j4(8)}{10} = j3.2\ \Omega, \qquad \mathbf{Z}_{cn} = \frac{8(2-j4)}{10} = (1.6-j3.2)\ \Omega$$

$$\mathbf{Z} = 12 + \mathbf{Z}_{an} + (\mathbf{Z}_{bn} - j3) \parallel (\mathbf{Z}_{cn} + j6 + 8)$$
$$= 12 + 1.6 + j0.8 + (j0.2) \parallel (9.6 + j2.8)$$
$$= 13.6 + j0.8 + \frac{j0.2(9.6+j2.8)}{9.6+j3}$$
$$= 13.6 + j1 = 13.64\angle 4.204^\circ\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{50\angle 0^\circ}{13.64\angle 4.204^\circ} = 3.666\angle -4.204^\circ\ \text{A}$$

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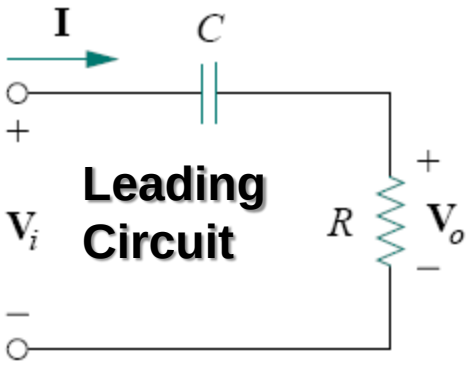
الإشارات الجيبية والمطاور

دارات إزاحة الطور: Phase-Shifters تستخدم دارات **RL** أو دارات **RC** لازاحة طور الاشارة المتناوبة نظراً لوجود المكثف الذي يقدم التيار المار بالدارة على الجهد المطبق عليها .

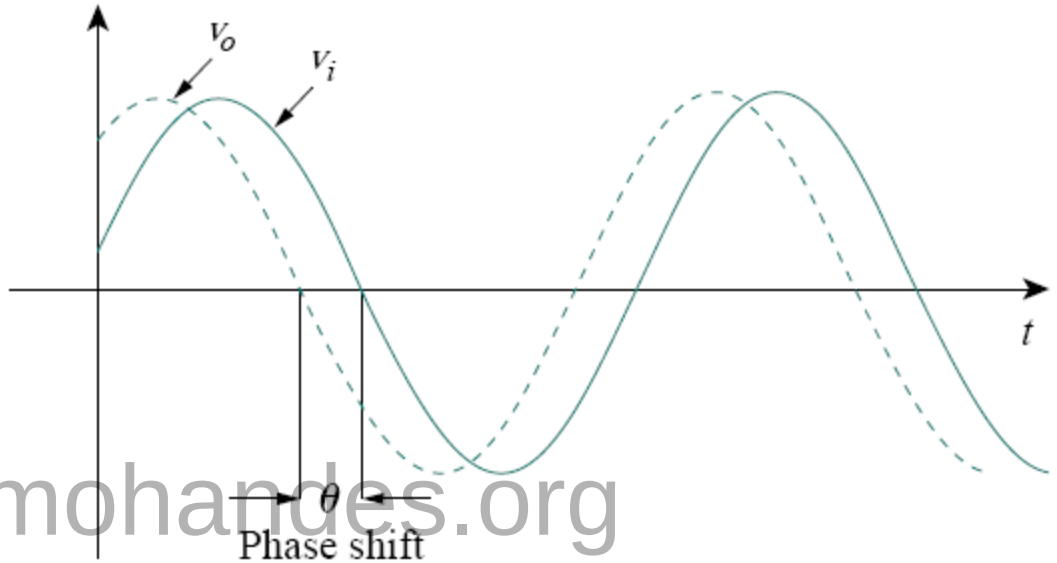


$$\theta = \tan^{-1}\frac{X_C}{R}$$

$$X_C = -1/\omega C$$

$$0 < \theta < 90^\circ$$

Since the output voltage $V_o(t)$ across the resistor is in phase with the current, $V_o(t)$ leads (positive phase shift) $V_i(t)$ as



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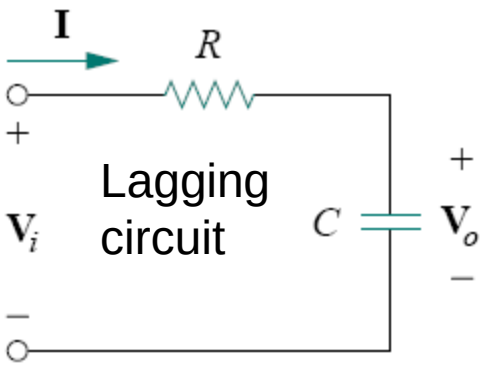
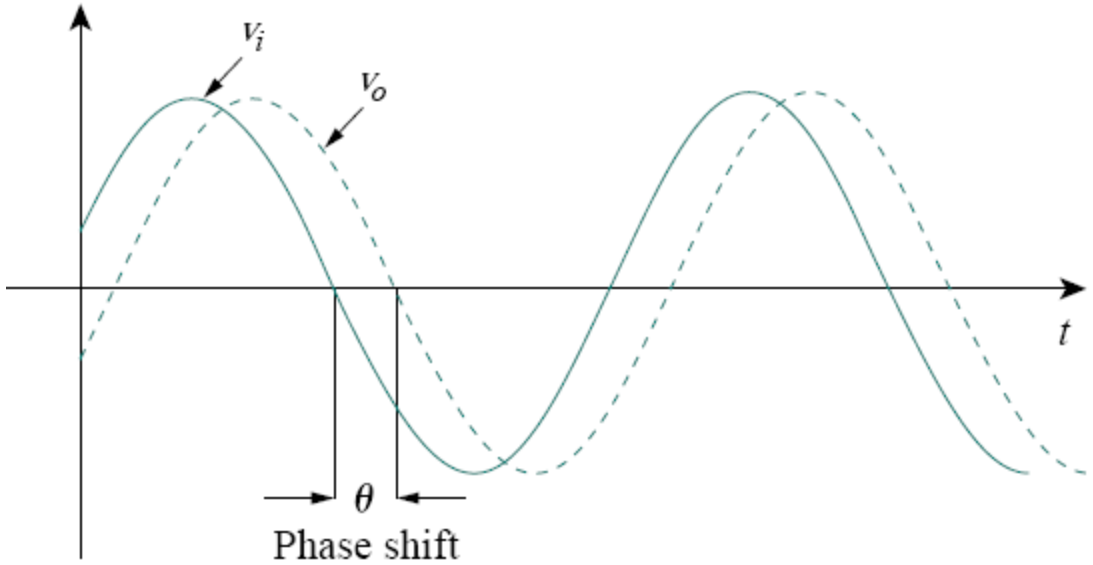
دارات إزاحة الطور: Phase-Shifters

the output is taken across the capacitor :

I leads the input voltage $v_i(t)$ by θ, but the output voltage $v_o(t)$ across the capacitor lags (negative phase shift) the input voltage $v_i(t)$ as shown:





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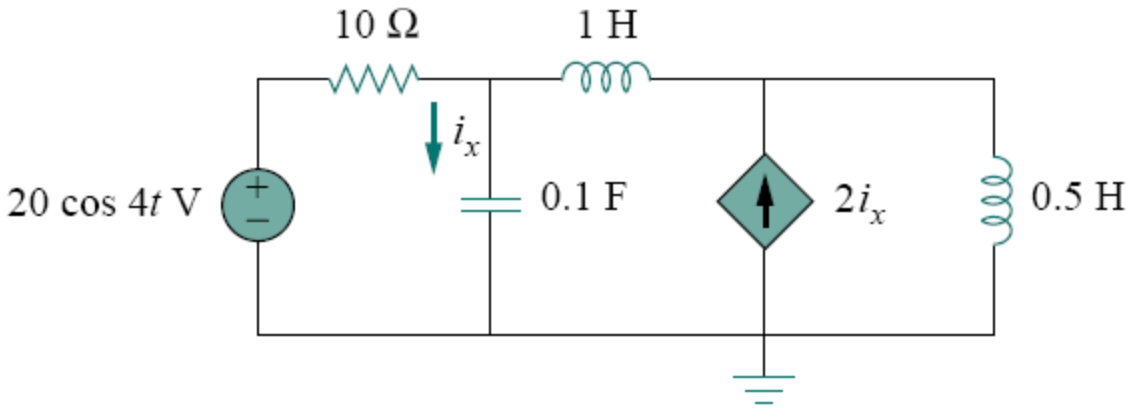
Steps to Analyze ac Circuits خطوات دراسة دارات التيار المتناوب:

1. Transform the circuit to the phasor or frequency domain. حول الدارة لى المجال التردد باستخدام المطاور
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).(..... حل الدارة باستخدام الطرق المختلفة (الحلقات , العقد, التراكب الخ
3. Transform the resulting phasor to the time domain. حول النتيجة إلى المجال الزمني

EXAMPLE

أوجد التيار i_x للدارة المبين بالشكل التالي :

NODAL ANALYSIS



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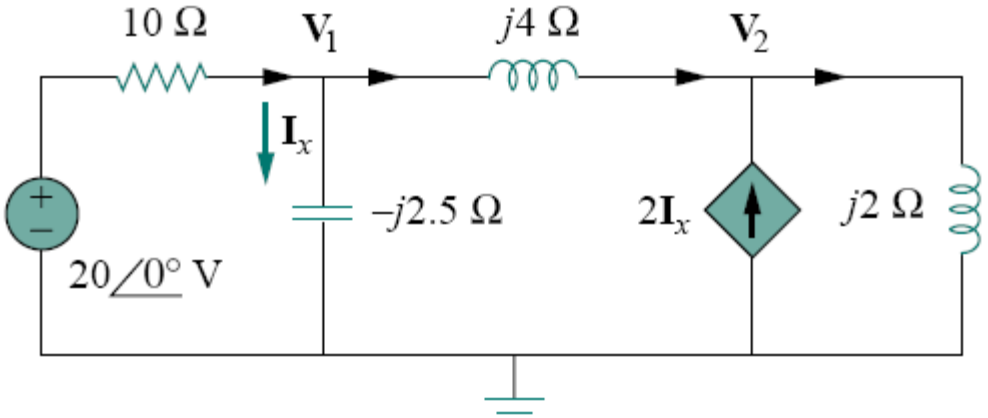
الفصل الرابع: دارات التيار المتناوب

SINUSOIDAL STEADY-STATE ANALYSIS

دراسة الحالة المستقرة بالتيار المتناوب

EXAMPLE

$$20\cos 4t \Longrightarrow 20\angle 0^\circ, \quad \omega = 4 \text{ rad/s}$$
$$1 \text{ H} \Longrightarrow j\omega L = j4$$
$$0.5 \text{ H} \Longrightarrow j\omega L = j2$$
$$0.1 \text{ F} \Longrightarrow \frac{1}{j\omega C} = -j2.5$$



$$\frac{20 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4}$$

$$(1 + j1.5)\mathbf{V}_1 + j2.5\mathbf{V}_2 = 20$$

$$2\mathbf{I}_x + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$\mathbf{I}_x = \mathbf{V}_1/-j2.5$$

$$\frac{2\mathbf{V}_1}{-j2.5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j4} = \frac{\mathbf{V}_2}{j2}$$

$$11\mathbf{V}_1 + 15\mathbf{V}_2 = 0$$

$$\mathbf{V}_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97\angle 18.43^\circ \text{ V}$$

$$\mathbf{V}_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91\angle 198.3^\circ \text{ V}$$

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{-j2.5} = \frac{18.97\angle 18.43^\circ}{2.5\angle -90^\circ} = 7.59\angle 108.4^\circ \text{ A}$$

$$i_x = 7.59\cos(4t + 108.4^\circ) \text{ A}$$

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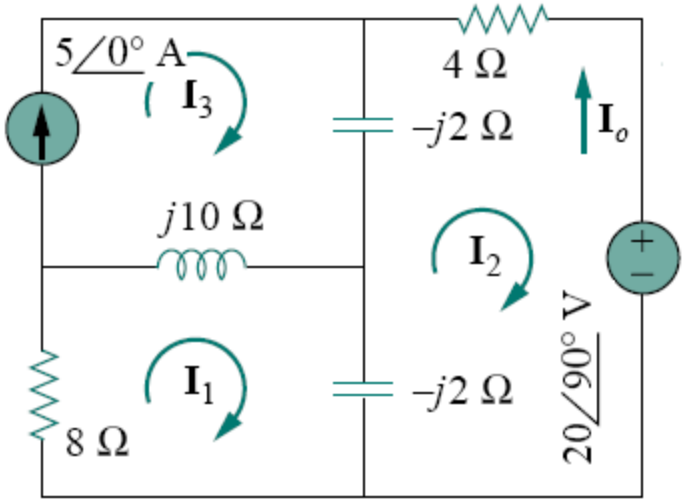
EXAMPLE

أوجد التيار $\mathbf{I}_o$ في الدارة المبين بالشكل التالي بطريقة الحلقات

MESH ANALYSIS



$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0$$

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\angle 90^\circ = 0$$

$$\mathbf{I}_3 = 5$$

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle -35.22^\circ}{68} = 6.12\angle -35.22^\circ \text{ A}$$

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle 144.78^\circ \text{ A}$$

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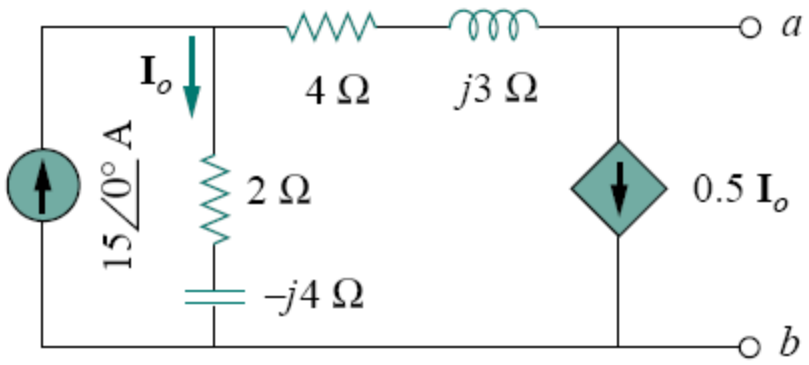
EXAMPLE

THEVENIN AND NORTON EQUIVALENT CIRCUITS

أوجد دارة ثفنن المكافئة المنظورة من القطبين a,b للدارة المبينة بالشكل التالي:

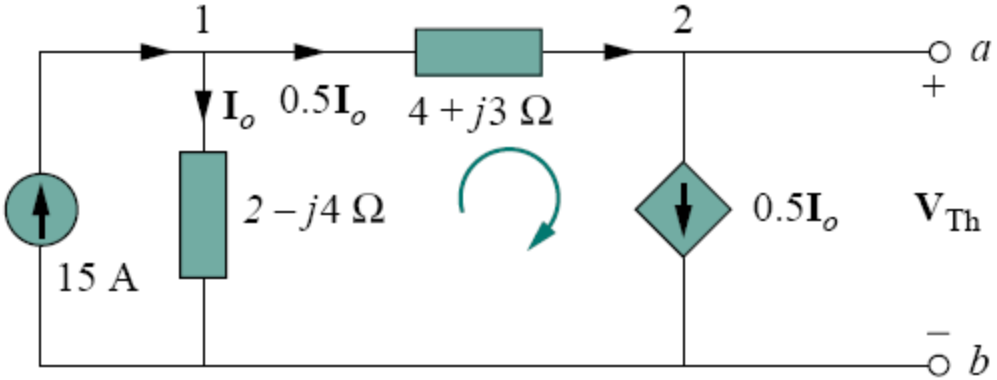


$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o \quad \Longrightarrow \quad \mathbf{I}_o = 10 \text{ A}$$

$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{\text{Th}} = 0$$

$$\mathbf{V}_{\text{Th}} = 10(2 - j4) - 5(4 + j3) = -j55$$

$$\mathbf{V}_{\text{Th}} = 55\angle -90° \text{ V}$$



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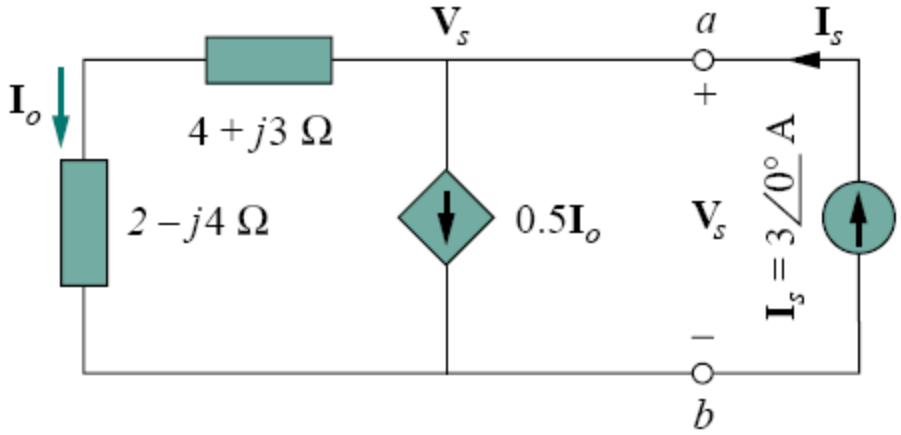
EXAMPLE

THEVENIN AND NORTON EQUIVALENT CIRCUITS

3 امبير قيمة لا على التعين قابلة للقسمة على مجموع تيارات العقدة V_S



$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \quad \Longrightarrow \quad \mathbf{I}_o = 2\ \text{A}$$

$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

$$\mathbf{Z}_{\text{Th}} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega$$

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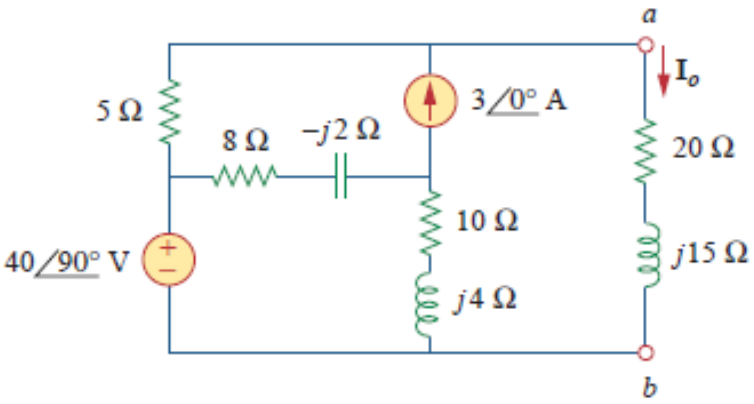
EXAMPLE

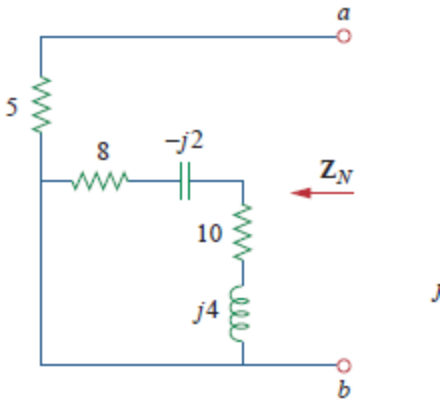
Obtain current **Io** ***in Fig. below using Norton's theorem.***



THEVENIN AND NORTON EQUIVALENT CIRCUITS

We set the sources to zero as shown

$Z_N = 5$



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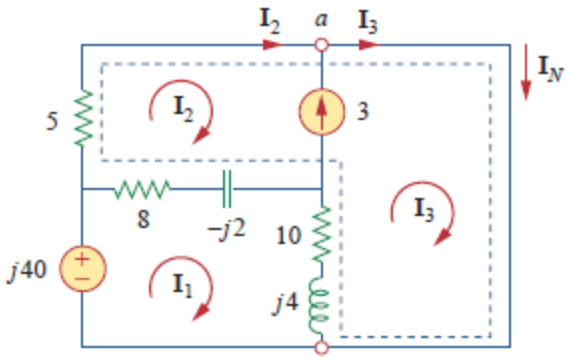
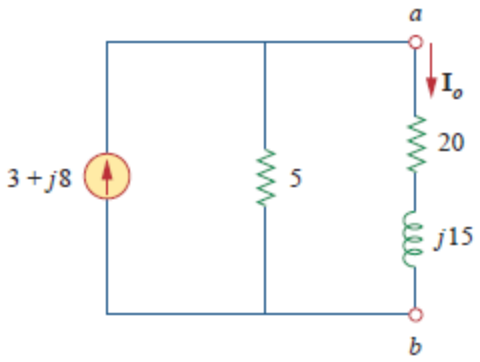
EXAMPLE

THEVENIN AND NORTON EQUIVALENT CIRCUITS

To get I_N we short-circuit terminals *a-b as in Fig Below*





$$\mathbf{I}_o = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = 1.465\angle 38.48^\circ \text{ A}$$

Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1:

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0$$

For the supermesh, $$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0$$

At node *a, due to the current source between meshes 2 and 3,* $\mathbf{I}_3 = \mathbf{I}_2 + 3$

Adding Eqs. gives $$-j40 + 5\mathbf{I}_2 = 0 \quad \Rightarrow \quad \mathbf{I}_2 = j8$$

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$

The Norton current is $\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)\text{ A}$

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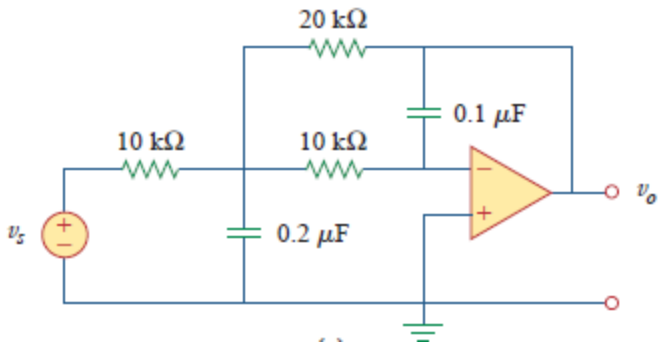
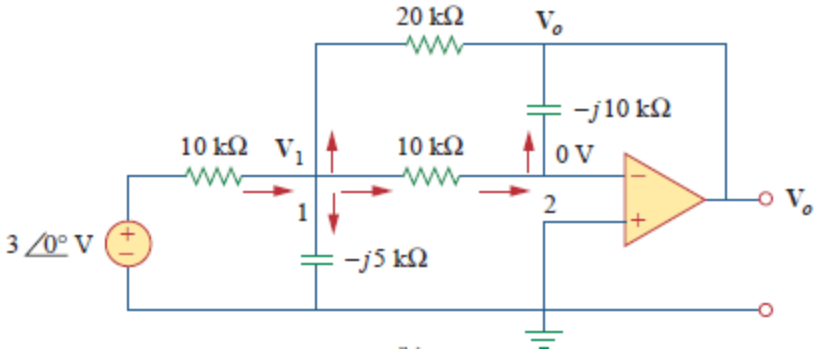
Op Amp AC Circuits

the key to analyzing op amp circuits is to keep two important properties of an ideal op amp in mind:

1. No current enters either of its input terminals.
2. The voltage across its input terminals is zero.

Determine V_o (t) for the op amp circuit in) if V_s=3 cos 1000*t V.*





$$\frac{3\angle 0° - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1}{-j5} + \frac{\mathbf{V}_1 - 0}{10} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$6 = (5 + j4)\mathbf{V}_1 - \mathbf{V}_o$$

$$\frac{\mathbf{V}_1 - 0}{10} = \frac{0 - \mathbf{V}_o}{-j10}$$

$$\mathbf{V}_1 = -j\mathbf{V}_o$$

$$6 = -j(5 + j4)\mathbf{V}_o - \mathbf{V}_o = (3 - j5)\mathbf{V}_o$$

$$\mathbf{V}_o = \frac{6}{3 - j5} = 1.029\angle 59.04°$$

$$v_o(t) = 1.029\cos(1000t + 59.04°)\ \text{V}$$

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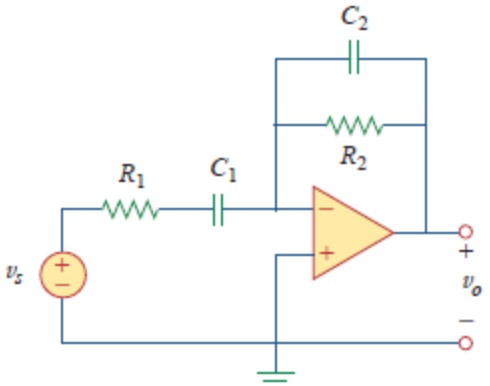
Op Amp AC Circuits

Compute the closed-loop gain and phase shift for the circuit in Fig.

Assume that $R_1 = R_2 = 10\ \text{k}\Omega$, $C_1 = 2\ \mu\text{F}$, $C_2 = 1\ \mu\text{F}$, and $\omega = 200$ rad/s.



$$\mathbf{Z}_f = R_2 \left\| \frac{1}{j\omega C_2} = \frac{R_2}{1 + j\omega R_2 C_2} \right.$$

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

$$\mathbf{G} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

$$\mathbf{G} = \frac{-j4}{(1 + j4)(1 + j2)} = 0.434 \angle 130.6^\circ$$

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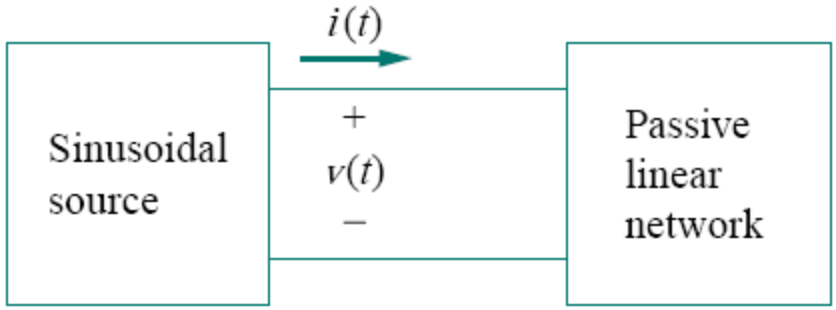
الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب

الاستطاعة اللحظية والوسطى: INSTANTANEOUS AND AVERAGE POWER



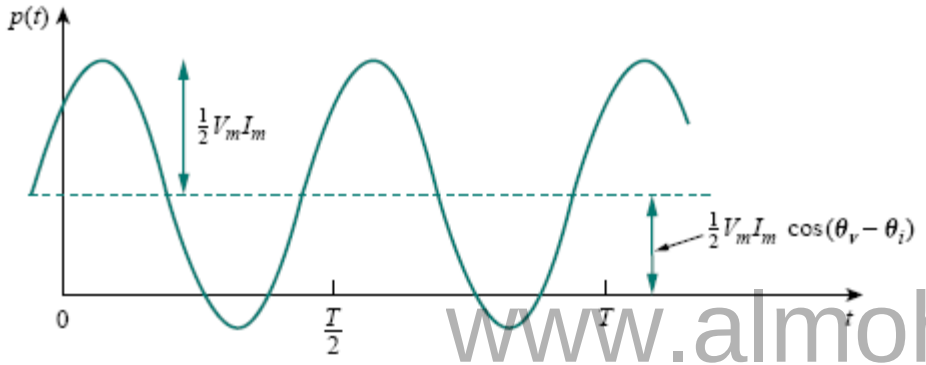
$$p(t) = v(t)i(t)$$
$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

الاستطاعة اللحظية :

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$



$$p(t) = p(t + T_0)$$
$$T_0 = T/2$$
$$T = 2\pi/\omega$$

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الفصل الرابع: دارات التيار المتناوب

AC POWER ANALYSIS

دراسة الاستطاعة بالتيار المتناوب

الاستطاعة الوسطى :

$$P = \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$P = \frac{1}{2}\text{Re}\left[\mathbf{V}\mathbf{I}^*\right] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

$$\mathbf{V} = V_m \angle\theta_v \text{ and } \mathbf{I} = I_m \angle\theta_i$$

$\theta_v = \theta_i$ resistive load

$$\frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}V_m I_m \angle\theta_v - \theta_i$$

$$= \frac{1}{2}V_m I_m \left[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)\right]$$

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$$

$$|\mathbf{I}|^2 = \mathbf{I} \times \mathbf{I}^*$$

$\theta_v - \theta_i = \pm 90°$ reactive circuit

$$P = \frac{1}{2}V_m I_m \cos 90° = 0$$

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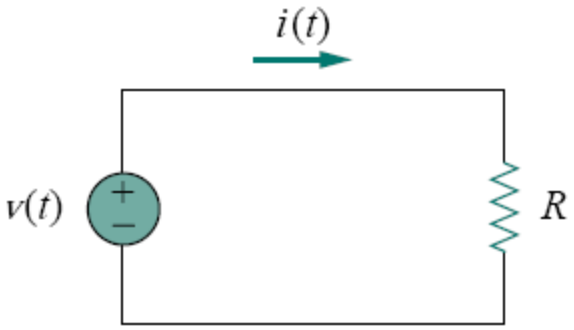
الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب

القيمة الفعالة : EFFECTIVE OR RMS VALUE

القيمة الفعالة لتيارمتناوبة هو التيار المستمر الذي يولد استطاعة وسطى في مقاومة مساوية للاستطاعة الوسطى للتيار المتناوب

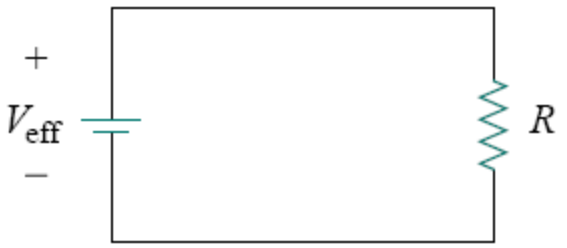
The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.



$$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt \qquad P = I_{\text{eff}}^2 R$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} \qquad V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

I_{eff}



$$I_{\text{eff}} = I_{\text{rms}}, \qquad V_{\text{eff}} = V_{\text{rms}} \qquad \textit{root-mean-square}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t\,dt} = \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1+\cos 2\omega t)\,dt} = \frac{I_m}{\sqrt{2}}$$

$$v(t) = V_m \cos \omega t \qquad V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \qquad P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

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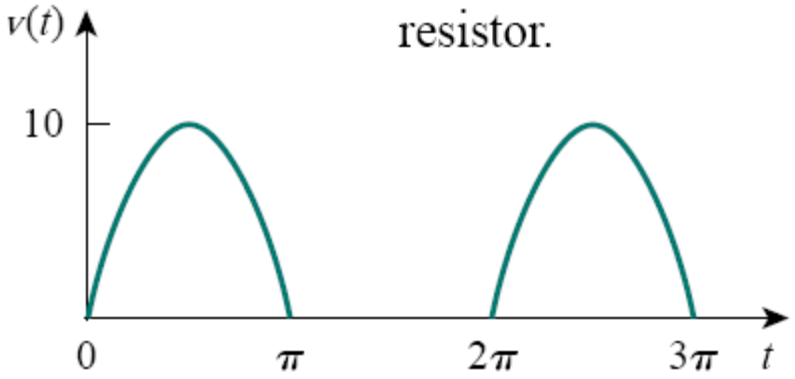
EXAMPLE Find the rms value and the amount of average power dissipated in a 10-Ω resistor.



$$T = 2\pi$$

$$v(t) = \begin{cases} 10\sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

$$\sin^2 t = \tfrac{1}{2}(1 - \cos 2t)$$

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v^2(t)\,dt = \frac{1}{2\pi}\left[\int_0^{\pi} (10\sin t)^2\,dt + \int_{\pi}^{2\pi} 0^2\,dt\right]$$

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^{\pi} \frac{100}{2}(1 - \cos 2t)\,dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\Bigg|_0^{\pi}$$

$$= \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25, \qquad V_{\text{rms}} = 5\ \text{V}$$

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5\ \text{W}$$

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APPARENT POWER AND POWER FACTOR: الاستطاعة الظاهرية ومعامل الاستطاعة

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad i(t) = I_m \cos(\omega t + \theta_i)$$

$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i) \qquad \textit{apparent power}$$

$$S = V_{\text{rms}} I_{\text{rms}}$$

The apparent power (in VA) is the product of the rms values of voltage and current.

$$\textit{power factor (pf)} = \cos(\theta_v - \theta_i)$$

$\theta_v - \theta_i$ is called the *power factor angle*, angle of the load impedance

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i$$

The power factor is the cosine of the phase difference between voltage and current. It is also the cosine of the angle of the load impedance.

معامل الاستطاعة هو تحيب زاوية ممانعة الحمل

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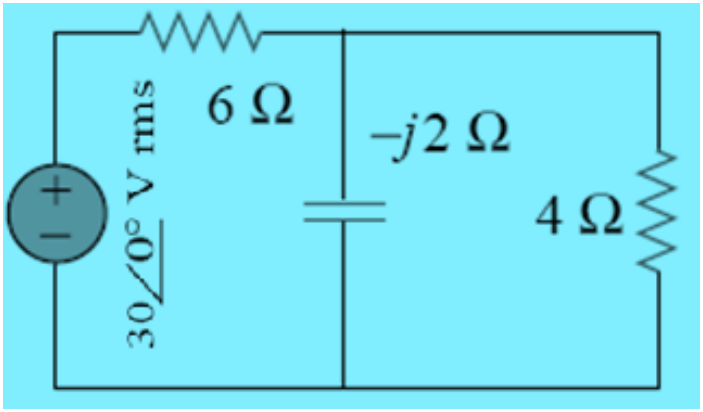
الفصل الرابع: دارات التيار المتناوب

AC POWER ANALYSIS

دراسة الاستطاعة بالتيار المتناوب

EXAMPLE

احسب معامل الاستطاعة للدارة المبينة بالشكل التالي ثم احسب الاستطاعة الوسطى الي يقدمها المولد للدارة



$$\mathbf{Z} = 6 + 4 \parallel (-j2)$$

$$= 6 + \frac{-j2 \times 4}{4 - j2}$$

$$= 6.8 - j1.6 = 7\angle -13.24\ \Omega$$

$$\text{pf} = \cos(-13.24) = 0.9734 \quad \text{(leading)} \quad \text{impedance is capacitive}$$

$$\mathbf{I}_{\text{rms}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{Z}} = \frac{30\angle 0°}{7\angle -13.24°} = 4.286\angle 13.24°\ \text{A}$$

$$P = V_{\text{rms}} I_{\text{rms}}\ \text{pf} = (30)(4.286)0.9734 = 125\ \text{W}$$

or

$$P = I_{\text{rms}}^2 R = (4.286)^2(6.8) = 125\ \text{W}$$ R is the resistive part of $\mathbf{Z}$.

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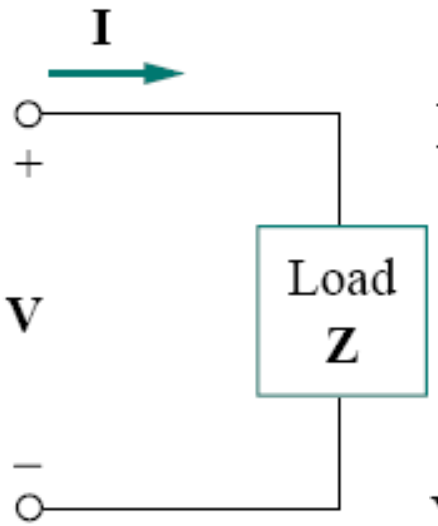
الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب

الاستطاعة العقدية: COMPLEX POWER



$$\mathbf{I} = I_m \angle \theta_i \qquad \mathbf{V} = V_m \angle \theta_v$$

$$\mathbf{S} = \mathbf{V}_{\text{rms}} \mathbf{I}^*_{\text{rms}} \qquad \mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^*$$

volt-amperes (VA)

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}} \angle \theta_v \qquad \mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}} \angle \theta_i$$

$$\mathbf{S} = V_{\text{rms}} I_{\text{rms}} \angle \theta_v - \theta_i$$

$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) + j V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \angle \theta_v - \theta_i$$

$$\mathbf{S} = I^2_{\text{rms}} \mathbf{Z} = \frac{V^2_{\text{rms}}}{\mathbf{Z}^*}$$

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الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب

الاستطاعة العقدية:COMPLEX POWER

$$\mathbf{S} = I_{\text{rms}}^2(R + jX) = P + jQ$$

الاستطاعة الظاهرية تقدر بـــ VA

$$P = \text{Re}(\mathbf{S}) = I_{\text{rms}}^2 R$$

الاستطاعة الحقيقية تقدر بـــ Watt

$$Q = \text{Im}(\mathbf{S}) = I_{\text{rms}}^2 X$$

الاستطاعة الردية تقدر بـــ VAR

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الاستطاعة العقدية:COMPLEX POWER

1. $Q = 0$ for resistive loads (unity pf).
2. $Q < 0$ for capacitive loads (leading pf).
3. $Q > 0$ for inductive loads (lagging pf).

Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.

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دراسة الاستطاعة بالتيار المتناوب

يمكن حساب الاستطاعة العقدية من مطاور الجهد والتيار وبالتالي يمكن منها الحصول على كل مركبات استطاعة الحمل :

$$\text{Complex Power} = \mathbf{S} = P + jQ = \frac{1}{2}\mathbf{V}\mathbf{I}^* = V_{\text{rms}} I_{\text{rms}} \angle \theta_v - \theta_i$$

$$\text{Apparent Power} = S = |\mathbf{S}| = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$

$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

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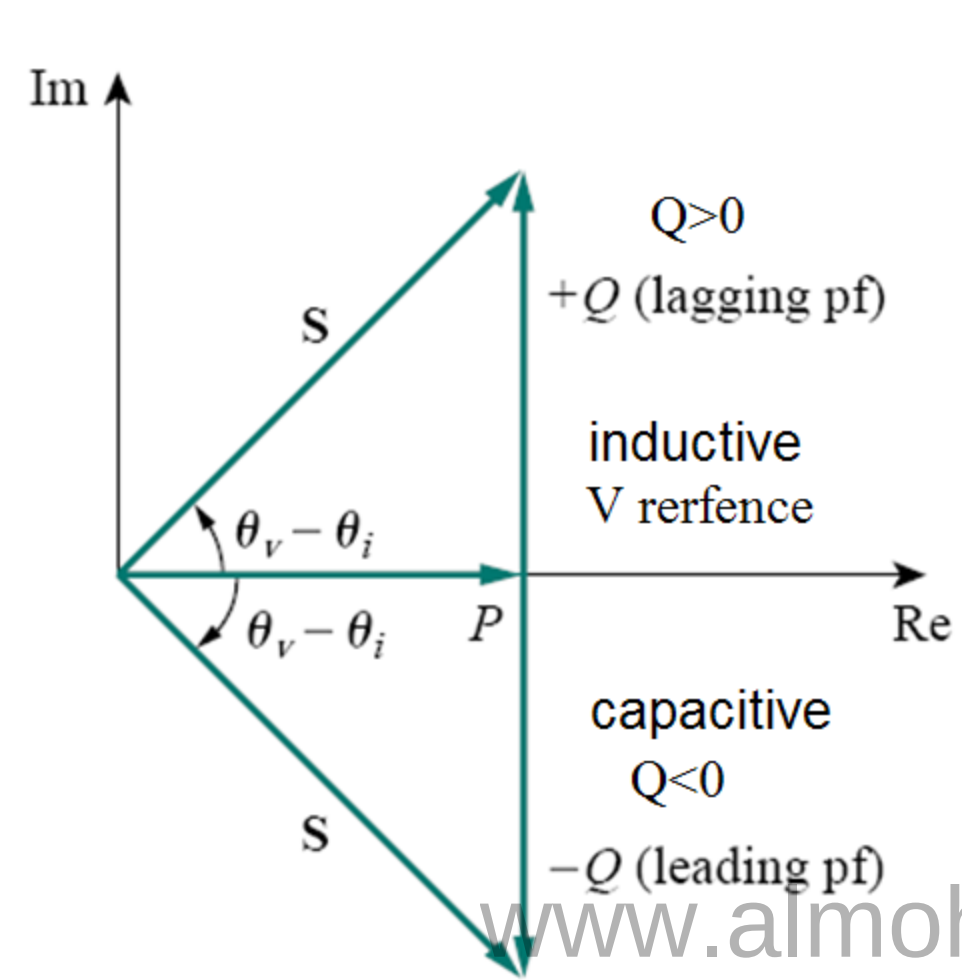
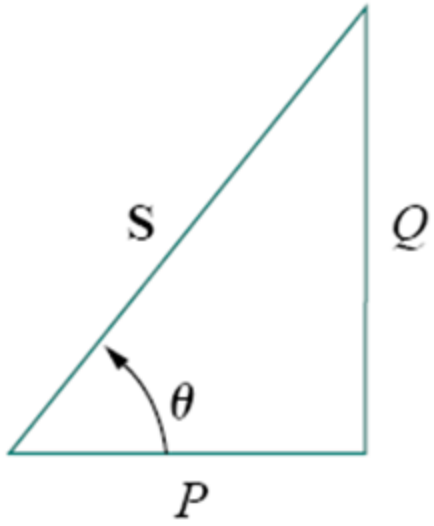
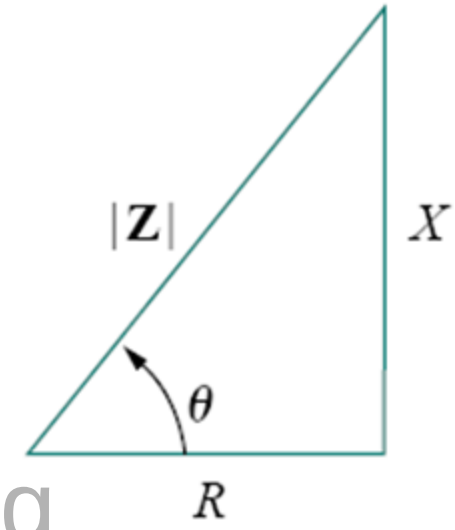
الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب

من الشائع عملياً تمثيل الاستطاعة العقدية باستخدام مثلث الاستطاعة على غرار مثلث الممانعة كما هو مبين بالشكل التالي:







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EXAMPLE A load Z draws 12 kVA at a power factor of 0.856 lagging from a 120-V rms sinusoidal source. Calculate:

(a) the average and reactive powers delivered to the load,
(b) the peak current, and
(c) the load impedance.

$S = 12{,}000 \text{ VA},$

$$\text{pf} = \cos\theta = 0.856$$

$$\theta = \cos^{-1} 0.856 = 31.13^\circ$$

$$P = S\cos\theta = 12{,}000 \times 0.856 = 10.272 \text{ kW}$$

$$Q = S\sin\theta = 12{,}000 \times 0.517 = 6.204 \text{ kVA}$$

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^*$$

$$\mathbf{I}_{\text{rms}}^* = \frac{\mathbf{S}}{\mathbf{V}_{\text{rms}}} = \frac{10\ 272 + j6204}{120\angle 0^\circ} = 85.6 + j51.7 \text{ A} = 100\angle 31.13^\circ \text{ A}$$

$$\mathbf{I}_{\text{rms}} = 100\angle -31.13^\circ$$

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EXAMPLE

$$I_m = \sqrt{2} I_{\text{rms}} = \sqrt{2}(100) = 141.4 \text{ A}$$

load impedance

$$\mathbf{Z} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{120 \angle 0^\circ}{100 \angle -31.13^\circ} = 1.2 \angle 31.13^\circ \ \Omega$$

an inductive impedance

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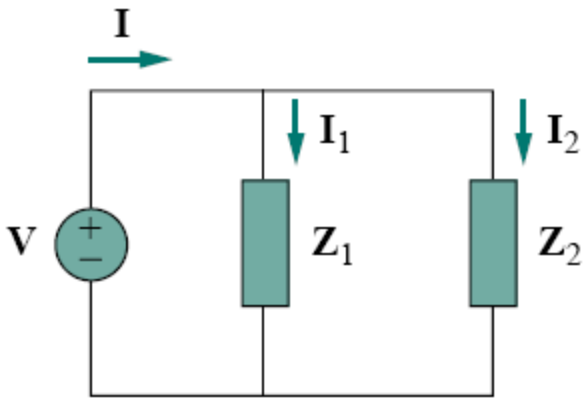
الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب

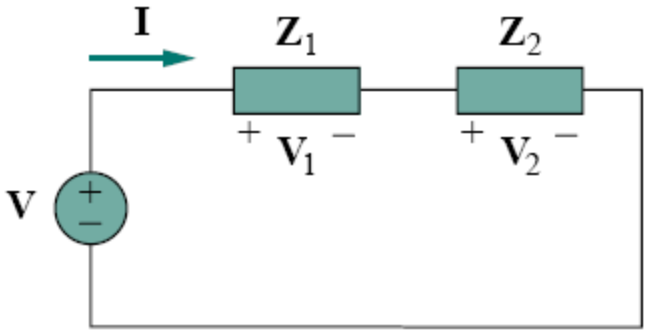
CONSERVATION OF AC POWER: حفظ الاستطاعة

يطبق هذا المبدأ بدارات التيار المستمر والمتناوب على السواء



$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}\mathbf{V}(\mathbf{I}_1^* + \mathbf{I}_2^*) = \frac{1}{2}\mathbf{V}\mathbf{I}_1^* + \frac{1}{2}\mathbf{V}\mathbf{I}_2^* = \mathbf{S}_1 + \mathbf{S}_2$$



The complex, real, and reactive powers of the sources equal the respective sums of the complex, real, and reactive powers of the individual loads.

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_1 + \cdots + \mathbf{S}_N$$

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \frac{1}{2}(\mathbf{V}_1 + \mathbf{V}_2)\mathbf{I}^* = \frac{1}{2}\mathbf{V}_1\mathbf{I}^* + \frac{1}{2}\mathbf{V}_2\mathbf{I}^* = \mathbf{S}_1 + \mathbf{S}_2$$

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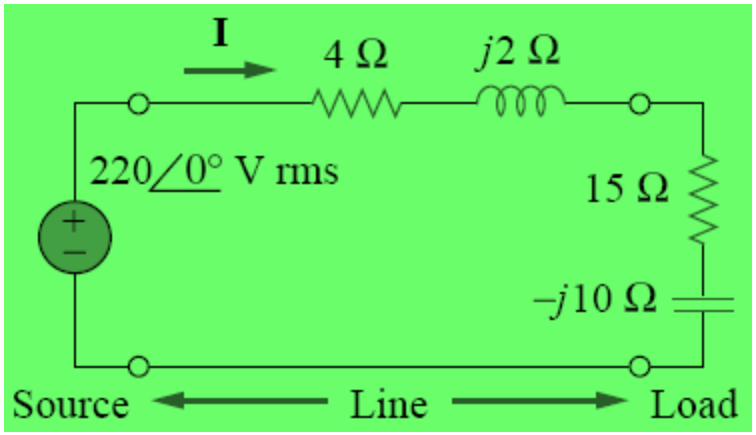
EXAMPLE

Find the real power and reactive power absorbed by: (a) the source, (b) the line, and (c) the load.



total impedance is

$$\mathbf{Z} = (4 + j2) + (15 - j10)$$
$$= 19 - j8 = 20.62\angle -22.83^\circ\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{220\angle 0^\circ}{20.62\angle -22.83^\circ}$$
$$= 10.67\angle 22.83^\circ\ \text{A rms}$$

source complex power is

$$\mathbf{S}_s = \mathbf{V}_s\mathbf{I}^* = (220\angle 0^\circ)(10.67\angle -22.83^\circ)$$
$$= 2347.4\angle -22.83^\circ = (2163.5 - j910.8)\ \text{VA}$$

$$\mathbf{V}_{\text{line}} = (4 + j2)\mathbf{I} = (4.472\angle 26.57^\circ)(10.67\angle 22.83^\circ)$$
$$= 47.72\angle 49.4^\circ\ \text{V rms}$$

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EXAMPLE

$$\mathbf{S}_{\text{line}} = \mathbf{V}_{\text{line}}\mathbf{I}^* = (47.72\angle 49.4^\circ)(10.67\angle -22.83^\circ)$$
$$= 509.2\angle 26.57^\circ = 455.4 + j227.7 \text{ VA}$$

or

$$\mathbf{S}_{\text{line}} = |\mathbf{I}|^2\mathbf{Z}_{\text{line}} = (10.67)^2(4 + j2) = 455.4 + j227.7 \text{ VA}$$

the load voltage

$$\mathbf{V}_L = (15 - j10)\mathbf{I} = (18.03\angle -33.7^\circ)(10.67\angle 22.83^\circ)$$
$$= 192.38\angle -10.87^\circ \text{ V rms}$$

load complex power

$$\mathbf{S}_L = \mathbf{V}_L\mathbf{I}^* = (192.38\angle -10.87^\circ)(10.67\angle -22.83^\circ)$$
$$= 2053\angle -33.7^\circ = (1708 - j1139) \text{ VA}$$

The real power is 1708 W and the reactive power is 1139 VAR (leading).
Note that $\mathbf{S}_s = \mathbf{S}_{\text{line}} + \mathbf{S}_L$, as expected.

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الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب

POWER FACTOR CORRECTION: تصحيح معامل الاستطاعة

إن معظم الاحمال الصناعية والمنزلية هي أحمال تحريضية بمعامل استطاعة تأخير بالطور منخفض القيمة , تدعى عملية زيادة معامل الاستطاعة دون تغيير جهد أو تيار الحمل بتصحيح معامل الاستطاعة . وذلك باضافة مكثفة تفرعية للحمل كما هو مبين بالشكل التالي:

The process of increasing the power factor without altering the voltage or current to the original load is known as power factor correction.

$$\text{pf} = \cos(\theta_v - \theta_i)$$

It is also the cosine of the angle of the load impedance or the ratio of real power to apparent power. The pf is lagging if the current lags voltage (inductive load) and is leading when the current leads voltage (capacitive load).

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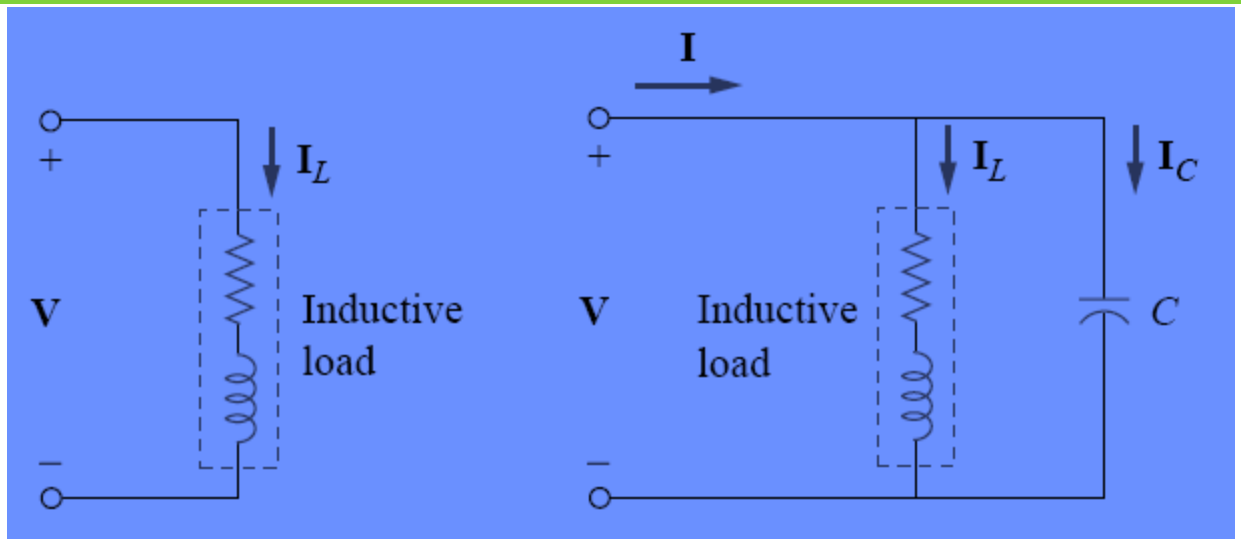
الفصل الرابع: دارات التيار المتناوب

AC POWER ANALYSIS

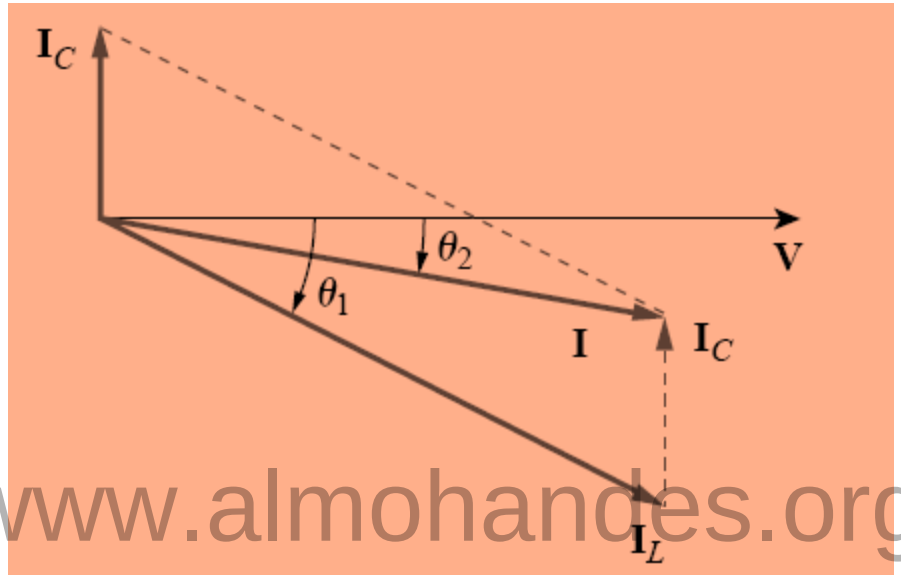
دراسة الاستطاعة بالتيار المتناوب

$Pf = \cos\theta_1$



$Pf = \cos\theta_2$

$$P = I_L^2 R$$



من الواضح أن التيار الكلي I أصغر من التيار الاصلي I_L قبل التصحيح وهذا يعود بالفائدة على المستهلك والمنتج كما ان الزاوية $\theta_1 < \theta_2$ وبالتالي أصبح معامل الاستطاعة أكبر

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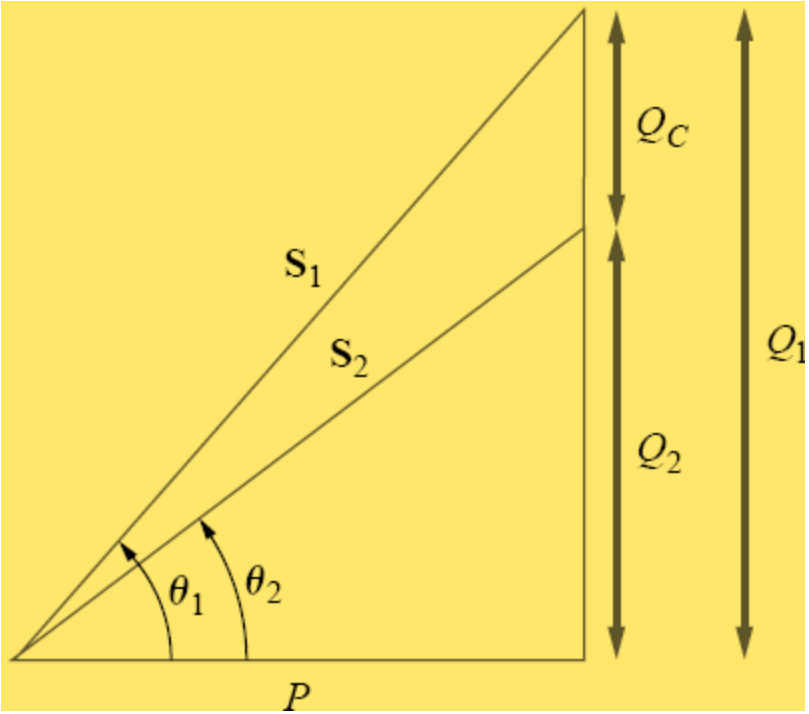
الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب



يمكن النظر إلى عملية التصحيح بطريقة أخرى:

$$P = S_1 \cos\theta_1, \qquad Q_1 = S_1 \sin\theta_1 = P \tan\theta_1$$

نرغب بزيادة معامل الاستطاعة من $\cos\theta_1$ إلى $\cos\theta_2$
دون تغيير الاستطاعة الحقيقية للحمل $P=S_2 \cos\theta_2$ وبالتالي
فان الاستطاعة الردية الجديدة هي:

average power due to the capacitance is zero

$$Q_2 = P \tan\theta_2$$

$$Q_C = Q_1 - Q_2 = P(\tan\theta_1 - \tan\theta_2)$$

$$Q_C = V_{\text{rms}}^2 / X_C = \omega C V_{\text{rms}}^2$$

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}$$

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الفصل الرابع: دارات التيار المتناوب
AC POWER ANALYSIS
دراسة الاستطاعة بالتيار المتناوب

في حالة الاحمال السعوية بمعامل استطاعة تقديم بالطور نحسب قيمة الملف التفرعي اللازم لتحسين معامل الاستطاعة من العلاقة التالية:

Although the most common situation in practice is that of an inductive load, it is also possible that the load is capacitive, that is, the load is operating at a leading power factor. In this case, an inductor should be connected across the load for power factor correction. The required shunt inductance L can be calculated from

$$Q_L = \frac{V_{\text{rms}}^2}{X_L} = \frac{V_{\text{rms}}^2}{\omega L} \qquad \Longrightarrow \qquad L = \frac{V_{\text{rms}}^2}{\omega Q_L}$$

where $Q_L = Q_1 - Q_2$, the difference between the new and old reactive powers.

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EXAMPLE **When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of 0.8.**
Find the value of capacitance necessary to raise the pf to 0.95.

$$\cos\theta_1 = 0.8 \quad \Longrightarrow \quad \theta_1 = 36.87^\circ \qquad S_1 = \frac{P}{\cos\theta_1} = \frac{4000}{0.8} = 5000 \text{ VA}$$

$$Q_1 = S_1 \sin\theta = 5000 \sin 36.87 = 3000 \text{ VAR}$$

$$\cos\theta_2 = 0.95 \quad \Longrightarrow \quad \theta_2 = 18.19^\circ \qquad S_2 = \frac{P}{\cos\theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

The real power P has not changed

$$Q_2 = S_2 \sin\theta_2 = 1314.4 \text{ VAR}$$

$$Q_C = Q_1 - Q_2 = 3000 - 1314.4 = 1685.6 \text{ VAR}$$

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5 \ \mu\text{F}$$

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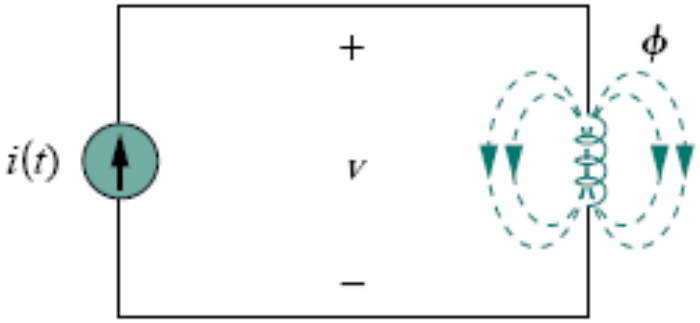
الفصل الرابع: دارات التيار المتناوب
Magnetically coupled circuits
الدارات المترابطة مغناطيسياً

INTRODUCTION: مقدمة

When two loops with or without contacts between them affect each other through the magnetic field generated by one of them ,they are said to be magnetically coupled.
The transformer is an electrical device designed on the basis of the concept of magnetic coupling

MUTUAL INDUCTANCE عامل التحريض المتبادل



$$v = N\frac{d\phi}{dt} \qquad v = N\frac{d\phi}{di}\frac{di}{dt}$$

$$v = L\frac{di}{dt} \qquad L = N\frac{d\phi}{di}$$

This inductance is commonly called *self-inductance, because it relates* the voltage induced in a coil by a time-varying current in the same coil.

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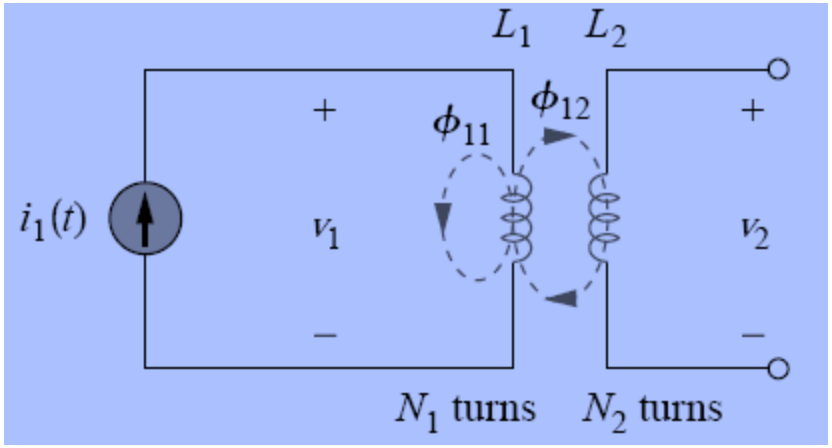
Mutual inductance M_{21} of coil 2 with respect to coil 1.



$$\phi_1 = \phi_{11} + \phi_{12}$$

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{12}}{dt}$$

$$v_1 = N_1 \frac{d\phi_1}{di_1}\frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$M_{21} = N_2 \frac{d\phi_{12}}{di_1}$$

$$v_2 = N_2 \frac{d\phi_{12}}{di_1}\frac{di_1}{dt} = M_{21}\frac{di_1}{dt}$$

$$v_2 = M_{21}\frac{di_1}{dt}$$

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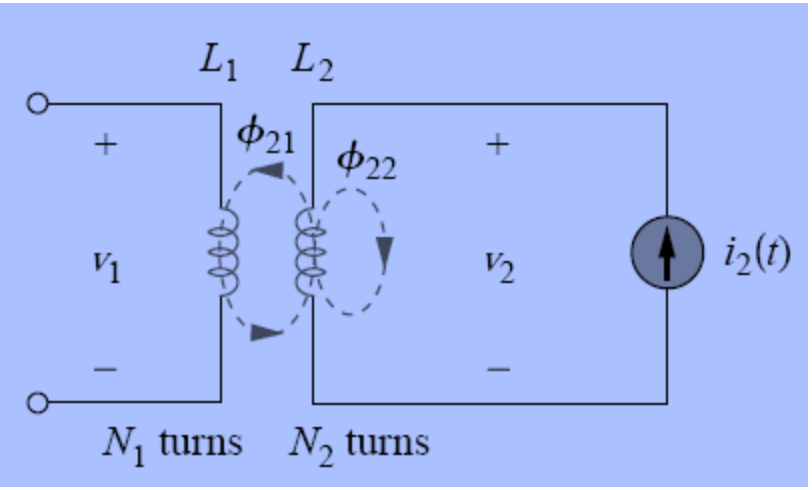
Mutual inductance M12 of coil 1 with respect to coil 2.

$$\phi_2 = \phi_{21} + \phi_{22}$$



$$v_2 = N_2\frac{d\phi_2}{dt} = N_2\frac{d\phi_2}{di_2}\frac{di_2}{dt} = L_2\frac{di_2}{dt}$$

$$v_1 = N_1\frac{d\phi_{21}}{dt} = N_1\frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12}\frac{di_2}{dt}$$

$$M_{12} = N_1\frac{d\phi_{21}}{di_2} \qquad v_1 = M_{12}\frac{di_2}{dt}$$

$$M_{12} = M_{21} = M$$

Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).

mutual inductance M is always a positive quantity, the mutual voltage M di/dt may be negative or positive, just like the self induced voltage Ldi/dt.

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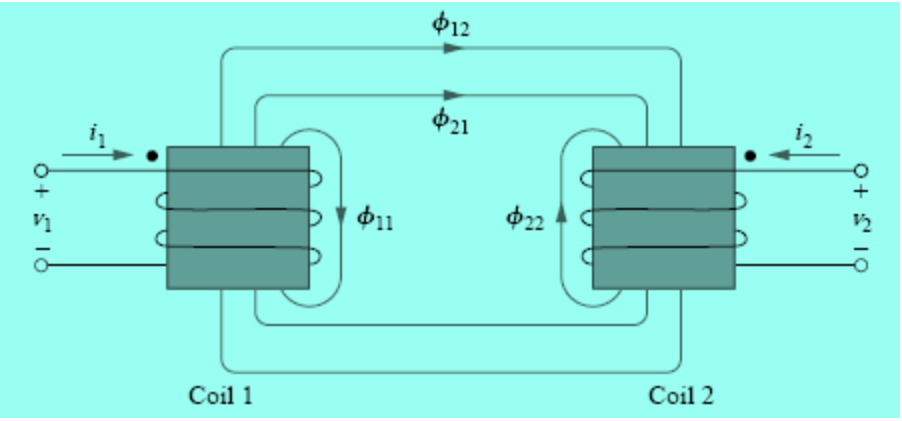
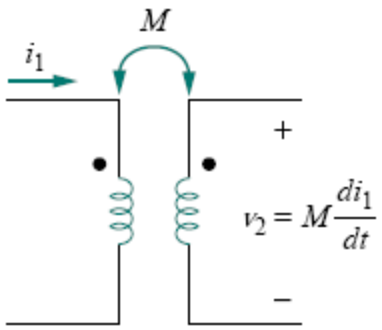
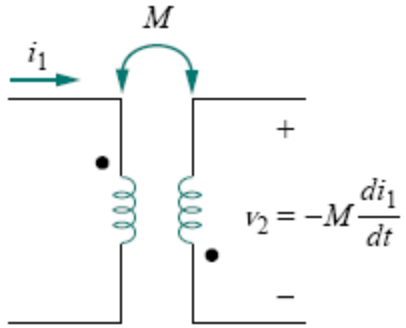
Dot convention:

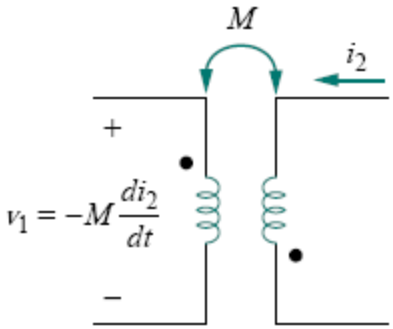


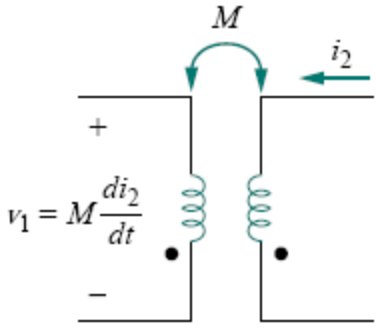
If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil.









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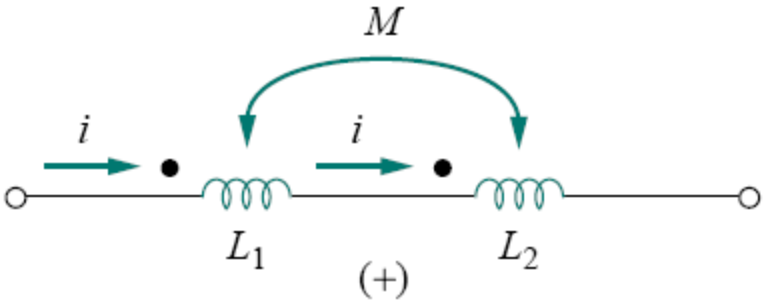
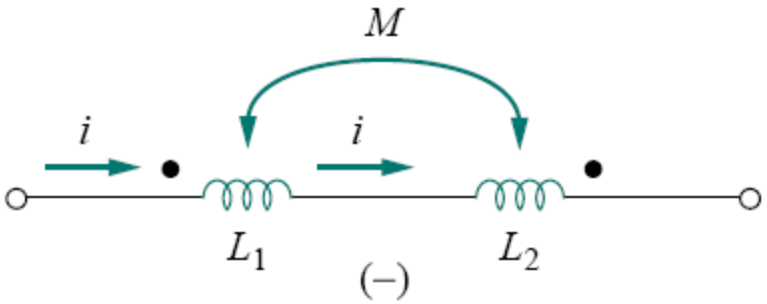


Dot convention for coils in series:

$$L = L_1 + L_2 + 2M \qquad \text{(Series-aiding connection)}$$



$$L = L_1 + L_2 - 2M \qquad \text{(Series-opposing connection)}$$

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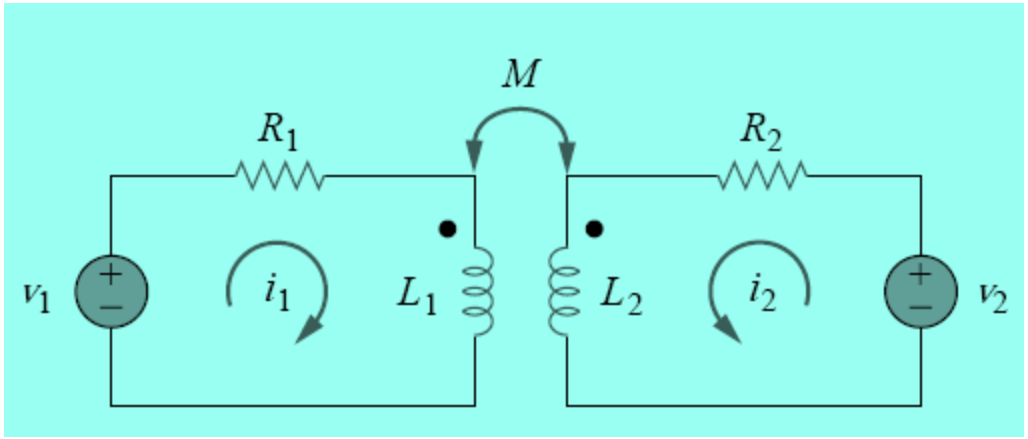
Time-domain analysis of a circuit containing coupled coils.



$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

in the frequency domain

$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M \mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2$$

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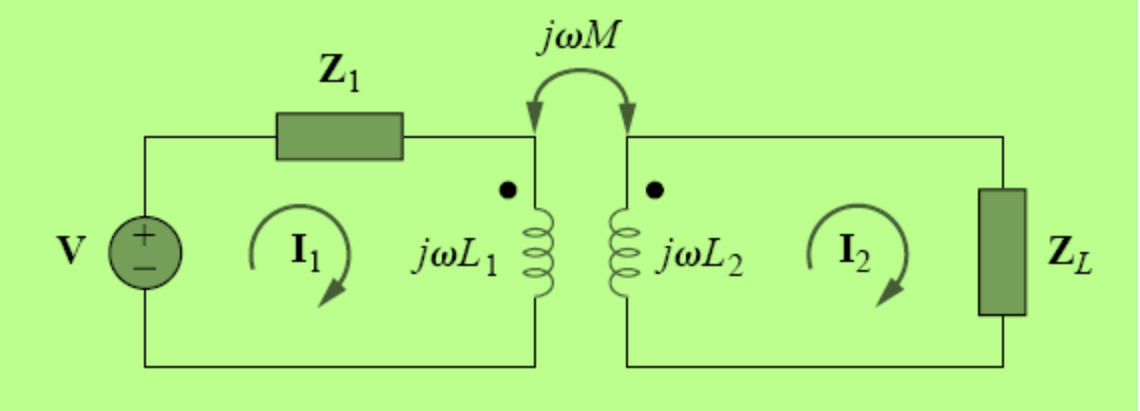
second example,



$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$

$$0 = -j\omega M\mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$$

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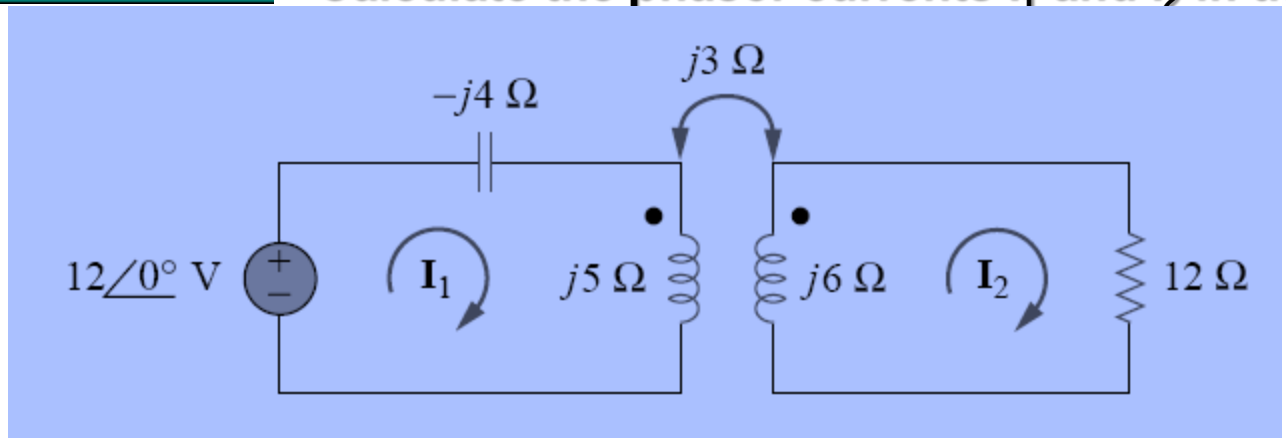
EXAMPLE

Calculate the phasor currents I_1 and I_2 in the following circuit



For coil 1, $-12 + (-j4 + j5)\mathbf{I}_1 - j3\mathbf{I}_2 = 0 \qquad j\mathbf{I}_1 - j3\mathbf{I}_2 = 12$

For coil 2, $-j3\mathbf{I}_1 + (12 + j6)\mathbf{I}_2 = 0 \qquad \mathbf{I}_1 = \dfrac{(12 + j6)\mathbf{I}_2}{j3} = (2 - j4)\mathbf{I}_2$

$$(j2 + 4 - j3)\mathbf{I}_2 = (4 - j)\mathbf{I}_2 = 12 \qquad \mathbf{I}_2 = \frac{12}{4 - j} = 2.91\angle 14.04^\circ \text{ A}$$

$$\mathbf{I}_1 = (2 - j4)\mathbf{I}_2 = (4.472\angle -63.43^\circ)(2.91\angle 14.04^\circ)$$

$$= 13.01\angle -49.39^\circ \text{ A}$$

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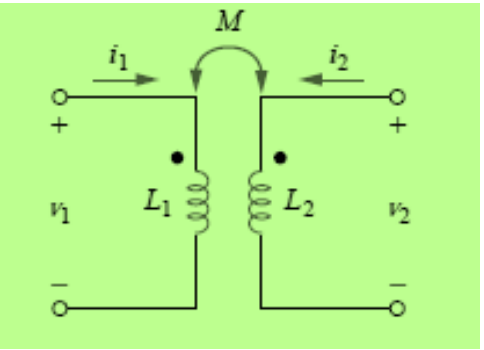
ENERGY IN A COUPLED CIRCUIT

energy stored in an inductor is given by:

$$w = \frac{1}{2}Li^2$$



Consider the circuit in Figure below. We assume that currents i_1 and i_2 are zero initially, so that the energy stored in the coils is zero. If we let i_1 increase from zero to I_1 while maintaining $i_2 = 0$, the power in coil 1 is

$$p_1(t) = v_1 i_1 = i_1 L_1 \frac{di_1}{dt}$$

and the energy stored in the circuit is

$$w_1 = \int p_1\, dt = L_1 \int_0^{I_1} i_1\, di_1 = \frac{1}{2}L_1 I_1^2$$

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If we now maintain $i_1 = I_1$ and increase i_2 from zero to I_2, the mutual voltage induced in coil 1 is $M_{12}\, di_2/dt$, while the mutual voltage induced in coil 2 is zero, since i1 does not change. The power in the coils is now

$$p_2(t) = i_1 M_{12}\frac{di_2}{dt} + i_2 v_2 = I_1 M_{12}\frac{di_2}{dt} + i_2 L_2\frac{di_2}{dt}$$

and the energy stored in the circuit is

$$w_2 = \int p_2\, dt = M_{12} I_1 \int_0^{I_2} di_2 + L_2 \int_0^{I_2} i_2\, di_2 = M_{12} I_1 I_2 + \frac{1}{2} L_2 I_2^2$$

$$w = w_1 + w_2 = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2 \qquad M_{12} = M_{21} = M$$

$$w = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

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instantaneous energy stored in the circuit the general expression

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$$

الإشارة الموجبة عندما تدخل التيارات إلى النقطة التي تحدد الاتجاه الموجب والإشارة السالبة عكس ذلك

$$\frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2 \geq 0$$

لا يمكن أن تكون القدرة المخزنة بالدارة سالبة لأن الدارة غير فعالة ومن ثم يمكن حساب القيمة العظمى لمعامل التحريض المتبادل للدارة

To complete the square, we both add and subtract the term $i_1 i_2 \sqrt{L1L2}$ on the right-hand side of last equation

$$\frac{1}{2}(i_1\sqrt{L_1} - i_2\sqrt{L_2})^2 + i_1 i_2(\sqrt{L_1 L_2} - M) \geq 0$$

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The squared term is never negative; at its least it is zero. Therefore, the second term on the right-hand side of last equation must be greater than zero:

$$\sqrt{L_1 L_2} - M \geq 0 \qquad M \leq \sqrt{L_1 L_2}$$

Coefficient of coupling k, معامل الترابط

$$k = \frac{M}{\sqrt{L_1 L_2}} \qquad \boxed{M = k\sqrt{L_1 L_2}} \qquad 0 \leq k \leq 1 \qquad 0 \leq M \leq \sqrt{L_1 L_2}$$

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{12}}{\phi_{11} + \phi_{12}} \qquad k = \frac{\phi_{21}}{\phi_2} = \frac{\phi_{21}}{\phi_{21} + \phi_{22}}$$

The coupling coefficient k is a measure of the magnetic coupling between two coils; $0 \leq k \leq 1$.

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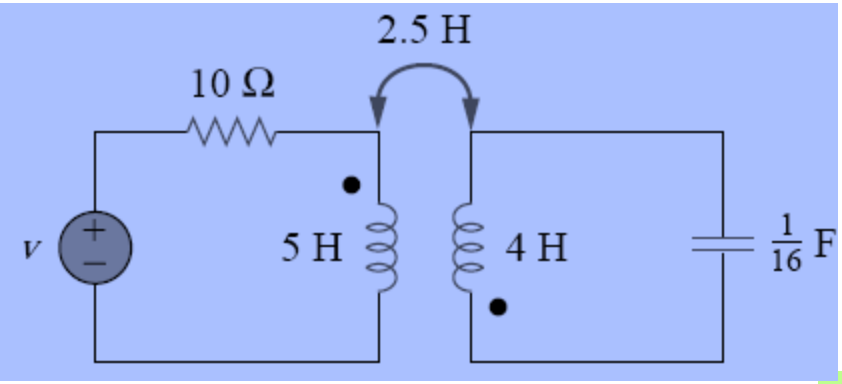
EXAMPLE **Determine the coupling coefficient . Calculate the energy stored in the coupled inductors at time t = 1 s if v = 60 cos (4t + 30°) V .**



$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{2.5}{\sqrt{20}} = 0.56$$

لحساب القدرة المخزنة بالدارة نقوم بتحويل الدارة إلى المجال الترددي

$$\begin{aligned}
60\cos(4t + 30^\circ) &\Longrightarrow 60\angle 30^\circ, \quad \omega = 4 \text{ rad/s} \\
5 \text{ H} &\Longrightarrow j\omega L_1 = j20\ \Omega \\
2.5 \text{ H} &\Longrightarrow j\omega M = j10\ \Omega \\
4 \text{ H} &\Longrightarrow j\omega L_2 = j16\ \Omega \\
\frac{1}{16} \text{ F} &\Longrightarrow \frac{1}{j\omega C} = -j4\ \Omega
\end{aligned}$$

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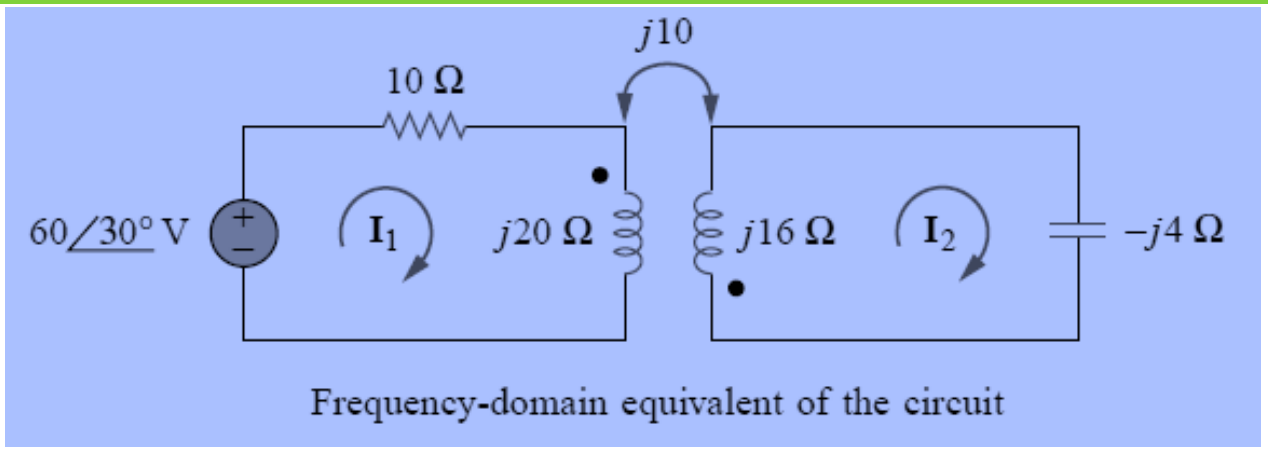
EXAMPLE



Frequency-domain equivalent of the circuit

$$(10 + j20)\mathbf{I}_1 + j10\mathbf{I}_2 = 60\angle 30^\circ$$

$$j10\mathbf{I}_1 + (j16 - j4)\mathbf{I}_2 = 0$$

$$\mathbf{I}_1 = -1.2\mathbf{I}_2$$

$$\mathbf{I}_2(-12 - j14) = 60\angle 30^\circ \quad \Longrightarrow \quad \mathbf{I}_2 = 3.254\angle 160.6^\circ \text{ A}$$

$$\mathbf{I}_1 = -1.2\mathbf{I}_2 = 3.905\angle -19.4^\circ \text{ A}$$

$$i_1 = 3.905\cos(4t - 19.4^\circ), \quad i_2 = 3.254\cos(4t + 160.6^\circ)$$

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At time $t = 1$ s, $4t = 4$ rad $= 229.2°$, and

$$i_1 = 3.905\cos(229.2° - 19.4°) = -3.389 \text{ A}$$

$$i_2 = 3.254\cos(229.2° + 160.6°) = 2.824 \text{ A}$$

The total energy stored in the coupled inductors is

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$= \frac{1}{2}(5)(-3.389)^2 + \frac{1}{2}(4)(2.824)^2 + 2.5(-3.389)(2.824) = 20.73 \text{ J}$$

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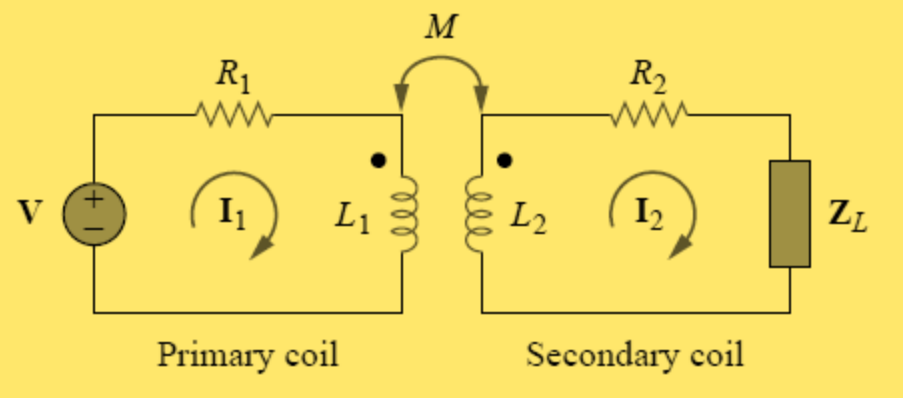
LINEAR TRANSFORMERS : المحولات الخطية

A transformer is generally a four-terminal device comprising two (or more) magnetically coupled coils .(*air-core transformers*)



input impedance Z_{in}

$$\mathbf{V} = (R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2$$
$$0 = -j\omega M\mathbf{I}_1 + (R_2 + j\omega L_2 + \mathbf{Z}_L)\mathbf{I}_2$$

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}}{\mathbf{I}_1} = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

the *reflected impedance :*

$$\mathbf{Z}_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + \mathbf{Z}_L}$$

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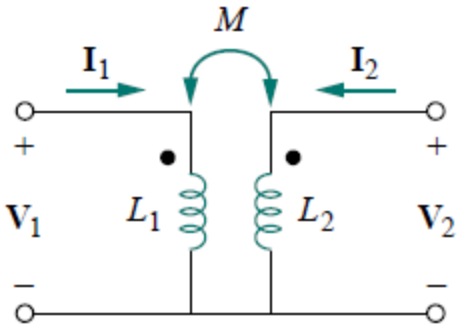
الدارة المكافئة للمحول equivalent T or Π circuit



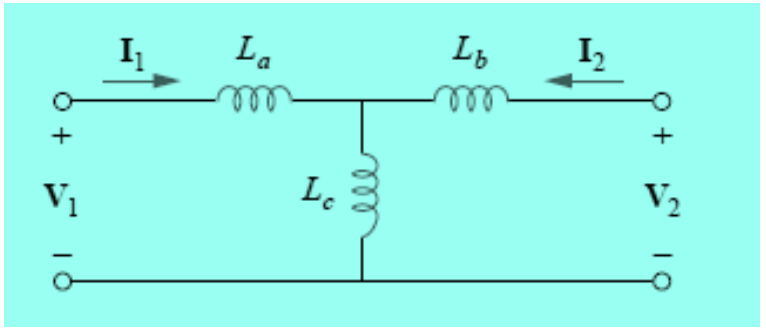
$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

بقلب المصفوفة نحصل على :

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{L_2}{j\omega(L_1L_2 - M^2)} & \dfrac{-M}{j\omega(L_1L_2 - M^2)} \\ \dfrac{-M}{j\omega(L_1L_2 - M^2)} & \dfrac{L_1}{j\omega(L_1L_2 - M^2)} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

For the T (or Y) network:

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} j\omega(L_a + L_c) & j\omega L_c \\ j\omega L_c & j\omega(L_b + L_c) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$



If the circuits are equivalent, Equations. must be identical. Equating terms in the impedance matrice lead to:

$$L_a = L_1 - M, \qquad L_b = L_2 - M, \qquad L_c = M$$

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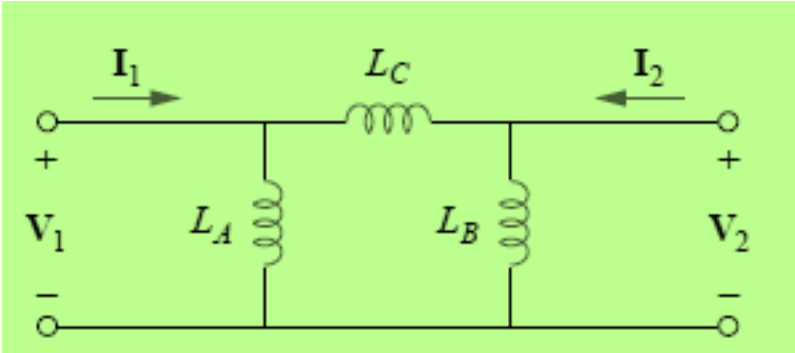
For theΠ *(or ∆) network*

$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{j\omega L_A} + \frac{1}{j\omega L_C} & -\frac{1}{j\omega L_C} \\ -\frac{1}{j\omega L_C} & \frac{1}{j\omega L_B} + \frac{1}{j\omega L_C} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$



$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \frac{L_2}{j\omega(L_1L_2 - M^2)} & \frac{-M}{j\omega(L_1L_2 - M^2)} \\ \frac{-M}{j\omega(L_1L_2 - M^2)} & \frac{L_1}{j\omega(L_1L_2 - M^2)} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

Equating terms in admittance matrices we obtain

$$L_A = \frac{L_1L_2 - M^2}{L_2 - M}, \qquad L_B = \frac{L_1L_2 - M^2}{L_1 - M}$$

$$L_C = \frac{L_1L_2 - M^2}{M}$$

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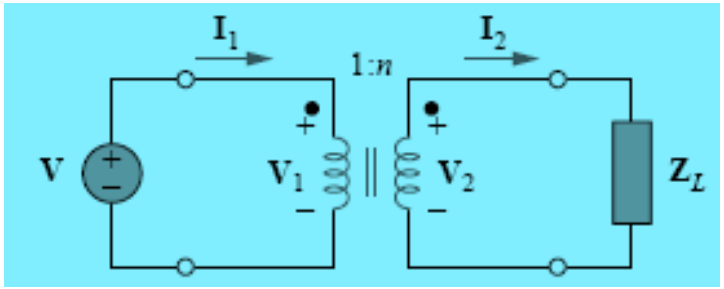
المحولات المثالية: IDEAL TRANSFORMERS

1. Coils have very large reactance's (L_1, L_2, $M \rightarrow \infty$).
2. Coupling coefficient is equal to unity ($k = 1$).
3. Primary and secondary coils are lossless ($R_1 = 0 = R_2$).

An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

بتطبيق إشارة متناوبة على إبتدائي المحول فإن نفس التدفق سيمر بكلا الملفين الابتدائي والثانوي ومن ثم فإن الجهد المتولد في لفات المحول يعطى حسب قانون فاراداي بالعلاقة :



$$v_1 = N_1 \frac{d\phi}{dt} \qquad v_2 = N_2 \frac{d\phi}{dt} \qquad \frac{v_2}{v_1} = \frac{N_2}{N_1} = n$$

Using phasor voltages V_1 and V_2

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = n$$ Equation 1

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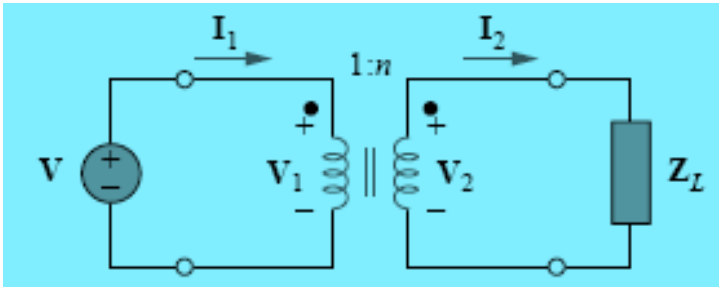
المحولات المثالية: IDEAL TRANSFORMERS



For the reason of power conservation, the energy supplied to the primary must equal the energy absorbed by the secondary, since there are no losses in an ideal transformer. This implies that :

$$v_1 i_1 = v_2 i_2$$

Equation 2

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

$$\frac{\mathbf{I}_2}{\mathbf{I}_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

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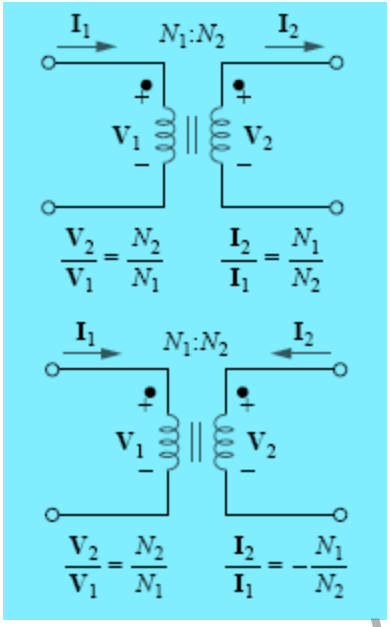
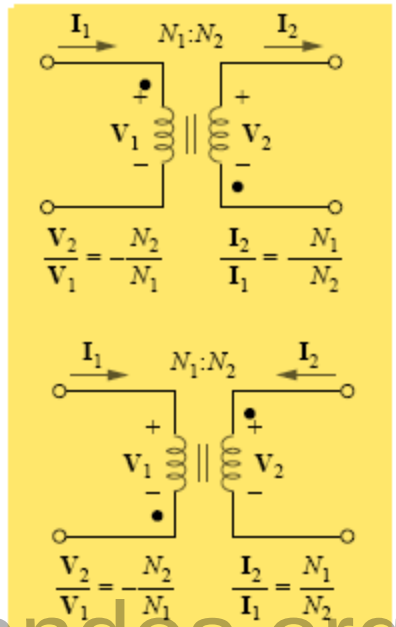
polarity of the voltages and the direction of the currents for the transformer :

1. If V_1 and V_2 are both positive or both negative at the dotted terminals, use +n in equation(1) Other wise, use −n.

2. If I_1 and I_2 both enter into or both leave the dotted terminals, use −n in Equation . (2). Otherwise, use +n.





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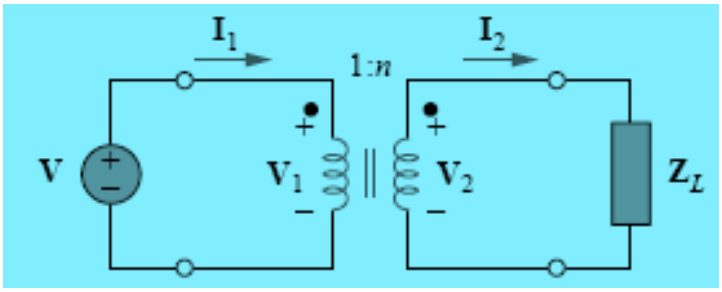
The input impedance as seen by the source:

$$\mathbf{V}_1 = \frac{\mathbf{V}_2}{n} \quad \text{or} \quad \mathbf{V}_2 = n\mathbf{V}_1$$

$$\mathbf{I}_1 = n\mathbf{I}_2 \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{I}_1}{n}$$



$$\mathbf{V}_2/\mathbf{I}_2 = \mathbf{Z}_L$$

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{1}{n^2}\frac{\mathbf{V}_2}{\mathbf{I}_2}$$

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}$$

The complex power in the primary winding is:

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1^* = \frac{\mathbf{V}_2}{n}(n\mathbf{I}_2)^* = \mathbf{V}_2\mathbf{I}_2^* = \mathbf{S}_2$$

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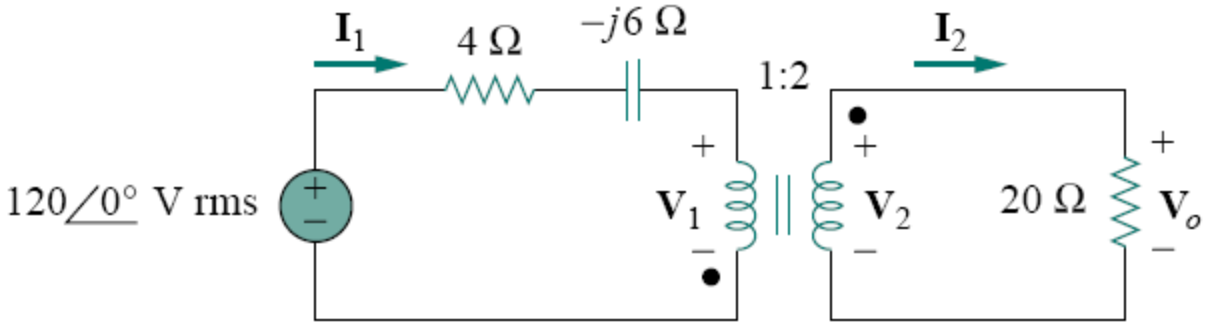
EXAMPLE

For the ideal transformer of the following circuit, find: (a) the source current I_1, (b) the output voltage Vo, and (c) the complex power supplied by the source.



$$\mathbf{Z}_R = \frac{20}{n^2} = \frac{20}{4} = 5\ \Omega$$

$$\mathbf{Z}_{\text{in}} = 4 - j6 + \mathbf{Z}_R = 9 - j6 = 10.82\angle -33.69^\circ\ \Omega$$

$$\mathbf{I}_1 = \frac{120\angle 0^\circ}{\mathbf{Z}_{\text{in}}} = \frac{120\angle 0^\circ}{10.82\angle -33.69^\circ} = 11.09\angle 33.69^\circ\ \text{A}$$

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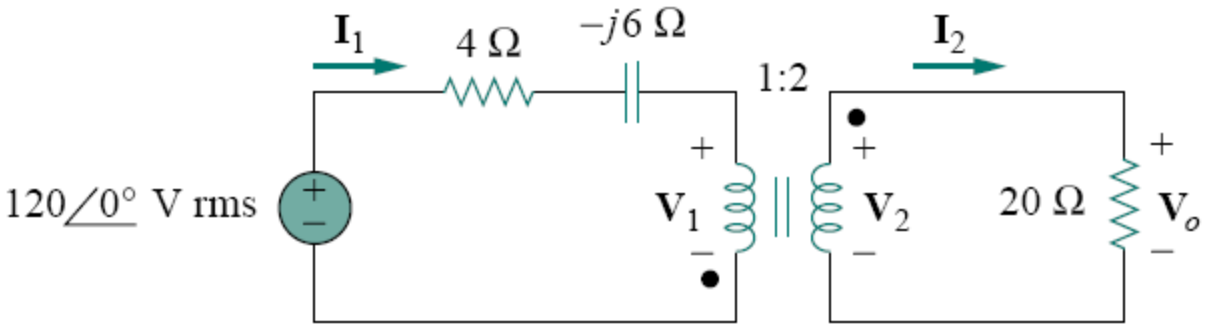

Since both I1 and I2 leave the dotted terminals,

$$\mathbf{I}_2 = -\frac{1}{n}\mathbf{I}_1 = -5.545\angle 33.69^\circ \text{ A}$$

$$\mathbf{V}_o = 20\mathbf{I}_2 = 110.9\angle 213.69^\circ \text{ V}$$

(c) The complex power supplied is

$$\mathbf{S} = \mathbf{V}_s\mathbf{I}_1^* = (120\angle 0^\circ)(11.09\angle -33.69^\circ) = 1330.8\angle -33.69^\circ \text{ VA}$$

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الفصل الرابع: دارات التيار المتناوب

FREQUENCY RESPONSE

الاستجابة الترددية

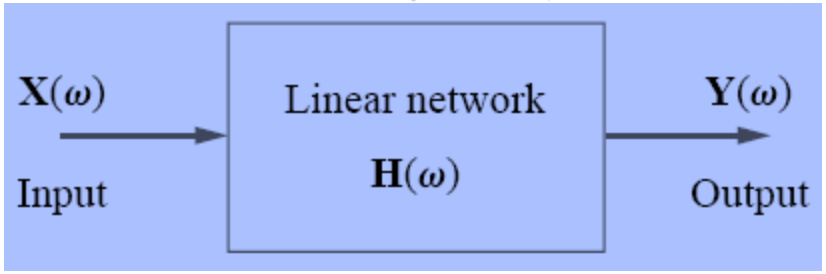
INTRODUCTION: مقدمة

يمكن دراسة الاستجابة الترددية للدارة بالحفاظ على مطال إشارة المنبع ثابتاً وتغيير التردد وبالتالي نكون قد حصلنا على وصف استجابة الحالة المستقرة للدارة كتابع للتردد .

The frequency response of a circuit is the variation in its behavior with change in signal frequency.

TRANSFER FUNCTION : تابع التحويل

ويدعى أيضاً تابع الشبكة Network Function H(ω) وهو أداة لدراسة الاستجابة الترددية للدارة , تغيرات مطال وزاوية تابع التحويل كتابع للمجال الترددي $0< \omega<\infty$, يمكن بشكل عام تمثيل أي دارة خطية بصندوق كما هو مبين بالشكل التالي:



$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)}$$

The transfer function $\mathbf{H}(\omega)$ of a circuit is the frequency-dependent ratio of a phasor output $\mathbf{Y}(\omega)$ (an element voltage or current) to a phasor input $\mathbf{X}(\omega)$ (source voltage or current).

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بفرض أن الشروط الابتدائية للدارة معدومة فإن لتابع التحويل إحدى الاشكال الأربع التالية كون أشارة الدخل والخرج للدارة إما جهد أو تيار :

$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)}$$

$$\mathbf{H}(\omega) = H(\omega) \angle \phi.$$

$$\mathbf{H}(\omega) = \frac{\mathbf{N}(\omega)}{\mathbf{D}(\omega)}$$

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A zero, as a *root* of the numerator polynomial, is a value that results in a zero value of the function. A pole, as a *root* of the denominator polynomial, is a value for which the function is infinite.

$$\mathbf{N}(\omega) = 0$$
$$j\omega = z_1, z_2, \ldots.$$
$$\mathbf{D}(\omega) = 0$$
$$j\omega = p_1, p_2, \ldots.$$

لتسهيل العمليات الحسابية نسبدل الحد (jω) بـــ S في التابع H(ω) ثم نعود ونستبدل S بـــ (jω)

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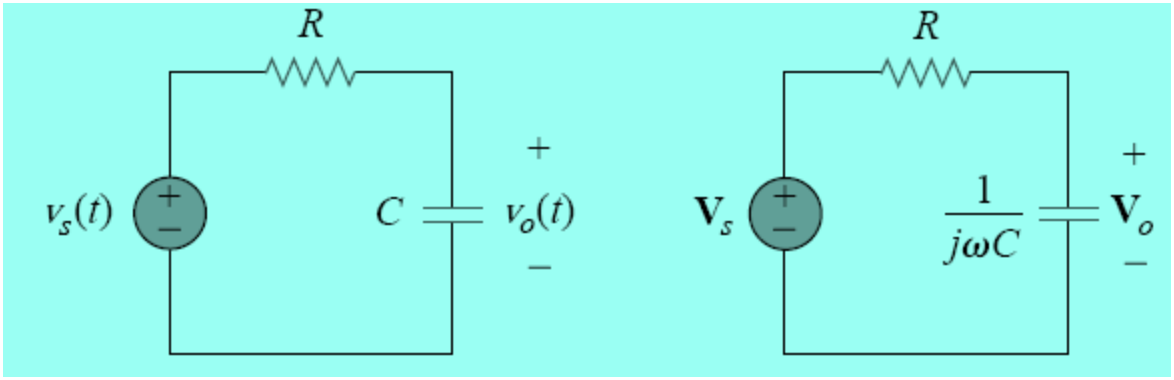
EXAMPLE

For the *RC* circuit in , obtain the transfer function $\mathbf{V}_o/\mathbf{V}_s$ and its frequency response. Let $v_s = V_m \cos \omega t$

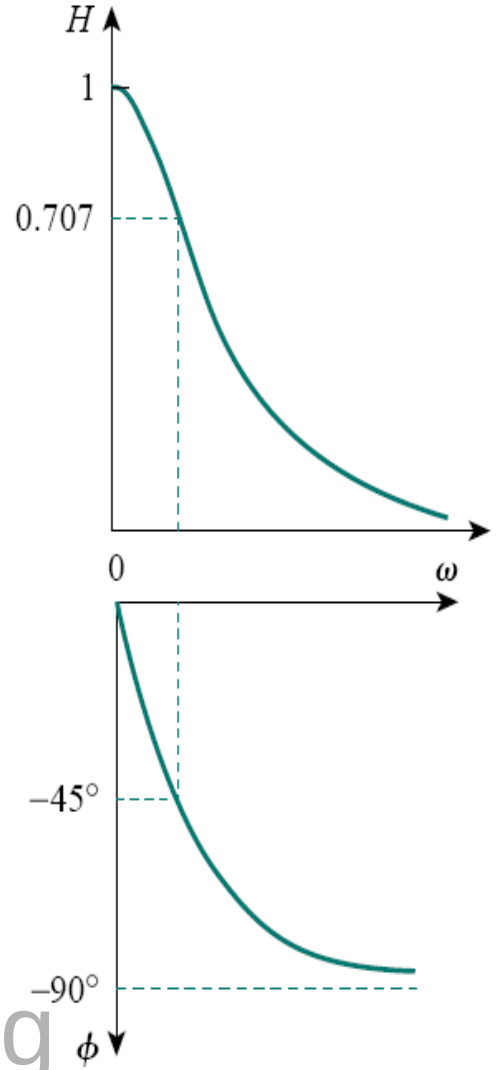


$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

$$H = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}, \qquad \phi = -\tan^{-1}\frac{\omega}{\omega_0} \qquad \omega_0 = 1/RC.$$

At $\omega = 0$, $H = 1$ and $\phi = 0$. At $\omega = \infty$, $H = 0$ and $\phi = -90°$

Also, at $\omega = \omega_0$, $H = 1/\sqrt{2}$ and $\phi = -45°$



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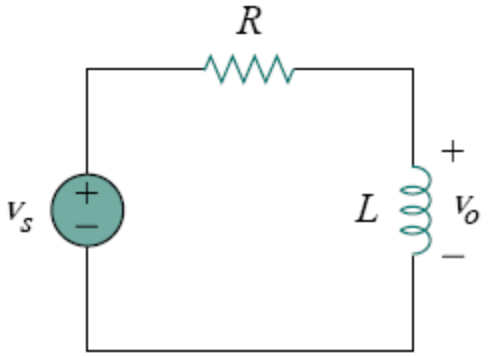
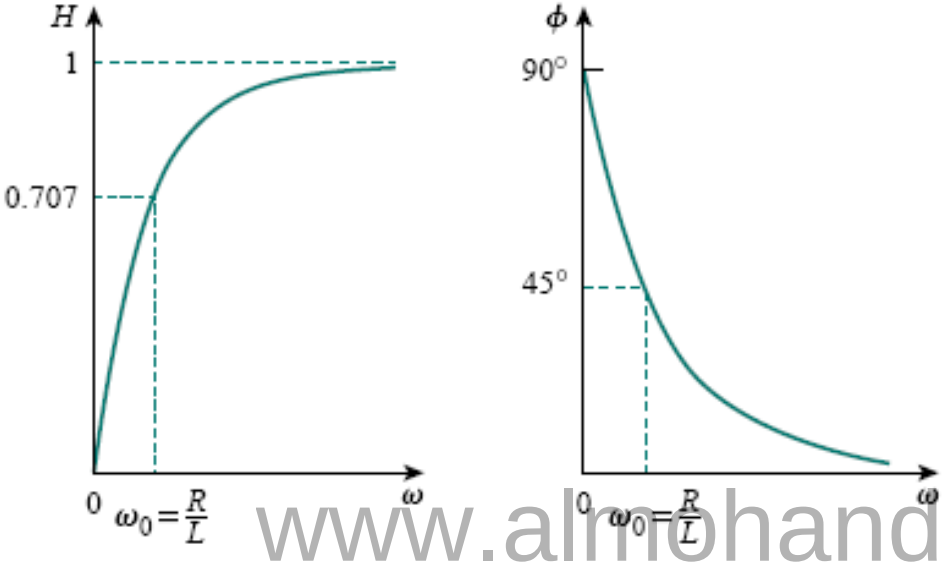
الفصل الرابع: دارات التيار المتناوب
FREQUENCY RESPONSE
الاستجابة الترددية



بنفس الطريقة يمكن إيجاد تابع التحويل للدارة RL

$$j\omega L/(R + j\omega L)$$



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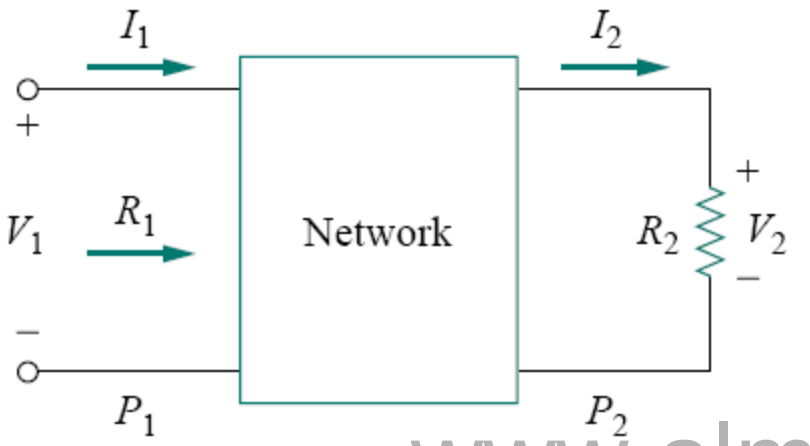
الديسيبل: THE DECIBEL SCALE

عملياً يمكن الحصول على منحنيات المطال والصفحة لتابع التحويل بسهولة باستخدام مخططات بود بما أن مخططات بود تستخدم محاور لوغاريتمية سنتعرف فيما يلي على هذا السلم .

$$G_{\text{dB}} = 10 \log_{10} \frac{P_2}{P_1}$$

$P_2 = 2P_1$ $\qquad G_{\text{dB}} = 10 \log_{10} 2 = 3 \text{ dB}$

$P_2 = 0.5P_1$ $\qquad G_{\text{dB}} = 10 \log_{10} 0.5 = -3 \text{ dB}$



$$G_{\text{dB}} = 20 \log_{10} \frac{V_2}{V_1}$$

$$G_{\text{dB}} = 20 \log_{10} \frac{I_2}{I_1}$$

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THE DECIBEL SCALE :الديسيبل

Specific gains and their decibel values.

Magnitude H	$20 \log_{10} H$ (dB)
0.001	−60
0.01	−40
0.1	−20
0.5	−6
$1/\sqrt{2}$	−3
1	0
$\sqrt{2}$	3
2	6
10	20
20	26
100	40
1000	60

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مخططات بود : BODE PLOTS

هي مخططات نصف لوغاريتمية مقدرة بالديسيبل لمطال تابع التحويل وبالدرجة بالنسبة للزاوية كتابع للتردد

Bode plots are semilog plots of the magnitude (in decibels) and phase (in degrees) of a transfer function versus frequency.

$$\mathbf{H} = H\angle\phi = He^{j\phi} \qquad \ln\mathbf{H} = \ln H + \ln e^{j\phi} = \ln H + j\phi$$

Thus, the real part of lnH is a function of the magnitude while the imaginary part is the phase.

Bode magnitude plot, the gain

$$H_{\text{dB}} = 20\log_{10} H$$

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standard form

$$\mathbf{H}(\omega) = \frac{K(j\omega)^{\pm 1}(1 + j\omega/z_1)[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]\cdots}{(1 + j\omega/p_1)[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]\cdots}$$

1. A gain K
2. A pole $(j\omega)^{-1}$ or zero $(j\omega)$ at the origin
3. A simple pole $1/(1 + j\omega/p_1)$ or zero $(1 + j\omega/z_1)$
4. A quadratic pole $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$ or zero $[1 + j2\zeta_1\omega/\omega_k + (j\omega/\omega_k)^2]$

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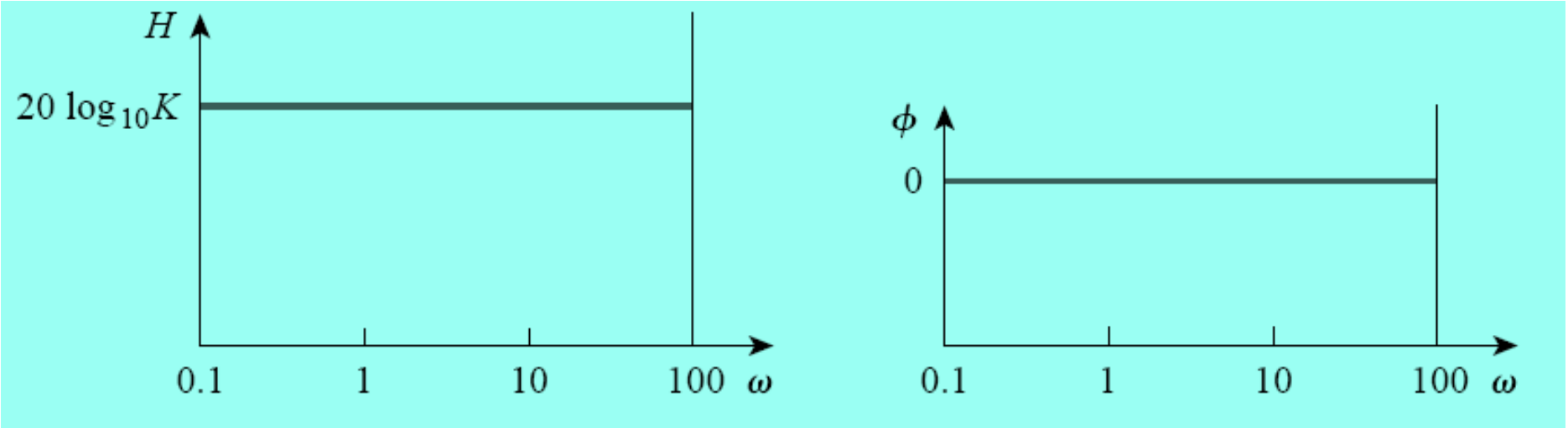
سنقوم فيما يلي برسم مكونات تابع التحويل بشكله العياري, باستخدام قطع مستقيمة:

Constant term: For the gain *K, the magnitude is 20 $\log_{10} K$ and the* phase is 0°; both are constant with frequency .



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Pole/zero at the origin: تمثيل قطب أو صفر عند المبدأ

zero $(j\omega)$ magnitude $20\log_{10}\omega$ and the phase is $90°$

pole $(j\omega)^{-1}$ magnitude plot is -20 dB/decade while the phase is $-90°$

for $(j\omega)^N$, where N is an integer,

the magnitude plot will have a slope of $20N$ dB/decade phase is $90N$ degrees

Simple pole/zero:

simple zero $(1 + j\omega/z_1)$ magnitude $20\log_{10}|1 + j\omega/z_1|$ and the phase is $\tan^{-1}\omega/z_1$

$$H_{\text{dB}} = 20\log_{10}\left|1 + \frac{j\omega}{z_1}\right| \quad \Longrightarrow \quad 20\log_{10} 1 = 0$$

as $\omega \to 0$

$$H_{\text{dB}} = 20\log_{10}\left|1 + \frac{j\omega}{z_1}\right| \quad \Longrightarrow \quad 20\log_{10}\frac{\omega}{z_1}$$

as $\omega \to \infty$

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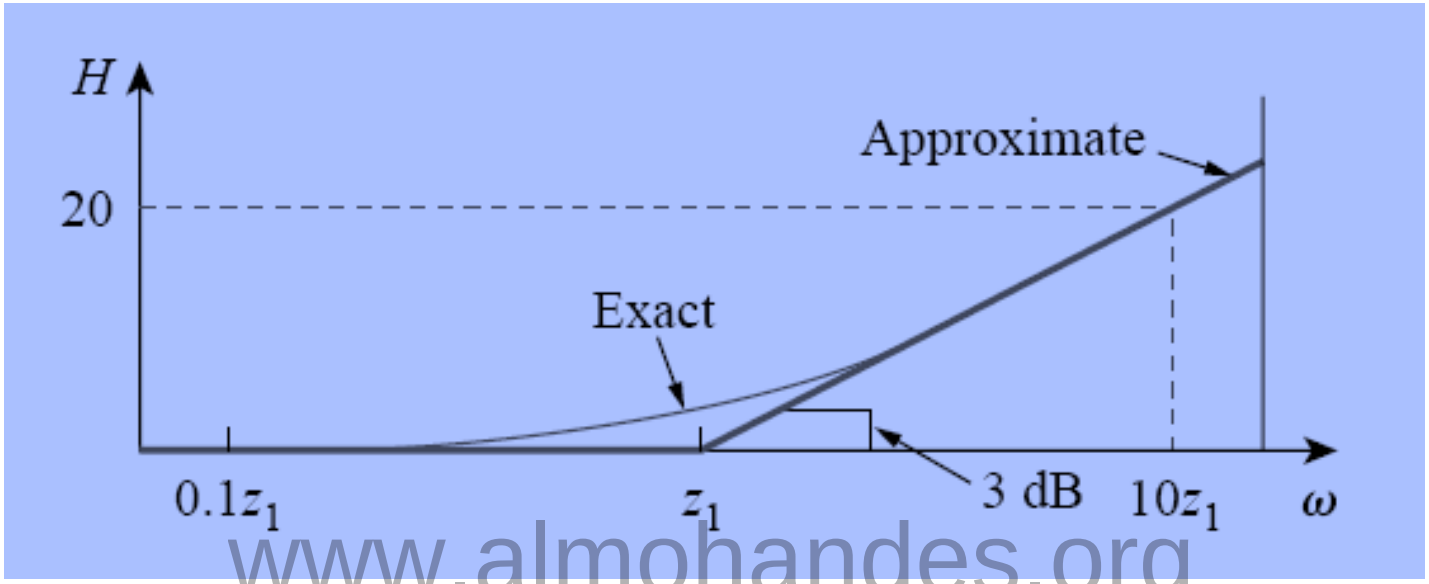
we can approximate the magnitude as zero (a straight line with zero slope) for small values of *ω and by a straight line with slope* 20 dB/decade for large values of *ω. The frequency ω = z_1 where the two* asymptotic lines meet is called the *corner frequency or break frequency.*

$$\omega = z_1$$

$$20 \log_{10} |(1 + j1)| = 20 \log_{10} \sqrt{2} = 3 \text{ dB}$$



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The phase $\tan^{-1}(\omega/z_1)$ can be expressed as

$$\phi = \tan^{-1}\left(\frac{\omega}{z_1}\right) = \begin{cases} 0, & \omega = 0 \\ 45^\circ, & \omega = z_1 \\ 90^\circ, & \omega \to \infty \end{cases}$$

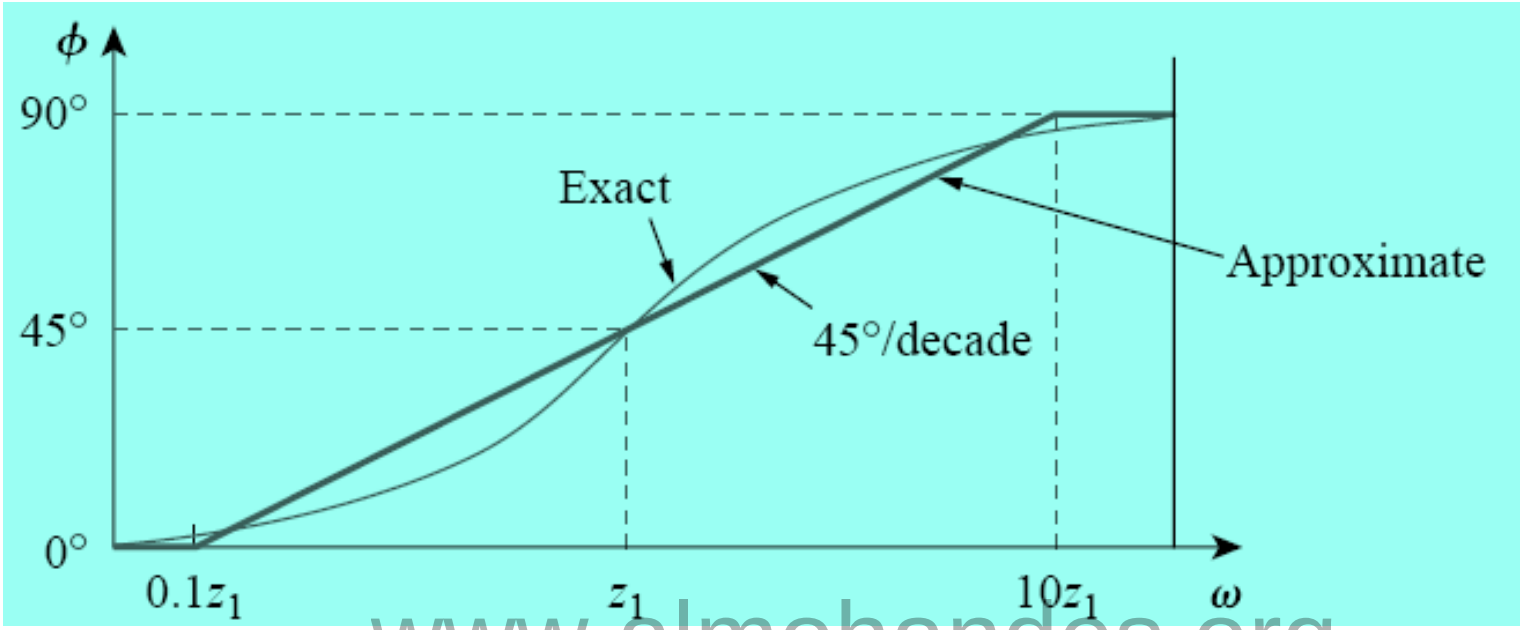
As a straight-line approximation,

we let $\phi \simeq 0$ for $\omega \leq z_1/10$,

$\phi \simeq 45^\circ$ for $\omega = z_1$

$\phi \simeq 90^\circ$ for $\omega \geq 10z_1$

straight-line plot has a slope of 45° per decade.



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The Bode plots for the pole $1/(1 + j\omega/p_1)$ *are similar to zero Boad plot* except that the corner frequency is at $\omega = p_1$, *the magnitude* has a slope of −20 dB/decade, and the phase has a slope of −45◦ per decade.

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تمثيل قطب أو صفر تربيعي : Quadratic pole/zero

The magnitude of the quadratic pole: $1/[1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2]$

$$-20\log_{10}|1 + j2\zeta_2\omega/\omega_n + (j\omega/\omega_n)^2|$$

the phase is : $-\tan^{-1}(2\zeta_2\omega/\omega_n)/(1 - \omega/\omega_n^2)$

$$H_{\text{dB}} = -20\log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right|_{\text{as}} \quad \omega \to 0 \quad \Longrightarrow \quad 0$$

$$H_{\text{dB}} = -20\log_{10}\left|1 + \frac{j2\zeta_2\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2\right|_{\text{as}} \quad \omega \to \infty \quad \Longrightarrow \quad -40\log_{10}\frac{\omega}{\omega_n}$$

Thus, the amplitude plot consists of two straight asymptotic lines: One with zero slope for $\omega < \omega_n$ and the other with slope −40 dB/decade for $\omega > \omega_n$, with ω_n as the corner frequency.

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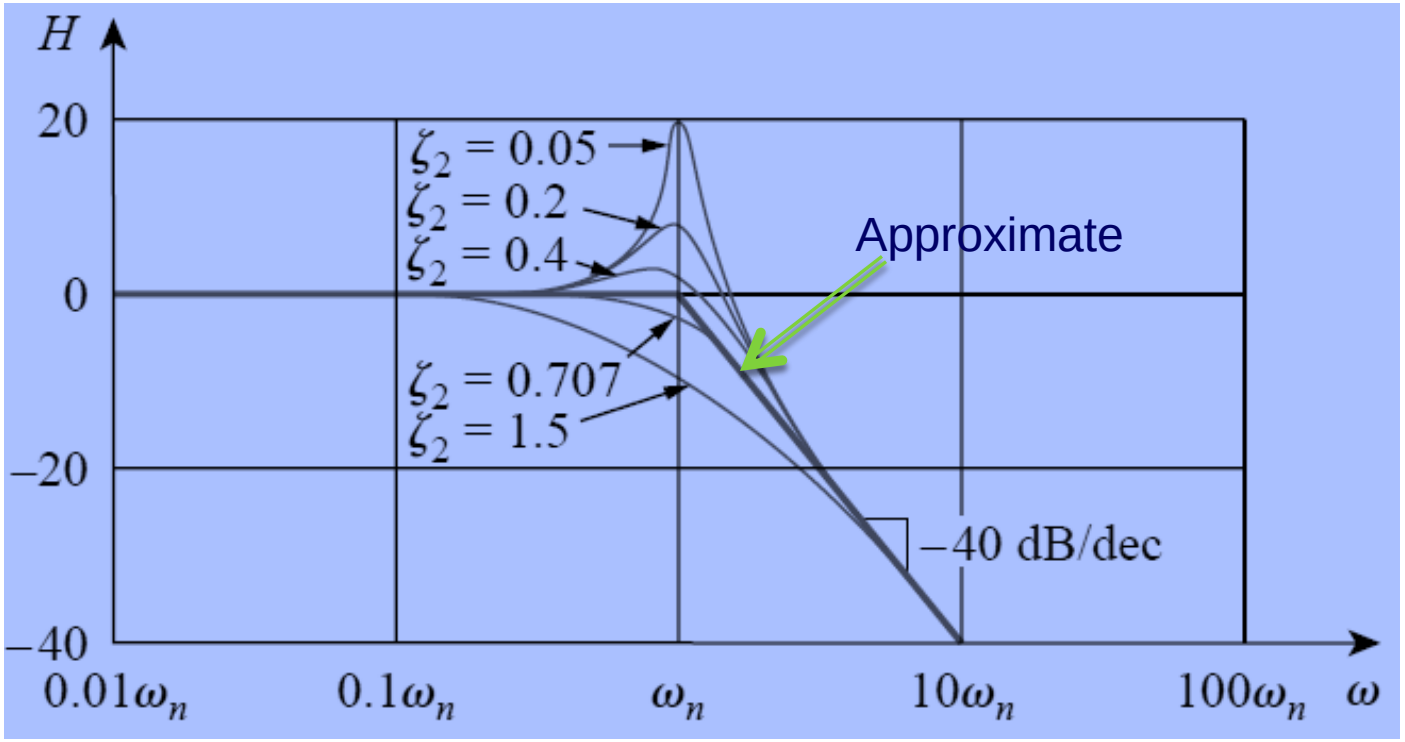
تمثيل قطب أو صفر تربيعي : Quadratic pole/zero

Note that the actual plot depends on the damping factor ζ_2 *as well as the corner frequency* ω_n



ω_n as the corner frequency

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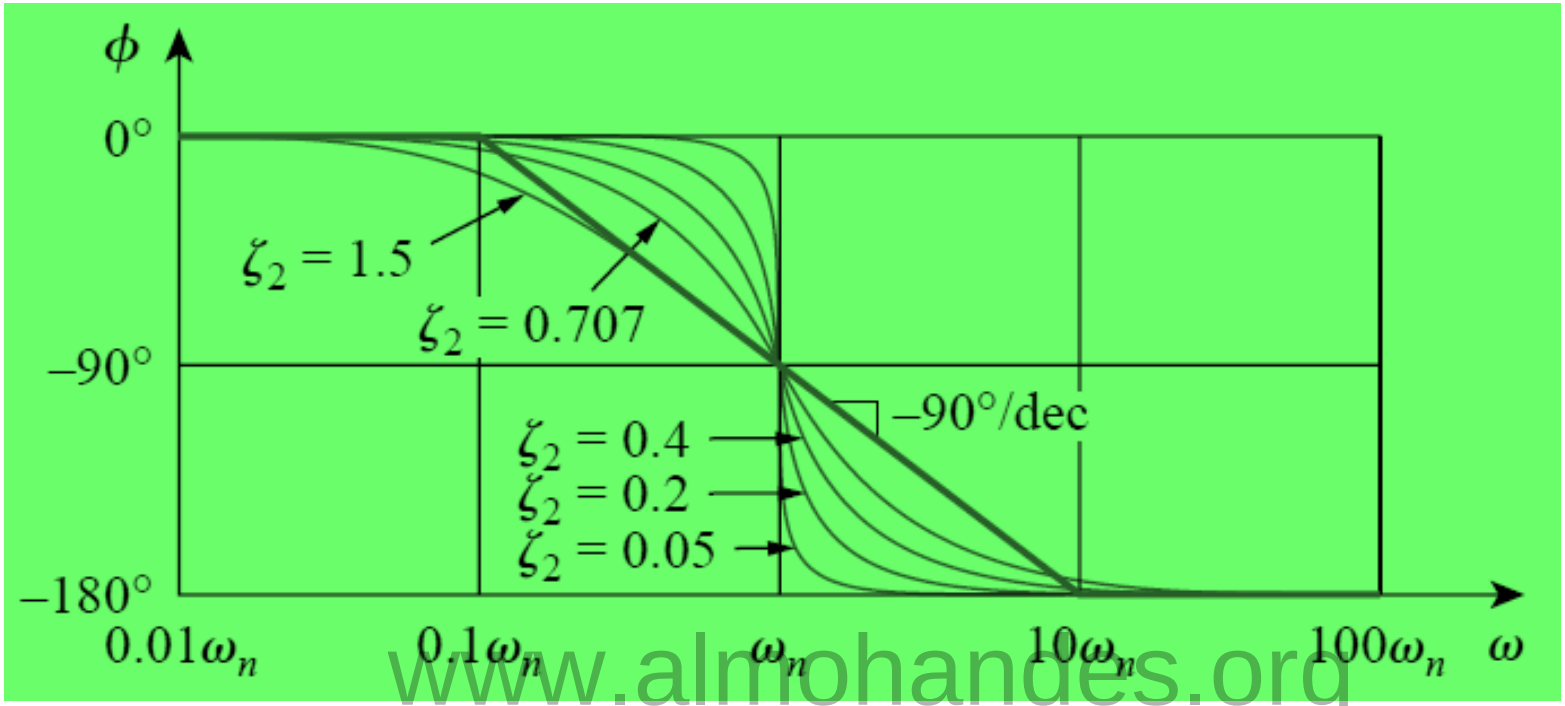
تمثيل قطب أو صفر تربيعي : Quadratic pole/zero

The phase can be expressed as:

$$\phi = -\tan^{-1}\frac{2\zeta_2\omega/\omega_n}{1-\omega^2/\omega_n^2} = \begin{cases} 0, & \omega = 0 \\ -90^\circ, & \omega = \omega_n \\ -180^\circ, & \omega \to \infty \end{cases}$$



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Notice that the straight-line approximations for both magnitude and phase plots for the quadratic pole are the same as those for a double pole, i.e. $(1 + j\omega/\omega_n)^{-2}$. We should expect this because the double pole $(1+j\omega/\omega n)^{-2}$ equals the quadratic pole $1/[1+j2\zeta_2\omega/\omega_n +(j\omega/\omega_n)^2]$ when $\zeta_2 = 1$.

For the quadratic zero $[1+j2\zeta_1\omega/\omega_k+(j\omega/\omega_k)^2]$, the plots are inverted because the magnitude plot has a slope of 40 dB/decade while the phase plot has a slope of 90° per decade

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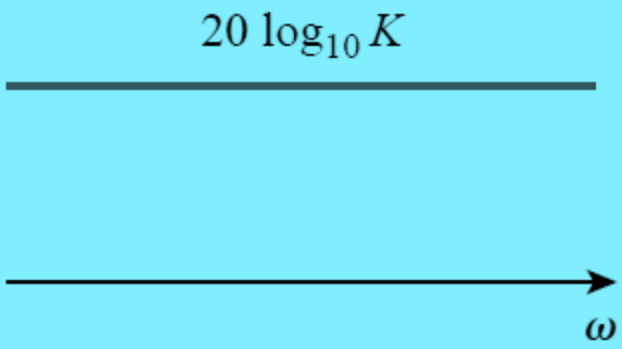
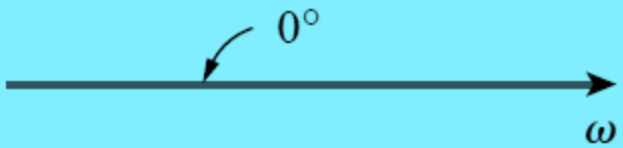
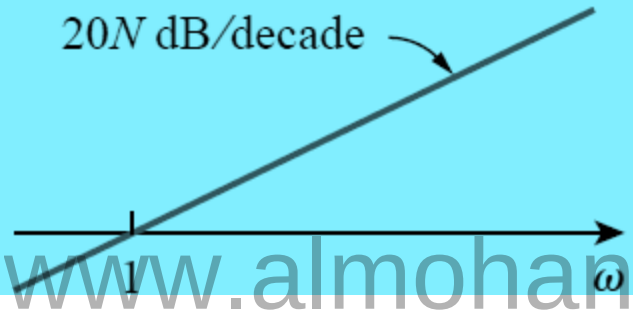
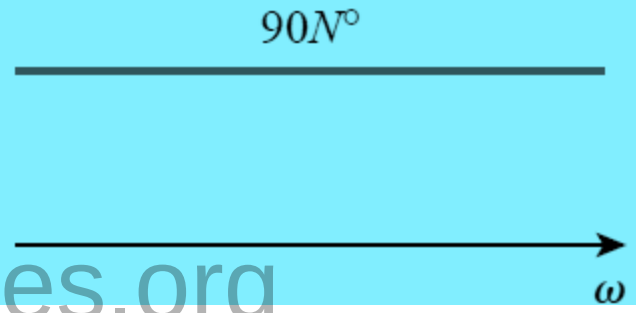
Summary of Bode straight-line magnitude and phase plots.

Factor	Magnitude	Phase
K		
$(j\omega)^N$		

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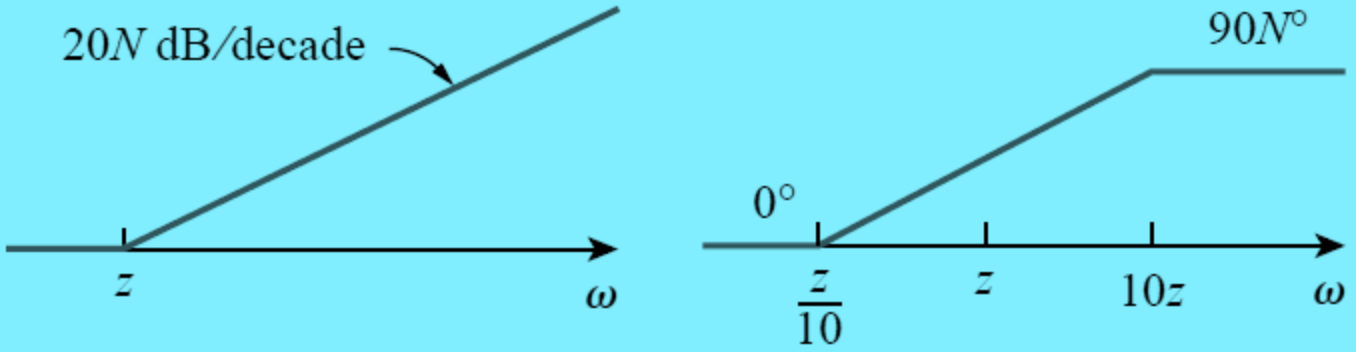
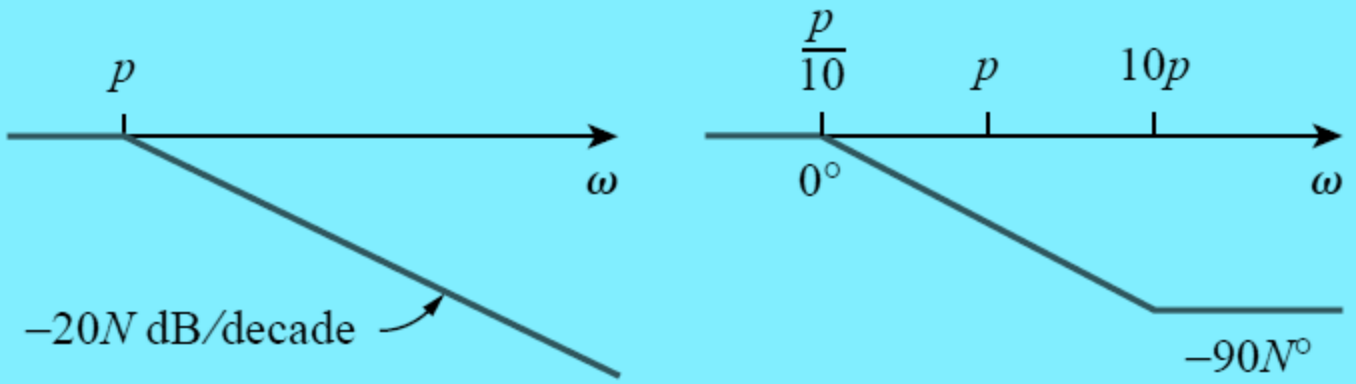
$$\left(1+\frac{j\omega}{z}\right)^N$$



$$\frac{1}{(1+j\omega/p)^N}$$



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EXAMPLE **Construct the Bode plots for the transfer function**

$$\mathbf{H}(\omega) = \frac{200\,j\omega}{(j\omega + 2)(j\omega + 10)}$$

We first put H(*ω*) *in the standard form*

$$\mathbf{H}(\omega) = \frac{10\,j\omega}{(1 + j\omega/2)(1 + j\omega/10)}$$

$$= \frac{10|j\omega|}{|1 + j\omega/2|\,|1 + j\omega/10|} \angle 90^\circ - \tan^{-1}\omega/2 - \tan^{-1}\omega/10$$

$$H_{\text{dB}} = 20\log_{10} 10 + 20\log_{10}|j\omega| - 20\log_{10}\left|1 + \frac{j\omega}{2}\right|$$

$$- 20\log_{10}\left|1 + \frac{j\omega}{10}\right|$$

$$\phi = 90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

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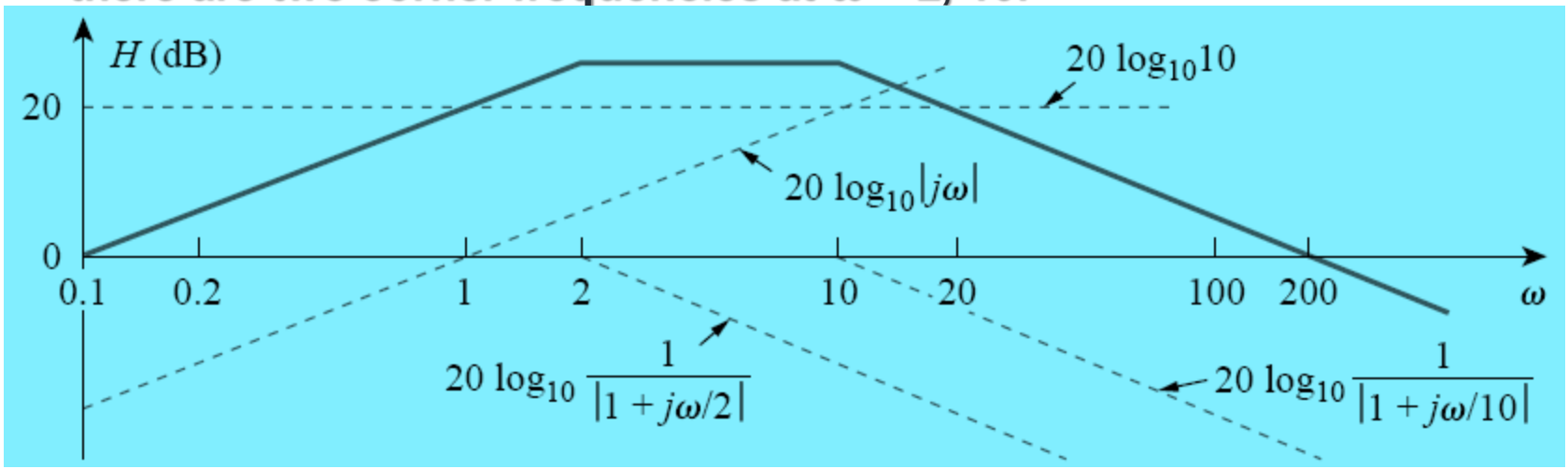
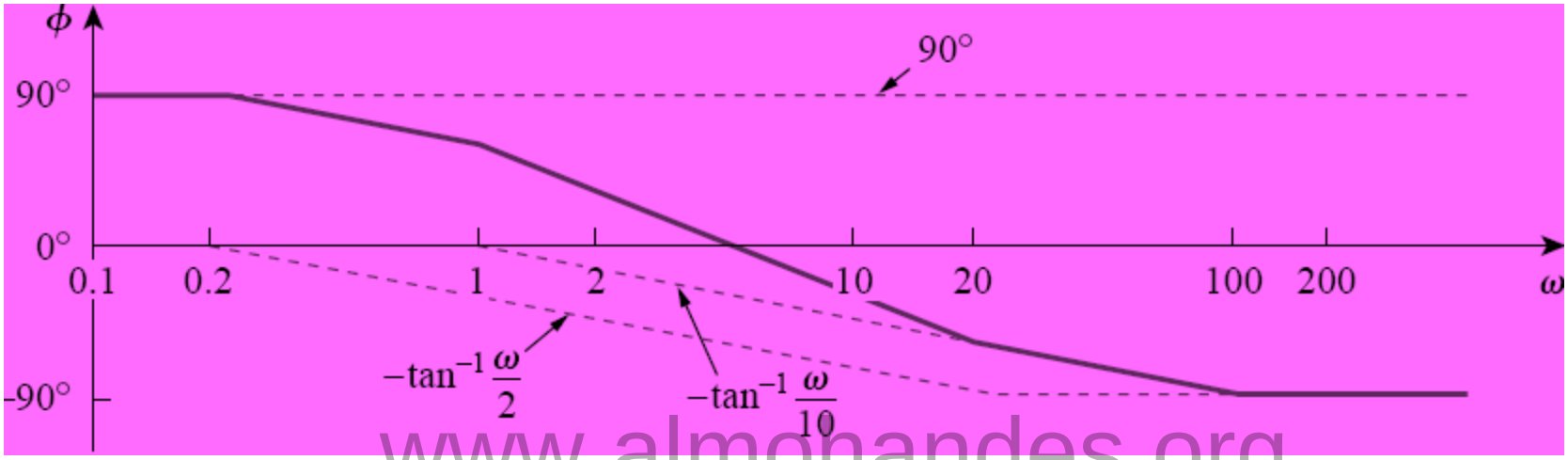
$$\phi = 90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

there are two corner frequencies at ω = 2, 10.





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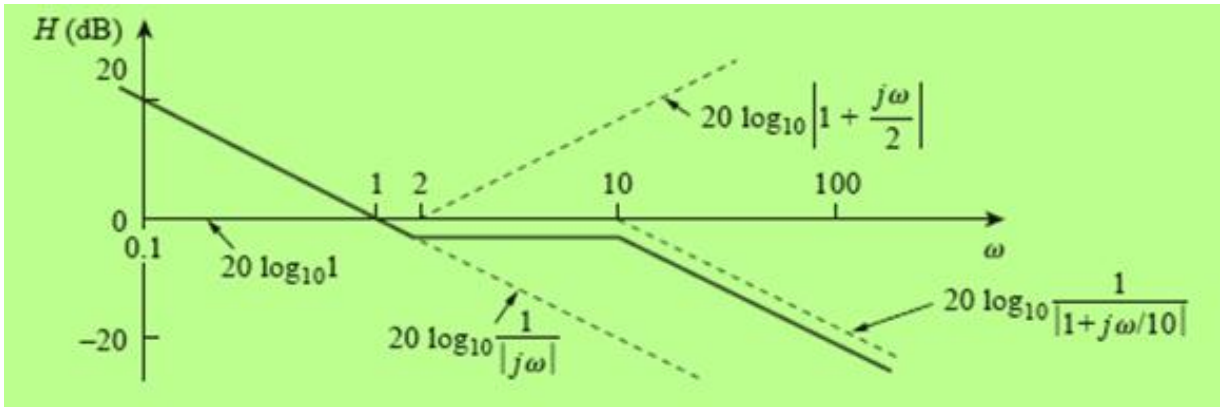
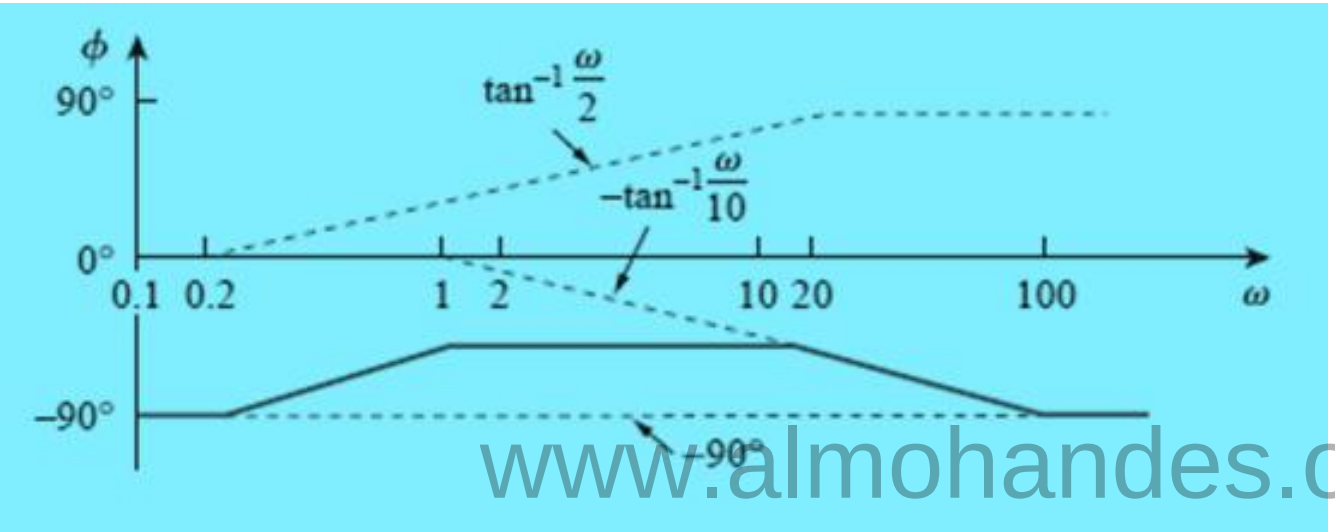
PRACTICE PROBLEM

Draw the Bode plots for the transfer function

$$\mathbf{H}(\omega) = \frac{5(j\omega + 2)}{j\omega(j\omega + 10)}$$





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EXAMPLE

Obtain the Bode plots for

$$\mathbf{H}(\omega) = \frac{j\omega + 10}{j\omega(j\omega + 5)^2}$$

Putting H(ω) *in the standard form,*

$$\mathbf{H}(\omega) = \frac{0.4\,(1 + j\omega/10)}{j\omega\,(1 + j\omega/5)^2}$$

$$H_{\text{dB}} = 20\log_{10} 0.4 + 20\log_{10}\left|1 + \frac{j\omega}{10}\right| - 20\log_{10}|j\omega|$$

$$- 40\log_{10}\left|1 + \frac{j\omega}{5}\right|$$

$$\phi = 0^\circ + \tan^{-1}\frac{\omega}{10} - 90^\circ - 2\tan^{-1}\frac{\omega}{5}$$

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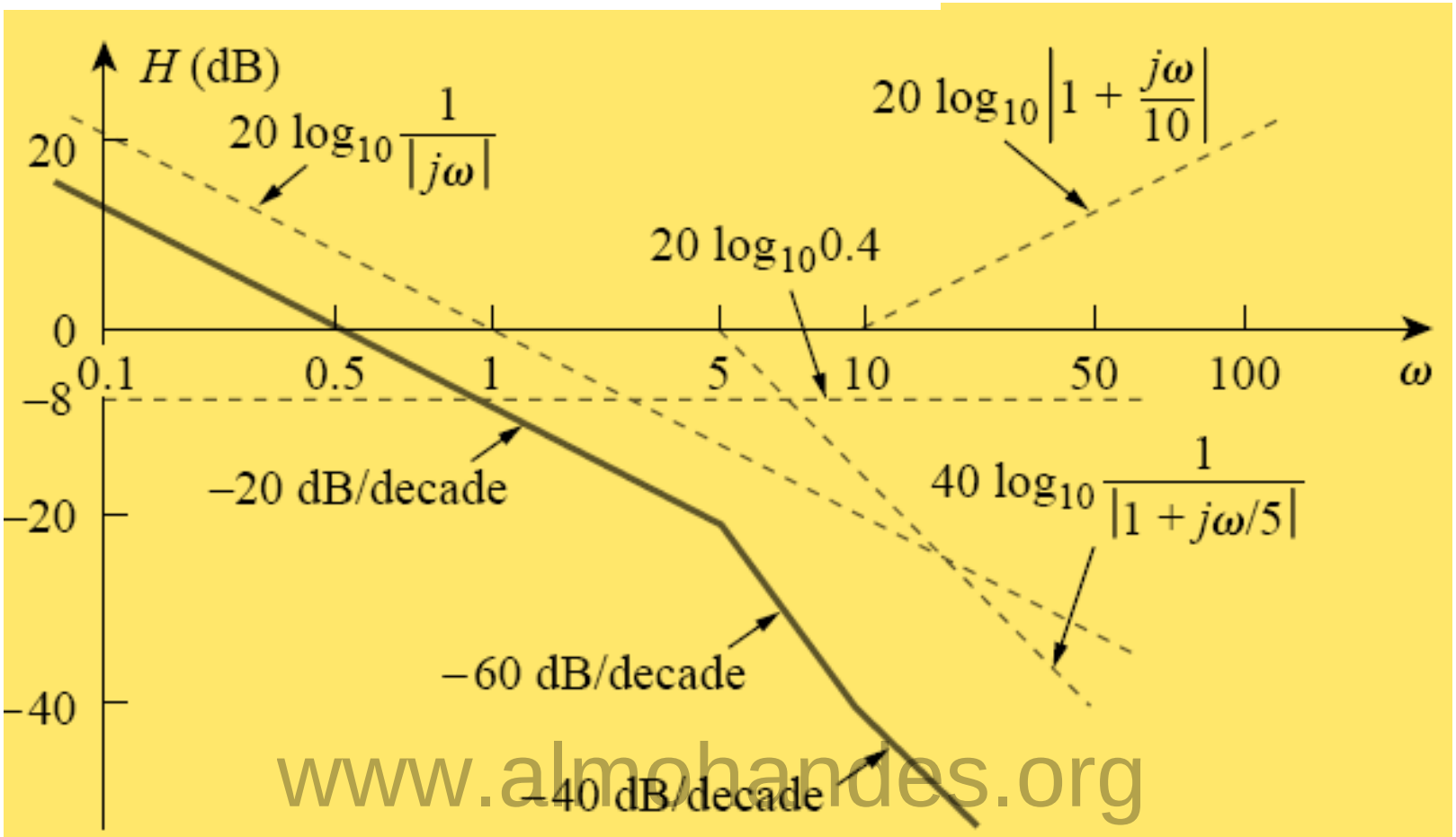
100

There are two corner frequencies at ω = 5, 10 rad/s.

$$H_{\text{dB}} = 20\log_{10}0.4 + 20\log_{10}\left|1+\frac{j\omega}{10}\right| - 20\log_{10}|j\omega| - 40\log_{10}\left|1+\frac{j\omega}{5}\right|$$



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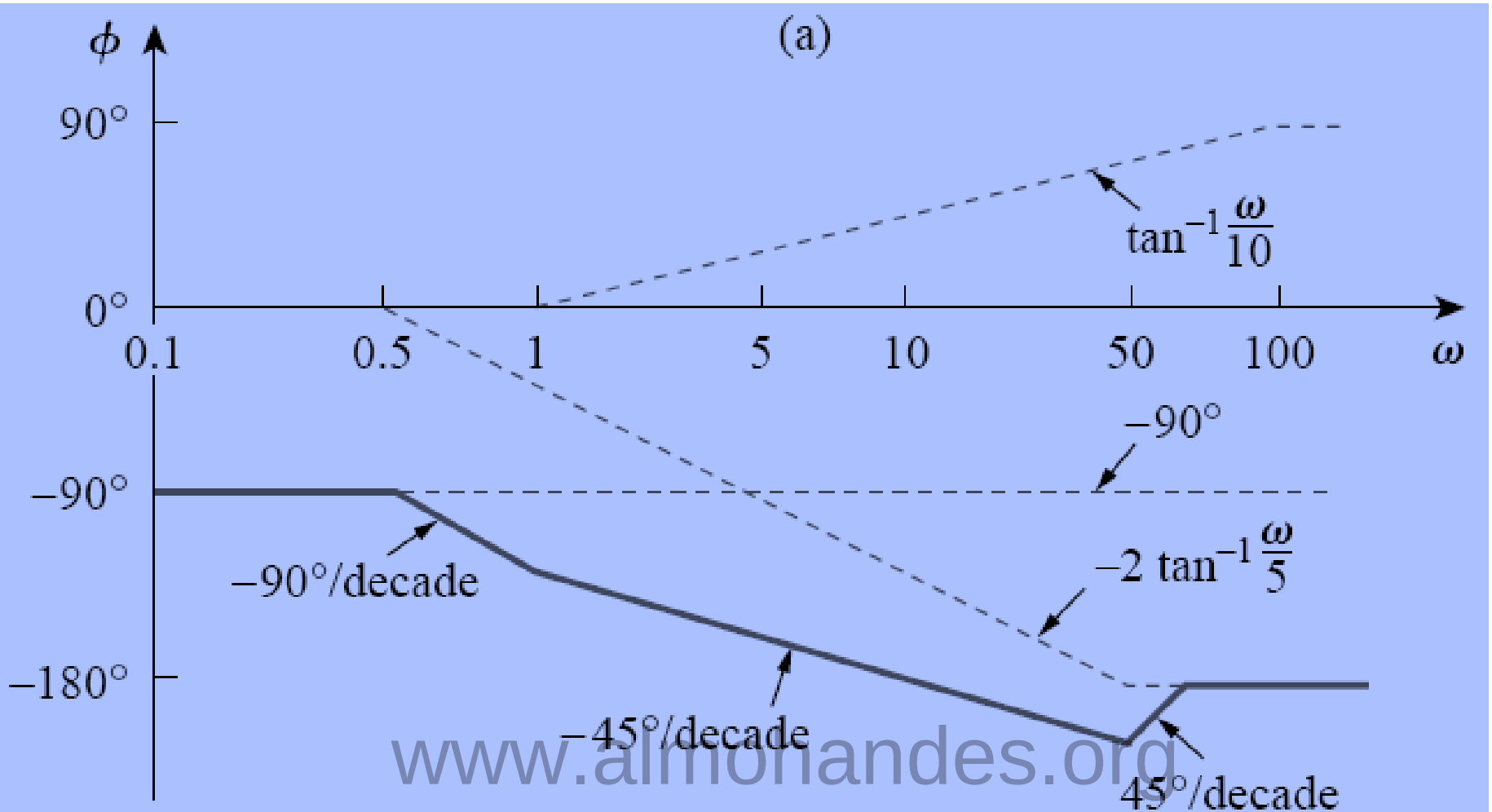
$$\phi = 0^\circ + \tan^{-1}\frac{\omega}{10} - 90^\circ - 2\tan^{-1}\frac{\omega}{5}$$



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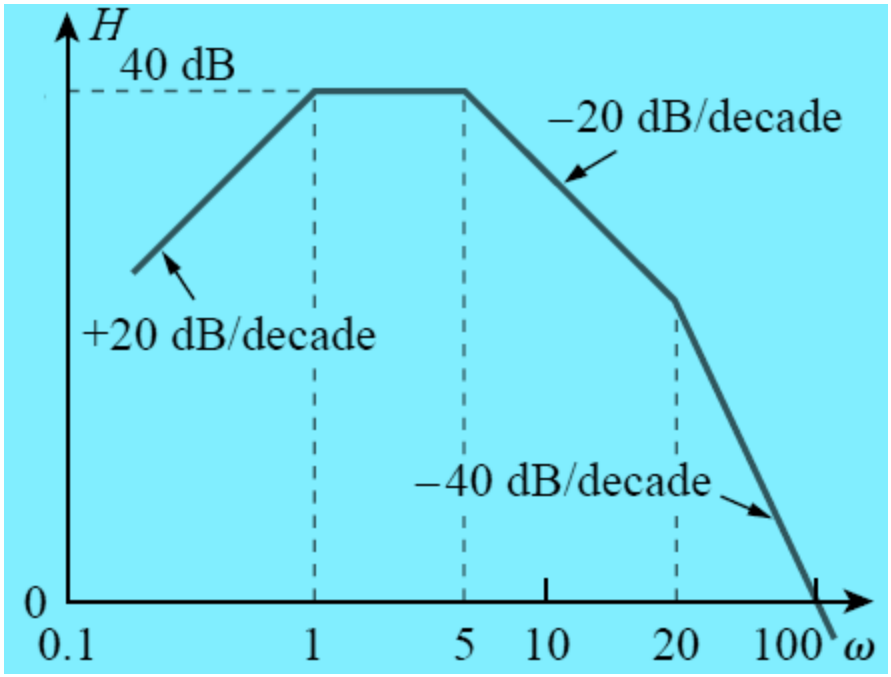
EXAMPLE 102

Given the Bode plot in the following figure, obtain the transfer function H(ω).

zero always causes an upward turn at a corner frequency, while a pole causes a downward turn



$$40 = 20\log_{10} K \Longrightarrow \log_{10} K = 2$$
$$\Longrightarrow K = 10^2 = 100$$

A pole at $p = 1$ $1/(1 + j\omega/1)$.

A pole at $p = 5$ $1/(1 + j\omega/5)$.

A pole at $p = 20$ $1/(1 + j\omega/20)$.

$$\mathbf{H}(\omega) = \frac{100\,j\omega}{(1 + j\omega/1)(1 + j\omega/5)(1 + j\omega/20)}$$
$$= \frac{j\omega 10^4}{(j\omega + 1)(j\omega + 5)(j\omega + 20)}$$

$$\mathbf{H}(s) = \frac{10^4 s}{(s+1)(s+5)(s+20)}, \qquad s = j\omega$$

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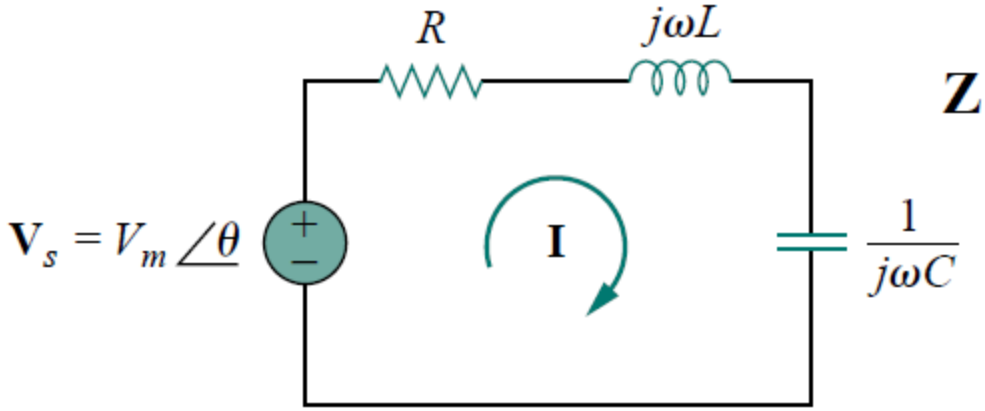
Resonance is a condition in an *RLC circuit in which the capacitive and inductive* reactance are equal in magnitude, there by resulting in a purely resistive impedance

يحدث الطنين التسلسلي أو التفرعي عندما تتساوى الممانعة السعوية مع الممانعة التحريضية بدارة RLC تسلسلية أو تفرعية وتصبح الممانعة الكلية للدارة أومية بحتة و تسخدم لبناء مرشح بانتقائية عالية للتردد , لنعتبر دارة الطنين التسلسية المبينة بالشكل التالي:



$$\mathbf{Z} = \mathbf{H}(\omega) = \frac{\mathbf{V}_s}{\mathbf{I}} = R + j\omega L + \frac{1}{j\omega C}$$

$$\mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\mathrm{Im}(\mathbf{Z}) = \omega L - \frac{1}{\omega C} = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

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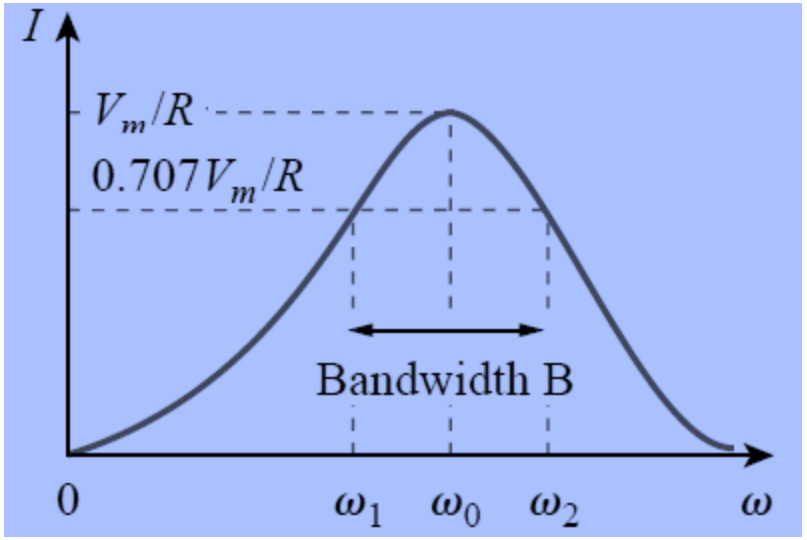
The frequency response of the circuit's current magnitude

$$I = |\mathbf{I}| = \frac{V_m}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

The average power dissipated by the *RLC circuit is*

$$P(\omega) = \frac{1}{2} I^2 R$$

$$P(\omega_0) = \frac{1}{2}\frac{V_m^2}{R}$$



At certain frequencies $\omega = \omega_1, \omega_2$, *the dissipated power is half the* maximum value,

$$P(\omega_1) = P(\omega_2) = \frac{(V_m/\sqrt{2})^2}{2R} = \frac{V_m^2}{4R}$$

ω_1 and ω_2 are called the half-power frequencies

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ترددات نصف الاستطاعة

ω_1 and ω_2 are called the half-power frequencies are obtained by setting $Z = \sqrt{2}R$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\Longrightarrow \quad \omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

bandwidth B,

$$B = \omega_2 - \omega_1$$

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The "sharpness" of the resonance in a resonant circuit is measured quantitatively by the *quality factor Q. At resonance, the reactive energy* in the circuit oscillates between the inductor and the capacitor. The quality factor relates the maximum or peak energy stored to the energy dissipated in the circuit per cycle of oscillation:

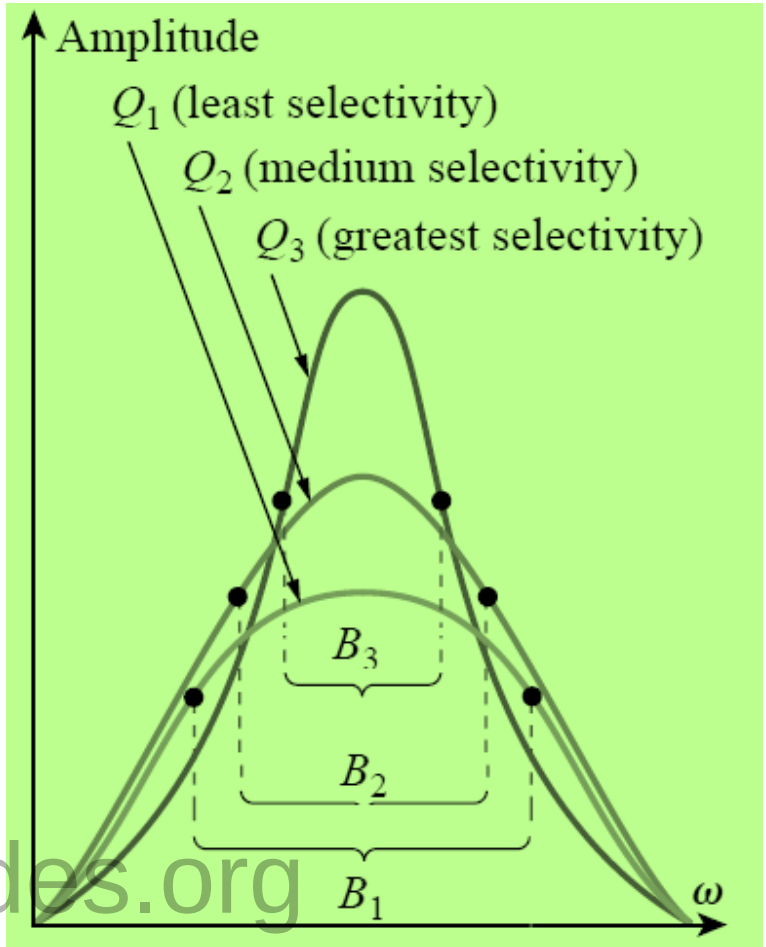
$$Q = 2\pi \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}}$$

$$Q = 2\pi \frac{\frac{1}{2}LI^2}{\frac{1}{2}I^2R(1/f)} = \frac{2\pi f L}{R}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} \Longrightarrow B = \frac{R}{L} = \frac{\omega_0}{Q}$$

$$B = \omega_0^2 C R$$

The quality factor of a resonant circuit is the ratio of its resonant frequency to its bandwidth



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For high-*Q circuits (Q ≥ 10),*

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

High-*Q circuits are used often in communications networks*

يمكن توصيف دارة الطنين بخمس محددات وهي:

- two half-power frequencies *ω1 and ω2,*
- *resonant frequency* ω_o,
- *bandwidth B,*
- *quality factor Q.*

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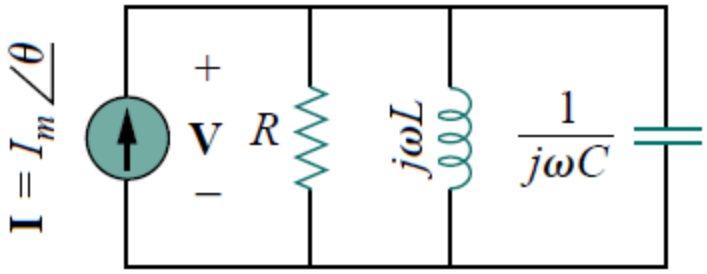
The parallel resonant circuit

$$\mathbf{Y} = H(\omega) = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

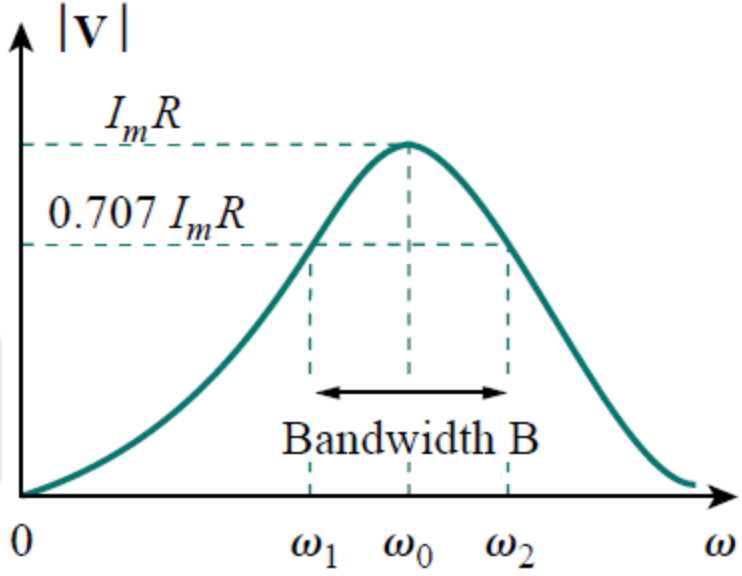


Resonance occurs when the imaginary part of **Y is zero,**

$$\mathbf{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

at resonance, the parallel *LC combination acts like an open circuit*



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علاقات الطنين التفرعي مشابه لعلاقات الطنين التسلسلي, للحصول عليها نستبدل R بـ 1/R و L بـ C

By replacing R with 1/R and L with C in the expressions for the series circuit ,we obtain for the parallel circuit expressions:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$\omega_1 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q},$$

$$\omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

high-*Q circuits* ($Q \geq 10$)

$$\omega_1 \simeq \omega_0 - \frac{B}{2}, \qquad \omega_2 \simeq \omega_0 + \frac{B}{2}$$

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Summary of the characteristics of resonant *RLC circuits*

Characteristic	Series circuit	Parallel circuit
Resonant frequency, ω_0	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality factor, Q	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$	$\frac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, B	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$
Half-power frequencies, ω_1, ω_2	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$	$\omega_0\sqrt{1+\left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$
For $Q \geq 10$, ω_1, ω_2	$\omega_0 \pm \frac{B}{2}$	$\omega_0 \pm \frac{B}{2}$

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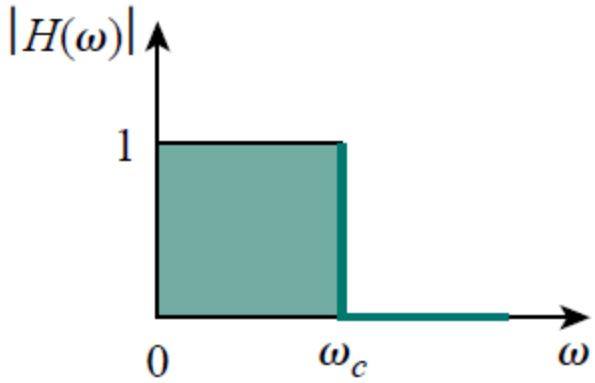
PASSIVE FILTERS

المرشحات غير الفعالة

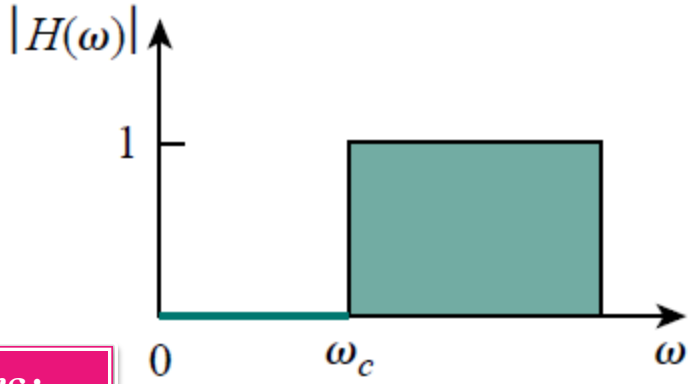
المرشح عبارة عن دارة مصممة لتقوم بتمرير الترددات المرغبة وحذف الترددات الأخرى ويوجد منها أربع أنواع

A filter is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others , there are four types of filters whether passive or active:

1- A low-pass filter passes low frequencies and stops high Frequencies.



2- A high-pass filter passes high frequencies and rejects low Frequencies.



ω_c *:cutoff frequency for low-pass and high-pass filters;*
ω_o *:center frequency for band-pass and band-stop filters.*

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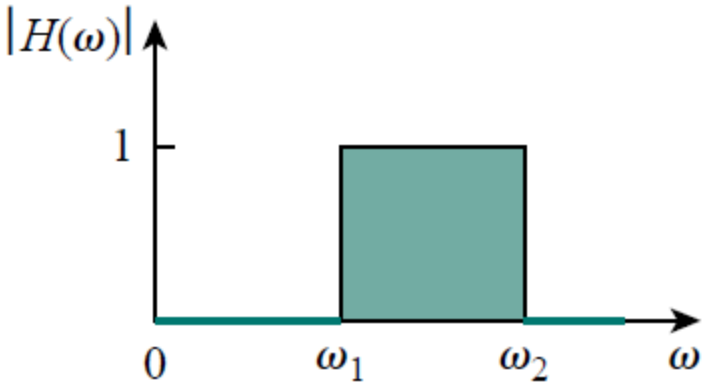
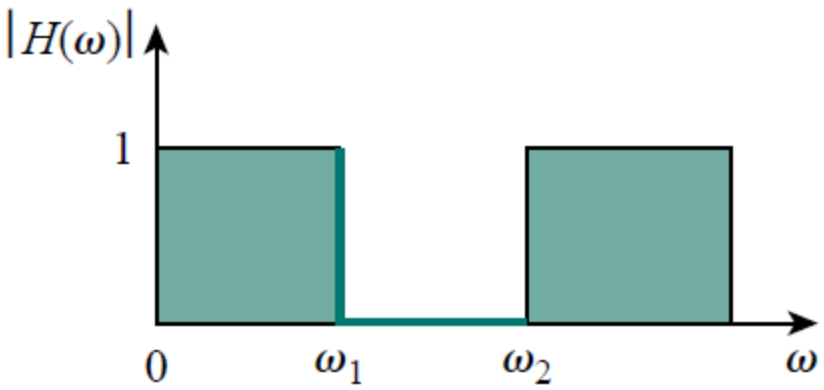
3-A band-pass filter passes frequencies within a frequency band and blocks or attenuates frequencies outside the band.



4-A band-stop filter passes frequencies outside a frequency band and blocks or attenuates frequencies within the band.



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Summary of the characteristics of filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

ω_c :cutoff frequency for low-pass and high-pass filters;
ω_o :center frequency for band-pass and band-stop filters.

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Low-pass Filter -1 مرشح تمرير منخفض

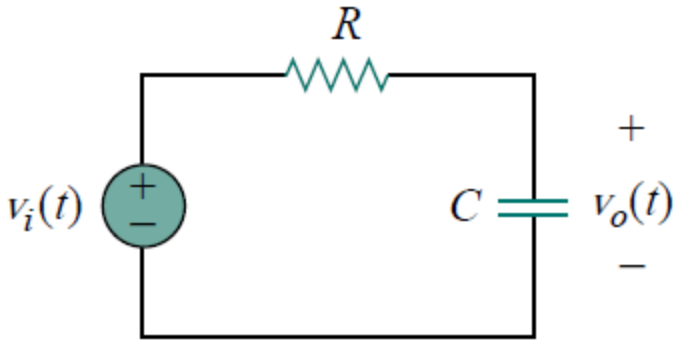
The transfer function

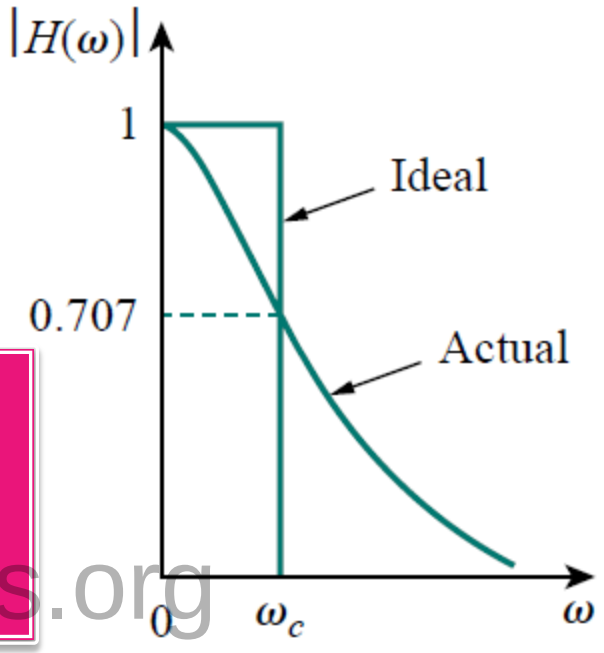
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

$$\mathbf{H}(0) = 1, \mathbf{H}(\infty) = 0$$

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}} \qquad \omega_c = \frac{1}{RC}$$





A low-pass filter is designed to pass only frequencies from dc up to the cut off frequency ω_c.

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High-pass Filter -2 مرشح تمرير مرتفع

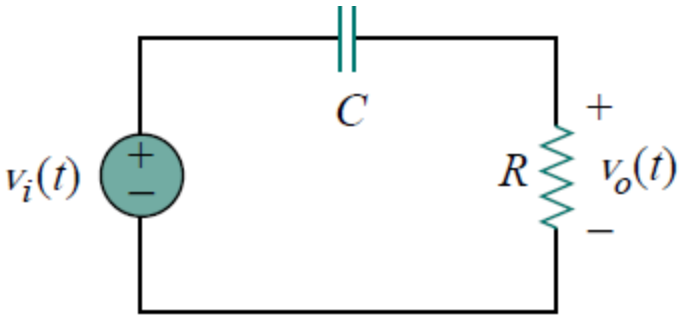
The transfer function

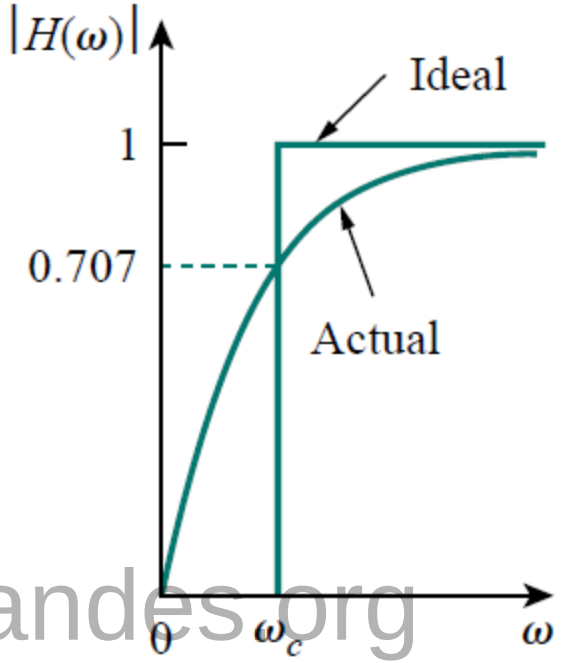
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC}$$

$$\mathbf{H}(0) = 0, \mathbf{H}(\infty) = 1.$$

$$\omega_c = \frac{1}{RC}$$





A high-pass filter is designed to pass all frequencies above its cutoff frequency ω_c.

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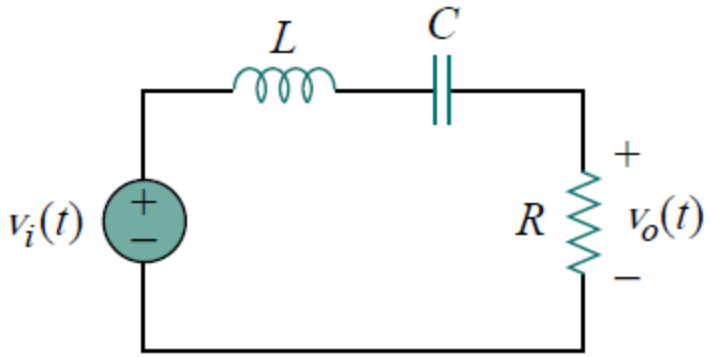
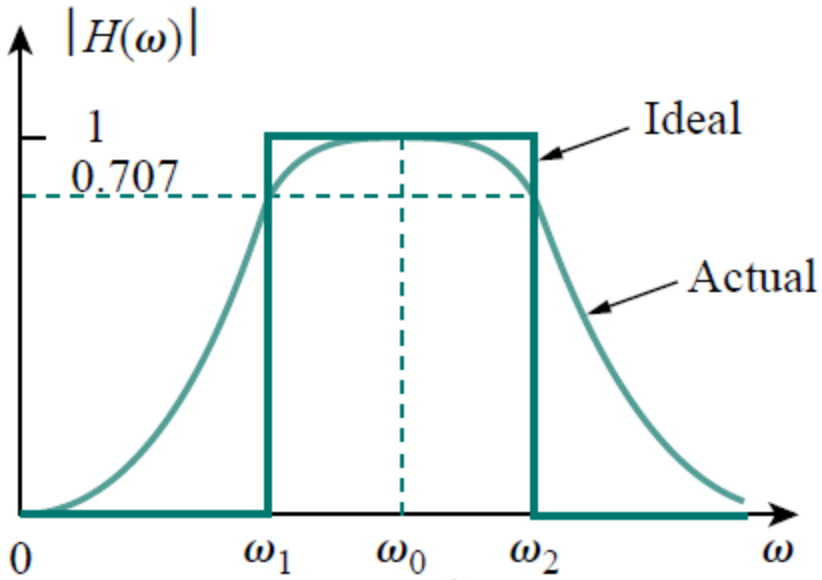
The transfer function

3- Band-pass Filter مرشح تمرير حزمة

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

$$\mathbf{H}(0) = 0, \mathbf{H}(\infty) = 0$$





The band-pass filter passes a band of frequencies ($\omega_1 < \omega < \omega_2$) centered on ω_o, *the center frequency, which is given by*

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

A band-pass filter can also be formed by cascading the low-pass filter (where $\omega_2 = \omega_c$) *with the high-pass filter* (where $\omega_1 = \omega_c$).

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Band-stop, band reject, or notch filter.

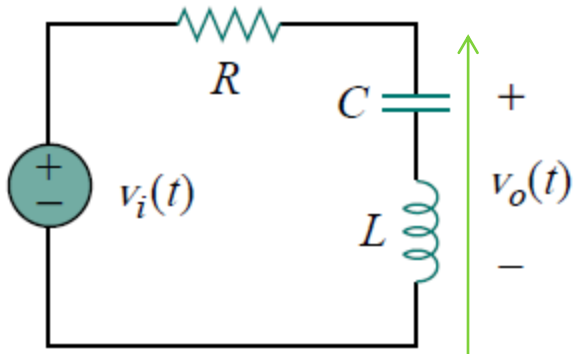
The transfer function

Band-stop Filter -4 مرشح حذف حزمة

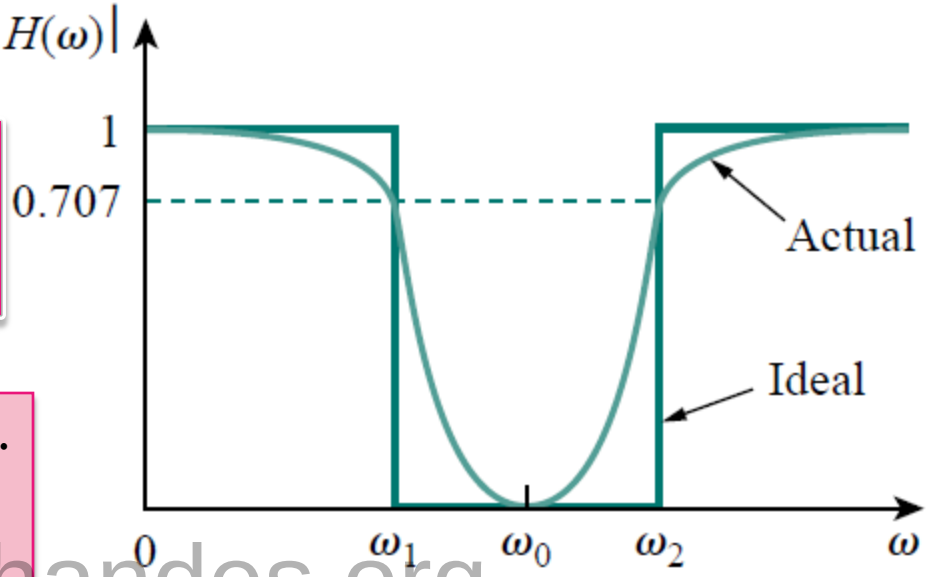
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

$$\mathbf{H}(0) = 1, \mathbf{H}(\infty) = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (B = \omega_2 - \omega_1)$$



A band-stop filter is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.



the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter

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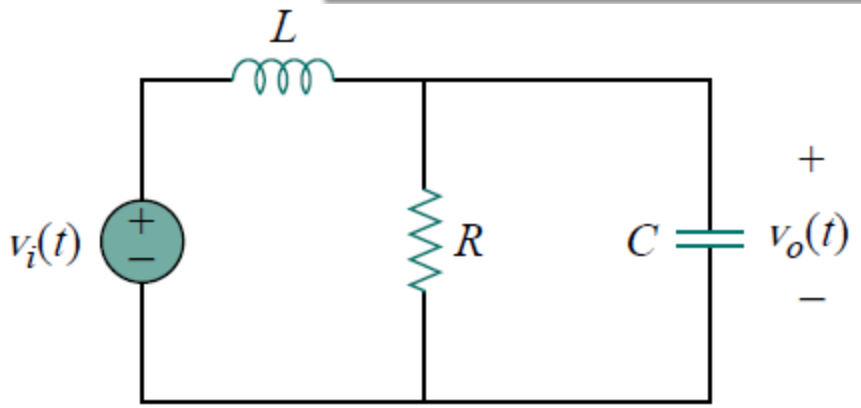
EXAMPLE

Determine what type of filter is shown in following circuit . Calculate the corner or cutoff frequency. Take $R = 2\ k\Omega$, $L = 2\ H$, *and* $C = 2\ \mu F$.



The transfer function

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}$$

$$s = j\omega$$

$$R \left\| \frac{1}{sC} = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC} \right.$$

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2 RLC + j\omega L + R}$$

$$\mathbf{H}(0) = 1 \text{ and } \mathbf{H}(\infty) = 0$$

second-order low-pass filter

$$H = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}}$$

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

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EXAMPLE

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of *R, L, and C,*

$$16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

$$\omega_c^2 = 0.5509$$

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

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