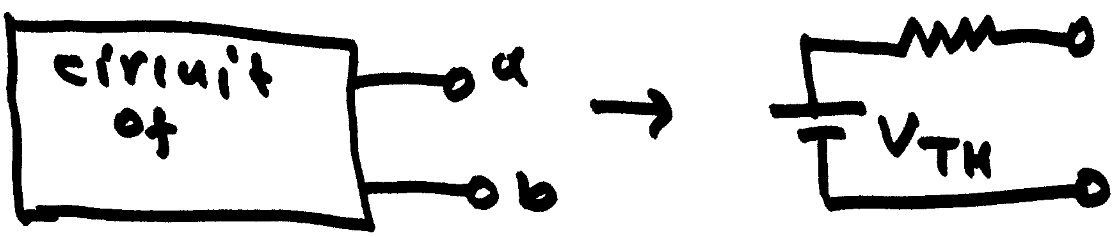
Thevenin's Theorem

1/28/02



arbitrary complexity

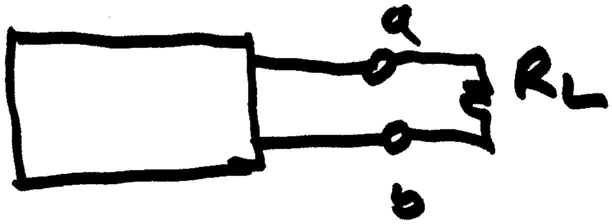
Assume that it contains Resistors and independent sources

Assume that it does **not** contain non-Linear components

It can be reduced to a voltage source with a Resistor in series

obtain V_{TH}, R_{TH}

Let



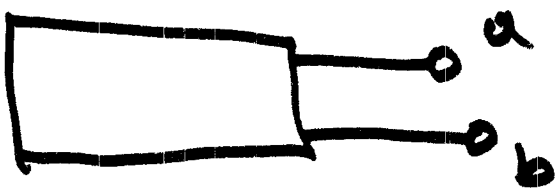
← Load Resitor

Find Thevenin equivalent as seen by R_L.

Step 1) Remove R_L

Step 2) V_{TH}: open circuit voltage between poits a b

$$V_{TH} = V_{ab}\Big|_{\text{open circuit}}$$



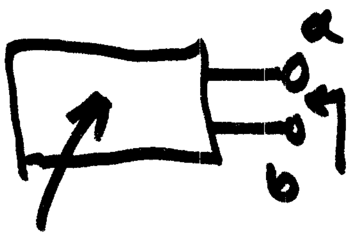
$$+ \; V_{ab}\Big|_{\text{open}} \equiv V_{TH} \; -$$

Step 3) Compute R_{TH}

a) Set all <u>independent</u> sources to zero

Note zero voltage → short
zero current → open

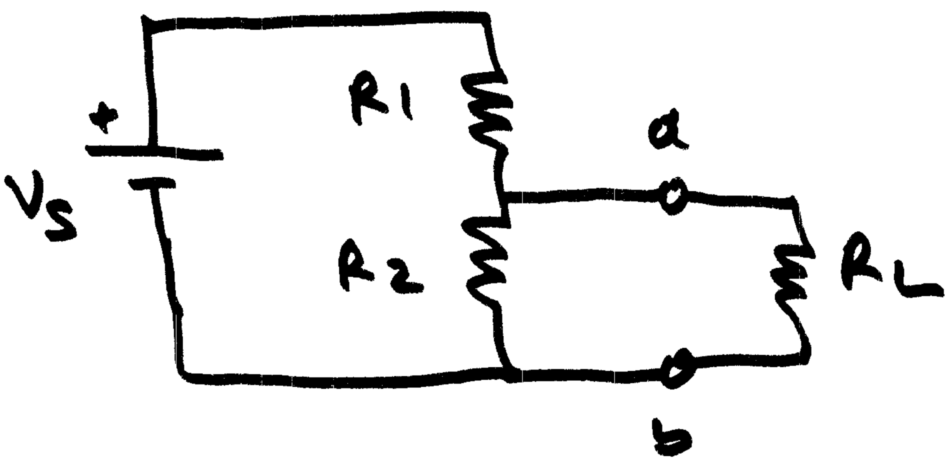
b) Compute the equivalent Resistance between points a b



source(s) = 0 $\quad R_{eq,a,b}$

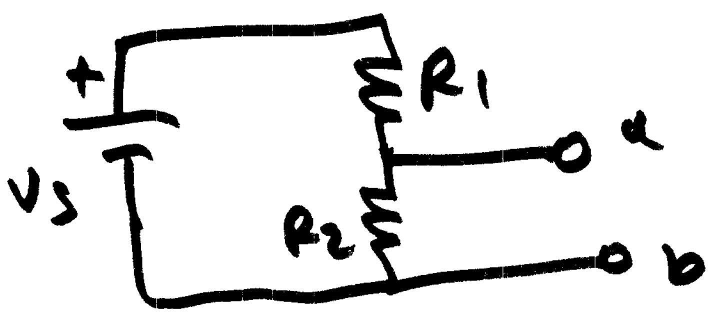
$$R_{TH} \equiv R_{eq\,a,b}\Big|_{\text{source(s)}=0}$$

Example



Find the Thevenin equivalent between points a, b, as seen by R_L

1) Remove R_L



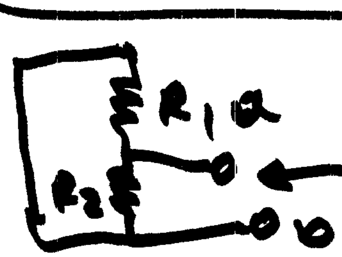
$V_{ab} = V_{TH}$

Voltage divider:

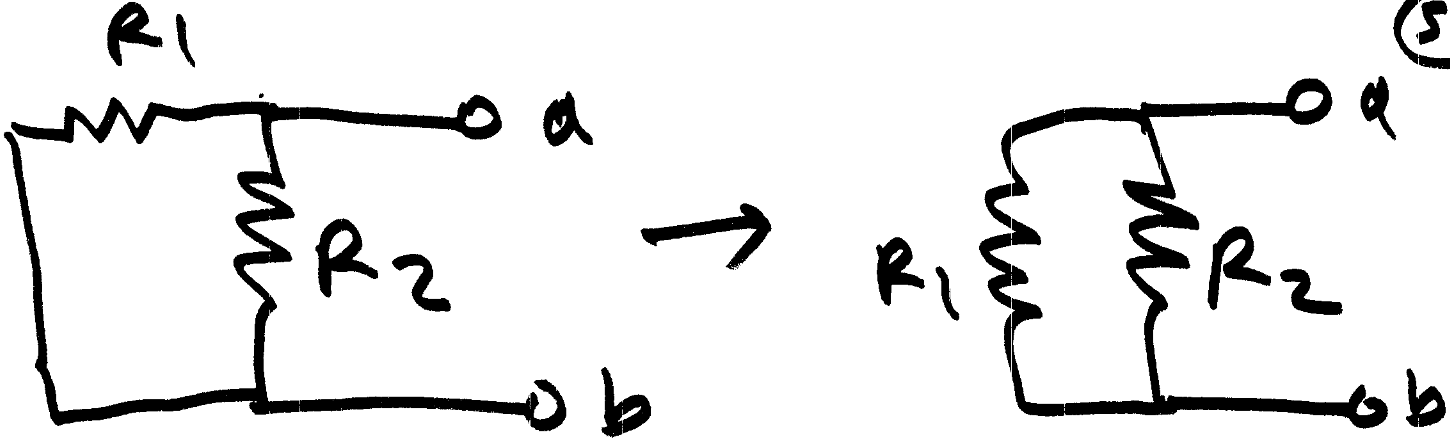
$$\boxed{V_{ab} = V_{TH} = V_S \frac{R_2}{R_1 + R_2}}$$

step 2

step 3 set $V_S = 0$

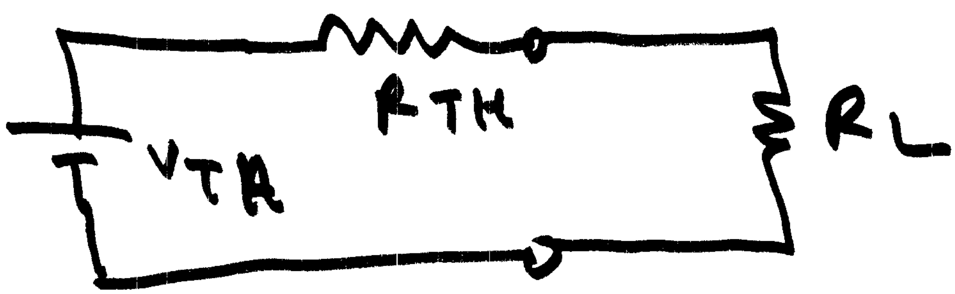


$R_{ab} = R_{TH}$



$R_{ab} = R_{TH} = R_1 \| R_2$

Thevenin equivalen



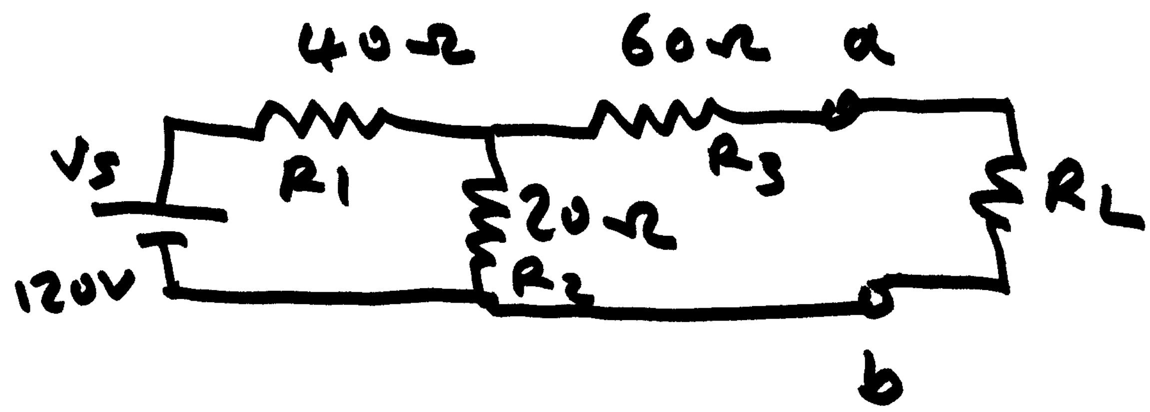
Example $V_S = 12V$ $R_2 = 1k\Omega$

$R_1 = 2k\Omega$

$\Rightarrow V_{TH} = 12 \times \frac{1}{3} = 4V$

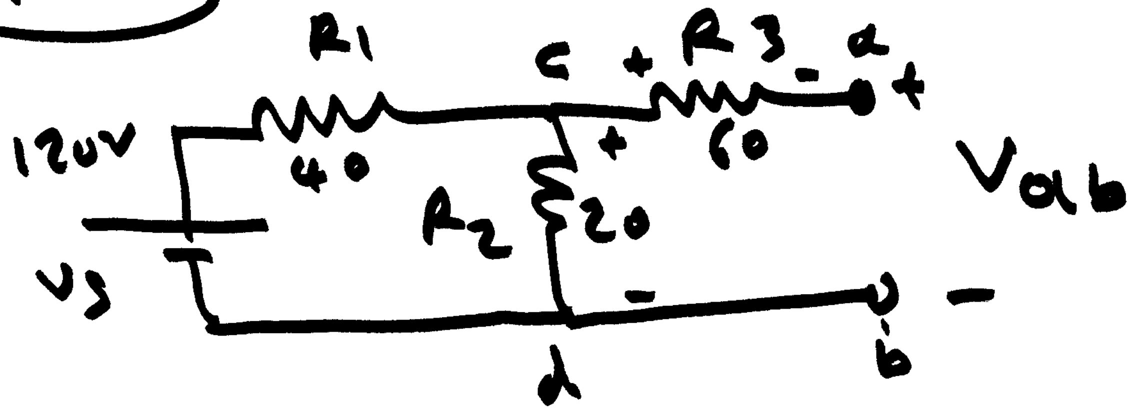
$R_{TH} = 1k \| 2k\Omega = \frac{2}{3}\, k\Omega \simeq 666.6\,\Omega$

Another example



Find Thevenin seen by R_L

step 1) Remove R_L



$$V_{cd} = V_S \frac{R_2}{R_1 + R_2}$$

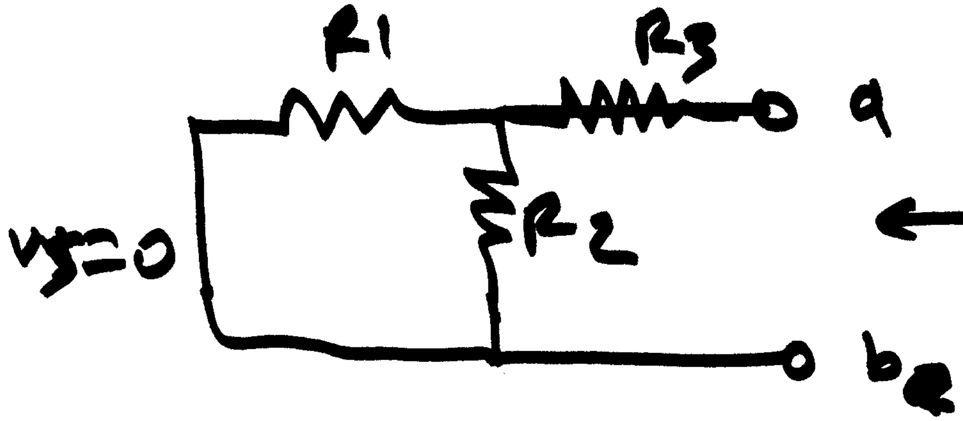
KVL: $V_{ca} + V_{ab} - V_{DC} = 0$

$$\Rightarrow V_{ab} = +V_{DC} - V_{CA}$$

$$V_{ca} = V_{R_3} = R_3 \times I_{R_3} \qquad I_{R_3} = 0$$

$$\Rightarrow V_{ab} = V_{cd} = V_S \frac{R_2}{R_1 + R_2}$$

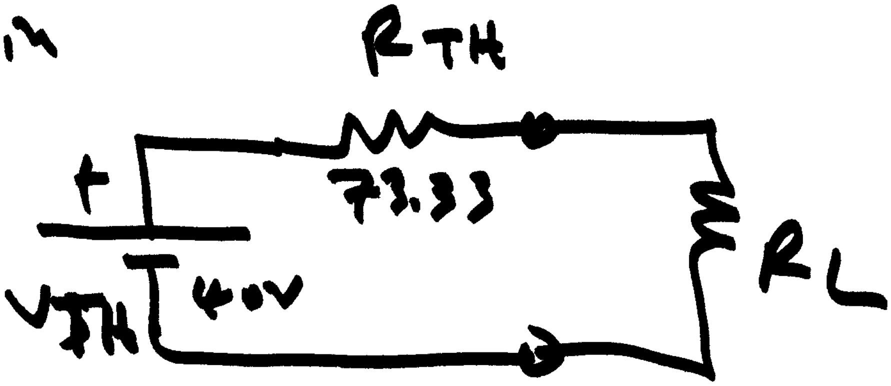
step3) S.C. $V_s = 0$



$\leftarrow R_{ab} = R_{TH}$

$$R_{ab} = R_{TH} = (R_1 \| R_2) + R_3$$

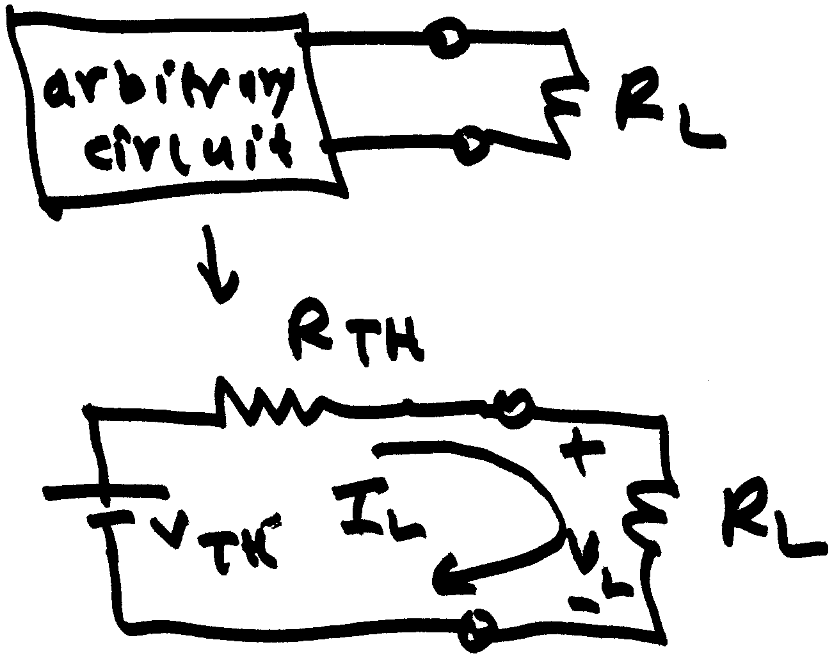
Thevenin



$$\Rightarrow V_{TH} = 120 \times \frac{20}{60} = 40V$$

$$R_{TH} = (20 \| 40) + 60 = 73.33\,\Omega$$

Theorem of maximum power transfer



Find R_L for maximum power dissipation across R_L

P_L = power lost at R_L

$$P_L = I_L^2 R_L = \frac{V_L^2}{R_L}$$

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}, \quad V_L = V_{TH} \frac{R_L}{R_L + R_{TH}}$$

$$P_L = V_{TH}^2 \cdot \frac{R_L}{(R_L + R_{TH})^2}$$

maximize

$$\frac{dP_L}{dR_L} = 0 \Rightarrow \dots V_{TH}^2 \frac{[R_{TH} - R_L]}{[R_{TH} + R_L]^3} = 0$$

$$\Rightarrow \boxed{R_L = R_{TH}}$$

Power is maximized when $R_L = R_{TH}$

$$\frac{d^2 P_L}{dR_L^2} = V_{TH}^2 \left[\frac{-(R_{TH}+R_L)^3 - 3(R_{TH}-R_L)(R_{TH}+R_L)^2}{(R_{TH}+R_L)^6} \right]$$

when $R_L = R_{TH} \Rightarrow \dfrac{d^2 P_L}{dR_L^2} < 0$

maximum

what is the value of the maximum power?

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maximum when $R_L = R_{TH}$

$$\Rightarrow P_{L,max} = V_{TH}^2 \frac{R_{TH}}{(R_{TH}+R_{TH})^2}$$

$$\boxed{P_{L,max} = \frac{V_{TH}^2}{4R_{TH}}}$$

$$\text{Efficiency} = \frac{\text{Productive (useful)}}{\text{total}}$$

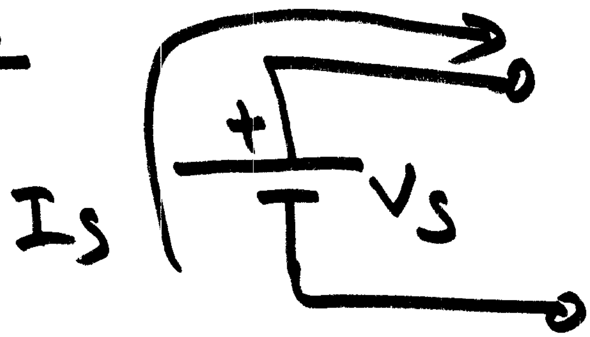
In this case:

Productive $= P_L$

Total = Delivered by Source

SP

Power delivered by source



$$P_{delivered} = V_S \cdot I_S$$

In previous circuit

$$I_L = I_S = \frac{V_{TH}}{R_{TH}+R_L} = \left.\frac{V_{TH}}{2R_{TH}}\right|_{R_L=R_{TH}}$$

P_S = Power from source

$$\boxed{P_S = V_{TH} \cdot I_L = \frac{V_{TH}^2}{2R_{TH}}}$$

% efficiency, η

$$\eta = \frac{P_L}{P_S} \times 100\%$$

$$\Rightarrow \eta = \frac{V_{TH}^2 / 4R_{TH}}{V_{TH}^2 / 2R_{TH}} = \frac{1}{2} = 50\%$$

at maximum power

<u>HW</u> a) Find the value of R_L that maximizes the efficiency

b) Find the maximum efficiency