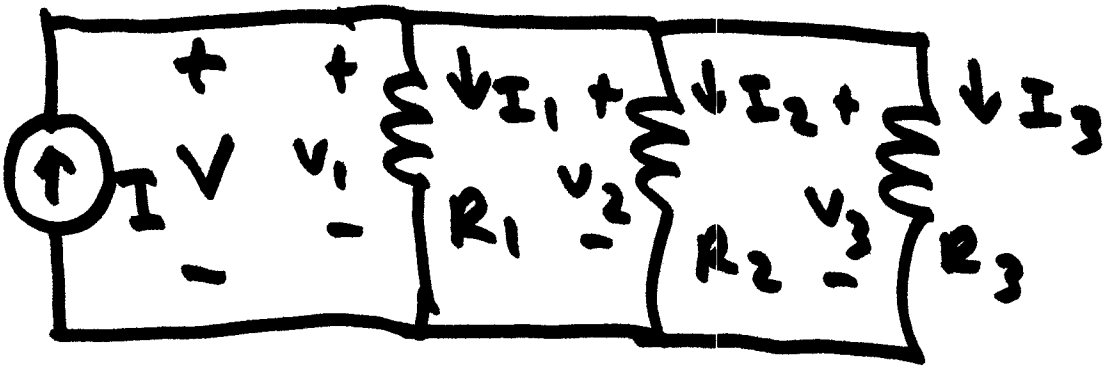
Example

11/14/02



You all agree that

$$v_1 = v_2 = v_3 = v$$

Since Resistors are connected in Parallel

By KCL $I = I_1 + I_2 + I_3$ (A)

By ohm's Law $I_1 = \frac{v_1}{R_1}$, where

$$I_2 = \frac{v_2}{R_2}, \quad I_3 = \frac{v_3}{R_3}$$

(A) $\Rightarrow$ $I = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3}$

$$I = v\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

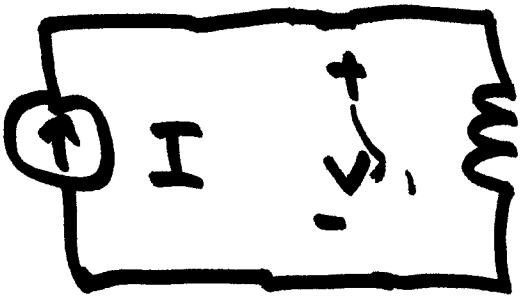
$$V = I \times \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \quad (B)$$

$$I_1 = \frac{V}{R_1} = \frac{I}{R_1} \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$I_2 = \frac{V}{R_2}$$

$$I_3 = \frac{V}{R_3}$$

Question



R_{eq} , $V = I . R_{eq}$

From (B)

$$\underbrace{I \times \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}}_{V} = \underbrace{I . R_{eq}}_{V}$$

$$\Rightarrow R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

or $$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Generalize for N Resistors

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N} = \sum_{k=1}^{N} \frac{1}{R_k}$$

Resistors in Parallel

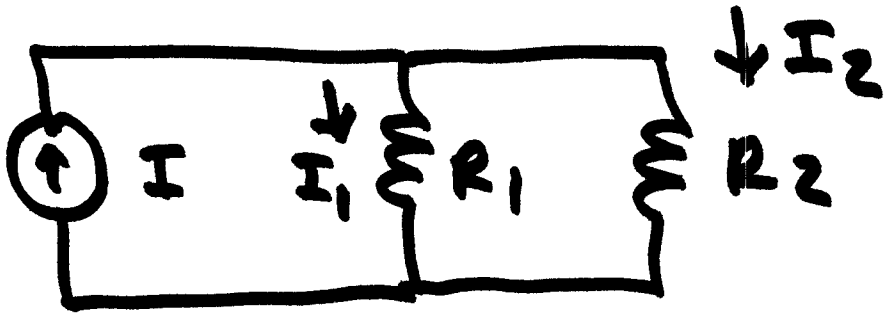
For two Resistors

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \boxed{R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}}$$

for 2 Resistors

Back to example

How about 2 Resistors only



Let $R_3 \to \infty$

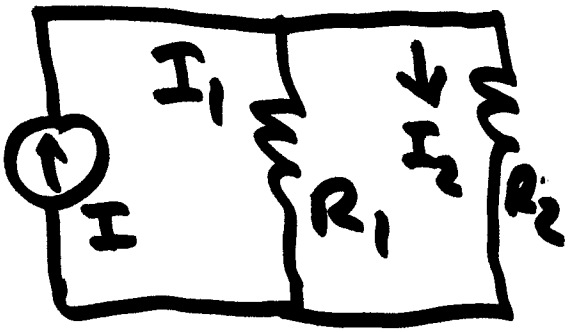
$$\Rightarrow I = \frac{V}{R_1} + \frac{V}{R_2} \Rightarrow V = I\left[\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right]$$

$$I_2 = \frac{V}{R_2} = I\,\frac{\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}}{R_2}$$

$$I_2 = I\,\frac{1}{R_2\left[\frac{1}{R_1}+\frac{1}{R_2}\right]}$$

$$I_2 = I\,\frac{1}{R_2\left[\frac{R_1+R_2}{R_1 \cdot R_2}\right]} = I\,\frac{R_1 R_2}{R_2\left[R_1+R_2\right]}$$

$$I_2 = I \frac{R_1}{R_1 + R_2}$$



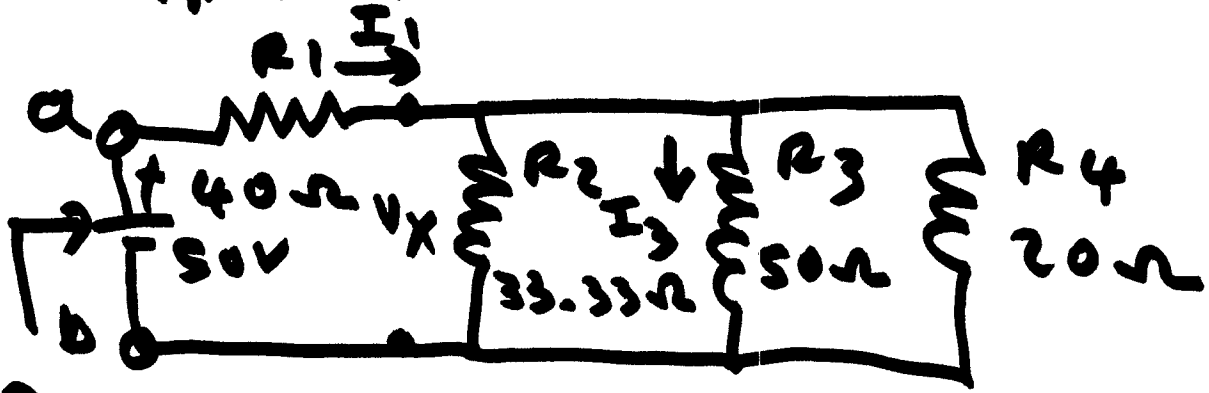
and

$$I_1 = I \frac{R_2}{R_1 + R_2}$$

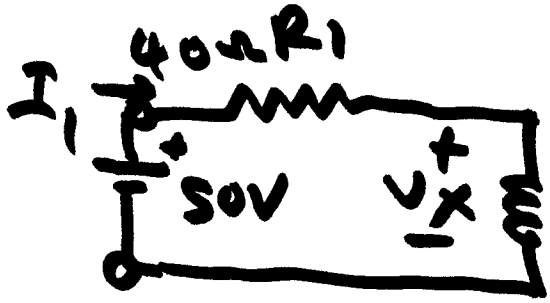
Current divider

Example

Example 1



Req Find Req between Point a, b



$R_2 \| R_3 \| R_4 = 10\Omega$

$$\Rightarrow R_{eq} = R_1 + [R_2 \| R_3 \| R_4]$$

$$R_2 \| R_3 \| R_4 = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} = 10\Omega$$

$$\Rightarrow R_{eq} = 40\Omega + 10\Omega = \underline{\underline{50\Omega}}$$

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expand example.

Compute current in R_3

if 50V source is connected

<u>method 1</u> use voltage divider

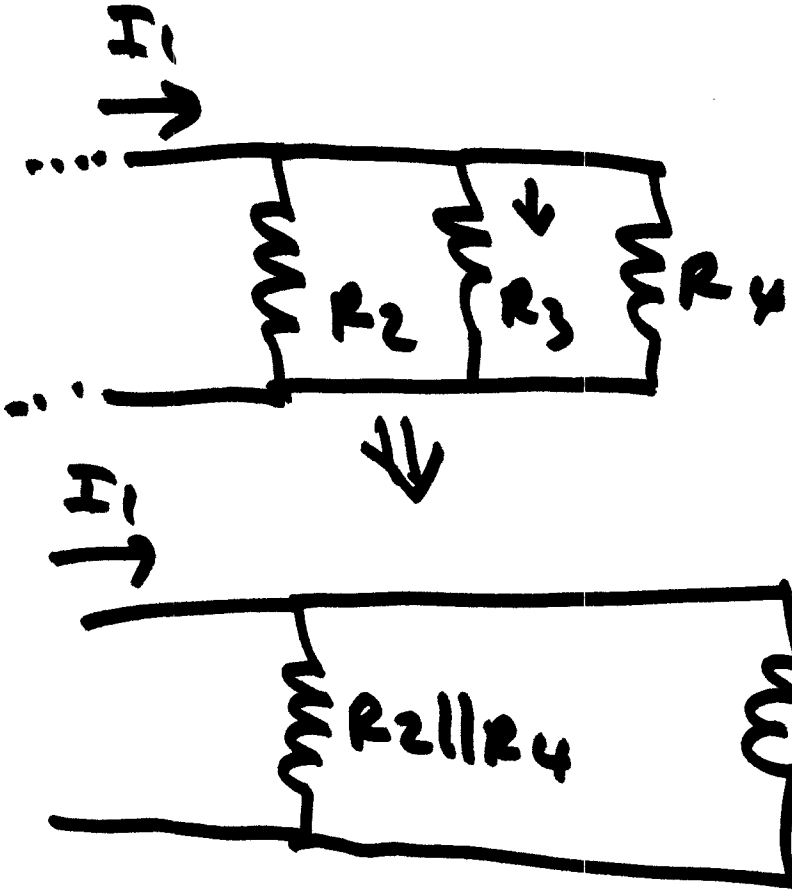
$$V_X = 50V \frac{10\Omega}{10+40} = 10V$$

$$V_{R_3} = V_X \quad \text{(voltage across } R_3\text{)}$$

$$I_3 = \frac{V_X}{R_3} = \frac{10V}{50\Omega} = \underline{\underline{0.2A}}$$

Method 2 use current divider

$$I_1 = \frac{50V}{R_{eq}} = \frac{50V}{50\Omega} = 1A$$



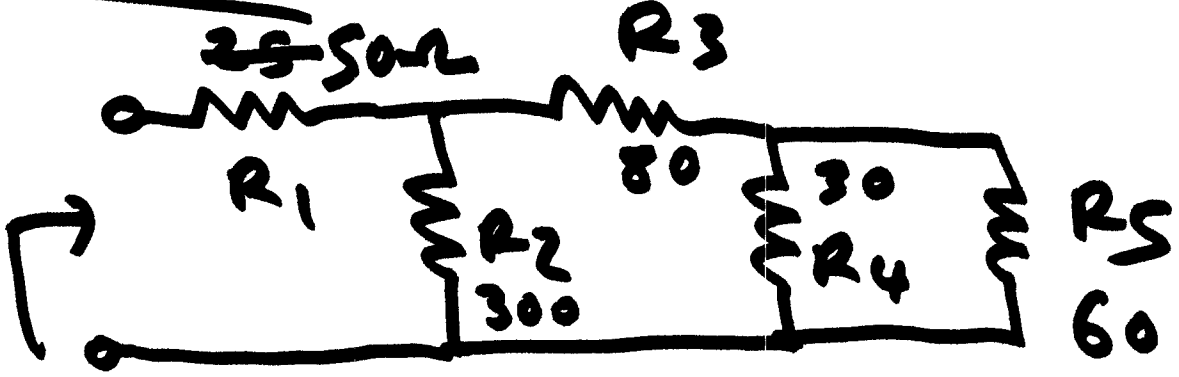
$$I_3 = I_1 \frac{R_2 \| R_4}{R_2 \| R_4 + R_3}$$

$$R_2 \| R_4 = \frac{R_2 \cdot R_4}{R_2 + R_4} = \frac{20 \times 33.33}{53.33} =$$

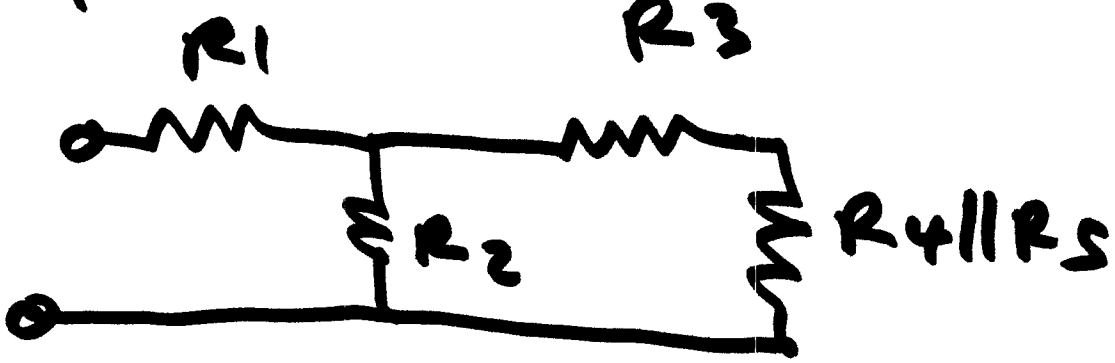
$$= 12.5\Omega$$

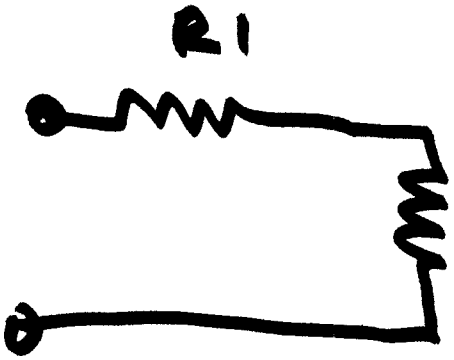
$$I_3 = 1A \frac{12.5}{12.5+50} = \frac{12.5}{62.5} = \underline{\underline{0.2A}}$$

Example 2



R_{eq}





$R_2 \| [R_3 + R_4 \| R_5]$

$$R_{eq} = R_1 + R_2 \| [R_3 + R_4 \| R_5]$$

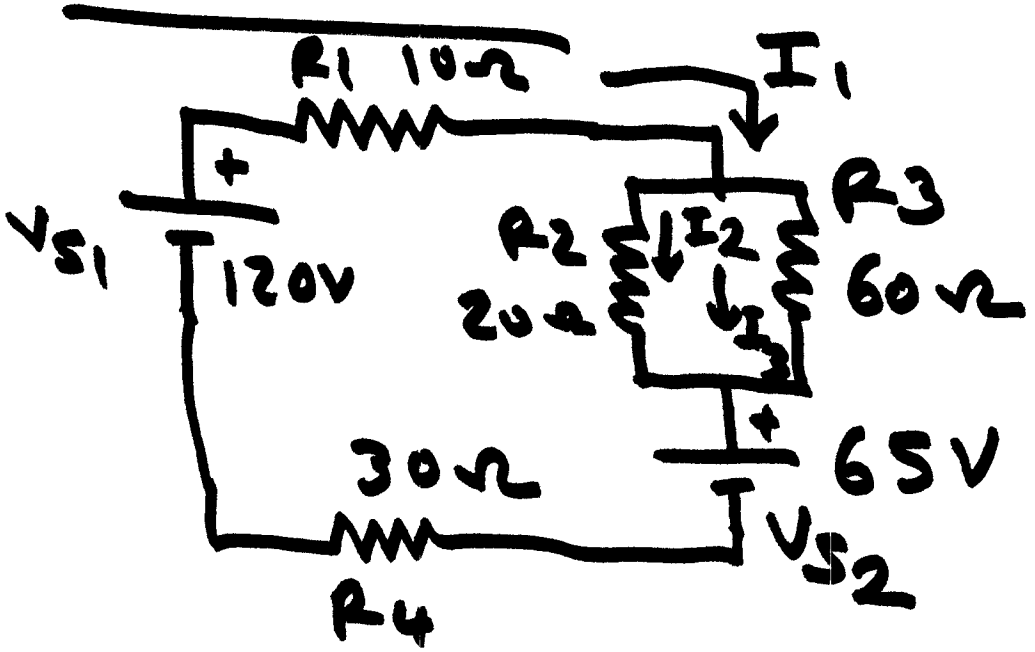
$R_4 \| R_5 = 30 \| 60 = 20$

$R_3 + R_4 \| R_5 = 80 + 20 = 100\ \Omega$

$R_{eq} = 25 + 300 \| 100 = 125\ \Omega$

(25 + 50 + 75 Ω)

Example 3



Find I_2, I_3

A method:. Compute I_1 and apply current divider

For computing I_1, combine $R_3 \| R_2$