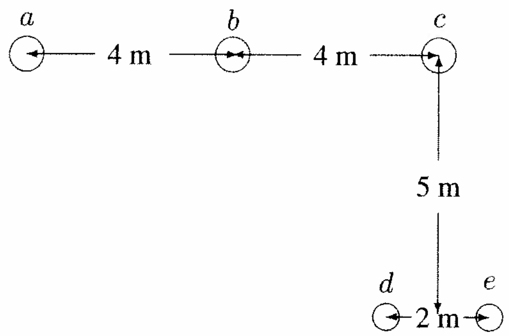
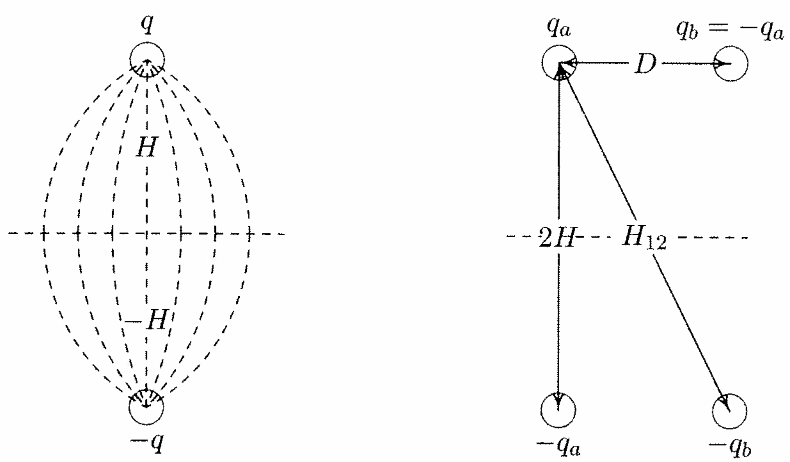



**FIGURE 4.36**
Conductor layout for Problem 4.14.

of the earth will produce the same field as a single charge and the earth surface. This imaginary conductor is called the image conductor. Figure 4.37(b) shows a single-phase line with its image conductors. Find the potential difference $V_{ab}$ and show that the equivalent capacitance to neutral is given by

$$C_{an} = C_{bn} = \frac{2\pi\varepsilon}{\ln(\frac{D}{r}\frac{2H}{H_{12}})}$$




(a) Earth plane replaced by image conductor

(b) Single-phase line and its image

**FIGURE 4.37**
Conductor layout for Problem 4.15.

# CHAPTER 5

# LINE MODEL AND PERFORMANCE

## 5.1 INTRODUCTION

In Chapter 4 the per-phase parameters of transmission lines were obtained. This chapter deals with the representation and performance of transmission lines under normal operating conditions. Transmission lines are represented by an equivalent model with appropriate circuit parameters on a "per-phase" basis. The terminal voltages are expressed from one line to neutral, the current for one phase and, thus, the three-phase system is reduced to an equivalent single-phase system.

The model used to calculate voltages, currents, and power flows depends on the length of the line. In this chapter the circuit parameters and voltage and current relations are first developed for "short" and "medium" lines. Problems relating to the regulation and losses of lines and their operation under conditions of fixed terminal voltages are then considered.

Next, long line theory is presented and expressions for voltage and current along the distributed line model are obtained. Propagation constant and characteristic impedance are defined, and it is demonstrated that the electrical power is being transmitted over the lines at approximately the speed of light. Since the terminal conditions at the two ends of the line are of primary importance, an equivalent

$\pi$ model is developed for the long lines. Several *MATLAB* functions are developed for calculation of line parameters and performance. Finally, line compensations are discussed for improving the line performance for unloaded and loaded transmission lines.

## 5.2 SHORT LINE MODEL

Capacitance may often be ignored without much error if the lines are less than about 80 km (50 miles) long, or if the voltage is not over 69 kV. The short line model is obtained by multiplying the series impedance per unit length by the line length.

$$\begin{aligned} Z &= (r + j\omega L)\ell \\ &= R + jX \end{aligned} \tag{5.1}$$

where $r$ and $L$ are the per-phase resistance and inductance per unit length, respectively, and $\ell$ is the line length. The short line model on a per-phase basis is shown in Figure 5.1. $V_S$ and $I_S$ are the phase voltage and current at the sending end of the line, and $V_R$ and $I_R$ are the phase voltage and current at the receiving end of the line.

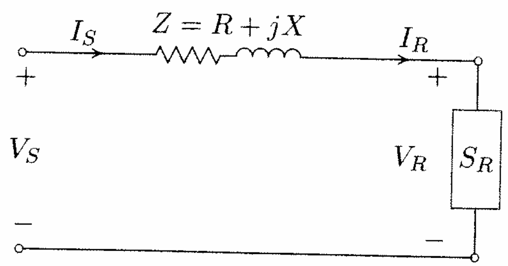



**FIGURE 5.1**
Short line model.

If a three-phase load with apparent power $S_{R(3\phi)}$ is connected at the end of the transmission line, the receiving end current is obtained by

$$I_R = \frac{S^*_{R(3\phi)}}{3V^*_R} \tag{5.2}$$

The phase voltage at the sending end is

$$V_S = V_R + ZI_R \tag{5.3}$$

and since the shunt capacitance is neglected, the sending end and the receiving end current are equal, i.e.,

$$I_S = I_R \tag{5.4}$$

The transmission line may be represented by a two-port network as shown in Figure 5.2, and the above equations can be written in terms of the generalized circuit constants commonly known as the $ABCD$ constants

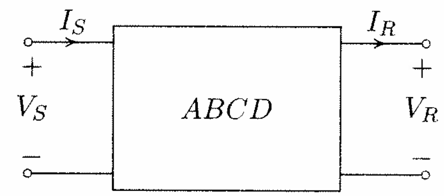



**FIGURE 5.2**
Two-port representation of a transmission line.

$$V_S = AV_R + BI_R \tag{5.5}$$
$$I_S = CV_R + DI_R \tag{5.6}$$

or in matrix form

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{5.7}$$

According to (5.3) and (5.4), for short line model

$$A = 1 \qquad B = Z \qquad C = 0 \qquad D = 1 \tag{5.8}$$

Voltage regulation of the line may be defined as the percentage change in voltage at the receiving end of the line (expressed as percent of full-load voltage) in going from no-load to full-load.

$$\text{Percent } VR = \frac{|V_{R(NL)}| - |V_{R(FL)}|}{|V_{R(FL)}|} \times 100 \tag{5.9}$$

At no-load $I_R = 0$ and from (5.5)

$$V_{R(NL)} = \frac{V_S}{A} \tag{5.10}$$

For a short line, $A = 1$ and $V_{R(NL)} = V_S$. Voltage regulation is a measure of line voltage drop and depends on the load power factor. Voltage regulation will be

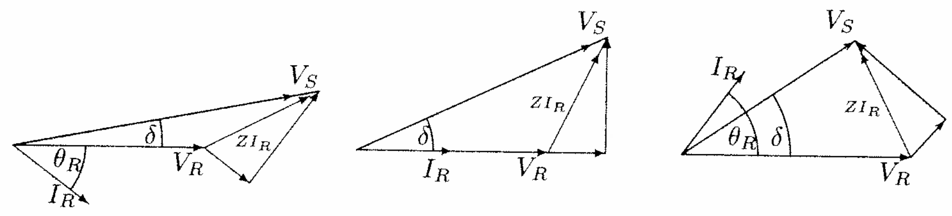



**FIGURE 5.3**
Phasor diagram for short line.

poorer at low lagging power factor loads. With capacitive loads, i.e., leading power factor loads, regulation may become negative. This is demonstrated by the phasor diagram of Figure 5.3.

Once the sending end voltage is calculated the sending-end power is obtained by

$$S_{S(3\phi)} = 3V_S I_S^* \tag{5.11}$$

The total line loss is then given by

$$S_{L(3\phi)} = S_{S(3\phi)} - S_{R(3\phi)} \tag{5.12}$$

and the transmission line efficiency is given by

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} \tag{5.13}$$

where $P_{R(3\phi)}$ and $P_{S(3\phi)}$ are the total real power at the receiving end and sending end of the line, respectively.

**Example 5.1** (chp5ex1)

A 220-kV, three-phase transmission line is 40 km long. The resistance per phase is 0.15 $\Omega$ per km and the inductance per phase is 1.3263 mH per km. The shunt capacitance is negligible. Use the short line model to find the voltage and power at the sending end and the voltage regulation and efficiency when the line is supplying a three-phase load of

(a) 381 MVA at 0.8 power factor lagging at 220 kV.
(b) 381 MVA at 0.8 power factor leading at 220 kV.

(a) The series impedance per phase is

$$Z = (r + j\omega L)\ell = (0.15 + j2\pi \times 60 \times 1.3263 \times 10^{-3})40 = 6 + j20 \ \Omega$$

The receiving end voltage per phase is

$$V_R = \frac{220\angle 0^\circ}{\sqrt{3}} = 127\angle 0^\circ \ \text{kV}$$

The apparent power is

$$S_{R(3\phi)} = 381\angle\cos^{-1} 0.8 = 381\angle 36.87^\circ = 304.8 + j228.6 \ \text{MVA}$$

The current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V_R^*} = \frac{381\angle -36.87^\circ \times 10^3}{3 \times 127\angle 0^\circ} = 1000\angle -36.87^\circ \ \text{A}$$

From (5.3) the sending end voltage is

$$\begin{aligned} V_S = V_R + ZI_R &= 127\angle 0^\circ + (6 + j20)(1000\angle -36.87^\circ)(10^{-3}) \\ &= 144.33\angle 4.93^\circ \ \text{kV} \end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 250 \ \text{kV}$$

The sending end power is

$$\begin{aligned} S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 144.33\angle 4.93 \times 1000\angle 36.87^\circ \times 10^{-3} \\ &= 322.8 \ \text{MW} + j288.6 \ \text{Mvar} \\ &= 433\angle 41.8^\circ \ \text{MVA} \end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{250 - 220}{220} \times 100 = 13.6\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{304.8}{322.8} \times 100 = 94.4\%$$

(b) The current for 381 MVA with 0.8 leading power factor is

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V_R^*} = \frac{381\angle 36.87^\circ \times 10^3}{3 \times 127\angle 0^\circ} = 1000\angle 36.87^\circ \ \text{A}$$

The sending end voltage is

$$V_S = V_R + ZI_R = 127\angle 0^\circ + (6 + j20)(1000\angle 36.87^\circ)(10^{-3})$$
$$= 121.39\angle 9.29^\circ \ \text{kV}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\, V_S = 210.26 \ \text{kV}$$

The sending end power is

$$S_{S(3\phi)} = 3V_S I_S^* = 3 \times 121.39\angle 9.29 \times 1000\angle -36.87^\circ \times 10^{-3}$$
$$= 322.8 \ \text{MW} - j168.6 \ \text{Mvar}$$
$$= 364.18\angle -27.58^\circ \ \text{MVA}$$

Voltage regulation is

$$\text{Percent } VR = \frac{210.26 - 220}{220} \times 100 = -4.43\%$$

Transmission line efficiency is

$$\eta = \frac{P_{R(3\phi)}}{P_{S(3\phi)}} = \frac{304.8}{322.8} \times 100 = 94.4\%$$

## 5.3 MEDIUM LINE MODEL

As the length of line increases, the line charging current becomes appreciable and the shunt capacitance must be considered. Lines above 80 km (50 miles) and below 250 km (150 miles) in length are termed as *medium length lines*. For medium length lines, half of the shunt capacitance may be considered to be lumped at each end of the line. This is referred to as the *nominal* $\pi$ model and is shown in Figure 5.4. $Z$ is the total series impedance of the line given by (5.1), and $Y$ is the total shunt admittance of the line given by

$$Y = (g + j\omega C)\ell \tag{5.14}$$

Under normal conditions, the shunt conductance per unit length, which represents the leakage current over the insulators and due to corona, is negligible and $g$ is assumed to be zero. $C$ is the line to neutral capacitance per km, and $\ell$ is the line length. The sending end voltage and current for the nominal $\pi$ model are obtained as follows:

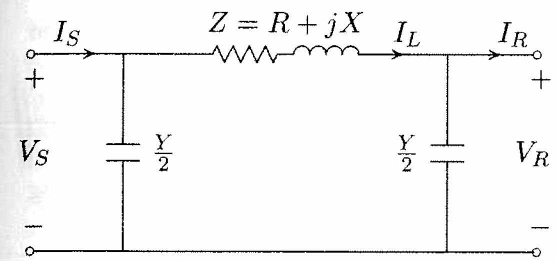



**FIGURE 5.4**
Nominal $\pi$ model for medium length line.

From KCL the current in the series impedance designated by $I_L$ is

$$I_L = I_R + \frac{Y}{2} V_R \tag{5.15}$$

From KVL the sending end voltage is

$$V_S = V_R + ZI_L \tag{5.16}$$

Substituting for $I_L$ from (5.15), we obtain

$$V_S = \left(1 + \frac{ZY}{2}\right) V_R + ZI_R \tag{5.17}$$

The sending end current is

$$I_S = I_L + \frac{Y}{2} V_S \tag{5.18}$$

Substituting for $I_L$ and $V_S$

$$I_S = Y\left(1 + \frac{ZY}{4}\right) V_R + \left(1 + \frac{ZY}{2}\right) I_R \tag{5.19}$$

Comparing (5.17) and (5.19) with (5.5) and (5.6), the $ABCD$ constants for the nominal $\pi$ model are given by

$$A = \left(1 + \frac{ZY}{2}\right) \qquad B = Z \tag{5.20}$$

$$C = Y\left(1 + \frac{ZY}{4}\right) \qquad D = \left(1 + \frac{ZY}{2}\right) \tag{5.21}$$

In general, the $ABCD$ constants are complex and since the $\pi$ model is a symmetrical two-port network, $A = D$. Furthermore, since we are dealing with a linear passive, bilateral two-port network, the determinant of the transmission matrix in (5.7) is unity, i.e.,

$$AD - BC = 1 \tag{5.22}$$

Solving (5.7), the receiving end quantities can be expressed in terms of the sending end quantities by

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_S \\ I_S \end{bmatrix} \tag{5.23}$$

Two *MATLAB* functions are written for computation of the transmission matrix. Function **[ Z, Y, ABCD ] = rlc2abcd(r, L, C, g, f, Length)** is used when resistance in ohm, inductance in mH and capacitance in $\mu$F per unit length are specified, and function **[Z, Y, ABCD ] = zy2abcd(z, y, Length)** is used when series impedance in ohm and shunt admittance in siemens per unit length are specified. The above functions provide options for the nominal $\pi$ model and the equivalent $\pi$ model discussed in Section 5.4.

**Example 5.2** (chp5ex2)

A 345-kV, three-phase transmission line is 130 km long. The resistance per phase is 0.036 $\Omega$ per km and the inductance per phase is 0.8 mH per km. The shunt capacitance is 0.0112 $\mu$F per km. The receiving end load is 270 MVA with 0.8 power factor lagging at 325 kV. Use the medium line model to find the voltage and power at the sending end and the voltage regulation.

The function **[Z, Y, ABCD] = rlc2abcd(r, L, C, g, f, Length)** is used to obtain the transmission matrix of the line. The following commands

```
r = .036; g = 0; f = 60;
L = 0.8;         % milli-Henry
C = 0.0112;      % micro-Farad
Length = 130;  VR3ph = 325;
VR = VR3ph/sqrt(3) + j*0;    % kV (receiving end phase voltage)
[Z, Y, ABCD] = rlc2abcd(r, L, C, g, f, Length);
AR = acos(0.8);
SR = 270*(cos(AR) + j*sin(AR));  %   MVA (receiving end power)
IR = conj(SR)/(3*conj(VR));      %  kA (receiving end current)
VsIs = ABCD* [VR; IR];           %      column vector [Vs; Is]
Vs = VsIs(1);
Vs3ph = sqrt(3)*abs(Vs);         % kV(sending end L-L voltage)
Is = VsIs(2); Ism = 1000*abs(Is);%    A (sending end current)
pfs= cos(angle(Vs)- angle(Is));  %  (sending end power factor)
Ss = 3*Vs*conj(Is);              %     MVA (sending end power)
REG = (Vs3ph/abs(ABCD(1,1)) - VR3ph)/VR3ph *100;
fprintf(' Is = %g A', Ism), fprintf('  pf = %g', pfs)
fprintf(' Vs = %g L-L kV', Vs3ph)
fprintf(' Ps = %g MW', real(Ss)),
fprintf('  Qs = %g Mvar', imag(Ss))
fprintf(' Percent voltage Reg. = %g', REG)
```

result in

```
Enter 1 for Medium line or 2 for long line → 1
Nominal π model
Z = 4.68 + j 39.2071 ohms
Y = 0 + j 0.000548899 siemens
```

$$\text{ABCD} = \begin{bmatrix} 0.98924 + j\,0.0012844 & 4.68 + j\,39.207 \\ -3.5251\text{e-}07 + j\,0.00054595 & 0.98924 + j\,0.0012844 \end{bmatrix}$$

```
Is = 421.132 A        pf = 0.869657
Vs = 345.002 L-L kV
Ps = 218.851 MW     Qs = 124.23 Mvar
Percent voltage Reg. = 7.30913
```

**Example 5.3** (chp5ex3)

A 345-kV, three-phase transmission line is 130 km long. The series impedance is $z = 0.036 + j0.3\ \Omega$ per phase per km, and the shunt admittance is $y = j4.22 \times 10^{-6}$ siemens per phase per km. The sending end voltage is 345 kV, and the sending end current is 400 A at 0.95 power factor lagging. Use the medium line model to find the voltage, current and power at the receiving end and the voltage regulation.

The function **[Z, Y, ABCD] = zy2abcd(z, y, Length)** is used to obtain the transmission matrix of the line. The following commands

```
z = .036 + j* 0.3;  y = j*4.22/1000000;  Length = 130;
Vs3ph = 345;  Ism = 0.4;  %kA;
As = -acos(0.95);
Vs = Vs3ph/sqrt(3) + j*0;      % kV (sending end phase voltage)
Is = Ism*(cos(As) + j*sin(As));
[Z,Y, ABCD] = zy2abcd(z, y, Length);
VrIr = inv(ABCD)* [Vs; Is];     %       column vector [Vr; Ir]
Vr = VrIr(1);
Vr3ph = sqrt(3)*abs(Vr);        % kV(receiving end L-L voltage)
Ir = VrIr(2); Irm = 1000*abs(Ir);  % A (receiving end current)
pfr= cos(angle(Vr)- angle(Ir));  %(receiving end power factor)
Sr = 3*Vr*conj(Ir);                % MVA (receiving end power)
```

```
REG = (Vs3ph/abs(ABCD(1,1)) - Vr3ph)/Vr3ph *100;
fprintf(' Ir = %g A', Irm), fprintf('  pf = %g', pfr)
fprintf(' Vr = %g L-L kV', Vr3ph)
fprintf(' Pr = %g MW', real(Sr))
fprintf('  Qr = %g Mvar', imag(Sr))
fprintf(' Percent voltage Reg. = %g', REG)
```

result in

```
Enter 1 for Medium line or 2 for long line → 1
Nominal π model
Z = 4.68 + j 39 ohms
Y = 0 + j 0.0005486 siemens

ABCD = [ 0.9893       + j 0.0012837   4.68   + j 39
         -3.5213e-07  + j 0.00054565  0.9893 + j 0.0012837 ]

Ir = 441.832 A        pf = 0.88750
Vr = 330.68 L-L kV
Pr = 224.592 MW     Qr = 116.612 Mvar
Percent voltage Reg. = 5.45863
```

## 5.4 LONG LINE MODEL

For the short and medium length lines reasonably accurate models were obtained by assuming the line parameters to be lumped. For lines 250 km (150 miles) and longer and for a more accurate solution the exact effect of the distributed parameters must be considered. In this section expressions for voltage and current at any point on the line are derived. Then, based on these equations an equivalent $\pi$ model is obtained for the long line. Figure 5.5 shows one phase of a distributed line of length $\ell$ km.

The series impedance per unit length is shown by the lowercase letter $z$, and the shunt admittance per phase is shown by the lowercase letter $y$, where $z = r + j\omega L$ and $y = g + j\omega C$. Consider a small segment of line $\Delta x$ at a distance $x$ from the receiving end of the line. The phasor voltages and currents on both sides of this segment are shown as a function of distance. From Kirchhoff's voltage law

$$V(x+\Delta x) = V(x) + z\,\Delta x\, I(x) \tag{5.24}$$

or

$$\frac{V(x+\Delta x) - V(x)}{\Delta x} = z\, I(x) \tag{5.25}$$

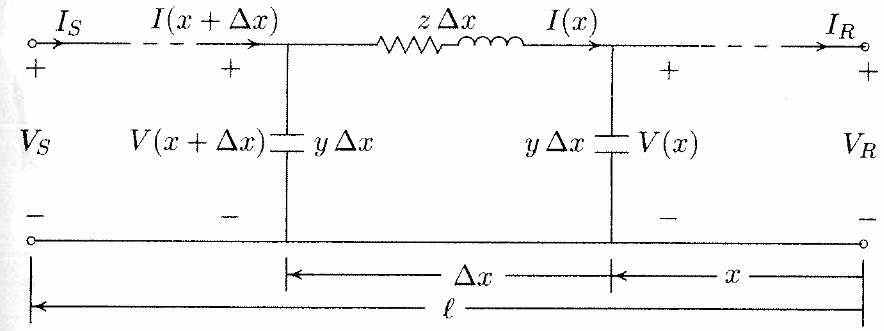



**FIGURE 5.5**
Long line with distributed parameters.

Taking the limit as $\Delta x \to 0$, we have

$$\frac{dV(x)}{dx} = z\, I(x) \tag{5.26}$$

Also, from Kirchhoff's current law

$$I(x+\Delta x) = I(x) + y\,\Delta x\, V(x+\Delta x) \tag{5.27}$$

or

$$\frac{I(x+\Delta x) - I(x)}{\Delta x} = y\, V(x+\Delta x) \tag{5.28}$$

Taking the limit as $\Delta x \to 0$, we have

$$\frac{dI(x)}{dx} = y\, V(x) \tag{5.29}$$

Differentiating (5.26) and substituting from (5.29), we get

$$\begin{aligned}\frac{d^2V(x)}{dx^2} &= z\,\frac{dI(x)}{dx}\\ &= zy\, V(x)\end{aligned} \tag{5.30}$$

Let

$$\gamma^2 = zy \tag{5.31}$$

The following second-order differential equation will result.

$$\frac{d^2V(x)}{dx^2} - \gamma^2\, V(x) = 0 \tag{5.32}$$

The solution of the above equation is

$$V(x) = A_1 e^{\gamma x} + A_2 e^{-\gamma x} \tag{5.33}$$

where $\gamma$, known as the *propagation constant*, is a complex expression given by (5.31) or

$$\gamma = \alpha + j\beta = \sqrt{zy} = \sqrt{(r + j\omega L)(g + j\omega C)} \tag{5.34}$$

The real part $\alpha$ is known as the *attenuation constant*, and the imaginary component $\beta$ is known as the *phase constant*. $\beta$ is measured in radian per unit length.

From (5.26), the current is

$$\begin{aligned} I(x) = \frac{1}{z}\frac{dV(x)}{dx} &= \frac{\gamma}{z}(A_1 e^{\gamma x} - A_2 e^{-\gamma x}) \\ &= \sqrt{\frac{y}{z}}(A_1 e^{\gamma x} - A_2 e^{-\gamma x}) \end{aligned} \tag{5.35}$$

or

$$I(x) = \frac{1}{Z_c}(A_1 e^{\gamma x} - A_2 e^{-\gamma x}) \tag{5.36}$$

where $Z_c$ is known as the *characteristic impedance*, given by

$$Z_c = \sqrt{\frac{z}{y}} \tag{5.37}$$

To find the constants $A_1$ and $A_2$ we note that when $x = 0$, $V(x) = V_R$, and $I(x) = I_R$. From (5.33) and (5.36) these constants are found to be

$$\begin{aligned} A_1 &= \frac{V_R + Z_c I_R}{2} \\ A_2 &= \frac{V_R - Z_c I_R}{2} \end{aligned} \tag{5.38}$$

Upon substitution in (5.33) and (5.36), the general expressions for voltage and current along a long transmission line become

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x} \tag{5.39}$$

$$I(x) = \frac{\frac{V_R}{Z_c} + I_R}{2} e^{\gamma x} - \frac{\frac{V_R}{Z_c} - I_R}{2} e^{-\gamma x} \tag{5.40}$$

The equations for voltage and currents can be rearranged as follows:

$$V(x) = \frac{e^{\gamma x} + e^{-\gamma x}}{2} V_R + Z_c \frac{e^{\gamma x} - e^{-\gamma x}}{2} I_R \tag{5.41}$$

$$I(x) = \frac{1}{Z_c}\frac{e^{\gamma x} - e^{-\gamma x}}{2} V_R + \frac{e^{\gamma x} + e^{-\gamma x}}{2} I_R \tag{5.42}$$

Recognizing the hyperbolic functions sinh, and cosh, the above equations are written as follows:

$$V(x) = \cosh \gamma x\, V_R + Z_c \sinh \gamma x\, I_R \tag{5.43}$$

$$I(x) = \frac{1}{Z_c} \sinh \gamma x\, V_R + \cosh \gamma x\, I_R \tag{5.44}$$

We are particularly interested in the relation between the sending end and the receiving end of the line. Setting $x = \ell$, $V(\ell) = V_s$ and $I(\ell) = I_s$, the result is

$$V_s = \cosh \gamma\ell\, V_R + Z_c \sinh \gamma\ell\, I_R \tag{5.45}$$

$$I_s = \frac{1}{Z_c} \sinh \gamma\ell\, V_R + \cosh \gamma\ell\, I_R \tag{5.46}$$

Rewriting the above equations in terms of the $ABCD$ constants as before, we have

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{5.47}$$

where

$$A = \cosh \gamma\ell \qquad B = Z_c \sinh \gamma\ell \tag{5.48}$$

$$C = \frac{1}{Z_c} \sinh \gamma\ell \qquad D = \cosh \gamma\ell \tag{5.49}$$

Note that, as before, $A = D$ and $AD - BC = 1$.

It is now possible to find an accurate equivalent $\pi$ model, shown in Figure 5.6, to replace the $ABCD$ constants of the two-port network. Similar to the expressions (5.17) and (5.19) obtained for the nominal $\pi$, for the equivalent $\pi$ model we have

$$V_S = \left(1 + \frac{Z'Y'}{2}\right) V_R + Z' I_R \tag{5.50}$$

$$I_S = Y'\left(1 + \frac{Z'Y'}{4}\right) V_R + \left(1 + \frac{Z'Y'}{2}\right) I_R \tag{5.51}$$

Comparing (5.50) and (5.51) with (5.45) and (5.46), respectively, and making use of the identity

$$\tanh \frac{\gamma\ell}{2} = \frac{\cosh \gamma\ell - 1}{\sinh \gamma\ell} \tag{5.52}$$

the parameters of the equivalent $\pi$ model are obtained.

$$Z' = Z_c \sinh \gamma\ell = Z\frac{\sinh \gamma\ell}{\gamma\ell} \tag{5.53}$$

$$\frac{Y'}{2} = \frac{1}{Z_c}\tanh\frac{\gamma\ell}{2} = \frac{Y}{2}\frac{\tanh \gamma\ell/2}{\gamma\ell/2} \tag{5.54}$$

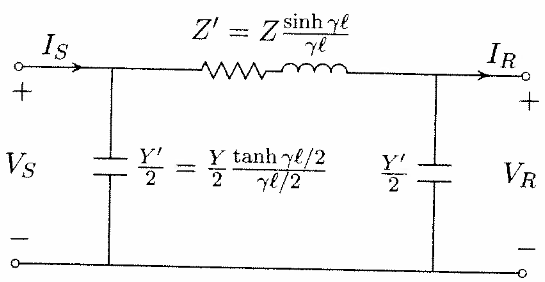



**FIGURE 5.6**
Equivalent $\pi$ model for long length line.

The functions **[Z, Y, ABCD ] = rlc2abcd(r, L, C, g, f, Length)** and **[Z, Y, ABCD ] = zy2abcd(z, y, Length)** with option 2 can be used for the evaluation of the transmission matrix and the equivalent $\pi$ parameters. However, Example 5.4 shows how these hyperbolic functions can be evaluated easily with simple *MATLAB* commands.

**Example 5.4** (chp5ex4)

A 500-kV, three-phase transmission line is 250 km long. The series impedance is $z = 0.045 + j0.4\ \Omega$ per phase per km and the shunt admittance is $y = j4 \times 10^{-6}$ siemens per phase per km. Evaluate the equivalent $\pi$ model and the transmission matrix

The following commands

```
z = 0.045 + j*.4;     y = j*4.0/1000000; Length = 250;
gamma = sqrt(z*y);  Zc = sqrt(z/y);
A = cosh(gamma*Length);  B = Zc*sinh(gamma*Length);
C = 1/Zc * sinh(gamma*Length);  D = A;
ABCD = [A  B; C  D]
Z = B; Y = 2/Zc * tanh(gamma*Length/2)
```

result in

```
ABCD =
       0.9504 + 0.0055i  10.8778 +98.3624i
      -0.0000 + 0.0010i   0.9504 + 0.0055i
Z =
      10.8778 +98.3624i
Y =
       0.0000 + 0.0010i
```

## 5.5 VOLTAGE AND CURRENT WAVES

The rms expression for the phasor value of voltage at any point along the line is given by (5.33). Substituting $\alpha + j\beta$ for $\gamma$, the phasor voltage is

$$V(x) = A_1 e^{\alpha x} e^{j\beta x} + A_2 e^{-\alpha x} e^{-j\beta x}$$

Transforming from phasor domain to time domain, the instantaneous voltage as a function of $t$ and $x$ becomes

$$v(t,x) = \sqrt{2}\,\Re\, A_1 e^{\alpha x} e^{j(\omega t + \beta x)} + \sqrt{2}\,\Re\, A_2 e^{-\alpha x} e^{j(\omega t - \beta x)} \tag{5.55}$$

As $x$ increases (moving away from the receiving end), the first term becomes larger because of $e^{\alpha x}$ and is called the *incident wave*. The second term becomes smaller because of $e^{-\alpha x}$ and is called the *reflected wave*. At any point along the line, voltage is the sum of these two components.

$$v(t,x) = v_1(t,x) + v_2(t,x) \tag{5.56}$$

where

$$v_1(t,x) = \sqrt{2}\,A_1 e^{\alpha x}\cos(\omega t + \beta x) \tag{5.57}$$

$$v_2(t,x) = \sqrt{2}\,A_2 e^{-\alpha x}\cos(\omega t - \beta x) \tag{5.58}$$

As the current expression is similar to the voltage, the current can also be considered as the sum of incident and reflected current waves.

Equations (5.57 ) and (5.58 ) behave like traveling waves as we move along the line. This is similar to the disturbance in the water at some sending point. To see this, consider the reflected wave $v_2(t,x)$ and imagine that we ride along with the wave. To observe the instantaneous value, for example the peak amplitude requires that

$$\omega t - \beta x = 2K\pi \quad \text{or} \quad x = \frac{\omega}{\beta}t - \frac{2K\pi}{\beta}$$

Thus, to keep up with the wave and observe the peak amplitude we must travel with the speed

$$\frac{dx}{dt} = \frac{\omega}{\beta} \tag{5.59}$$

Thus, the velocity of propagation is given by

$$v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} \tag{5.60}$$

The wavelength $\lambda$ or distance $x$ on the wave which results in a phase shift of $2\pi$ radian is

$$\beta\lambda = 2\pi$$

or

$$\lambda = \frac{2\pi}{\beta} \tag{5.61}$$

When line losses are neglected, i.e., when $g = 0$ and $r = 0$, the real part of the propagation constant $\alpha = 0$, and from (5.34) the phase constant becomes

$$\beta = \omega\sqrt{LC} \tag{5.62}$$

Also, the characteristic impedance is purely resistive and (5.37) becomes

$$Z_c = \sqrt{\frac{L}{C}} \tag{5.63}$$

which is commonly referred to as the *surge impedance*. Substituting for $\beta$ in (5.60) and (5.61), for a lossless line the velocity of propagation and the wavelength become

$$v = \frac{1}{\sqrt{LC}} \tag{5.64}$$

$$\lambda = \frac{1}{f\sqrt{LC}} \tag{5.65}$$

The expressions for the inductance per unit length $L$ and capacitance per unit length $C$ of a transmission line were derived in Chapter 4, given by (4.58) and (4.91). When the internal flux linkage of a conductor is neglected $GMR_L = GMR_C$, and upon substitution (5.64) and (5.65) become

$$v \simeq \frac{1}{\sqrt{\mu_0\varepsilon_0}} \tag{5.66}$$

$$\lambda \simeq \frac{1}{f\sqrt{\mu_0\varepsilon_0}} \tag{5.67}$$

Substituting for $\mu_0 = 4\pi \times 10^{-7}$ and $\varepsilon_0 = 8.85 \times 10^{-12}$, the velocity of the wave is obtained to be approximately $3 \times 10^8$ m/sec, i.e., the velocity of light. At 60 Hz, the wavelength is 5000 km. Similarly, substituting for $L$ and $C$ in (5.63), we have

$$Z_c \simeq \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\ln\frac{GMD}{GMR_c}$$

$$\simeq 60\ln\frac{GMD}{GMR_c} \tag{5.68}$$

For typical transmission lines the surge impedance varies from approximately $400\Omega$ for 69-kV lines down to around $250\ \Omega$ for double-circuit 765-kV transmission lines.

For a lossless line $\gamma = j\beta$ and the hyperbolic functions $\cosh\gamma x = \cosh j\beta x = \cos\beta x$ and $\sinh\gamma x = \sinh j\beta x = j\sin\beta x$, the equations for the rms voltage and current along the line, given by (5.43) and (5.44), become

$$V(x) = \cos\beta x\, V_R + jZ_c\sin\beta x\, I_R \tag{5.69}$$

$$I(x) = j\frac{1}{Z_c}\sin\beta x\, V_R + \cos\beta x\, I_R \tag{5.70}$$

At the sending end $x = \ell$

$$V_S = \cos\beta\ell\, V_R + jZ_c\sin\beta\ell\, I_R \tag{5.71}$$

$$I_S = j\frac{1}{Z_c}\sin\beta\ell\, V_R + \cos\beta\ell\, I_R \tag{5.72}$$

For hand calculation it is easier to use (5.71) and (5.72), and for more accurate calculations (5.47) through (5.49) can be used in *MATLAB*. The terminal conditions are readily obtained from the above equations. For example, for the open-circuited line $I_R = 0$, and from (5.71) the no-load receiving end voltage is

$$V_{R(nl)} = \frac{V_S}{\cos\beta\ell} \tag{5.73}$$

At no-load, the line current is entirely due to the line charging capacitive current and the receiving end voltage is higher than the sending end voltage. This is evident from (5.73), which shows that as the line length increases $\beta\ell$ increases and $\cos\beta\ell$ decreases, resulting in a higher no-load receiving end voltage.

For a solid short circuit at the receiving end, $V_R = 0$ and (5.71) and (5.72) reduce to

$$V_S = jZ_c\sin\beta\ell\, I_R \tag{5.74}$$

$$I_S = \cos\beta\ell\, I_R \tag{5.75}$$

The above equations can be used to find the short circuit currents at both ends of the line.

## 5.6 SURGE IMPEDANCE LOADING

When the line is loaded by being terminated with an impedance equal to its characteristic impedance, the receiving end current is

$$I_R = \frac{V_R}{Z_c} \tag{5.76}$$

For a lossless line $Z_c$ is purely resistive. The load corresponding to the surge impedance at rated voltage is known as the *surge impedance loading* ($SIL$), given by

$$SIL = 3V_R I_R^* = \frac{3|V_R|^2}{Z_c} \tag{5.77}$$

Since $V_R = V_{Lrated}/\sqrt{3}$, $SIL$ in MW becomes

$$SIL = \frac{(kV_{Lrated})^2}{Z_c} \quad \text{MW} \tag{5.78}$$

Substituting for $I_R$ in (5.69) and $V_R$ in (5.70) will result in

$$V(x) = (\cos\beta x + j\sin\beta x)V_R \quad \text{or} \quad V(x) = V_R\angle\beta x \tag{5.79}$$

$$I(x) = (\cos\beta x + j\sin\beta x)I_R \quad \text{or} \quad I(x) = I_R\angle\beta x \tag{5.80}$$

Equations (5.79) and (5.80) show that in a lossless line under surge impedance loading the voltage and current at any point along the line are constant in magnitude and are equal to their sending end values. Since $Z_c$ has no reactive component, there is no reactive power in the line, $Q_S = Q_R = 0$. This indicates that for $SIL$, the reactive losses in the line inductance are exactly offset by reactive power supplied by the shunt capacitance or $\omega L|I_R|^2 = \omega C|V_R|^2$. From this relation, we find that $Z_c = V_R/I_R = \sqrt{L/C}$, which verifies the result in (5.63). $SIL$ for typical transmission lines varies from approximately 150 MW for 230-kV lines to about 2000 MW for 765-kV lines. $SIL$ is a useful measure of transmission line capacity as it indicates a loading where the line's reactive requirements are small. For loads significantly above $SIL$, shunt capacitors may be needed to minimize voltage drop along the line, while for light loads significantly below $SIL$, shunt inductors may be needed. Generally the transmission line full-load is much higher than $SIL$. The voltage profile for various loading conditions is illustrated in Figure 5.11 (page 182) in Example 5.9(h).

**Example 5.5** (chp5ex5)

A three-phase, 60-Hz, 500-kV transmission line is 300 km long. The line inductance is 0.97 mH/km per phase and its capacitance is 0.0115 $\mu$F/km per phase. Assume a lossless line.

(a) Determine the line phase constant $\beta$, the surge impedance $Z_C$, velocity of propagation $v$ and the line wavelength $\lambda$.
(b) The receiving end rated load is 800 MW, 0.8 power factor lagging at 500 kV. Determine the sending end quantities and the voltage regulation.

(a) For a lossless line, from (5.62) we have

$$\beta = \omega\sqrt{LC} = 2\pi \times 60\sqrt{0.97 \times 0.0115 \times 10^{-9}} = 0.001259 \ \ \text{rad/km}$$

and from (5.63)

$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.97 \times 10^{-3}}{0.0115 \times 10^{-6}}} = 290.43 \ \ \Omega$$

Velocity of propagation is

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.97 \times 0.0115 \times 10^{-9}}} = 2.994 \times 10^5 \ \ \text{km/s}$$

and the line wavelength is

$$\lambda = \frac{v}{f} = \frac{1}{60}(2.994 \times 10^5) = 4990 \ \ \text{km}$$

(b) $\beta\ell = 0.001259 \times 300 = 0.3777 \ \ \text{rad} = 21.641^\circ$

The receiving end voltage per phase is

$$V_R = \frac{500\angle 0^\circ}{\sqrt{3}} = 288.675\angle 0^\circ \ \ \text{kV}$$

The receiving end apparent power is

$$S_{R(3\phi)} = \frac{800}{0.8}\angle\cos^{-1}0.8 = 1000\angle 36.87^\circ = 800 + j600 \ \ \text{MVA}$$

The receiving end current per phase is given by

$$I_R = \frac{S^*_{R(3\phi)}}{3\,V_R^*} = \frac{1000\angle -36.87^\circ \times 10^3}{3 \times 288.675\angle 0^\circ} = 1154.7\angle -36.87^\circ \ \ \text{A}$$

From (5.71) the sending end voltage is

$$\begin{aligned}V_S &= \cos\beta\ell\, V_R + jZ_c \sin\beta\ell\, I_R\\ &= (0.9295)288.675\angle 0^\circ + j(290.43)(0.3688)(1154.7\angle -36.87^\circ)(10^{-3})\\ &= 356.53\angle 16.1^\circ \text{ kV}\end{aligned}$$

The sending end line-to-line voltage magnitude is

$$|V_{S(L-L)}| = \sqrt{3}\,|V_S| = 617.53 \text{ kV}$$

From (5.72) the sending end current is

$$\begin{aligned}I_S &= j\frac{1}{Z_c}\sin\beta\ell\, V_R + \cos\beta\ell\, I_R\\ &= j\frac{1}{290.43}(0.3688)(288.675\angle 0^\circ)(10^3) + (0.9295)(1154.7\angle -36.87^\circ)\\ &= 902.3\angle -17.9^\circ \text{ A}\end{aligned}$$

The sending end power is

$$\begin{aligned}S_{S(3\phi)} = 3V_S I_S^* &= 3 \times 356.53\angle 16.1 \times 902.3\angle{-17.9^\circ} \times 10^{-3}\\ &= 800 \text{ MW} + j539.672 \text{ Mvar}\\ &= 965.1\angle 34^\circ \text{ MVA}\end{aligned}$$

Voltage regulation is

$$\text{Percent } VR = \frac{356.53/0.9295 - 288.675}{288.675} \times 100 = 32.87\%$$

The line performance of the above transmission line including the line resistance is obtained in Example 5.9 using the **lineperf** program. When a line is operating at the rated load, the exact solution results in $V_{S(L-L)} = 623.5\angle 15.57^\circ$ kV, and $I_s = 903.1\angle{-17.7^\circ}$ A. This shows that the lossless assumption yields acceptable results and is suitable for hand calculation.

## 5.7 COMPLEX POWER FLOW THROUGH TRANSMISSION LINES

Specific expressions for the complex power flow on a line may be obtained in terms of the sending end and receiving end voltage magnitudes and phase angles and the $ABCD$ constants. Consider Figure 5.2 where the terminal relations are given by (5.5) and (5.6). Expressing the $ABCD$ constants in polar form as $A = |A|\angle\theta_A$, $B = |B|\angle\theta_B$, the sending end voltage as $V_S = |V_S|\angle\delta$, and the receiving end voltage as reference $V_R = |V_R|\angle 0$, from (5.5) $I_R$ can be written as

$$\begin{aligned}I_R &= \frac{|V_S|\angle\delta - |A|\angle\theta_A|V_R|\angle 0}{|B|\angle\theta_B}\\ &= \frac{|V_S|}{|B|}\angle\delta - \theta_B - \frac{|A||V_R|}{|B|}\angle\theta_A - \theta_B\end{aligned} \qquad (5.81)$$

The receiving end complex power is

$$S_{R(3\phi)} = P_{R(3\phi)} + jQ_{R(3\phi)} = 3V_R I_R^* \qquad (5.82)$$

Substituting for $I_R$ from (5.81), we have

$$S_{R(3\phi)} = 3\frac{|V_S||V_R|}{|B|}\angle\theta_B - \delta - 3\frac{|A||V_R|^2}{|B|}\angle\theta_B - \theta_A \qquad (5.83)$$

or in terms of the line-to-line voltages, we have

$$S_{R(3\phi)} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\angle\theta_B - \delta - \frac{|A||V_{R(L-L)}|^2}{|B|}\angle\theta_B - \theta_A \qquad (5.84)$$

The real and reactive power at the receiving end of the line are

$$P_{R(3\phi)} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\cos(\theta_B - \delta) - \frac{|A||V_{R(L-L)}|^2}{|B|}\cos(\theta_B - \theta_A) \qquad (5.85)$$

$$Q_{R(3\phi)} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\sin(\theta_B - \delta) - \frac{|A||V_{R(L-L)}|^2}{|B|}\sin(\theta_B - \theta_A) \qquad (5.86)$$

The sending end power is

$$S_{S(3\phi)} = P_{S(3\phi)} + jQ_{S(3\phi)} = 3V_S I_S^* \qquad (5.87)$$

From (5.23), $I_S$ can be written as

$$I_S = \frac{|A|\angle\theta_A|V_S|\angle\delta - |V_R|\angle 0}{|B|\angle\theta_B} \qquad (5.88)$$

Substituting for $I_S$ in (5.87) yields

$$P_{S(3\phi)} = \frac{|A||V_{S(L-L)}|^2}{|B|}\cos(\theta_B - \theta_A) - \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\cos(\theta_B + \delta) \qquad (5.89)$$

$$Q_{S(3\phi)} = \frac{|A||V_{S(L-L)}|^2}{|B|}\sin(\theta_B - \theta_A) - \frac{|V_{S(L-L)}||V_{R(L-L)}|}{|B|}\sin(\theta_B + \delta) \qquad (5.90)$$

The real and reactive transmission line losses are

$$P_{L(3\phi)} = P_{S(3\phi)} - P_{R(3\phi)} \tag{5.91}$$

$$Q_{L(3\phi)} = Q_{S(3\phi)} - Q_{R(3\phi)} \tag{5.92}$$

The locus of all points obtained by plotting $Q_{R(3\phi)}$ versus $P_{R(3\phi)}$ for fixed line voltages and varying load angle $\delta$ is a circle known as the *receiving end power circle diagram*. A family of such circles with fixed receiving end voltage and varying sending end voltage is extremely useful in assessing the performance characteristics of the transmission line. A function called **pwrcirc(ABCD)** is developed for the construction of the receiving end power circle diagram, and its use is demonstrated in Example 5.9(g).

For a lossless line $B = jX'$, $\theta_A = 0$, $\theta_B = 90°$, and $A = \cos\beta\ell$, and the real power transferred over the line is given by

$$P_{3\phi} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'}\sin\delta \tag{5.93}$$

and the receiving end reactive power is

$$Q_{R3\phi} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X'}\cos\delta - \frac{|V_{R(L-L)}|^2}{X'}\cos\beta\ell \tag{5.94}$$

For a given system operating at constant voltage, the power transferred is proportional to the sine of the power angle $\delta$. As the load increases, $\delta$ increases. For a lossless line, the maximum power that can be transmitted under stable steady-state condition occurs for an angle of 90°. However, a transmission system with its connected synchronous machines must also be able to withstand, without loss of stability, sudden changes in generation, load, and faults. To assure an adequate margin of stability, the practical operating load angle is usually limited to 35 to 45°.

## 5.8 POWER TRANSMISSION CAPABILITY

The power handling ability of a line is limited by the thermal loading limit and the stability limit. The increase in the conductor temperature, due to the real power loss, stretches the conductors. This will increase the sag between transmission towers. At higher temperatures this may result in irreversible stretching. The thermal limit is specified by the current-carrying capacity of the conductor and is available in the manufacturer's data. If the current-carrying capacity is denoted by $I_{thermal}$, the thermal loading limit of a line is

$$S_{thermal} = 3V_{\phi rated}I_{thermal} \tag{5.95}$$



The expression for real power transfer over the line for a lossless line is given by (5.93). The theoretical maximum power transfer is when $\delta = 90°$. The practical operating load angle for the line alone is limited to no more than 30 to 45°. This is because of the generator and transformer reactances which, when added to the line, will result in a larger $\delta$ for a given load. For planning and other purposes, it is very useful to express the power transfer formula in terms of $SIL$, and construct the line loadability curve. For a lossless line $X' = Z_c\sin\beta\ell$, and (5.93) may be written as

$$P_{3\phi} = \left(\frac{|V_{S(L-L)}|}{V_{rated}}\right)\left(\frac{|V_{R(L-L)}|}{V_{rated}}\right)\left(\frac{V_{rated}^2}{Z_c}\right)\frac{\sin\delta}{\sin\beta\ell} \tag{5.96}$$

The first two terms within parenthesis are the per-unit voltages denoted by $V_{Spu}$ and $V_{Rpu}$, and the third term is recognized as $SIL$. Equation (5.96) may be written as

$$P_{3\phi} = \frac{|V_{Spu}||V_{Rpu}|SIL}{\sin\beta\ell}\sin\delta$$

$$= \frac{|V_{Spu}||V_{Rpu}|SIL}{\sin(\frac{2\pi\ell}{\lambda})}\sin\delta \tag{5.97}$$

The function **loadabil(L, C, f)** obtains the loadability curve and thermal limit curve of the line. The loadability curve as obtained in Figure 5.12 (page 182) for Example 5.9(i) shows that for short and medium lines the thermal limit dictates the maximum power transfer. Whereas, for longer lines the limit is set by the practical line loadability curve. As we see in the next section, for longer lines it may be necessary to use series capacitors in order to increase the power transfer over the line.

**Example 5.6** (chp5ex6)

A three-phase power of 700-MW is to be transmitted to a substation located 315 km from the source of power. For a preliminary line design assume the following parameters:

$V_S = 1.0$ per unit, $V_R = 0.9$ per unit, $\lambda = 5000$ km, $Z_c = 320\ \Omega$, and $\delta = 36.87°$

(a) Based on the practical line loadability equation determine a nominal voltage level for the transmission line.

(b) For the transmission voltage level obtained in (a) calculate the theoretical maximum power that can be transferred by the transmission line.

(a) From (5.61), the line phase constant is

$$\beta\ell = \frac{2\pi}{\lambda}\ell \text{ rad}$$

$$= \frac{360}{\lambda}\ell = \frac{360}{5000}(315) = 22.68°$$

From the practical line loadability given by (5.97), we have

$$700 = \frac{(1.0)(0.9)(SIL)}{\sin(22.68^\circ)} \sin(36.87^\circ)$$

Thus

$$SIL = 499.83 \text{ MW}$$

From (5.78)

$$kV_L = \sqrt{(Z_c)(SIL)} = \sqrt{(320)(499.83)} = 400 \text{ kV}$$

(b) The equivalent line reactance for a lossless line is given by

$$X' = Z_c \sin\beta\ell = 320\sin(22.68) = 123.39 \ \Omega$$

For a lossless line, the maximum power that can be transmitted under steady state condition occurs for a load angle of 90°. Thus, from (5.93), assuming $|V_S| = 1.0$ pu and $|V_R| = 0.9$ pu, the theoretical maximum power is

$$P_{3\phi(max)} = \frac{(400)(0.9)(400)}{123.39}(1) = 1167 \text{ MW}$$

## 5.9 LINE COMPENSATION

We have noted that a transmission line loaded to its surge impedance loading has no net reactive power flow into or out of the line and will have approximately a flat voltage profile along its length. On long transmission lines, light loads appreciably less than $SIL$ result in a rise of voltage at the receiving end, and heavy loads appreciably greater than $SIL$ will produce a large dip in voltage. The voltage profile of a long line for various loading conditions is shown in Figure 5.11 (page 182). Shunt reactors are widely used to reduce high voltages under light load or open line conditions. If the transmission system is heavily loaded, shunt capacitors, static var control, and synchronous condensers are used to improve voltage, increase power transfer, and improve the system stability.

### 5.9.1 SHUNT REACTORS

Shunt reactors are applied to compensate for the undesirable voltage effects associated with line capacitance. The amount of reactor compensation required on a transmission line to maintain the receiving end voltage at a specified value can be obtained as follows.



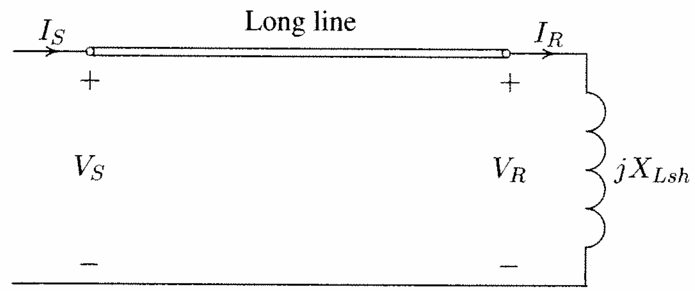



**FIGURE 5.7**
Shunt reactor compensation.

Consider a reactor of reactance $X_{Lsh}$, connected at the receiving end of a long transmission line as shown in Figure 5.7. The receiving end current is

$$I_R = \frac{V_R}{jX_{Lsh}} \tag{5.98}$$

Substituting $I_R$ into (5.71) results in

$$V_S = V_R(\cos\beta\ell + \frac{Z_c}{X_{Lsh}}\sin\beta\ell)$$

Note that $V_S$ and $V_R$ are in phase, which is consistent with the fact that no real power is being transmitted over the line. Solving for $X_{Lsh}$ yields

$$X_{Lsh} = \frac{\sin\beta\ell}{\frac{V_S}{V_R} - \cos\beta\ell}Z_c \tag{5.99}$$

For $V_S = V_R$, the required inductor reactance is

$$X_{Lsh} = \frac{\sin\beta\ell}{1 - \cos\beta\ell}Z_c \tag{5.100}$$

To find the relation between $I_S$ and $I_R$, we substitute for $V_R$ from (5.98) into (5.72)

$$I_S = \left(-\frac{1}{Z_c}\sin\beta\ell\, X_{Lsh} + \cos\beta\ell\right) I_R$$

Substituting for $X_{Lsh}$ from (5.100) for the case when $V_S = V_R$ results in

$$I_S = -I_R \tag{5.101}$$

With one reactor only at the receiving end, the voltage profile will not be uniform, and the maximum rise occurs at the midspan. It is left as an exercise to show that for $V_S = V_R$, the voltage at the midspan is given by

$$V_m = \frac{V_R}{\cos\frac{\beta\ell}{2}} \tag{5.102}$$

Also, the current at the midspan is zero. The function **openline(ABCD)** is used to find the receiving end voltage of an open line and to determine the Mvar of the reactor required to maintain the no-load receiving end voltage at a specified value. Example 5.9(d) illustrates the reactor compensation. Installing reactors at both ends of the line will improve the voltage profile and reduce the tension at midspan.

**Example 5.7** (chp5ex7)

For the transmission line of Example 5.5:
(a) Calculate the receiving end voltage when line is terminated in an open circuit and is energized with 500 kV at the sending end.
(b) Determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end to keep the no-load receiving end voltage at the rated value.

(a) The line is energized with 500 kV at the sending end. The sending end voltage per phase is

$$V_S = \frac{500\angle 0^\circ}{\sqrt{3}} = 288.675 \text{ kV}$$

From Example 5.5, $Z_c = 290.43$ and $\beta\ell = 21.641°$.

When the line is open $I_R = 0$ and from (5.71) the no-load receiving end voltage is given by

$$V_{R(nl)} = \frac{V_S}{\cos\beta\ell} = \frac{288.675}{0.9295} = 310.57 \text{ kV}$$

The no-load receiving end line-to-line voltage is

$$V_{R(L-L)(nl)} = \sqrt{3}\,V_{R(nl)} = 537.9 \text{ kV}$$

(b) For $V_S = V_R$, the required inductor reactance given by (5.100) is

$$X_{Lsh} = \frac{\sin(21.641°)}{1 - \cos(21.641°)}(290.43) = 1519.5 \ \Omega$$

The three-phase shunt reactor rating is

$$Q_{3\phi} = \frac{(kV_{Lrated})^2}{X_{Lsh}} = \frac{(500)^2}{1519.5} = 164.53 \text{ Mvar}$$



## 5.9.2 SHUNT CAPACITOR COMPENSATION

Shunt capacitors are used for lagging power factor circuits created by heavy loads. The effect is to supply the requisite reactive power to maintain the receiving end voltage at a satisfactory level. Capacitors are connected either directly to a bus bar or to the tertiary winding of a main transformer and are disposed along the route to minimize the losses and voltage drops. Given $V_S$ and $V_R$, (5.85) and (5.86) can be used conveniently to compute the required capacitor Mvar at the receiving end for a specified load. A function called **shntcomp(ABCD)** is developed for this purpose, and its use is demonstrated in Example 5.9(f).

## 5.9.3 SERIES CAPACITOR COMPENSATION

Series capacitors are connected in series with the line, usually located at the midpoint, and are used to reduce the series reactance between the load and the supply point. This results in improved transient and steady-state stability, more economical loading, and minimum voltage dip on load buses. Series capacitors have the good characteristics that their reactive power production varies concurrently with the line loading. Studies have shown that the addition of series capacitors on EHV transmission lines can more than double the transient stability load limit of long lines at a fraction of the cost of a new transmission line.

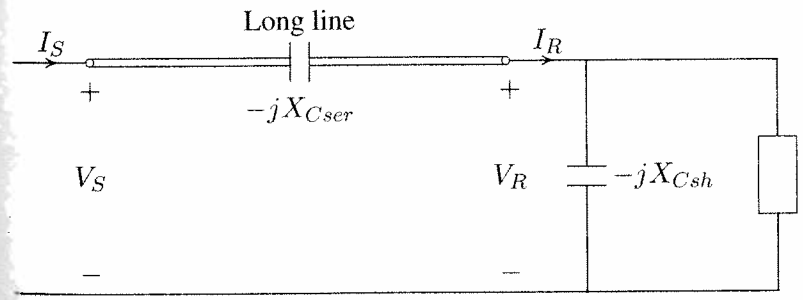



**FIGURE 5.8**
Shunt and series capacitor compensation.

With the series capacitor switched on as shown in Figure 5.8, from (5.93), the power transfer over the line for a lossless line becomes

$$P_{3\phi} = \frac{|V_{S(L-L)}||V_{R(L-L)}|}{X' - X_{Cser}} \sin\delta \tag{5.103}$$

Where $X_{Cser}$ is the series capacitor reactance. The ratio $X_{Cser}/X'$ expressed as a percentage is usually referred to as the *percentage compensation*. The percentage compensation is in the range of 25 to 70 percent.

One major drawback with series capacitor compensation is that special protective devices are required to protect the capacitors and bypass the high current produced when a short circuit occurs. Also, inclusion of series capacitors establishes a resonant circuit that can oscillate at a frequency below the normal synchronous frequency when stimulated by a disturbance. This phenomenon is referred to as *subsynchronous resonance* (SSR). If the synchronous frequency minus the electrical resonant frequency approaches the frequency of one of the turbine-generator natural torsional modes, considerable damage to the turbine-generator may result. If $L'$ is the lumped line inductance corrected for the effect of distribution and $C_{ser}$ is the capacitance of the series capacitor, the subsynchronous resonant frequency is

$$f_r = f_s\sqrt{\frac{1}{L'C_{ser}}} \tag{5.104}$$

where $f_s$ is the synchronous frequency. The function **sercomp(ABCD)** can be used to obtain the line performance for a specified percentage compensation. Finally, when line is compensated with both series and shunt capacitors, for the specified terminal voltages, the function **srshcomp(ABCD)** is used to obtain the line performance and the required shunt capacitor. These compensations are also demonstrated in Example 5.9(f).

**Example 5.8** (chp5ex8)

The transmission line in Example 5.5 supplies a load of 1000 MVA, 0.8 power factor lagging at 500 kV.
(a) Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 500 kV when the line is energized with 500 kV at the sending end.
(b) Only series capacitors are installed at the midpoint of the line providing 40 percent compensation. Find the sending end voltage and voltage regulation.

(a) From Example 5.5, $Z_c = 290.43$ and $\beta\ell = 21.641°$. Thus, the equivalent line reactance for a lossless line is given by

$$X' = Z_c \sin\beta\ell = 290.43\sin(21.641°) = 107.11 \ \Omega$$

The receiving end power is

$$S_{R(3\phi)} = 1000\angle\cos^{-1}(0.8) = 800 + j600 \ \text{MVA}$$

For the above operating condition, the power angle $\delta$ is obtained from (5.93)

$$800 = \frac{(500)(500)}{107.11}\sin\delta$$

which results in $\delta = 20.044°$. Using the approximate relation given by (5.94), the net reactive power at the receiving end is

$$Q_{R(3\phi)} = \frac{(500)(500)}{107.11}\cos(20.044°) - \frac{(500)^2}{107.11}\cos(21.641°) = 23.15 \ \text{Mvar}$$

Thus, the required capacitor Mvar is $S_C = j23.15 - j600 = -j576.85$

The capacitive reactance is given by

$$X_C = \frac{|V_L|^2}{S_C^*} = \frac{(500)^2}{j576.85} = -j433.38 \ \Omega$$

or

$$C = \frac{10^6}{2\pi(60)(433.38)} = 6.1 \ \mu\text{F}$$

The shunt compensation for the above transmission line including the line resistance is obtained in Example 5.9(f) using the **lineperf** program. The exact solution results in 613.8 Mvar for capacitor reactive power as compared to 576.85 Mvar obtained from the approximate formula for the lossless line. This represents approximately an error of 6 percent.

(b) For 40 percent compensation, the series capacitor reactance per phase is

$$X_{ser} = 0.4X' = 0.4(107.1) = 42.84 \ \Omega$$

The new equivalent $\pi$ circuit parameters are given by

$$Z' = j(X' - X_{ser}) = j(107.1 - 42.84) = j64.26 \ \Omega$$
$$Y' = = j\frac{2}{Z_c}\tan(\beta\ell/2) = j\frac{2}{290.43}\tan(21.641°/2) = j0.001316 \ \text{siemens}$$

The new $B$ constant is $B = j64.26$ and the new $A$ constant is given by

$$A = 1 + \frac{Z'Y'}{2} = 1 + \frac{(j64.26)(j0.001316)}{2} = 0.9577$$

The receiving end voltage per phase is

$$V_R = \frac{500}{\sqrt{3}} = 288.675 \ \text{kV}$$

and the receiving end current is

$$I_R = \frac{S^*_{R(3\phi)}}{3V_R^*} = \frac{1000\angle{-36.87°}}{3\times 288.675\angle 0°} = 1.1547\angle{-36.87°} \ \text{kA}$$

Thus, the sending end voltage is

$$\begin{aligned} V_S = AV_R + BI_R &= 0.9577 \times 288.675 + j64.26 \times 1.1547\angle{-36.87^\circ} \\ &= 326.4\angle 10.47^\circ \text{ kV} \end{aligned}$$

and the line-to-line voltage magnitude is $|V_{S(L-L)}| = \sqrt{3}\,V_S = 565.4$ kV. Voltage regulation is

$$\text{Percent } VR = \frac{565.4/0.958 - 500}{500} \times 100 = 18\%$$

The exact solution obtained in Example 5.9(f) results in $V_{S(L-L)} = 571.9$ kV. This represents an error of 1.0 percent.

## 5.10 LINE PERFORMANCE PROGRAM

A program called **lineperf** is developed for the complete analysis and compensation of a transmission line. The command **lineperf** displays a menu with five options for the computation of the parameters of the $\pi$ models and the transmission constants. Selection of these options will call upon the following functions.

**[Z, Y, ABCD] = rlc2abcd(r, L, C, g, f, Length)** computes and returns the $\pi$ model parameters and the transmission constants when $r$ in ohm, $L$ in mH, and $C$ in $\mu$F per unit length, frequency, and line length are specified.

**[Z, Y, ABCD] = zy2abcd(z, y, Length)** computes and returns the $\pi$ model parameters and the transmission constants when impedance and admittance per unit length are specified.

**[Z, Y, ABCD] = pi2abcd(Z, Y)** returns the ABCD constants when the $\pi$ model parameters are specified.

**[Z, Y, ABCD] = abcd2pi(A, B, C)** returns the $\pi$ model parameters when the transmission constants are specified.

**[L , C] = gmd2lc** computes and returns the inductance and capacitance per phase when the line configuration and conductor dimensions are specified.

**[r, L, C, f] = abcd2rlc(ABCD)** returns the line parameters per unit length and frequency when the transmission constants are specified.



Any of the above functions can be used independently when the arguments of the functions are defined in the *MATLAB* environment. If the above functions are typed without the parenthesis and the arguments, the user will be prompted to enter the required data. Next the **lineperf** loads the program **listmenu** which displays a list of eight options for transmission line analysis and compensation. Selection of these options will call upon the following functions.

**givensr(ABCD)** prompts the user to enter $V_R$, $P_R$ and $Q_R$. This function computes $V_S$, $P_S$, $Q_S$, line losses, voltage regulation, and transmission efficiency.

**givenss(ABCD)** prompts the user to enter $V_S$, $P_S$ and $Q_S$. This function computes $V_R$, $P_R$, $Q_R$, line losses, voltage regulation, and transmission efficiency.

**givenzl(ABCD)** prompts the user to enter $V_R$ and the load impedance. This function computes $V_S$, $P_S$, $Q_S$, line losses, voltage regulation, and transmission efficiency.

**openline(ABCD)** prompts the user to enter $V_S$. This function computes $V_R$ for the open-ended line. Also, the reactance and the Mvar of the necessary reactor to maintain the receiving end voltage at a specified value are obtained. In addition, the function plots the voltage profile of the line.

**shcktlin(ABCD)** prompts the user to enter $V_S$. This function computes the current at both ends of the line for a solid short circuit at the receiving end.

Option 6 is for capacitive compensation and calls upon **compmenu** which displays three options. Selection of these options will call upon the following functions.

**shntcomp(ABCD)** prompts the user to enter $V_S$, $P_R$, $Q_R$ and the desired $V_R$. This function computes the capacitance and the Mvar of the shunt capacitor bank to be installed at the receiving end in order to maintain the specified $V_R$. Then, $V_S$, $P_S$, $Q_S$, line losses, voltage regulation, and transmission efficiency are found.

**sercomp(ABCD)** prompts the user to enter $V_R$, $P_R$, $Q_R$, power, and the percentage compensation (i.e., $X_{Cser}/X_{line} \times 100$ ). This function computes the Mvar of the specified series capacitor and $V_S$, $P_S$, $Q_S$, line losses, voltage regulation, and transmission efficiency for the compensated line.

**srshcomp(ABCD)** prompts the user to enter $V_S$, $P_R$, $Q_R$, the desired $V_R$ and the percentage series capacitor compensation. This function computes the capacitance and the Mvar of a shunt capacitor to be installed at the receiving end in order to maintain the specified $V_R$. Also, $V_S$, $P_S$, $Q_S$, line losses, voltage regulation, and transmission efficiency are obtained for the compensated line.

Option 7 loads the **pwrcirc(ABCD)** which prompts for the receiving end voltage. This function constructs the receiving end power circle diagram for various values of $V_S$ from $V_R$ up to $1.3V_R$.

Option 8 calls upon **profmenu** which displays two options. Selection of these options will call upon the following functions:

**vprofile(r, L, C, f)** prompts the user to enter $V_S$, rated MVA, power factor, $V_R$, $P_R$, and $Q_R$. This function displays a graph consisting of voltage profiles for line length up to $1/8$ of the line wavelength for the following cases: open-ended line, line terminated in $SIL$, short-circuited line, and full-load.

**loadabil(L, C, f)** prompts the user for $V_S$, $V_R$, rated line voltage, and current-carrying capacity of the line. This function displays a graph consisting of the practical line loadability curve for $\delta = 30°$, the theoretical stability limit curve, and the thermal limit. This function assumes a lossless line and the plots are obtained for a line length up to $1/4$ of the line wavelength.

Any of the above functions can be used independently when the arguments of the functions are defined in the *MATLAB* environment. The $ABCD$ constant is entered as a matrix. If the above functions are typed without the parenthesis and the arguments, the user will be prompted to enter the required data.

A new GUI program named **lineperfgui** is developed for the analysis and compensation of the transmission line. This interactive easy-to-use GUI program provides maximum flexibility for entering different types of data. It includes eight options for transmission line performance and compensation.

**Example 5.9 (Run lineperfgui)**

A three-phase, 60-Hz, 550-kV transmission line is 300 km long. The line parameters per phase per unit length are found to be

$$r = 0.016\ \Omega/\text{km} \quad L = 0.97\ \text{mH/km} \quad C = 0.0115\ \mu\text{F/km}$$

(a) Determine the line performance when load at the receiving end is 800 MW, 0.8 power factor lagging at 500 kV.

The command:

```
lineperf
```

displays the following menu

```
Type of parameters for input                          Select
Parameters per unit length
r (Ω), g (siemens), L (mH), C (μF)                       1

Complex z and y per unit length
r + j*x (Ω), g + j*b (siemens)                           2

Nominal π or Eq. π model                                 3

A, B, C, D constants                                     4

Conductor configuration and dimension                    5

To quit                                                  0
Select number of menu → 1
Enter line length = 300
Enter frequency in Hz = 60
Enter line resistance/phase in Ω/unit length, r = 0.016
Enter line inductance in mH per unit length, L = 0.97
Enter line capacitance in μF per unit length, C = .0115
Enter line conductance in siemens per unit length, g = 0
Enter 1 for medium line or 2 for long line → 2

Equivalent π model
Z' = 4.57414 + j 107.119 ohms
Y' = 6.9638e-07 + j 0.00131631 siemens
Zc = 290.496 + j -6.35214 ohms
αℓ = 0.00826172 neper βℓ = 0.377825 radian = 21.6478°
```

$$\text{ABCD} = \begin{bmatrix} 0.9295 & + & j0.0030478 & 4.5741 & + & j107.12 \\ -1.3341e-06 & + & j0.0012699 & 0.9295 & + & j0.0030478 \end{bmatrix}$$

At this point the program **listmenu** is automatically loaded and displays the following menu.

```
      Transmission line performance
              Analysis                              Select

To calculate sending end quantities
for specified receiving end MW, Mvar                  1
```

```
To calculate receiving end quantities
for specified sending end MW, Mvar                    2

To calculate sending end quantities
when load impedance is specified                      3

Open-end line and reactive compensation               4

Short-circuited line                                  5

Capacitive compensation                               6

Receiving end circle diagram                          7

Loadability curve and voltage profile                 8


To quit                                               0

Select number of menu → 1

Enter receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 800
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 600

Line performance for specified receiving end quantities
--------------------------------------------------------

Vr = 500 kV (L-L) at 0°
Pr = 800 MW Qr = 600 Mvar
Ir = 1154.7 A at -36.8699° PFr = 0.8 lagging
Vs = 623.511 kV (L-L) at 15.5762°
Is = 903.113 A at -17.6996°, PFs = 0.836039 lagging
Ps = 815.404 MW,     Qs = 535.129 Mvar
PL = 15.4040 MW,     QL = -64.871 Mvar
Percent Voltage Regulation = 34.1597
Transmission line efficiency = 98.1108
```

At the end of this analysis the **listmenu** (Analysis Menu) is displayed.

(b) Determine the receiving end quantities and the line performance when 600 MW and 400 Mvar are being transmitted at 525 kV from the sending end.

Selecting option 2 of the **listmenu** results in

```
Enter sending end line-line voltage kV = 525
Enter sending end voltage phase angle° = 0
Enter sending end 3-phase power MW = 600
Enter sending end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 400

Line performance for specified sending end quantities
------------------------------------------------------

Vs = 525 kV (L-L) at 0°
Ps = 600 MW,     Qs = 400 Mvar
Is = 793.016 A at -33.6901°, PFs = 0.83205 lagging
Vr = 417.954 kV (L-L) at -16.3044°
Ir = 1002.6 A at -52.16° PFr = 0.810496 lagging
Pr = 588.261 MW,     Qr = 425.136 Mvar
PL = 11.7390 MW,     QL = -25.136 Mvar
Percent Voltage Regulation = 35.1383
Transmission line efficiency = 98.0435
```

(c) Determine the sending end quantities and the line performance when the receiving end load impedance is 290 Ω at 500 kV.

Selecting option 3 of the listmenu results in

```
Enter receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end complex load impedance 290 + j * 0

Line performance for specified load impedance
----------------------------------------------

Vr = 500 kV (L-L) at 0°
Ir = 995.431 A at 0° PFr = 1
Pr = 862.069 MW,     Qr = 0 Mvar
Vs = 507.996 kV (L-L) at 21.5037°
Is = 995.995 A at 21.7842°, PFs = 0.999988 leading
Ps = 876.341 MW Qs = -4.290 Mvar
PL = 14.272 MW QL = -4.290 Mvar
Percent Voltage Regulation = 9.30464
Transmission line efficiency = 98.3714
```

(d) Find the receiving end voltage when the line is terminated in an open circuit and is energized with 500 kV at the sending end. Also, determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 500 kV.

Selecting option 4 of the **listmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter sending end voltage phase angle° = 0

Open line and shunt reactor compensation

Vs = 500 kV (L-L) at 0°
Vr = 537.92 kV (L-L) at -0.00327893°
Is = 394.394 A at 89.8723°, PFs = 0.0022284 leading
Desired no load receiving end voltage = 500 kV
Shunt reactor reactance = 1519.4 Ω
Shunt reactor rating = 164.538 Mvar
```

The voltage profile for the uncompensated and the compensated line is also found as shown in Figure 5.9.

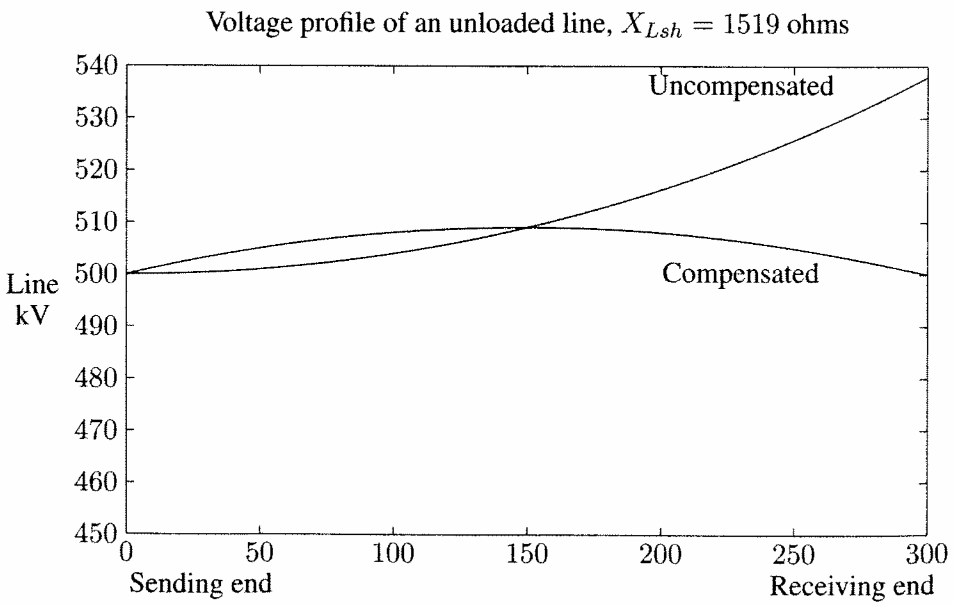



**FIGURE 5.9**
Compensated and uncompensated voltage profile of open-ended line.

(e) Find the receiving end and the sending end currents when the line is terminated in a short circuit.



Selecting option 5 of the **listmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter sending end voltage phase angle° = 0

Line short-circuited at the receiving end

Vs = 500 kV (L-L) at 0°
Ir = 2692.45 A at -87.5549°
Is = 2502.65 A at -87.367°
```

(f) The line loading in part (a) resulted in a voltage regulation of 34.16 percent, which is unacceptably high. To improve the line performance, the line is compensated with series and shunt capacitors. For the loading condition in (a):

(1) Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 500 kV when the line is energized with 500 kV at the sending end.

Selecting option 6 will display the **compmenu** as follows:

```
        Capacitive compensation
               Analysis                    Select

Shunt capacitive compensation                1

Series capacitive compensation               2

Series and shunt capacitive compensation     3

To quit                                      0
```

Selecting option 1 of the **compmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter desired receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 800
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 600
```

```
Shunt capacitive compensation
-----------------------------
Vs = 500 kV (L-L) at 20.2479°
Vr = 500 kV (L-L) at 0°
Pload = 800 MW,     Qload = 600 Mvar
Load current = 1154.7 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 407.267 Ω, 6.51314 μF,613.849 Mvar
Shunt capacitor current = 708.811 A at 90°
Pr = 800.000 MW,     Qr = -13.849 Mvar
Ir = 923.899 A at 0.991732°, PFr = 0.99985 leading
Is = 940.306 A at 24.121° PFs = 0.997716 leading
Ps = 812.469 MW,     Qs = -55.006 Mvar
PL = 12.469 MW,     QL = -41.158 Mvar
Percent Voltage Regulation = 7.58405
Transmission line efficiency = 98.4653
```

(2) Determine the line performance when the line is compensated by series capacitors for 40 percent compensation with the load condition in (a) at 500 kV.

Selecting option 2 of the **compmenu** results in

```
Enter receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 800
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 600
Enter percent compensation for series capacitor
(Recommended range 25 to 75% of the line reactance) = 40

Series capacitor compensation
-----------------------------
Vr = 500 kV (L-L) at 0°
Pr = 800 MW,     Qr = 600 Mvar
Required series capacitor: 42.8476 Ω, 61.9074 μF, 47.4047 Mvar
Subsynchronous resonant frequency = 37.9473 Hz
Ir = 1154.7 A at -36.8699°, PFr = 0.8 lagging
Vs = 571.904 kV (L-L) at 9.95438°
Is = 932.258 A at -18.044°, PFs = 0.882961 lagging
Ps = 815.383 MW,     Qs = 433.517 Mvar
PL = 15.383 MW,     QL = -166.483 Mvar
Percent Voltage Regulation = 19.4322
Transmission line efficiency = 98.1134
```



(3) The line has 40 percent series capacitor compensation and supplies the load in (a). Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 500 kV when line is energized with 500 kV at the sending end.

Selecting option 3 of the **compmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter desired receiving end line-line voltage kV = 500
Enter receiving end voltage phase angle° = 0
Enter receiving end 3-phase power MW = 800
Enter receiving end 3-phase reactive power
(+ for lagging and - for leading power factor) Mvar = 600
Enter percent compensation for series capacitor
(Recommended range 25 to 75% of the line reactance) = 40

Series and shunt capacitor compensation
---------------------------------------
Vs = 500 kV (L-L) at 12.0224°
Vr = 500 kV (L-L) at 0°
Pload = 800 MW,     Qload = 600 Mvar
Load current = 1154.7 A at -36.8699°, PFl = 0.8 lagging
Required shunt capacitor: 432.736 Ω, 6.1298 μF, 577.72 Mvar
Shunt capacitor current = 667.093 A at 90°
Required series capacitor: 42.8476 Ω, 61.9074 μF,37.7274 Mvar
Subsynchronous resonant frequency = 37.9473 Hz
Pr = 800 MW,     Qr = 22.2804 Mvar
Ir = 924.119 A at -1.5953°, PFr = 0.999612 lagging
Is = 951.165 A at 21.5977°, PFs = 0.986068 leading
Ps = 812.257 MW,     Qs = -137.023 Mvar
PL = 12.257 MW,     QL = -159.304 Mvar
Percent Voltage Regulation = 4.41619
Transmission line efficiency = 98.491
```

(g) Construct the receiving end circle diagram.

Selecting option 7 of the **listmenu** results in

```
Enter receiving end line-line voltage kV = 500
```

A plot of the receiving end circle diagram is obtained as shown in Figure 5.10.

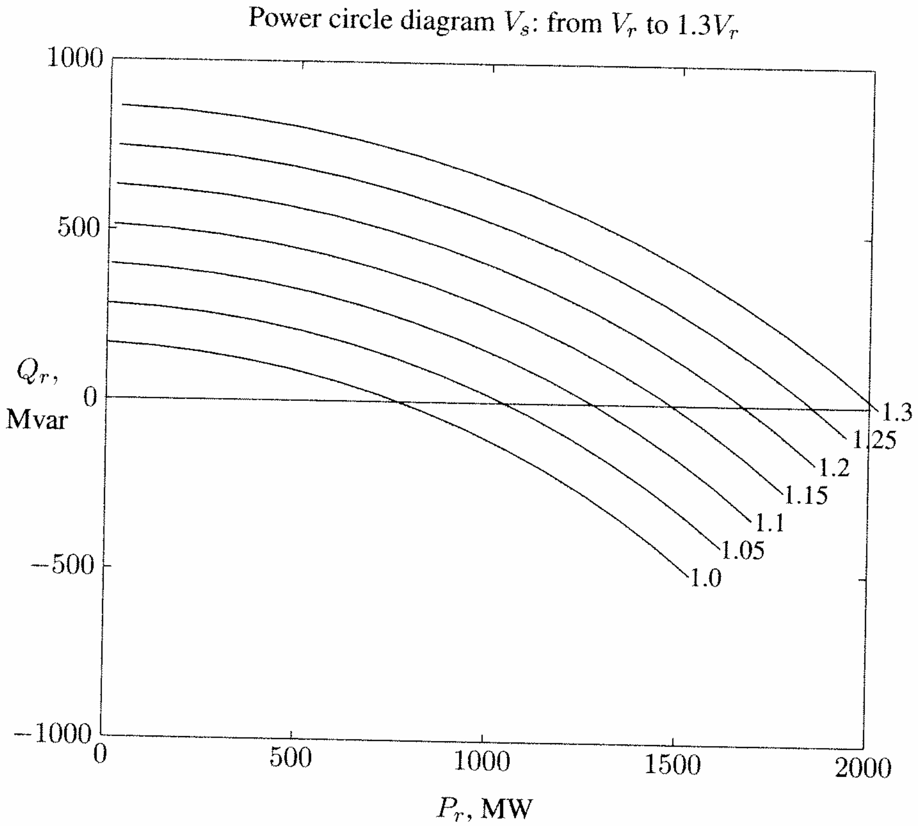



**FIGURE 5.10**
Receiving end circle diagram.

(h) Determine the line voltage profile for the following cases: no-load, rated load, line terminated in the $SIL$, and short-circuited line.

Selecting option 8 of the **listmenu** results in

```
        Voltage profile and line loadability
                Analysis                         Select

    Voltage profile curves                          1
    Line loadability curve                          2
    To quit                                         0
```

Selecting option 1 of the **profmenu** results in



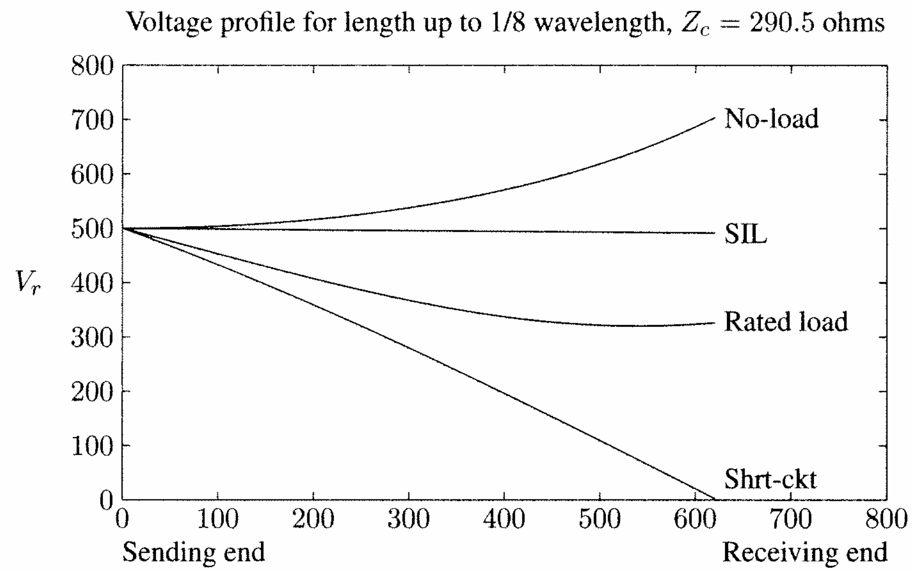



**FIGURE 5.11**
Voltage profile for length up to 1/8 wavelength.

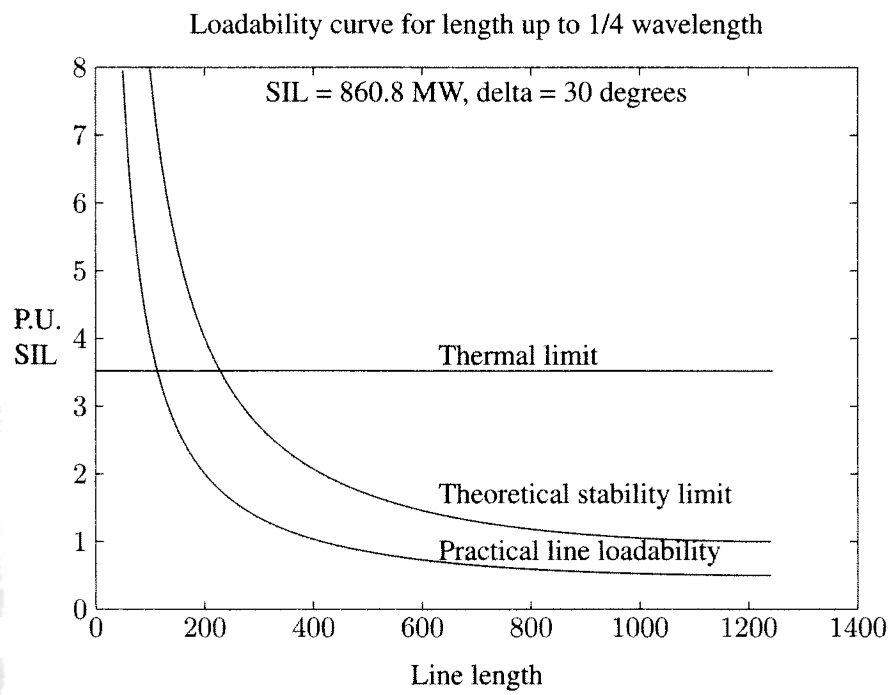



**FIGURE 5.12**
Line loadability curve for length up to 1/4 wavelength.

```
Enter sending end line-line voltage kV = 500
Enter rated sending end power, MVA = 1000
Enter power factor = 0.8
```

A plot of the voltage profile is obtained as shown in Figure 5.11 (page 182).

(i) Obtain the line loadability curves.
Selecting option 2 of the **profmenu** results in

```
Enter sending end line-line voltage kV = 500
Enter receiving end line-line voltage kV = 500
Enter rated line-line voltage kV = 500
Enter line current-carrying capacity, Amp/phase = 3500
```

The line loadability curve is obtained as shown in Figure 5.12 (page 182).

## PROBLEMS

**5.1.** A 69-kV, three-phase short transmission line is 16 km long. The line has a per phase series impedance of $0.125+j0.4375\ \Omega$ per km. Determine the sending end voltage, voltage regulation, the sending end power, and the transmission efficiency when the line delivers

(a) 70 MVA, 0.8 lagging power factor at 64 kV.
(b) 120 MW, unity power factor at 64 kV.

Use **lineperf** program to verify your results.

**5.2.** Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.1. The line delivers 70 MVA, 0.8 lagging power factor at 64 kV. Determine the total Mvar and the capacitance per phase of the Y-connected capacitors when the sending end voltage is

(a) 69 kV.
(b) 64 kV.

*Hint*: Use (5.85) and (5.86) to compute the power angle $\delta$ and the receiving end reactive power.

(c) Use **lineperf** to obtain the compensated line performance.

**5.3.** A 230-kV, three-phase transmission line has a per phase series impedance of $z = 0.05 + j0.45\ \Omega$ per km and a per phase shunt admittance of $y = j3.4 \times 10^{-6}$ siemens per km. The line is 80 km long. Using the nominal $\pi$ model, determine

(a) The transmission line ABCD constants.



Find the sending end voltage and current, voltage regulation, the sending end power and the transmission efficiency when the line delivers

(b) 200 MVA, 0.8 lagging power factor at 220 kV.
(c) 306 MW, unity power factor at 220 kV.

Use **lineperf** program to verify your results.

**5.4.** Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.3. The line delivers 200 MVA, 0.8 lagging power factor at 220 kV.

(a)Determine the total Mvar and the capacitance per phase of the Y-connected capacitors when the sending end voltage is 220 kV. *Hint*: Use (5.85) and (5.86) to compute the power angle $\delta$ and the receiving end reactive power.
(b) Use **lineperf** to obtain the compensated line performance.

**5.5.** A three-phase, 345-kV, 60-Hz transposed line is composed of two ACSR, 1,113,000-cmil, 45/7 Bluejay conductors per phase with flat horizontal spacing of 11 m. The conductors have a diameter of 3.195 cm and a $GMR$ of 1.268 cm. The bundle spacing is 45 cm. The resistance of each conductor in the bundle is $0.0538\ \Omega$ per km and the line conductance is negligible. The line is 150 km long. Using the nominal $\pi$ model, determine the ABCD constant of the line. Use **lineperf** and option 5 to verify your results.

**5.6.** The ABCD constants of a three-phase, 345-kV transmission line are

$$A = D = 0.98182 + j0.0012447$$
$$B = 4.035 + j58.947$$
$$C = j0.00061137$$

The line delivers 400 MVA at 0.8 lagging power factor at 345 kV. Determine the sending end quantities, voltage regulation, and transmission efficiency.

**5.7.** Write a *MATLAB* function named **[ABCD] = abcdm(z, y, Lngt)** to evaluate and return the ABCD transmission matrix for a medium-length transmission line where **z** is the per phase series impedance per unit length, **y** is the shunt admittance per unit length, and **Lngt** is the line length. Then, write a program that uses the above function and computes the receiving end quantities, voltage regulation, and the line efficiency when sending end quantities are specified. The program should prompt for the following quantities:

The sending end line-to-line voltage magnitude in kV
The sending end voltage phase angle in degrees

The three-phase sending end real power in MW
The three-phase sending end reactive power in Mvar

Use your program to obtain the solution for the following case.

A three-phase transmission line has a per phase series impedance of $z = 0.03 + j0.4\ \Omega$ per km and a per phase shunt admittance of $y = j4.0 \times 10^{-6}$ siemens per km. The line is 125 km long. Obtain the ABCD transmission matrix. Determine the receiving end quantities, voltage regulation, and the line efficiency when the line is sending 407 MW, 7.833 Mvar at 350 kV.

**5.8.** Obtain the solution for Problems 5.8 through 5.13 using the **lineperf** program. Then, solve each problem using hand calculations.

A three-phase, 765-kV, 60-Hz transposed line is composed of four ACSR, 1,431,000-cmil, 45/7 Bobolink conductors per phase with flat horizontal spacing of 14 m. The conductors have a diameter of 3.625 cm and a $GMR$ of 1.439 cm. The bundle spacing is 45 cm. The line is 400 km long, and for the purpose of this problem, a lossless line is assumed.

(a) Determine the transmission line surge impedance $Z_c$, phase constant $\beta$, wavelength $\lambda$, the surge impedance loading SIL, and the ABCD constant.
(b) The line delivers 2000 MVA at 0.8 lagging power factor at 735 kV. Determine the sending end quantities and voltage regulation.
(c) Determine the receiving end quantities when 1920 MW and 600 Mvar are being transmitted at 765 kV at the sending end.
(d) The line is terminated in a purely resistive load. Determine the sending end quantities and voltage regulation when the receiving end load resistance is 264.5 $\Omega$ at 735 kV.

**5.9.** The transmission line in Problem 5.8 is energized with 765 kV at the sending end when the load at the receiving end is removed.

(a) Find the receiving end voltage.
(b) Determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 735 kV.

**5.10.** The transmission line in Problem 5.8 is energized with 765 kV at the sending end when a three-phase short-circuit occurs at the receiving end. Determine the receiving end current and the sending end current.

**5.11.** Shunt capacitors are installed at the receiving end to improve the line performance of Problem 5.8. The line delivers 2000 MVA, 0.8 lagging power factor. Determine the total Mvar and the capacitance per phase of the Y-connected capacitors to keep the receiving end voltage at 735 kV when the sending end voltage is 765 kV. *Hint*: Use (5.93) and (5.94) to compute the power angle $\delta$ and the receiving end reactive power. Find the sending end quantities and voltage regulation for the compensated line.

**5.12.** Series capacitors are installed at the midpoint of the line in Problem 5.8, providing 40 percent compensation. Determine the sending end quantities and the voltage regulation when the line delivers 2000 MVA at 0.8 lagging power factor at 735 kV.

**5.13.** Series capacitors are installed at the midpoint of the line in Problem 5.8, providing 40 percent compensation. In addition, shunt capacitors are installed at the receiving end. The line delivers 2000 MVA, 0.8 lagging power factor. Determine the total Mvar and the capacitance per phase of the series and shunt capacitors to keep the receiving end voltage at 735 kV when the sending end voltage is 765 kV. Find the sending end quantities and voltage regulation for the compensated line.

**5.14.** The transmission line in Problem 5.8 has a per phase resistance of 0.011 $\Omega$ per km. Using the **lineperf** or **lineperfgui** program, perform the following analysis and present a summary of the calculation along with your conclusions and recommendations.

(a) Determine the sending end quantities for the specified receiving end quantities of $735\angle 0^\circ$ kV, 1600 MW, 1200 Mvar.
(b) Determine the receiving end quantities for the specified sending end quantities of $765\angle 0^\circ$ kV, 1920 MW, 600 Mvar.
(c) Determine the sending end quantities for a load impedance of $282.38 + j0\ \Omega$ at 735 kV.
(d) Find the receiving end voltage when the line is terminated in an open circuit and is energized with 765 kV at the sending end. Also, determine the reactance and the Mvar of a three-phase shunt reactor to be installed at the receiving end in order to limit the no-load receiving end voltage to 765 kV. Obtain the voltage profile for the uncompensated and the compensated line.
(e) Find the receiving end and the sending end current when the line is terminated in a three-phase short circuit.
(f) For the line loading of part (a), determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 735 kV when line is energized with 765 kV. Obtain the line performance of the compensated line.
(g) Determine the line performance when the line is compensated by series capacitor for 40 percent compensation with the load condition in part (a) at 735 kV.

(h) The line has 40 percent series capacitor compensation and supplies the load in part (a). Determine the Mvar and the capacitance of the shunt capacitors to be installed at the receiving end to keep the receiving end voltage at 735 kV when line is energized with 765 kV at the sending end.
(i) Obtain the receiving end circle diagram.
(j) Obtain the line voltage profile for a sending end voltage of 765 kV.
(k) Obtain the line loadability curves when the sending end voltage is 765 kV, and the receiving end voltage is 735 kV. The current-carrying capacity of the line is 5000 A per phase.

**5.15.** The ABCD constants of a lossless three-phase, 500-kV transmission line are

$$A = D = 0.86 + j0$$
$$B = 0 + j130.2$$
$$C = j0.002$$

(a) Obtain the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

To improve the line performance, series capacitors are installed at both ends in each phase of the transmission line. As a result of this, the compensated ABCD constants become

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2}jX_c \\ 0 & 1 \end{bmatrix}$$

where $X_c$ is the total reactance of the series capacitor. If $X_c = 100\ \Omega$

(b) Determine the compensated ABCD constants.
(c) Determine the sending end quantities and the voltage regulation when line delivers 1000 MVA at 0.8 lagging power factor at 500 kV.

**5.16.** A three-phase 420-kV, 60-HZ transmission line is 463 km long and may be assumed lossless. The line is energized with 420 kV at the sending end. When the load at the receiving end is removed, the voltage at the receiving end is 700 kV, and the per phase sending end current is $646.6\angle 90°$ A.

(a) Find the phase constant $\beta$ in radians per km and the surge impedance $Z_c$ in $\Omega$.
(b) Ideal reactors are to be installed at the receiving end to keep $|V_S| = |V_R| = 420$ kV when load is removed. Determine the reactance per phase and the required three-phase kvar.

**5.17.** A three-phase power of 3600 MW is to be transmitted via four identical 60-Hz transmission lines for a distance of 300 km. From a preliminary line design, the line phase constant and surge impedance are given by $\beta = 9.46 \times 10^{-4}$ radian/km and $Z_c = 343\ \Omega$, respectively.

Based on the practical line loadability criteria determine the suitable nominal voltage level in kV for each transmission line. Assume $V_S = 1.0$ per unit, $V_R = 0.9$ per unit, and the power angle $\delta = 36.87°$.

**5.18.** Power system studies on an existing system have indicated that 2400 MW are to be transmitted for a distance of 400 km. The voltage levels being considered include 345 kV, 500 kV, and 765 kV. For a preliminary design based on the practical line loadability, you may assume the following surge impedances

345 kV $Z_C = 320\ \Omega$
500 kV $Z_C = 290\ \Omega$
765 kV $Z_C = 265\ \Omega$

The line wavelength may be assumed to be 5000 km. The practical line loadability may be based on a load angle $\delta$ of 35°. Assume $|V_S| = 1.0$ pu and $|V_R| = 0.9$ pu. Determine the number of three-phase transmission circuits required for each voltage level. Each transmission tower may have up to two circuits. To limit the corona loss, all 500-kV lines must have at least two conductors per phase, and all 765-kV lines must have at least four conductors per phase. The bundle spacing is 45 cm. The conductor size should be such that the line would be capable of carrying current corresponding to at least 5000 MVA. Use **acsrgui** command in *MATLAB* to find a suitable conductor size. Following are the minimum recommended spacings between adjacent phase conductors at various voltage levels.

| Voltage level, kV | Spacing meter |
|---|---|
| 345 | 7.0 |
| 500 | 9.0 |
| 765 | 12.5 |

(a) Select a suitable voltage level, and conductor size, and tower structure. Use **lineperfgui** program and the first analysis to obtain the voltage regulation and transmission efficiency based on a receiving end power of 3000 MVA at 0.8 power factor lagging at the selected rated voltage. Modify your design and select a conductor size for a line efficiency of at least 94 percent for the above specified load.
(b) Obtain the line performance including all analysis of the **lineperfgui** program for your final selection. Summarize the line characteristics and the required line compensation.

CHAPTER

# 6

# POWER FLOW ANALYSIS

## 6.1 INTRODUCTION

In the previous chapters, modeling of the major components of an electric power system was discussed. This chapter deals with the steady-state analysis of an interconnected power system during normal operation. The system is assumed to be operating under balanced condition and is represented by a single-phase network. The network contains hundreds of nodes and branches with impedances specified in per unit on a common MVA base.

Network equations can be formulated systematically in a variety of forms. However, the node-voltage method, which is the most suitable form for many power system analyses, is commonly used. The formulation of the network equations in the nodal admittance form results in complex linear simultaneous algebraic equations in terms of node currents. When node currents are specified, the set of linear equations can be solved for the node voltages. However, in a power system, powers are known rather than currents. Thus, the resulting equations in terms of power, known as the *power flow equation*, become nonlinear and must be solved by iterative techniques. Power flow studies, commonly referred to as *load flow*, are the backbone of power system analysis and design. They are necessary for planning, operation, economic scheduling and exchange of power between utilities. In addition, power flow analysis is required for many other analyses such as transient stability and contingency studies.





In this chapter, the bus admittance matrix of the node-voltage equation is formulated, and two *MATLAB* function named **ybus1** and **lfybus** are developed for the systematic formation of the bus admittance matrix. Next, two commonly used iterative techniques, namely Gauss-Seidel and Newton-Raphson methods for the solution of nonlinear algebraic equations, are discussed. These techniques are employed in the solution of power flow problems. Three programs **lfgauss**, **lfnewton**, and **decouple** are developed for the solution of power flow problems by Gauss-Seidel, Newton-Raphson, and the fast decoupled power flow, respectively.

## 6.2 BUS ADMITTANCE MATRIX

In order to obtain the node-voltage equations, consider the simple power system shown in Figure 6.1 where impedances are expressed in per unit on a common MVA base and for simplicity resistances are neglected. Since the nodal solution is based upon Kirchhoff's current law, impedances are converted to admittance, i.e.,

$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$

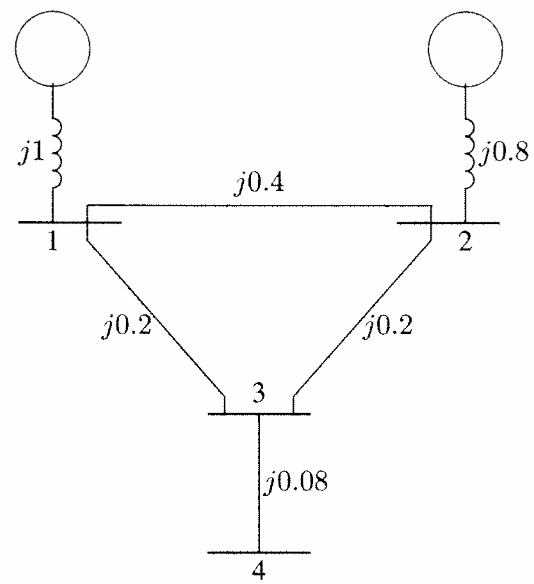



**FIGURE 6.1**
The impedance diagram of a simple system.

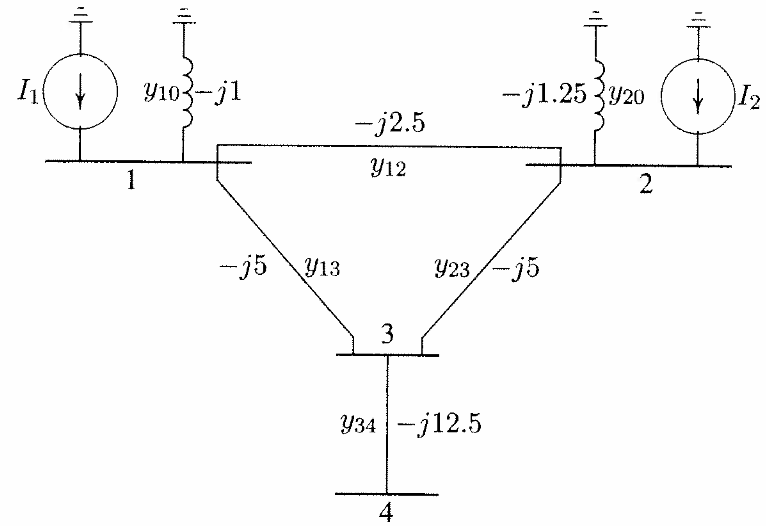



**FIGURE 6.2**
The admittance diagram for system of Figure 6.1.

The circuit has been redrawn in Figure 6.2 in terms of admittances and transformation to current sources. Node 0 (which is normally ground) is taken as reference. Applying KCL to the independent nodes 1 through 4 results in

$$\begin{aligned} I_1 &= y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3) \\ I_2 &= y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3) \\ 0 &= y_{23}(V_3 - V_2) + y_{13}(V_3 - V_1) + y_{34}(V_3 - V_4) \\ 0 &= y_{34}(V_4 - V_3) \end{aligned}$$

Rearranging these equations yields

$$\begin{aligned} I_1 &= (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3 \\ I_2 &= -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3 \\ 0 &= -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4 \\ 0 &= -y_{34}V_3 + y_{34}V_4 \end{aligned}$$

We introduce the following admittances

$$\begin{aligned} Y_{11} &= y_{10} + y_{12} + y_{13} \\ Y_{22} &= y_{20} + y_{12} + y_{23} \\ Y_{33} &= y_{13} + y_{23} + y_{34} \\ Y_{44} &= y_{34} \\ Y_{12} &= Y_{21} = -y_{12} \\ Y_{13} &= Y_{31} = -y_{13} \\ Y_{23} &= Y_{32} = -y_{23} \\ Y_{34} &= Y_{43} = -y_{34} \end{aligned}$$

The node equation reduces to

$$\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 + Y_{13}V_3 + Y_{14}V_4 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 \\ I_3 &= Y_{31}V_1 + Y_{32}V_2 + Y_{33}V_3 + Y_{34}V_4 \\ I_4 &= Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{44}V_4 \end{aligned}$$

In the above network, since there is no connection between bus 1 and 4, $Y_{14} = Y_{41} = 0$; similarly $Y_{24} = Y_{42} = 0$.

Extending the above relation to an $n$ bus system, the node-voltage equation in matrix form is

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_i \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1i} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2i} & \cdots & Y_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{i1} & Y_{i2} & \cdots & Y_{ii} & \cdots & Y_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{ni} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \tag{6.1}$$

or

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\,\mathbf{V}_{bus} \tag{6.2}$$

where $\mathbf{I}_{bus}$ is the vector of the injected bus currents (i.e., external current sources). The current is positive when flowing towards the bus, and it is negative if flowing away from the bus. $\mathbf{V}_{bus}$ is the vector of bus voltages measured from the reference node (i.e., node voltages). $\mathbf{Y}_{bus}$ is known as the *bus admittance matrix*. The diagonal element of each node is the sum of admittances connected to it. It is known as the *self-admittance* or *driving point admittance*, i.e.,

$$Y_{ii} = \sum_{j=0}^{n} y_{ij} \qquad j \neq i \tag{6.3}$$

The off-diagonal element is equal to the negative of the admittance between the nodes. It is known as the *mutual admittance* or *transfer admittance*, i.e.,

$$Y_{ij} = Y_{ji} = -y_{ij} \tag{6.4}$$

When the bus currents are known, (6.2) can be solved for the $n$ bus voltages.

$$\mathbf{V}_{bus} = \mathbf{Y}_{bus}^{-1}\,\mathbf{I}_{bus} \tag{6.5}$$

The inverse of the bus admittance matrix is known as the *bus impedance matrix* $Z_{bus}$. The admittance matrix obtained with one of the buses as reference is nonsingular. Otherwise the nodal matrix is singular.

Inspection of the bus admittance matrix reveals that the matrix is symmetric along the leading diagonal, and we need to store the upper triangular nodal admittance matrix only. In a typical power system network, each bus is connected to only a few nearby buses. Consequently, many off-diagonal elements are zero. Such a matrix is called *sparse*, and efficient numerical techniques can be applied to compute its inverse. By means of an appropriately ordered triangular decomposition, the inverse of a sparse matrix can be expressed as a product of sparse matrix factors, thereby giving an advantage in computational speed, storage and reduction of round-off errors. However, $Z_{bus}$, which is required for short-circuit analysis, can be obtained directly by the method of *building algorithm* without the need for matrix inversion. This technique is discussed in Chapter 9.

Based on (6.3) and (6.4), the bus admittance matrix for the network in Figure 6.2 obtained by inspection is

$$Y_{bus} = \begin{bmatrix} -j8.50 & j2.50 & j5.00 & 0 \\ j2.50 & -j8.75 & j5.00 & 0 \\ j5.00 & j5.00 & -j22.50 & j12.50 \\ 0 & 0 & j12.50 & -j12.50 \end{bmatrix}$$

A function called **Y = ybus1(zdata)** is written for the formation of the bus admittance matrix. **zdata** is the line data input and contains four columns. The first two columns are the line bus numbers and the remaining columns contain the line resistance and reactance in per unit. The function returns the bus admittance matrix. The algorithm for the bus admittance program is very simple and basic to power system programming. Therefore, it is presented here for the reader to study and understand the method of solution. In the program, the line impedances are first converted to admittances. $Y$ is then initialized to zero. In the first loop, the line data is searched, and the off-diagonal elements are entered. Finally, in a nested loop, line data is searched to find the elements connected to a bus, and the diagonal elements are thus formed.

The following is a program for building the bus admittance matrix:

```
function[Y] = ybus1(zdata)
nl=zdata(:,1); nr=zdata(:,2); R=zdata(:,3); X=zdata(:,4);
nbr=length(zdata(:,1)); nbus = max(max(nl), max(nr));
Z = R + j*X;                                   %branch impedance
```



```
y= ones(nbr,1)./Z;                             %branch admittance
Y = zeros(nbus,nbus);                       % initialize Y to zero
for k = 1:nbr;    % formation of the off diagonal elements
    if nl(k) > 0 & nr(k) > 0
    Y(nl(k),nr(k)) = Y(nl(k),nr(k)) - y(k);
    Y(nr(k),nl(k)) = Y(nl(k),nr(k));
    end
end
for n = 1:nbus        % formation of the diagonal elements
    for k = 1:nbr
    if nl(k) == n | nr(k) == n
    Y(n,n) = Y(n,n) + y(k);
    else, end
    end
end
```

**Example 6.1** (chp6ex1)

The emfs shown in Figure 6.1 are $E_1 = 1.1\angle 0°$ and $E_2 = 1.0\angle 0°$. Use the function **Y = ybus1(zdata)** to obtain the bus admittance matrix. Find the bus impedance matrix by inversion, and solve for the bus voltages.

With source transformation, the equivalent current sources are

$$I_1 = \frac{1.1}{j1.0} = -j1.1 \text{ pu}$$

$$I_2 = \frac{1.0}{j0.8} = -j1.25 \text{ pu}$$

The following commands

```
%    From  To   R    X
z = [ 0    1    0   1.0
      0    2    0   0.8
      1    2    0   0.4
      1    3    0   0.2
      2    3    0   0.2
      3    4    0   0.08];
Y = ybus1(z)                           % bus admittance matrix
Ibus = [-j*1.1; -j*1.25; 0; 0];  % vector of bus currents
Zbus = inv(Y)                            % bus impedance matrix
Vbus = Zbus*Ibus
```

result in

```
Y =
      0 - 8.50i   0 + 2.50i   0 +  5.00i   0 +  0.00i
      0 + 2.50i   0 - 8.75i   0 +  5.00i   0 +  0.00i
      0 + 5.00i   0 + 5.00i   0 - 22.50i   0 + 12.50i
      0 + 0.00i   0 + 0.00i   0 + 12.50i   0 - 12.50i
Zbus =
      0 + 0.50i   0 + 0.40i   0 + 0.450i   0 + 0.450i
      0 + 0.40i   0 + 0.48i   0 + 0.440i   0 + 0.440i
      0 + 0.45i   0 + 0.44i   0 + 0.545i   0 + 0.545i
      0 + 0.45i   0 + 0.44i   0 + 0.545i   0 + 0.625i
Vbus =
      1.0500
      1.0400
      1.0450
      1.0450
```

The solution of equation $\mathbf{I}_{bus} = \mathbf{Y}_{bus}\mathbf{V}_{bus}$ by inversion is very inefficient. It is not necessary to obtain the inverse of $\mathbf{Y}_{bus}$. Instead, direct solution is obtained by optimally ordered triangular factorization. In *MATLAB*, the solution of linear simultaneous equations $AX = B$ is obtained by using the matrix division operator \ (i.e., $X = A \setminus B$), which is based on the triangular factorization and Gaussian elimination. This technique is superior in both execution time and numerical accuracy. It is two to three times as fast and produces residuals on the order of machine accuracy.

In Example 6.1, obtain the direct solution by replacing the statements `Zbus = inv(Y)` and `Vbus = Zbus*Ibus` with `Vbus = Y\ Ibus`.

## 6.3 SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS

The most common techniques used for the iterative solution of nonlinear algebraic equations are Gauss-Seidel, Newton-Raphson, and Quasi-Newton methods. The Gauss-Seidel and Newton-Raphson methods are discussed for one-dimensional equation, and are then extended to $n$-dimensional equations.

### 6.3.1 GAUSS-SEIDEL METHOD

The Gauss-Seidel method is also known as the method of successive displacements. To illustrate the technique, consider the solution of the nonlinear equation given by

$$f(x) = 0 \tag{6.6}$$

The above function is rearranged and written as

$$x = g(x) \tag{6.7}$$

If $x^{(k)}$ is an initial estimate of the variable $x$, the following iterative sequence is formed.

$$x^{(k+1)} = g(x^{(k)}) \tag{6.8}$$

A solution is obtained when the difference between the absolute value of the successive iteration is less than a specified accuracy, i.e.,

$$|x^{(k+1)} - x^{(k)}| \leq \epsilon \tag{6.9}$$

where $\epsilon$ is the desired accuracy.

**Example 6.2** (chp6ex2)

Use the Gauss-Seidel method to find a root of the following equation

$$f(x) = x^3 - 6x^2 + 9x - 4 = 0$$

Solving for $x$, the above expression is written as

$$\begin{aligned} x &= -\frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9} \\ &= g(x) \end{aligned}$$

The *MATLAB* **plot** command is used to plot $g(x)$ and $x$ over a range of 0 to 4.5, as shown in Figure 6.3. The intersections of $g(x)$ and $x$ results in the roots of $f(x)$. From Figure 6.3 two of the roots are found to be 1 and 4. Actually, there is a repeated root at $x = 1$. Apply the Gauss-Seidel algorithm, and use an initial estimate of

$$x^{(0)} = 2$$

From (6.8), the first iteration is

$$x^{(1)} = g(2) = -\frac{1}{9}(2)^3 + \frac{6}{9}(2)^2 + \frac{4}{9} = 2.2222$$

The second iteration is

$$x^{(2)} = g(2.2222) = -\frac{1}{9}(2.2222)^3 + \frac{6}{9}(2.2222)^2 + \frac{4}{9} = 2.5173$$

The subsequent iterations result in 2.8966, 3.3376, 3.7398, 3.9568, 3.9988 and 4.0000. The process is repeated until the change in variable is within the desired

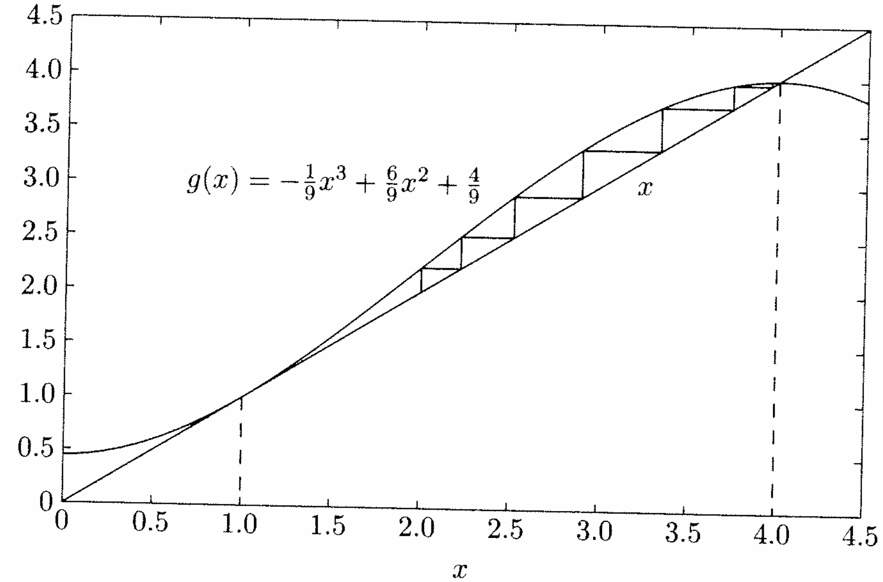



**FIGURE 6.3**
Graphical illustration of the Gauss-Seidel method.

accuracy. It can be seen that the Gauss-Seidel method needs many iterations to achieve the desired accuracy, and there is no guarantee for the convergence. In this example, since the initial estimate was within a "boxed in" region, the solution converged in a zigzag fashion to one of the roots. In fact, if the initial estimate was outside this region, say $x^{(0)} = 6$, the process would diverge. A test of convergence, especially for the $n$-dimensional case, is difficult, and no general methods are known.

The following commands show the procedure for the solution of the given equation starting with an initial estimate of $x^{(0)} = 2$.

```
dx=1;          % Change in variable is set to a high value
x=2;                                    % Initial estimate
iter = 0;                              % Iteration counter
disp('Iter    g         dx       x')%Heading for results
while abs(dx) >= 0.001 & iter < 100 %Test for convergence
iter = iter + 1;                        % No. of iterations
g = -1/9*x^3+6/9*x^2+4/9 ;
dx = g-x;                              % Change in variable
x = x + dx;                  % Successive approximation
fprintf('%g', iter), disp([g, dx, x])
end
```

The result is

```
Iter     g        dx        x
 1     2.2222   0.2222   2.2222
 2     2.5173   0.2951   2.5173
 3     2.8966   0.3793   2.8966
 4     3.3376   0.4410   3.3376
 5     3.7398   0.4022   3.7398
 6     3.9568   0.2170   3.9568
 7     3.9988   0.0420   3.9988
 8     4.0000   0.0012   4.0000
 9     4.0000   0.0000   4.0000
```

In some cases, an acceleration factor can be used to improve the rate of convergence. If $\alpha > 1$ is the acceleration factor, the Gauss-Seidel algorithm becomes

$$x^{(k+1)} = x^{(k)} + \alpha[g(x^{(k)}) - x^{(k)}] \tag{6.10}$$

**Example 6.3** (chp6ex3)

Find a root of the equation in Example 6.2, using the Gauss-Seidel method with an acceleration factor of $\alpha = 1.25$:

Starting with an initial estimate of $x^{(0)} = 2$ and using (6.10), the first iteration is

$$\begin{aligned} g(2) &= -\frac{1}{9}(2)^3 + \frac{6}{9}(2)^2 + \frac{4}{9} = 2.2222 \\ x^{(1)} &= 2 + 1.25[2.2222 - 2] = 2.2778 \end{aligned}$$

The second iteration is

$$\begin{aligned} g(2.2778) &= -\frac{1}{9}(2.2778)^3 + \frac{6}{9}(2.2778)^2 + \frac{4}{9} = 2.5902 \\ x^{(2)} &= 2.2778 + 1.25[2.5902 - 2.2778] = 2.6683 \end{aligned}$$

The subsequent iterations result in 3.0801, 3.1831, 3.7238, 4.0084, 3.9978 and 4.0005. The effect of acceleration is shown graphically in Figure 6.4. Care must be taken not to use a very large acceleration factor since the larger step size may result in an overshoot. This can cause an increase in the number of iterations or even result in divergence. In the *MATLAB* command of Example 6.2, replace the command before the end statement by $x = x + 1.25 * dx$ to reflect the effect of the acceleration factor and run the program.

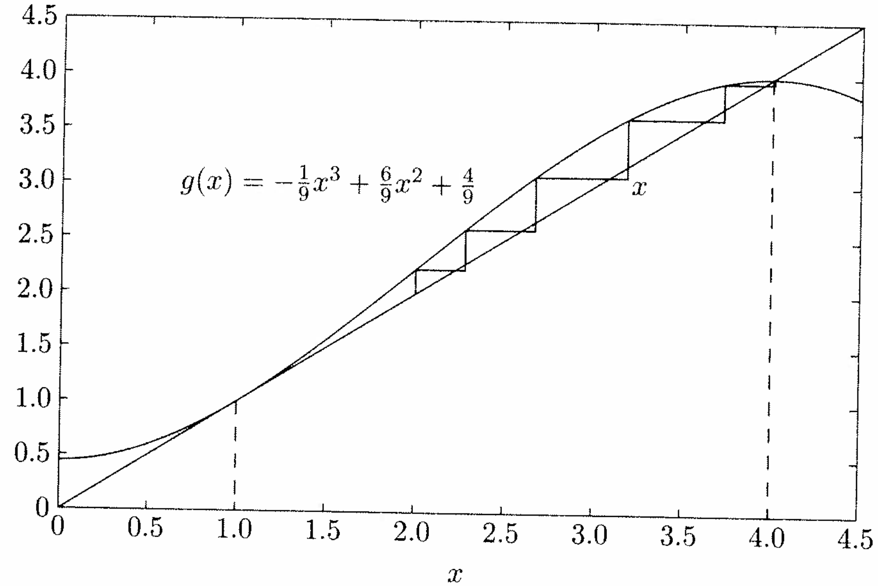



**FIGURE 6.4**
Graphical illustration of the Gauss-Seidel method using acceleration factor.

We now consider the system of $n$ equations in $n$ variables

$$\begin{aligned} f_1(x_1, x_2, \cdots, x_n) &= c_1 \\ f_2(x_1, x_2, \cdots, x_n) &= c_2 \\ \cdots\cdots\cdots\cdots & \\ f_n(x_1, x_2, \cdots, x_n) &= c_n \end{aligned} \tag{6.11}$$

Solving for one variable from each equation, the above functions are rearranged and written as

$$\begin{aligned} x_1 &= c_1 + g_1(x_1, x_2, \cdots, x_n) \\ x_2 &= c_2 + g_2(x_1, x_2, \cdots, x_n) \\ \cdots\cdots\cdots\cdots & \\ x_n &= c_n + g_n(x_1, x_2, \cdots, x_n) \end{aligned} \tag{6.12}$$

The iteration procedure is initiated by assuming an approximate solution for each of the independent variables $(x_1^{(0)}, x_2^{(0)} \cdots, x_n^{(0)})$. Equation (6.12) results in a new approximate solution $(x_1^{(1)}, x_2^{(1)} \cdots, x_n^{(1)})$. In the Gauss-Seidel method, the updated values of the variables calculated in the preceding equations are immediately used in the solution of the subsequent equations. At the end of each iteration, the calculated values of all variables are tested against the previous values. If all changes in the variables are within the specified accuracy, a solution has converged, otherwise another iteration must be performed. The rate of convergence can often be increased by using a suitable acceleration factor $\alpha$, and the iterative sequence becomes

$$x_i^{(k+1)} = x_i^{(k)} + \alpha(x_{i\,cal}^{(k+1)} - x_i^{(k)}) \tag{6.13}$$

### 6.3.2 NEWTON-RAPHSON METHOD

The most widely used method for solving simultaneous nonlinear algebraic equations is the Newton-Raphson method. Newton's method is a successive approximation procedure based on an initial estimate of the unknown and the use of Taylor's series expansion. Consider the solution of the one-dimensional equation given by

$$f(x) = c \tag{6.14}$$

If $x^{(0)}$ is an initial estimate of the solution, and $\Delta x^{(0)}$ is a small deviation from the correct solution, we must have

$$f(x^{(0)} + \Delta x^{(0)}) = c$$

Expanding the left-hand side of the above equation in Taylor's series about $x^{(0)}$ yields

$$f(x^{(0)}) + \left(\frac{df}{dx}\right)^{(0)} \Delta x^{(0)} + \frac{1}{2!}\left(\frac{d^2 f}{dx^2}\right)^{(0)} (\Delta x^{(0)})^2 + \cdots = c$$

Assuming the error $\Delta x^{(0)}$ is very small, the higher-order terms can be neglected, which results in

$$\Delta c^{(0)} \simeq \left(\frac{df}{dx}\right)^{(0)} \Delta x^{(0)}$$

where

$$\Delta c^{(0)} = c - f(x^{(0)})$$

Adding $\Delta x^{(0)}$ to the initial estimate will result in the second approximation

$$x^{(1)} = x^{(0)} + \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}}$$

Successive use of this procedure yields the Newton-Raphson algorithm

$$\Delta c^{(k)} = c - f(x^{(k)}) \tag{6.15}$$

$$\Delta x^{(k)} = \frac{\Delta c^{(k)}}{\left(\frac{df}{dx}\right)^{(k)}} \tag{6.16}$$

$$x^{(k+1)} = x^{(k)} + \Delta x^{(k)} \tag{6.17}$$

(6.16) can be rearranged as

$$\Delta c^{(k)} = j^{(k)} \Delta x^{(k)} \tag{6.18}$$

where

$$j^{(k)} = \left(\frac{df}{dx}\right)^{(k)}$$

The relation in (6.18) demonstrates that the nonlinear equation $f(x) - c = 0$ is approximated by the tangent line on the curve at $x^{(k)}$. Therefore, a linear equation is obtained in terms of the small changes in the variable. The intersection of the tangent line with the $x$-axis results in $x^{(k+1)}$. This idea is demonstrated graphically in Example 6.4.

**Example 6.4** (chp6ex4)

Use the Newton-Raphson method to find a root of the equation given in Example 6.2. Assume an initial estimate of $x^{(0)} = 6$.

The *MATLAB* **plot** command is used to plot $f(x) = x^3 - 6x^2 + 9x - 4$ over a range of 0 to 6 as shown in Figure 6.5. The intersections of $f(x)$ with the $x$-axis results in the roots of $f(x)$. From Figure 6.5, two of the roots are found to be 1 and 4. Actually, there is a repeated root at $x = 1$.

Also, Figure 6.5 gives a graphical description of the Newton-Raphson method. Starting with an initial estimate of $x^{(0)} = 6$, we extrapolate along the tangent to its intersection with the $x$-axis and take that as the next approximation. This is continued until successive $x$-values are sufficiently close.

The analytical solution given by the Newton-Raphson algorithm is

$$\frac{df(x)}{dx} = 3x^2 - 12x + 9$$

$$\Delta c^{(0)} = c - f(x^{(0)}) = 0 - [(6)^3 - 6(6)^2 + 9(6) - 4] = -50$$

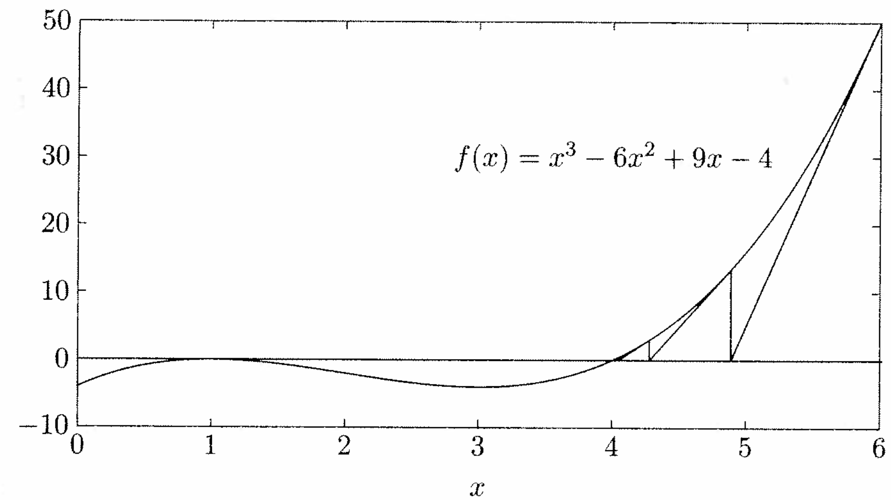



**FIGURE 6.5**
Graphical illustration of the Newton-Raphson algorithm.

$$\left(\frac{df}{dx}\right)^{(0)} = 3(6)^2 - 12(6) + 9 = 45$$

$$\Delta x^{(0)} = \frac{\Delta c^{(0)}}{\left(\frac{df}{dx}\right)^{(0)}} = \frac{-50}{45} = -1.1111$$

Therefore, the result at the end of the first iteration is

$$x^{(1)} = x^{(0)} + \Delta x^{(0)} = 6 - 1.1111 = 4.8889$$

The subsequent iterations result in

$$x^{(2)} = x^{(1)} + \Delta x^{(1)} = 4.8889 - \frac{13.4431}{22.037} = 4.2789$$

$$x^{(3)} = x^{(2)} + \Delta x^{(2)} = 4.2789 - \frac{2.9981}{12.5797} = 4.0405$$

$$x^{(4)} = x^{(3)} + \Delta x^{(3)} = 4.0405 - \frac{0.3748}{9.4914} = 4.0011$$

$$x^{(5)} = x^{(4)} + \Delta x^{(4)} = 4.0011 - \frac{0.0095}{9.0126} = 4.0000$$

We see that Newton's method converges considerably more rapidly than the Gauss-Seidel method. The method may converge to a root different from the expected one or diverge if the starting value is not close enough to the root.

The following commands show the procedure for the solution of the given equation by the Newton-Raphson method.

```
dx=1;          % Change in variable is set to a high value
x=input('Enter initial estimate -> '); % Initial estimate
iter = 0;                            % Iteration counter
disp('iter   Dc        J          dx        x') % Heading
while abs(dx) >= 0.001 & iter < 100  Test for convergence
iter = iter + 1;                     % No. of iterations
Dc = 0 - (x^3 - 6*x^2 + 9*x - 4);               % Residual
J = 3*x^2 - 12*x + 9;                         % Derivative
dx= Dc/J;                            %Change in variable
x=x + dx;                           % Successive solution
fprintf('%g', iter), disp([Dc, J, dx, x])
end
```

The result is

```
Enter the initial estimate -> 6
iter   Dc         J         dx         x
 1  -50.0000   45.0000   -1.1111    4.8889
 2  -13.4431   22.0370   -0.6100    4.2789
 3   -2.9981   12.5797   -0.2383    4.0405
 4   -0.3748    9.4914   -0.0395    4.0011
 5   -0.0095    9.0126   -0.0011    4.0000
 6   -0.0000    9.0000   -0.0000    4.0000
```

Now consider the $n$-dimensional equations given by (6.11). Expanding the left-hand side of the equations (6.11) in the Taylor's series about the initial estimates and neglecting all higher order terms, leads to the expression

$$(f_1)^{(0)} + \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \cdots + \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_1$$

$$(f_2)^{(0)} + \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \cdots + \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_2$$

$$\vdots$$

$$(f_n)^{(0)} + \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} \Delta x_1^{(0)} + \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} \Delta x_2^{(0)} + \cdots + \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \Delta x_n^{(0)} = c_n$$

or in matrix form

$$\begin{bmatrix} c_1 - (f_1)^{(0)} \\ c_2 - (f_2)^{(0)} \\ \vdots \\ c_n - (f_n)^{(0)} \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(0)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(0)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(0)} & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(0)} \\ \vdots & \vdots & \ddots & \cdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(0)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(0)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(0)} \end{bmatrix} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \\ \vdots \\ \Delta x_n^{(0)} \end{bmatrix}$$

In short form, it can be written as

$$\Delta C^{(k)} = J^{(k)} \Delta X^{(k)}$$

or

$$\Delta X^{(k)} = [J^{(k)}]^{-1} \Delta C^{(k)} \tag{6.19}$$

and the Newton-Raphson algorithm for the $n$-dimensional case becomes

$$X^{(k+1)} = X^{(k)} + \Delta X^{(k)} \tag{6.20}$$

where

$$\Delta X^{(k)} = \begin{bmatrix} \Delta x_1^{(k)} \\ \Delta x_2^{(k)} \\ \vdots \\ \Delta x_n^{(k)} \end{bmatrix} \quad \text{and} \quad \Delta C^{(k)} = \begin{bmatrix} c_1 - (f_1)^{(k)} \\ c_2 - (f_2)^{(k)} \\ \vdots \\ c_n - (f_n)^{(k)} \end{bmatrix} \tag{6.21}$$

$$J^{(k)} = \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_1}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^{(k)} \\ \left(\frac{\partial f_2}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_2}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^{(k)} \\ \vdots & \vdots & \ddots & \cdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^{(k)} & \left(\frac{\partial f_n}{\partial x_2}\right)^{(k)} & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right)^{(k)} \end{bmatrix} \tag{6.22}$$

$J^{(k)}$ is called the *Jacobian matrix*. Elements of this matrix are the partial derivatives evaluated at $X^{(k)}$. It is assumed that $J^{(k)}$ has an inverse during each iteration. Newton's method, as applied to a set of nonlinear equations, reduces the problem to solving a set of linear equations in order to determine the values that improve the accuracy of the estimates.

The solution of (6.19) by inversion is very inefficient. It is not necessary to obtain the inverse of $J^{(k)}$. Instead, a direct solution is obtained by optimally ordered triangular factorization. In *MATLAB*, the solution of linear simultaneous equations $\Delta C = J\Delta X$ is obtained by using the matrix division operator \ (i.e., $\Delta X = J \setminus \Delta C$) which is based on the triangular factorization and Gaussian elimination.

**Example 6.5** (chp6ex5)

Use the Newton-Raphson method to find the intersections of the curves

$$x_1^2 + x_2^2 = 4$$
$$e^{x_1} + x_2 = 1$$

Graphically, the solution to this system is represented by the intersections of the circle $x_1^2 + x_2^2 = 4$ with the curve $e^{x_1} + x_2 = 1$. Figure 6.6 shows that these are near (1, −1.7) and (−1.8, 0.8).

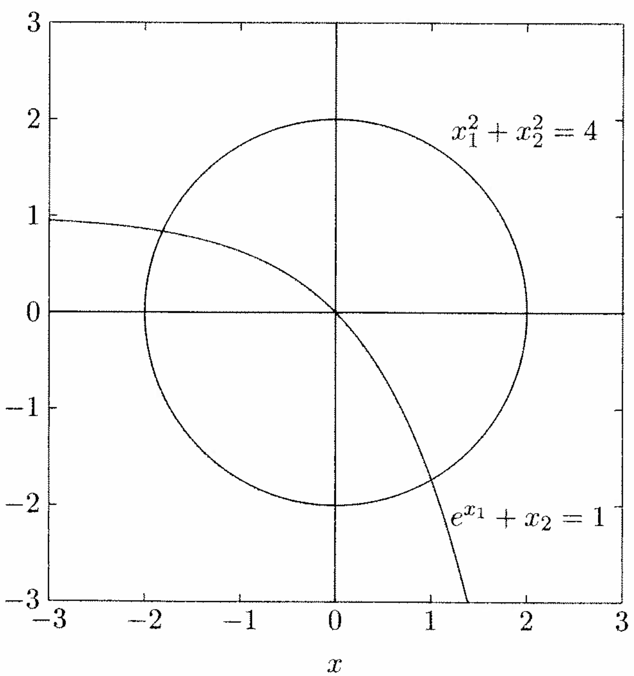



**FIGURE 6.6**
Graphs of Example 6.5.

Taking partial derivatives of the above functions results in the Jacobian matrix

$$J = \begin{bmatrix} 2x_1 & 2x_2 \\ e^{x_1} & 1 \end{bmatrix}$$

The Newton-Raphson algorithm for the above system is presented in the following statements.

```
iter = 0;                                % Iteration counter
x=input('Enter initial estimates, col. vector[x1;x2]->');
Dx = [1; 1];  % Change in variable is set to a high value
C=[4; 1];
disp('Iter   DC       Jacobian matrix        Dx         x');
                                              % Heading for results
while max(abs(Dx)) >= 0.0001 & iter <10 %Convergence test
iter=iter+1;                                   % Iteration counter
f = [x(1)^2+x(2)^2;  exp(x(1))+x(2)];                 % Functions
DC = C - f;                                           % Residuals
J = [2*x(1)      2*x(2)                          % Jacobian matrix
     exp(x(1))       1];
Dx=J\DC;                                      % Change in variables
x=x+Dx;                                     % Successive solutions
fprintf('%g', iter), disp([DC, J, Dx, x])             % Results
end
```

When the program is run, the user is prompted to enter the initial estimate. Let us try an initial estimate given by [0.5; -1].

Enter Initial estimates, col. vector $[x_1;\ x_2] \rightarrow [0.5;\ -1]$

| Iter | $\Delta C$ | Jacobian matrix | | $\Delta x$ | $x$ |
|---|---|---|---|---|---|
| 1 | 2.7500 | 1.0000 | -2.0000 | 0.8034 | 1.3034 |
| | 0.3513 | 1.6487 | 1.0000 | -0.9733 | -1.9733 |
| 2 | -1.5928 | 2.6068 | -3.9466 | -0.2561 | 1.0473 |
| | -0.7085 | 3.6818 | 1.0000 | 0.2344 | -1.7389 |
| 3 | -0.1205 | 2.0946 | -3.4778 | -0.0422 | 1.0051 |
| | -0.1111 | 2.8499 | 1.0000 | 0.0092 | -1.7296 |
| 4 | -0.0019 | 2.0102 | -3.4593 | -0.0009 | 1.0042 |
| | -0.0025 | 2.7321 | 1.0000 | 0.0000 | -1.7296 |
| 5 | -0.0000 | 2.0083 | -3.4593 | -0.0000 | 1.0042 |
| | -0.0000 | 2.7296 | 1.0000 | -0.0000 | -1.7296 |

After five iterations, the solution converges to $x_1 = 1.0042$ and $x_2 = -1.7296$ accurate to four decimal places. Starting with an initial value of [−0.5; 1], which is closer to the other intersection, results in $x_1 = -1.8163$ and $x_2 = 0.8374$.

**Example 6.6** (chp6ex6)

Starting with the initial values, $x_1 = 1$, $x_2 = 1$, and $x_3 = 1$, solve the following system of equations by the Newton-Raphson method.

$$x_1^2 - x_2^2 + x_3^2 = 11$$
$$x_1x_2 + x_2^2 - 3x_3 = 3$$
$$x_1 - x_1x_3 + x_2x_3 = 6$$

Taking partial derivatives of the above functions results in the Jacobian matrix

$$J = \begin{bmatrix} 2x_1 & -2x_2 & 2x_3 \\ x_2 & x_1 + 2x_2 & -3 \\ 1 - x_3 & x_3 & -x_1 + x_2 \end{bmatrix}$$

The following statements solve the given system of equations by the Newton-Raphson algorithm

```
Dx=[10;10;10]; %Change in variable is set to a high value
x=[1; 1; 1];                              % Initial estimate
C=[11; 3; 6];
iter = 0;                                 % Iteration counter
while max(abs(Dx))>=.0001 & iter<10;%Test for convergence
iter = iter + 1                           % No. of iterations
F = [x(1)^2-x(2)^2+x(3)^2                 % Functions
     x(1)*x(2)+x(2)^2-3*x(3)
     x(1)-x(1)*x(3)+x(2)*x(3)];
DC =C - F                                 % Residuals
J = [2*x(1)  -2*x(2)       2*x(3)        % Jacobian matrix
     x(2)    x(1)+2*x(2)   -3
     1-x(3)  x(3)          -x(1)+x(2)]
Dx=J\DC                                   %Change in variable
x=x+Dx                                    % Successive solution
end
```

The program results for the first iteration are

```
DC =                J =
      10                  2    -2     2
       4                  1     3    -3
       5                  0     1     0
Dx =                x =
     4.750               5.750
     5.000               6.000
     5.250               6.250
```

After six iterations, the solution converges to $x_1 = 2.0000$, $x_2 = 3.0000$, and $x_3 = 4.0000$.

Newton's method has the advantage of converging quadratically when we are near a root. However, more functional evaluations are required during each iteration. A very important limitation is that it does not generally converge to a solution from an arbitrary starting point.

## 6.4 POWER FLOW SOLUTION

Power flow studies, commonly known as *load flow*, form an important part of power system analysis. They are necessary for planning, economic scheduling, and control of an existing system as well as planning its future expansion. The problem consists of determining the magnitudes and phase angle of voltages at each bus and active and reactive power flow in each line.

In solving a power flow problem, the system is assumed to be operating under balanced conditions and a single-phase model is used. Four quantities are associated with each bus. These are voltage magnitude $|V|$, phase angle $\delta$, real power $P$, and reactive power $Q$. The system buses are generally classified into three types.

**Slack bus** One bus, known as *slack* or *swing bus*, is taken as reference where the magnitude and phase angle of the voltage are specified. This bus makes up the difference between the scheduled loads and generated power that are caused by the losses in the network.

**Load buses** At these buses the active and reactive powers are specified. The magnitude and the phase angle of the bus voltages are unknown. These buses are called P-Q buses.

**Regulated buses** These buses are the *generator buses*. They are also known as *voltage-controlled buses*. At these buses, the real power and voltage magnitude are specified. The phase angles of the voltages and the reactive power are to be determined. The limits on the value of the reactive power are also specified. These buses are called P-V buses.

### 6.4.1 POWER FLOW EQUATION

Consider a typical bus of a power system network as shown in Figure 6.7. Transmission lines are represented by their equivalent $\pi$ models where impedances have been converted to per unit admittances on a common MVA base.

Application of KCL to this bus results in

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n) \\ &= (y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n \quad (6.23) \end{aligned}$$

or

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i \qquad (6.24)$$

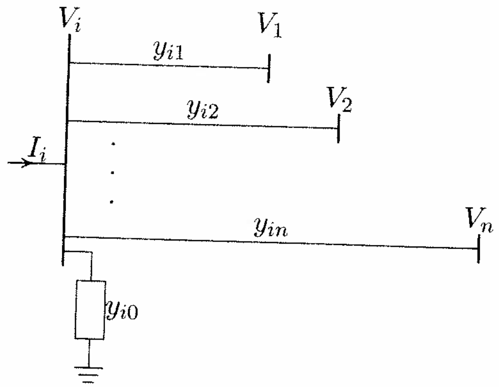



**FIGURE 6.7**
A typical bus of the power system.

The real and reactive power at bus $i$ is

$$P_i + jQ_i = V_i I_i^* \tag{6.25}$$

or

$$I_i = \frac{P_i - jQ_i}{V_i^*} \tag{6.26}$$

Substituting for $I_i$ in (6.24) yields

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i \tag{6.27}$$

From the above relation, the mathematical formulation of the power flow problem results in a system of algebraic nonlinear equations which must be solved by iterative techniques.

## 6.5 GAUSS-SEIDEL POWER FLOW SOLUTION

In the power flow study, it is necessary to solve the set of nonlinear equations represented by (6.27) for two unknown variables at each node. In the Gauss-Seidel method (6.27) is solved for $V_i$, and the iterative sequence becomes

$$V_i^{(k+1)} = \frac{\frac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} + \sum_{j=1}^{n} y_{ij} V_j^{(k)}}{\sum_{j=0}^{n} y_{ij}} \qquad j \neq i \tag{6.28}$$

where $y_{ij}$ shown in lowercase letters is the actual admittance in per unit. $P_i^{sch}$ and $Q_i^{sch}$ are the net real and reactive powers expressed in per unit. In writing the KCL, current entering bus $i$ was assumed positive. Thus, for buses where real and reactive powers are injected into the bus, such as generator buses, $P_i^{sch}$ and $Q_i^{sch}$ have positive values. For load buses where real and reactive powers are flowing away from the bus, $P_i^{sch}$ and $Q_i^{sch}$ have negative values. If (6.27) is solved for $P_i$ and $Q_i$, we have

$$P_i^{(k+1)} = \Re\{V_i^{*(k)} [V_i^{(k)} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{(k)}]\} \qquad j \neq i \tag{6.29}$$

$$Q_i^{(k+1)} = -\Im\{V_i^{*(k)} [V_i^{(k)} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{(k)}]\} \qquad j \neq i \tag{6.30}$$

The power flow equation is usually expressed in terms of the elements of the bus admittance matrix. Since the off-diagonal elements of the bus admittance matrix $Y_{bus}$, shown by uppercase letters, are $Y_{ij} = -y_{ij}$, and the diagonal elements are $Y_{ii} = \sum y_{ij}$, (6.28) becomes

$$V_i^{(k+1)} = \frac{\frac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} - \sum_{j \neq i} Y_{ij} V_j^{(k)}}{Y_{ii}} \tag{6.31}$$

and

$$P_i^{(k+1)} = \Re\{V_i^{*(k)} [V_i^{(k)} Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j^{(k)}]\} \quad j \neq i \tag{6.32}$$

$$Q_i^{(k+1)} = -\Im\{V_i^{*(k)} [V_i^{(k)} Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j^{(k)}]\} \quad j \neq i \tag{6.33}$$

$Y_{ii}$ includes the admittance to ground of line charging susceptance and any other fixed admittance to ground. In Section 6.7, a model is presented for transformers containing off-nominal ratio, which includes the effect of transformer tap setting.

Since both components of voltage are specified for the slack bus, there are $2(n-1)$ equations which must be solved by an iterative method. Under normal operating conditions, the voltage magnitude of buses are in the neighborhood of 1.0 per unit or close to the voltage magnitude of the slack bus. Voltage magnitude at load buses are somewhat lower than the slack bus value, depending on the reactive power demand, whereas the scheduled voltage at the generator buses are somewhat higher. Also, the phase angle of the load buses are below the reference angle in accordance to the real power demand, whereas the phase angle of the generator buses may be above the reference value depending on the amount of real power flowing into the bus. Thus, for the Gauss-Seidel method, an initial voltage estimate of $1.0 + j0.0$ for unknown voltages is satisfactory, and the converged solution correlates with the actual operating states.

For P-Q buses, the real and reactive powers $P_i^{sch}$ and $Q_i^{sch}$ are known. Starting with an initial estimate, (6.31) is solved for the real and imaginary components of voltage. For the voltage-controlled buses (P-V buses) where $P_i^{sch}$ and $|V_i|$ are specified, first (6.33) is solved for $Q_i^{(k+1)}$, and then is used in (6.31) to solve for $V_i^{(k+1)}$. However, since $|V_i|$ is specified, only the imaginary part of $V_i^{(k+1)}$ is retained, and its real part is selected in order to satisfy

$$(e_i^{(k+1)})^2 + (f_i^{(k+1)})^2 = |V_i|^2 \tag{6.34}$$

or

$$e_i^{(k+1)} = \sqrt{|V_i|^2 - (f_i^{(k+1)})^2} \tag{6.35}$$

where $e_i^{(k+1)}$ and $f_i^{(k+1)}$ are the real and imaginary components of the voltage $V_i^{(k+1)}$ in the iterative sequence.

The rate of convergence is increased by applying an acceleration factor to the approximate solution obtained from each iteration.

$$V_i^{(k+1)} = V_i^{(k)} + \alpha(V_{i\,cal}^{(k)} - V_i^{(k)}) \tag{6.36}$$

where $\alpha$ is the acceleration factor. Its value depends upon the system. The range of 1.3 to 1.7 is found to be satisfactory for typical systems.

The updated voltages immediately replace the previous values in the solution of the subsequent equations. The process is continued until changes in the real and imaginary components of bus voltages between successive iterations are within a specified accuracy, i.e.,

$$|e_i^{(k+1)} - e_i^{(k)}| \le \epsilon$$
$$|f_i^{(k+1)} - f_i^{(k)}| \le \epsilon \tag{6.37}$$

For the power mismatch to be reasonably small and acceptable, a very tight tolerance must be specified on both components of the voltage. A voltage accuracy in the range of 0.00001 to 0.00005 pu is satisfactory. In practice, the method for determining the completion of a solution is based on an accuracy index set up on the power mismatch. The iteration continues until the magnitude of the largest element in the $\Delta P$ and $\Delta Q$ columns is less than the specified value. A typical power mismatch accuracy is 0.001 pu

Once a solution is converged, the net real and reactive powers at the slack bus are computed from (6.32) and (6.33).

## 6.6 LINE FLOWS AND LOSSES

After the iterative solution of bus voltages, the next step is the computation of line flows and line losses. Consider the line connecting the two buses $i$ and $j$ in Figure 6.8. The line current $I_{ij}$, measured at bus $i$ and defined positive in the direction

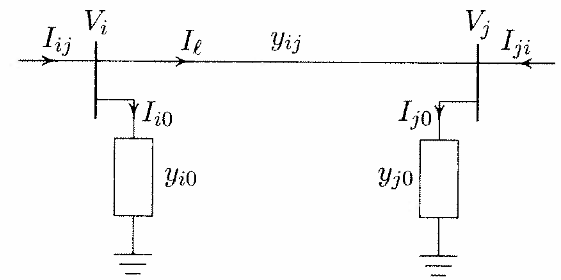



**FIGURE 6.8**
Transmission line model for calculating line flows.

$i \rightarrow j$ is given by

$$I_{ij} = I_\ell + I_{i0} = y_{ij}(V_i - V_j) + y_{i0}V_i \tag{6.38}$$

Similarly, the line current $I_{ji}$ measured at bus $j$ and defined positive in the direction $j \rightarrow i$ is given by

$$I_{ji} = -I_\ell + I_{j0} = y_{ij}(V_j - V_i) + y_{j0}V_j \tag{6.39}$$

The complex powers $S_{ij}$ from bus $i$ to $j$ and $S_{ji}$ from bus $j$ to $i$ are

$$S_{ij} = V_i I_{ij}^* \tag{6.40}$$
$$S_{ji} = V_j I_{ji}^* \tag{6.41}$$

The power loss in line $i - j$ is the algebraic sum of the power flows determined from (6.40) and (6.41), i.e.,

$$S_{L\,ij} = S_{ij} + S_{ji} \tag{6.42}$$

The power flow solution by the Gauss-Seidel method is demonstrated in the following two examples.

**Example 6.7** (chp6ex7)

Figure 6.9 shows the one-line diagram of a simple three-bus power system with generation at bus 1. The magnitude of voltage at bus 1 is adjusted to 1.05 per unit. The scheduled loads at buses 2 and 3 are as marked on the diagram. Line impedances are marked in per unit on a 100-MVA base and the line charging susceptances are neglected.

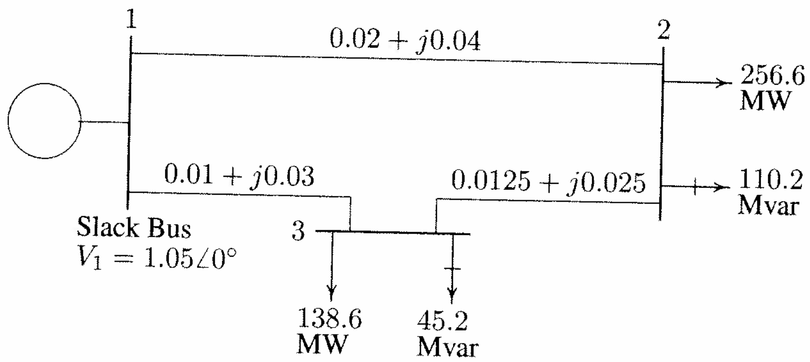



**FIGURE 6.9**
One-line diagram of Example 6.7 (impedances in pu on 100-MVA base).

(a) Using the Gauss-Seidel method, determine the phasor values of the voltage at the load buses 2 and 3 (P-Q buses) accurate to four decimal places.
(b) Find the slack bus real and reactive power.
(c) Determine the line flows and line losses. Construct a power flow diagram showing the direction of line flow.

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The admittances are marked on the network shown in Figure 6.10.
At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(256.6 + j110.2)}{100} = -2.566 - j1.102 \quad \text{pu}$$

$$S_3^{sch} = -\frac{(138.6 + j45.2)}{100} = -1.386 - j0.452 \quad \text{pu}$$

Since the actual admittances are readily available in Figure 6.10, for hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, $V_2$ and $V_3$ are computed from (6.28) as follows

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$

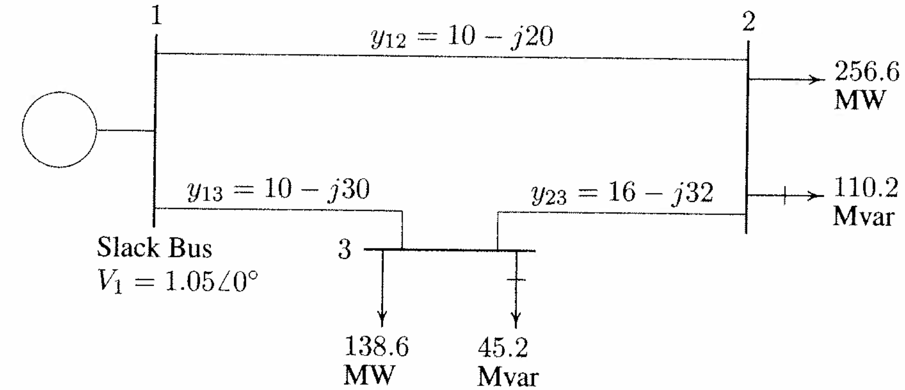



**FIGURE 6.10**
One-line diagram of Example 6.7 (admittances in pu on 100-MVA base).

$$= \frac{\frac{-2.566 + j1.102}{1.0 - j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.0 + j0)}{(26 - j52)}$$

$$= 0.9825 - j0.0310$$

and

$$V_3^{(1)} = \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}}$$

$$= \frac{\frac{-1.386 + j0.452}{1 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.9825 - j0.0310)}{(26 - j62)}$$

$$= 1.0011 - j0.0353$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-2.566 + j1.102}{0.9825 + j0.0310} + (10 - j20)(1.05 + j0) + (16 - j32)(1.0011 - j0.0353)}{(26 - j52)}$$

$$= 0.9816 - j0.0520$$

and

$$V_3^{(2)} = \frac{\frac{-1.386 + j0.452}{1.0011 + j0.0353} + (10 - j30)(1.05 + j0) + (16 - j32)(0.9816 - j0.052)}{(26 - j62)}$$

$$= 1.0008 - j0.0459$$

The process is continued and a solution is converged with an accuracy of $5 \times 10^{-5}$ per unit in seven iterations as given below.

$$V_2^{(3)} = 0.9808 - j0.0578 \qquad V_3^{(3)} = 1.0004 - j0.0488$$

$$V_2^{(4)} = 0.9803 - j0.0594 \qquad V_3^{(4)} = 1.0002 - j0.0497$$
$$V_2^{(5)} = 0.9801 - j0.0598 \qquad V_3^{(5)} = 1.0001 - j0.0499$$
$$V_2^{(6)} = 0.9801 - j0.0599 \qquad V_3^{(6)} = 1.0000 - j0.0500$$
$$V_2^{(7)} = 0.9800 - j0.0600 \qquad V_3^{(7)} = 1.0000 - j0.0500$$

The final solution is

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle{-3.5035^\circ} \quad \text{pu}$$
$$V_3 = 1.0000 - j0.0500 = 1.00125\angle{-2.8624^\circ} \quad \text{pu}$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$\begin{aligned} P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\ &= 1.05[1.05(20 - j50) - (10 - j20)(0.98 - j.06) - \\ &\quad (10 - j30)(1.0 - j0.05)] \\ &= 4.095 - j1.890 \end{aligned}$$

or the slack bus real and reactive powers are $P_1 = 4.095$ pu $= 409.5$ MW and $Q_1 = 1.890$ pu $= 189$ Mvar.

(c) To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1.05 + j0) - (0.98 - j0.06)] = 1.9 - j0.8$$
$$I_{21} = -I_{12} = -1.9 + j0.8$$
$$I_{13} = y_{13}(V_1 - V_3) = (10 - j30)[(1.05 + j0) - (1.0 - j0.05)] = 2.0 - j1.0$$
$$I_{31} = -I_{13} = -2.0 + j1.0$$
$$I_{23} = y_{23}(V_2 - V_3) = (16 - j32)[(0.98 - j0.06) - (1 - j0.05)] = -.64 + j.48$$
$$I_{32} = -I_{23} = 0.64 - j0.48$$

The line flows are

$$\begin{aligned} S_{12} = V_1 I_{12}^* &= (1.05 + j0.0)(1.9 + j0.8) = 1.995 + j0.84 \quad \text{pu} \\ &= 199.5 \ \text{MW} + j84.0 \ \text{Mvar} \\ S_{21} = V_2 I_{21}^* &= (0.98 - j0.06)(-1.9 - j0.8) = -1.91 - j0.67 \quad \text{pu} \\ &= -191.0 \ \text{MW} - j67.0 \ \text{Mvar} \\ S_{13} = V_1 I_{13}^* &= (1.05 + j0.0)(2.0 + j1.0) = 2.1 + j1.05 \quad \text{pu} \\ &= 210.0 \ \text{MW} + j105.0 \ \text{Mvar} \end{aligned}$$



$$\begin{aligned} S_{31} = V_3 I_{31}^* &= (1.0 - j0.05)(-2.0 - j1.0) = -2.05 - j0.90 \quad \text{pu} \\ &= -205.0 \ \text{MW} - j90.0 \ \text{Mvar} \\ S_{23} = V_2 I_{23}^* &= (0.98 - j0.06)(-0.656 + j0.48) = -0.656 - j0.432 \quad \text{pu} \\ &= -65.6 \ \text{MW} - j43.2 \ \text{Mvar} \\ S_{32} = V_3 I_{32}^* &= (1.0 - j0.05)(0.64 + j0.48) = 0.664 + j0.448 \quad \text{pu} \\ &= 66.4 \ \text{MW} + j44.8 \ \text{Mvar} \end{aligned}$$

and the line losses are

$$S_{L\,12} = S_{12} + S_{21} = 8.5 \ \text{MW} + j17.0 \ \text{Mvar}$$
$$S_{L\,13} = S_{13} + S_{31} = 5.0 \ \text{MW} + j15.0 \ \text{Mvar}$$
$$S_{L\,23} = S_{23} + S_{32} = 0.8 \ \text{MW} + j1.60 \ \text{Mvar}$$

The power flow diagram is shown in Figure 6.11, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

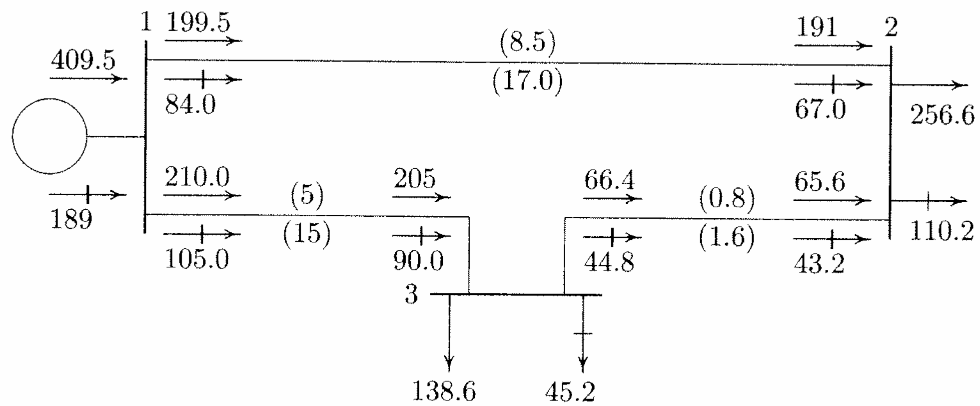



**FIGURE 6.11**
Power flow diagram of Example 6.7 (powers in MW and Mvar).

**Example 6.8** (chp6ex8)

Figure 6.12 shows the one-line diagram of a simple three-bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100 MVA base, and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-Seidel method including line flows and line losses.

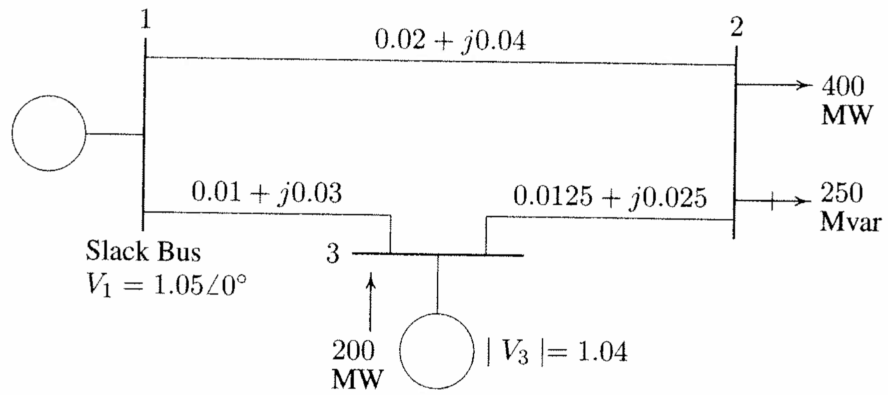



**FIGURE 6.12**
One-line diagram of Example 6.8 (impedances in pu on 100-MVA base).

Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$
$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

Bus 1 is taken as the reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.04 + j0.0$, $V_2$ and $V_3$ are computed from (6.28).

$$\begin{aligned} V_2^{(1)} &= \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0 + j2.5}{1.0 - j0} + (10 - j20)(1.05 + j0) + (16 - j32)(1.04 + j0)}{(26 - j52)} \\ &= 0.97462 - j0.042307 \end{aligned}$$

Bus 3 is a regulated bus where voltage magnitude and real power are specified. For the voltage-controlled bus, first the reactive power is computed from (6.30)

$$\begin{aligned} Q_3^{(1)} &= -\Im\{V_3^{*(0)}[V_3^{(0)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(1)}]\} \\ &= -\Im\{(1.04 - j0)[(1.04 + j0)(26 - j62) - (10 - j30)(1.05 + j0) - \\ &\quad (16 - j32)(0.97462 - j0.042307)]\} \\ &= 1.16 \end{aligned}$$

The value of $Q_3^{(1)}$ is used as $Q_3^{sch}$ for the computation of voltage at bus 3. The complex voltage at bus 3, denoted by $V_{c3}^{(1)}$, is calculated

$$\begin{aligned} V_{c3}^{(1)} &= \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13} + y_{23}} \\ &= \frac{\frac{2.0 - j1.16}{1.04 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.97462 - j0.042307)}{(26 - j62)} \\ &= 1.03783 - j0.005170 \end{aligned}$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(1)}$ is retained, i.e, $f_3^{(1)} = -0.005170$, and its real part is obtained from

$$e_3^{(1)} = \sqrt{(1.04)^2 - (0.005170)^2} = 1.039987$$

Thus

$$V_3^{(1)} = 1.039987 - j0.005170$$

For the second iteration, we have

$$\begin{aligned} V_2^{(2)} &= \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(1)}} + y_{12}V_1 + y_{23}V_3^{(1)}}{y_{12} + y_{23}} \\ &= \frac{\frac{-4.0 + j2.5}{.97462 + j.042307} + (10 - j20)(1.05) + (16 - j32)(1.039987 + j0.005170)}{(26 - j52)} \\ &= 0.971057 - j0.043432 \end{aligned}$$

$$\begin{aligned} Q_3^{(2)} &= -\Im\{V_3^{*(1)}[V_3^{(1)}(y_{13} + y_{23}) - y_{13}V_1 - y_{23}V_2^{(2)}]\} \\ &= -\Im\{(1.039987 + j0.005170)[(1.039987 - j0.005170)(26 - j62) - \\ &\quad (10 - j30)(1.05 + j0) - (16 - j32)(0.971057 - j0.043432)]\} \\ &= 1.38796 \end{aligned}$$

$$\begin{aligned} V_{c3}^{(2)} &= \frac{\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{*(1)}} + y_{13}V_1 + y_{23}V_2^{(2)}}{y_{13} + y_{23}} \\ &= \frac{\frac{2.0 - j1.38796}{1.039987 + j0.00517} + (10 - j30)(1.05) + (16 - j32)(.971057 - j.043432)}{(26 - j62)} \\ &= 1.03908 - j0.00730 \end{aligned}$$

Since $|V_3|$ is held constant at 1.04 pu, only the imaginary part of $V_{c3}^{(2)}$ is retained, i.e, $f_3^{(2)} = -0.00730$, and its real part is obtained from

$$e_3^{(2)} = \sqrt{(1.04)^2 - (0.00730)^2} = 1.039974$$

or

$$V_3^{(2)} = 1.039974 - j0.00730$$

The process is continued and a solution is converged with an accuracy of $5 \times 10^{-5}$ pu in seven iterations as given below.

$$V_2^{(3)} = 0.97073 - j0.04479 \quad Q_3^{(3)} = 1.42904 \quad V_3^{(3)} = 1.03996 - j0.00833$$
$$V_2^{(4)} = 0.97065 - j0.04533 \quad Q_3^{(4)} = 1.44833 \quad V_3^{(4)} = 1.03996 - j0.00873$$
$$V_2^{(5)} = 0.97062 - j0.04555 \quad Q_3^{(5)} = 1.45621 \quad V_3^{(5)} = 1.03996 - j0.00893$$
$$V_2^{(6)} = 0.97061 - j0.04565 \quad Q_3^{(6)} = 1.45947 \quad V_3^{(6)} = 1.03996 - j0.00900$$
$$V_2^{(7)} = 0.97061 - j0.04569 \quad Q_3^{(7)} = 1.46082 \quad V_3^{(7)} = 1.03996 - j0.00903$$

The final solution is

$$V_2 = 0.97168\angle{-2.6948^\circ} \quad \text{pu}$$

$$S_3 = 2.0 + j1.4617 \quad \text{pu}$$
$$V_3 = 1.04\angle{-.498^\circ} \quad \text{pu}$$
$$S_1 = 2.1842 + j1.4085 \quad \text{pu}$$

Line flows and line losses are computed as in Example 6.7, and the results expressed in MW and Mvar are

$$S_{12} = 179.36 + j118.734 \quad S_{21} = -170.97 - j101.947 \quad S_{L\,12} = 8.39 + j16.79$$
$$S_{13} = 39.06 + j22.118 \quad S_{31} = -38.88 - j\,21.569 \quad S_{L\,13} = 0.18 + j0.548$$
$$S_{23} = -229.03 - j148.05 \quad S_{32} = 238.88 + j167.746 \quad S_{L\,23} = 9.85 + j19.69$$

The power flow diagram is shown in Figure 6.13, where real power direction is indicated by $\rightarrow$ and the reactive power direction is indicated by $\mapsto$. The values within parentheses are the real and reactive losses in the line.

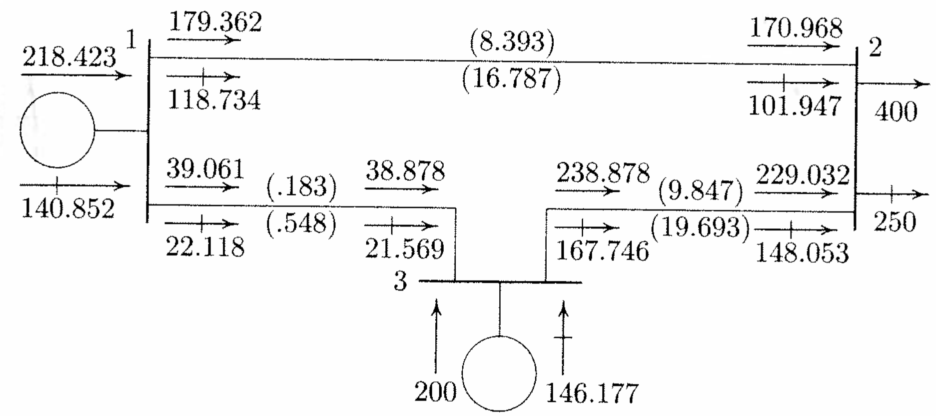



**FIGURE 6.13**
Power flow diagram of Example 6.8 (powers in MW and Mvar).

## 6.7 TAP CHANGING TRANSFORMERS

In Section 2.6 it was shown that the flow of real power along a transmission line is determined by the angle difference of the terminal voltages, and the flow of reactive power is determined mainly by the magnitude difference of terminal voltages. Real and reactive powers can be controlled by use of tap changing transformers and regulating transformers.

In a tap changing transformer, when the ratio is at the nominal value, the transformer is represented by a series admittance $y_t$ in per unit. With off-nominal ratio, the per unit admittance is different from both sides of the transformer, and the admittance must be modified to include the effect of the off-nominal ratio. Consider a transformer with admittance $y_t$ in series with an ideal transformer representing the off-nominal tap ratio $1{:}a$ as shown in Figure 6.14. $y_t$ is the admittance in per unit based on the nominal turn ratio and $a$ is the per unit off-nominal tap position allowing for small adjustment in voltage of usually $\pm 10$ percent. In the case of phase shifting transformers, $a$ is a complex number. Consider a fictitious bus $x$ between the turn ratio and admittance of the transformer. Since the complex power on either side of the ideal transformer is the same, it follows that if the voltage goes through a positive phase angle shift, the current will go through a negative phase angle shift. Thus, for the assumed direction of currents, we have

$$V_x = \frac{1}{a}V_j \qquad (6.43)$$
$$I_i = -a^* I_j \qquad (6.44)$$

The current $I_i$ is given by

$$I_i = y_t(V_i - V_x)$$

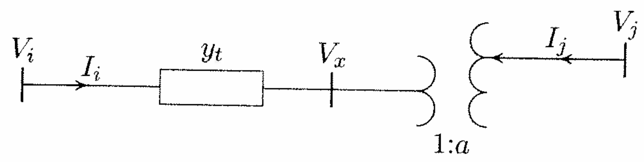



**FIGURE 6.14**
Transformer with tap setting ratio $a$:1

Substituting for $V_x$, we have

$$I_i = y_t V_i - \frac{y_t}{a} V_j \tag{6.45}$$

Also, from (6.44) we have

$$I_j = -\frac{1}{a^*} I_i$$

substituting for $I_i$ from (6.45) we have

$$I_j = -\frac{y_t}{a^*} V_i + \frac{y_t}{|a|^2} V_j \tag{6.46}$$

writing (6.45) and (6.46) in matrix form results in

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_t & -\frac{y_t}{a} \\ -\frac{y_t}{a^*} & \frac{y_t}{|a|^2} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \tag{6.47}$$

For the case when $a$ is real, the $\pi$ model shown in Figure 6.15 represents the admittance matrix in (6.47). In the $\pi$ model, the left side corresponds to the non-tap side and the right side corresponds to the tap side of the transformer.

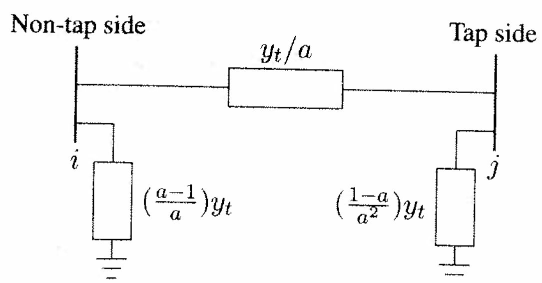



**FIGURE 6.15**
Equivalent circuit for a tap changing transformer.

## 6.8 POWER FLOW PROGRAMS

Several computer programs have been developed for the power flow solution of practical systems. Each method of solution consists of four programs. The program for the Gauss-Seidel method is **lfgauss**, which is preceded by **lfybus**, and is followed by **busout** and **lineflow**. Programs **lfybus**, **busout**, and **lineflow** are designed to be used with two more power flow programs. These are **lfnewton** for the Newton-Raphson method and **decouple** for the fast decoupled method. The following is a brief description of the programs used in the Gauss-Seidel method.

**lfybus** This program requires the line and transformer parameters and transformer tap settings specified in the input file named **linedata**. It converts impedances to admittances and obtains the bus admittance matrix. The program is designed to handle parallel lines.

**lfgauss** This program obtains the power flow solution by the Gauss-Seidel method and requires the files named **busdata** and **linedata**. It is designed for the direct use of load and generation in MW and Mvar, bus voltages in per unit, and angle in degrees. Loads and generation are converted to per unit quantities on the base MVA selected. A provision is made to maintain the generator reactive power of the voltage-controlled buses within their specified limits. The violation of reactive power limit may occur if the specified voltage is either too high or too low. After a few iterations ($10^{th}$ iteration in the Gauss method), the var calculated at the generator buses are examined. If a limit is reached, the voltage magnitude is adjusted in steps of 0.5 percent up to $\pm 5$ percent to bring the var demand within the specified limits.

**busout** This program produces the bus output result in a tabulated form. The bus output result includes the voltage magnitude and angle, real and reactive power of generators and loads, and the shunt capacitor/reactor Mvar. Total generation and total load are also included as outlined in the sample case.

**lineflow** This program prepares the line output data. It is designed to display the active and reactive power flow entering the line terminals and line losses as well as the net power at each bus. Also included are the total real and reactive losses in the system. The output of this portion is also shown in the sample case.

## 6.9 DATA PREPARATION

In order to perform a power flow analysis by the Gauss-Seidel method in the *MATLAB* environment, the following variables must be defined: power system base MVA, power mismatch accuracy, acceleration factor, and maximum number of iterations. The name (in lowercase letters) reserved for these variables are **basemva**, **accuracy**, **accel**, and **maxiter**, respectively. Typical values are as follows:

```
basemva = 100;    accuracy = 0.001;
accel   = 1.6;    maxiter = 80;
```

The initial step in the preparation of input file is the numbering of each bus. Buses are numbered sequentially. Although the numbers are sequentially assigned, the buses need not be entered in sequence. In addition, the following data files are required.

**BUS DATA FILE – busdata** The format for the bus entry is chosen to facilitate the required data for each bus in a single row. The information required must be included in a matrix called **busdata**. Column 1 is the bus number. Column 2 contains the bus code. Columns 3 and 4 are voltage magnitude in per unit and phase angle in degrees. Columns 5 and 6 are load MW and Mvar. Column 7 through 10 are MW, Mvar, minimum Mvar and maximum Mvar of generation, in that order. The last column is the injected Mvar of shunt capacitors. The bus code entered in column 2 is used for identifying load, voltage-controlled, and slack buses as outlined below:

**1** This code is used for the slack bus. The only necessary information for this bus is the voltage magnitude and its phase angle.

**0** This code is used for load buses. The loads are entered positive in megawatts and megavars. For this bus, initial voltage estimate must be specified. This is usually 1 and 0 for voltage magnitude and phase angle, respectively. If voltage magnitude and phase angle for this type of bus are specified, they will be taken as the initial starting voltage for that bus instead of a flat start of 1 and 0.

**2** This code is used for the voltage-controlled buses. For this bus, voltage magnitude, real power generation in megawatts, and the minimum and maximum limits of the megavar demand must be specified.

**LINE DATA FILE – linedata** Lines are identified by the node-pair method. The information required must be included in a matrix called **linedata**. Columns 1 and 2 are the line bus numbers. Columns 3 through 5 contain the line resistance, reactance, and one-half of the total line charging susceptance in per unit on the specified MVA base. The last column is for the transformer tap setting; for lines, 1 must be entered in this column. The lines may be entered in any sequence or order with the only restriction being that if the entry is a transformer, the left bus number is assumed to be the tap side of the transformer.

The IEEE 30 bus system is used to demonstrate the data preparation and the use of the power flow programs by the Gauss-Seidel method.

**Example 6.9** (chp6ex9)

Figure 6.16 is part of the American Electric Power Service Corporation network which is being made available to the electric utility industry as a standard test case for evaluating various analytical methods and computer programs for the solution of power system problems. Use the **lfgauss** program to obtain the power solution by the Gauss-Seidel method. Bus 1 is taken as the slack bus with its voltage adjusted to $1.06\angle 0^\circ$ pu. The data for the voltage-controlled buses is

**Regulated Bus Data**

| Bus No. | Voltage Magnitude | Min. Mvar Capacity | Max. Mvar Capacity |
|---|---|---|---|
| 2 | 1.043 | -40 | 50 |
| 5 | 1.010 | -40 | 40 |
| 8 | 1.010 | -10 | 40 |
| 11 | 1.082 | -6 | 24 |
| 13 | 1.071 | -6 | 24 |

Transformer tap setting are given in the table below. The left bus number is assumed to be the tap side of the transformer.

**Transformer Data**

| Transformer Designation | Tap Setting pu |
|---|---|
| 4 – 12 | 0.932 |
| 6 – 9 | 0.978 |
| 6 – 10 | 0.969 |
| 28 – 27 | 0.968 |

The data for the injected $Q$ due to shunt capacitors is

**Injected Q due to Capacitors**

| Bus No. | Mvar |
|---|---|
| 10 | 19 |
| 24 | 4.3 |

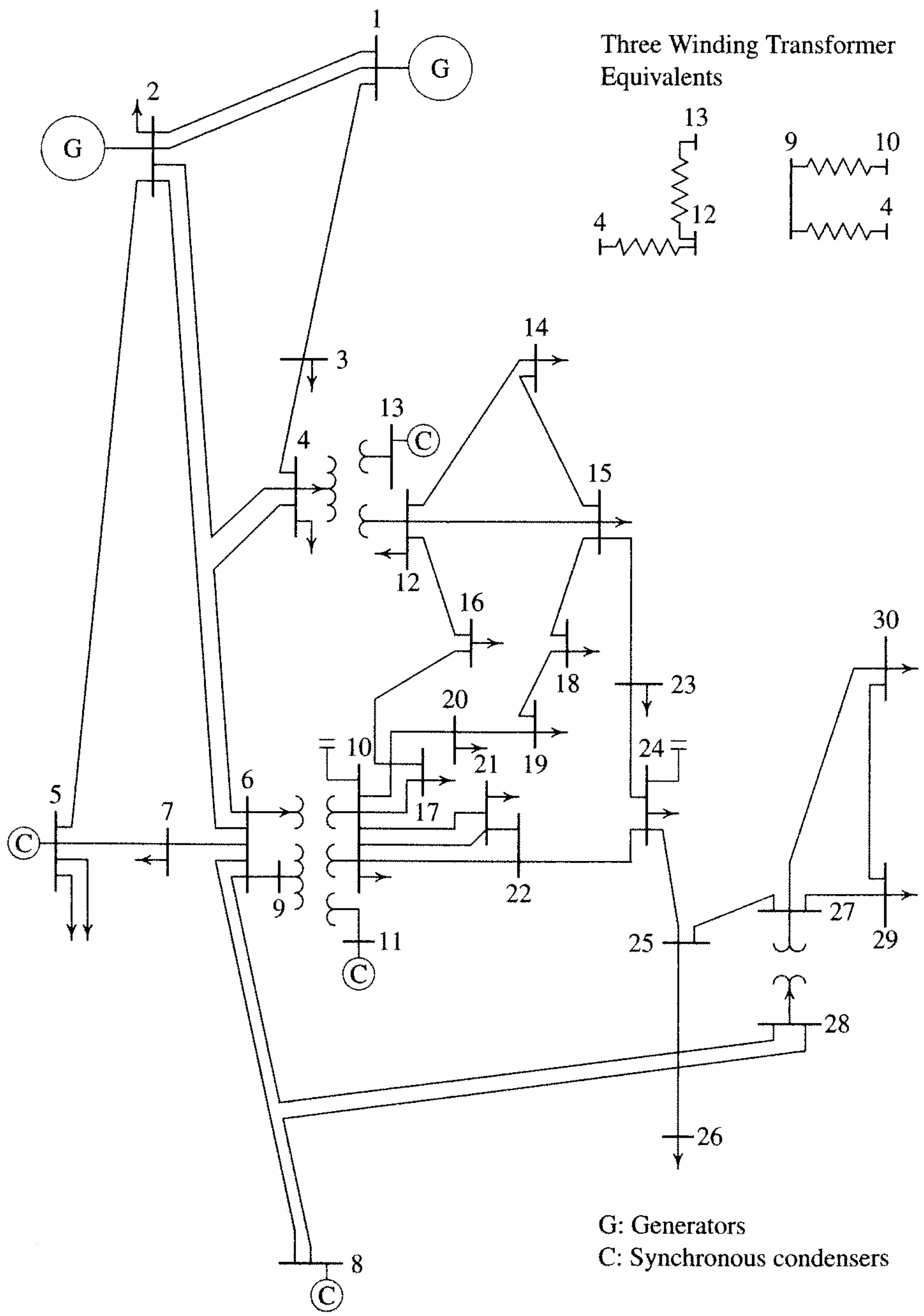



**FIGURE 6.16**
30-bus IEEE sample system.

Generation and loads are as given in the data prepared for use in the *MATLAB* environment in the matrix defined as **busdata**. Code 0, code 1, and code 2 are used for the load buses, the slack bus and the voltage-controlled buses, respectively. Values for **basemva**, **accuracy**, **accel** and **maxiter** must be specified. Line data are as given in the matrix called **linedata**. The last column of this data must contain 1 for lines, or the tap setting values for transformers with off-nominal turn ratio. The control commands required are **lfybus**, **lfgauss** and **lineflow**. A **diary** command may be used to save the output to the specified file name. The power flow data and the commands required are as follows.

```
clear        % clears all variables from workspace.
basemva = 100;  accuracy = 0.001; accel = 1.8; maxiter = 100;
%   IEEE 30-BUS TEST SYSTEM (American Electric Power)
%       Bus Bus Voltage Angle --Load--  ---Generator---Injected
%        No code Mag. Degree MW   Mvar   MW  Mvar Qmin Qmax Mvar
busdata=[1  1  1.06   0      0.0  0.0   0.0  0.0   0   0      0
         2  2  1.043  0    21.70 12.7  40.0  0.0 -40  50      0
         3  0  1.0    0      2.4  1.2   0.0  0.0   0   0      0
         4  0  1.06   0      7.6  1.6   0.0  0.0   0   0      0
         5  2  1.01   0     94.2 19.0   0.0  0.0 -40  40      0
         6  0  1.0    0      0.0  0.0   0.0  0.0   0   0      0
         7  0  1.0    0     22.8 10.9   0.0  0.0   0   0      0
         8  2  1.01   0     30.0 30.0   0.0  0.0 -10  40      0
         9  0  1.0    0      0.0  0.0   0.0  0.0   0   0      0
        10  0  1.0    0      5.8  2.0   0.0  0.0   0   0     19
        11  2  1.082  0      0.0  0.0   0.0  0.0  -6  24      0
        12  0  1.0    0     11.2  7.5   0    0     0   0      0
        13  2  1.071  0      0.0  0.0   0    0    -6  24      0
        14  0  1.0    0      6.2  1.6   0    0     0   0      0
        15  0  1.0    0      8.2  2.5   0    0     0   0      0
        16  0  1.0    0      3.5  1.8   0    0     0   0      0
        17  0  1.0    0      9.0  5.8   0    0     0   0      0
        18  0  1.0    0      3.2  0.9   0    0     0   0      0
        19  0  1.0    0      9.5  3.4   0    0     0   0      0
        20  0  1.0    0      2.2  0.7   0    0     0   0      0
        21  0  1.0    0     17.5 11.2   0    0     0   0      0
        22  0  1.0    0      0.0  0.0   0    0     0   0      0
        23  0  1.0    0      3.2  1.6   0    0     0   0      0
        24  0  1.0    0      8.7  6.7   0    0     0   0    4.3
        25  0  1.0    0      0.0  0.0   0    0     0   0      0
        26  0  1.0    0      3.5  2.3   0    0     0   0      0
        27  0  1.0    0      0.0  0.0   0    0     0   0      0
        28  0  1.0    0      0.0  0.0   0    0     0   0      0
        29  0  1.0    0      2.4  0.9   0    0     0   0      0
        30  0  1.0    0     10.6  1.9   0    0     0   0     0];
```

```
% Line Data
%
%         Bus  bus   R        X         1/2 B  1 for Line code or
%         nl   nr    pu       pu        pu     tap setting value
linedata=[1    2    0.0192   0.0575    0.02640    1
          1    3    0.0452   0.1852    0.02040    1
          2    4    0.0570   0.1737    0.01840    1
          3    4    0.0132   0.0379    0.00420    1
          2    5    0.0472   0.1983    0.02090    1
          2    6    0.0581   0.1763    0.01870    1
          4    6    0.0119   0.0414    0.00450    1
          5    7    0.0460   0.1160    0.01020    1
          6    7    0.0267   0.0820    0.00850    1
          6    8    0.0120   0.0420    0.00450    1
          6    9    0.0      0.2080    0.0      0.978
          6   10    0.0      0.5560    0.0      0.969
          9   11    0.0      0.2080    0.0        1
          9   10    0.0      0.1100    0.0        1
          4   12    0.0      0.2560    0.0      0.932
         12   13    0.0      0.1400    0.0        1
         12   14    0.1231   0.2559    0.0        1
         12   15    0.0662   0.1304    0.0        1
         12   16    0.0945   0.1987    0.0        1
         14   15    0.2210   0.1997    0.0        1
         16   17    0.0824   0.1923    0.0        1
         15   18    0.1073   0.2185    0.0        1
         18   19    0.0639   0.1292    0.0        1
         19   20    0.0340   0.0680    0.0        1
         10   20    0.0936   0.2090    0.0        1
         10   17    0.0324   0.0845    0.0        1
         10   21    0.0348   0.0749    0.0        1
         10   22    0.0727   0.1499    0.0        1
         21   22    0.0116   0.0236    0.0        1
         15   23    0.1000   0.2020    0.0        1
         22   24    0.1150   0.1790    0.0        1
         23   24    0.1320   0.2700    0.0        1
         24   25    0.1885   0.3292    0.0        1
         25   26    0.2544   0.3800    0.0        1
         25   27    0.1093   0.2087    0.0        1
         28   27    0.0000   0.3960    0.0      0.968
         27   29    0.2198   0.4153    0.0        1
         27   30    0.3202   0.6027    0.0        1
         29   30    0.2399   0.4533    0.0        1
          8   28    0.0636   0.2000    0.0214     1
          6   28    0.0169   0.0599    0.065      1];
%
    lfybus                     % Forms the bus admittance matrix
    lfgauss        % Power flow solution by Gauss-Seidel method
    busout        % Prints the power flow solution on the screen
    lineflow  % Computes and displays the line flow and losses
```

The **lfgauss**, **busout** and the **lineflow** produce the following tabulated results.

```
                Power Flow Solution by Gauss-Seidel Method
                  Maximum Power mismatch = 0.000951884
                         No. of iterations = 34

Bus Voltage  Angle    ------Load------    --Generation--  Injected
No.  Mag.    Degree     MW       Mvar       MW      Mvar     Mvar

 1   1.060    0.000    0.000    0.000   260.950  -17.010    0.00
 2   1.043   -5.496   21.700   12.700    40.000   48.826    0.00
 3   1.022   -8.002    2.400    1.200     0.000    0.000    0.00
 4   1.013   -9.659    7.600    1.600     0.000    0.000    0.00
 5   1.010  -14.380   94.200   19.000     0.000   35.995    0.00
 6   1.012  -11.396    0.000    0.000     0.000    0.000    0.00
 7   1.003  -13.149   22.800   10.900     0.000    0.000    0.00
 8   1.010  -12.114   30.000   30.000     0.000   30.759    0.00
 9   1.051  -14.432    0.000    0.000     0.000    0.000    0.00
10   1.044  -16.024    5.800    2.000     0.000    0.000   19.00
11   1.082  -14.432    0.000    0.000     0.000   16.113    0.00
12   1.057  -15.301   11.200    7.500     0.000    0.000    0.00
13   1.071  -15.300    0.000    0.000     0.000   10.406    0.00
14   1.043  -16.190    6.200    1.600     0.000    0.000    0.00
15   1.038  -16.276    8.200    2.500     0.000    0.000    0.00
16   1.045  -15.879    3.500    1.800     0.000    0.000    0.00
17   1.039  -16.187    9.000    5.800     0.000    0.000    0.00
18   1.028  -16.881    3.200    0.900     0.000    0.000    0.00
19   1.025  -17.049    9.500    3.400     0.000    0.000    0.00
20   1.029  -16.851    2.200    0.700     0.000    0.000    0.00
21   1.032  -16.468   17.500   11.200     0.000    0.000    0.00
22   1.033  -16.455    0.000    0.000     0.000    0.000    0.00
23   1.027  -16.660    3.200    1.600     0.000    0.000    0.00
24   1.022  -16.829    8.700    6.700     0.000    0.000    4.30
25   1.019  -16.423    0.000    0.000     0.000    0.000    0.00
26   1.001  -16.835    3.500    2.300     0.000    0.000    0.00
27   1.026  -15.913    0.000    0.000     0.000    0.000    0.00
28   1.011  -12.056    0.000    0.000     0.000    0.000    0.00
```

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 29 | 1.006 | -17.133 | 2.400 | 0.900 | 0.000 | 0.000 | 0.00 |
| 30 | 0.994 | -18.016 | 10.600 | 1.900 | 0.000 | 0.000 | 0.00 |
| Total | | | 283.400 | 126.200 | 300.950 | 125.089 | 23.30 |

Line Flow and Losses

| --Line-- from | to | Power at bus & line flow MW | Mvar | MVA | --Line loss-- MW | Mvar | Transformer tap |
|---|---|---|---|---|---|---|---|
| 1 | | 260.950 | -17.010 | 261.504 | | | |
| | 2 | 177.743 | -22.140 | 179.117 | 5.461 | 10.517 | |
| | 3 | 83.197 | 5.125 | 83.354 | 2.807 | 7.079 | |
| 2 | | 18.300 | 36.126 | 40.497 | | | |
| | 1 | -172.282 | 32.657 | 175.350 | 5.461 | 10.517 | |
| | 4 | 45.702 | 2.720 | 45.783 | 1.106 | -0.519 | |
| | 5 | 82.990 | 1.704 | 83.008 | 2.995 | 8.178 | |
| | 6 | 61.905 | -0.966 | 61.913 | 2.047 | 2.263 | |
| 3 | | -2.400 | -1.200 | 2.683 | | | |
| | 1 | -80.390 | 1.954 | 80.414 | 2.807 | 7.079 | |
| | 4 | 78.034 | -3.087 | 78.095 | 0.771 | 1.345 | |
| 4 | | -7.600 | -1.600 | 7.767 | | | |
| | 2 | -44.596 | -3.239 | 44.713 | 1.106 | -0.519 | |
| | 3 | -77.263 | 4.432 | 77.390 | 0.771 | 1.345 | |
| | 6 | 70.132 | -17.624 | 72.313 | 0.605 | 1.181 | |
| | 12 | 44.131 | 14.627 | 46.492 | 0.000 | 4.686 | 0.932 |
| 5 | | -94.200 | 16.995 | 95.721 | | | |
| | 2 | -79.995 | 6.474 | 80.256 | 2.995 | 8.178 | |
| | 7 | -14.210 | 10.467 | 17.649 | 0.151 | -1.687 | |
| 6 | | 0.000 | 0.000 | 0.000 | | | |
| | 2 | -59.858 | 3.229 | 59.945 | 2.047 | 2.263 | |
| | 4 | -69.527 | 18.805 | 72.026 | 0.605 | 1.181 | |
| | 7 | 37.537 | -1.915 | 37.586 | 0.368 | -0.598 | |
| | 8 | 29.534 | -3.712 | 29.766 | 0.103 | -0.558 | |
| | 9 | 27.687 | -7.318 | 28.638 | 0.000 | 1.593 | 0.978 |
| | 10 | 15.828 | 0.656 | 15.842 | 0.000 | 1.279 | 0.969 |
| | 28 | 18.840 | -9.575 | 21.134 | 0.060 | -13.085 | |
| 7 | | -22.800 | -10.900 | 25.272 | | | |
| | 5 | 14.361 | -12.154 | 18.814 | 0.151 | -1.687 | |
| | 6 | -37.170 | 1.317 | 37.193 | 0.368 | -0.598 | |
| 8 | | -30.000 | 0.759 | 30.010 | | | |
| | 6 | -29.431 | 3.154 | 29.599 | 0.103 | -0.558 | |
| | 28 | -0.570 | -2.366 | 2.433 | 0.000 | -4.368 | |
| 9 | | 0.000 | 0.000 | 0.000 | | | |
| | 6 | -27.687 | 8.911 | 29.086 | 0.000 | 1.593 | |
| | 11 | 0.003 | -15.653 | 15.653 | -0.000 | 0.461 | |
| | 10 | 27.731 | 6.747 | 28.540 | 0.000 | 0.811 | |
| 10 | | -5.800 | 17.000 | 17.962 | | | |
| | 6 | -15.828 | 0.623 | 15.840 | 0.000 | 1.279 | |
| | 9 | -27.731 | -5.936 | 28.359 | 0.000 | 0.811 | |
| | 20 | 9.018 | 3.569 | 9.698 | 0.081 | 0.180 | |
| | 17 | 5.347 | 4.393 | 6.920 | 0.014 | 0.037 | |
| | 21 | 15.723 | 9.846 | 18.551 | 0.110 | 0.236 | |
| | 22 | 7.582 | 4.487 | 8.811 | 0.052 | 0.107 | |
| 11 | | 0.000 | 16.113 | 16.113 | | | |
| | 9 | -0.003 | 16.114 | 16.114 | -0.000 | 0.461 | |
| 12 | | -11.200 | -7.500 | 13.479 | | | |
| | 4 | -44.131 | -9.941 | 45.237 | 0.000 | 4.686 | |
| | 13 | -0.021 | -10.274 | 10.274 | 0.000 | 0.132 | |
| | 14 | 7.852 | 2.428 | 8.219 | 0.074 | 0.155 | |
| | 15 | 17.852 | 6.968 | 19.164 | 0.217 | 0.428 | |
| | 16 | 7.206 | 3.370 | 7.955 | 0.053 | 0.112 | |
| 13 | | 0.000 | 10.406 | 10.406 | | | |
| | 12 | 0.021 | 10.406 | 10.406 | 0.000 | 0.132 | |
| 14 | | -6.200 | -1.600 | 6.403 | | | |
| | 12 | -7.778 | -2.273 | 8.103 | 0.074 | 0.155 | |
| | 15 | 1.592 | 0.708 | 1.742 | 0.006 | 0.006 | |
| 15 | | -8.200 | -2.500 | 8.573 | | | |
| | 12 | -17.634 | -6.540 | 18.808 | 0.217 | 0.428 | |
| | 14 | -1.586 | -0.702 | 1.734 | 0.006 | 0.006 | |
| | 18 | 6.009 | 1.741 | 6.256 | 0.039 | 0.079 | |
| | 23 | 5.004 | 2.963 | 5.815 | 0.031 | 0.063 | |
| 16 | | -3.500 | -1.800 | 3.936 | | | |
| | 12 | -7.152 | -3.257 | 7.859 | 0.053 | 0.112 | |
| | 17 | 3.658 | 1.440 | 3.931 | 0.012 | 0.027 | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 17 | | -9.000 | -5.800 | 10.707 | | | |
| | 16 | -3.646 | -1.413 | 3.910 | 0.012 | 0.027 | |
| | 10 | -5.332 | -4.355 | 6.885 | 0.014 | 0.037 | |
| 18 | | -3.200 | -0.900 | 3.324 | | | |
| | 15 | -5.970 | -1.661 | 6.197 | 0.039 | 0.079 | |
| | 19 | 2.779 | 0.787 | 2.888 | 0.005 | 0.010 | |
| 19 | | -9.500 | -3.400 | 10.090 | | | |
| | 18 | -2.774 | -0.777 | 2.881 | 0.005 | 0.010 | |
| | 20 | -6.703 | -2.675 | 7.217 | 0.017 | 0.034 | |
| 20 | | -2.200 | -0.700 | 2.309 | | | |
| | 19 | 6.720 | 2.709 | 7.245 | 0.017 | 0.034 | |
| | 10 | -8.937 | -3.389 | 9.558 | 0.081 | 0.180 | |
| 21 | | -17.500 | -11.200 | 20.777 | | | |
| | 10 | -15.613 | -9.609 | 18.333 | 0.110 | 0.236 | |
| | 22 | -1.849 | -1.627 | 2.463 | 0.001 | 0.001 | |
| 22 | | 0.000 | 0.000 | 0.000 | | | |
| | 10 | -7.531 | -4.380 | 8.712 | 0.052 | 0.107 | |
| | 21 | 1.850 | 1.628 | 2.464 | 0.001 | 0.001 | |
| | 24 | 5.643 | 2.795 | 6.297 | 0.043 | 0.067 | |
| 23 | | -3.200 | -1.600 | 3.578 | | | |
| | 15 | -4.972 | -2.900 | 5.756 | 0.031 | 0.063 | |
| | 24 | 1.771 | 1.282 | 2.186 | 0.006 | 0.012 | |
| 24 | | -8.700 | -2.400 | 9.025 | | | |
| | 22 | -5.601 | -2.728 | 6.230 | 0.043 | 0.067 | |
| | 23 | -1.765 | -1.270 | 2.174 | 0.006 | 0.012 | |
| | 25 | -1.322 | 1.604 | 2.079 | 0.008 | 0.014 | |
| 25 | | 0.000 | 0.000 | 0.000 | | | |
| | 24 | 1.330 | -1.590 | 2.073 | 0.008 | 0.014 | |
| | 26 | 3.520 | 2.372 | 4.244 | 0.044 | 0.066 | |
| | 27 | -4.866 | -0.786 | 4.929 | 0.026 | 0.049 | |
| 26 | | -3.500 | -2.300 | 4.188 | | | |
| | 25 | -3.476 | -2.306 | 4.171 | 0.044 | 0.066 | |
| 27 | | 0.000 | 0.000 | 0.000 | | | |
| | 25 | 4.892 | 0.835 | 4.963 | 0.026 | 0.049 | |
| | 28 | -18.192 | -4.152 | 18.660 | -0.000 | 1.310 | |
| | 29 | 6.178 | 1.675 | 6.401 | 0.086 | 0.162 | |
| | 30 | 7.093 | 1.663 | 7.286 | 0.162 | 0.304 | |
| 28 | | 0.000 | 0.000 | 0.000 | | | |
| | 27 | 18.192 | 5.463 | 18.994 | -0.000 | 1.310 | 0.968 |
| | 8 | 0.570 | -2.003 | 2.082 | 0.000 | -4.368 | |
| | 6 | -18.780 | -3.510 | 19.106 | 0.060 | -13.085 | |
| 29 | | -2.400 | -0.900 | 2.563 | | | |
| | 27 | -6.093 | -1.513 | 6.278 | 0.086 | 0.162 | |
| | 30 | 3.716 | 0.601 | 3.764 | 0.034 | 0.063 | |
| 30 | | -10.600 | -1.900 | 10.769 | | | |
| | 27 | -6.932 | -1.359 | 7.064 | 0.162 | 0.304 | |
| | 29 | -3.683 | -0.537 | 3.722 | 0.034 | 0.063 | |
| Total loss | | | | | 17.594 | 22.233 | |

## 6.10 NEWTON-RAPHSON POWER FLOW SOLUTION

Because of its quadratic convergence, Newton's method is mathematically superior to the Gauss-Seidel method and is less prone to divergence with ill-conditioned problems. For large power systems, the Newton-Raphson method is found to be more efficient and practical. The number of iterations required to obtain a solution is independent of the system size, but more functional evaluations are required at each iteration. Since in the power flow problem real power and voltage magnitude are specified for the voltage-controlled buses, the power flow equation is formulated in polar form. For the typical bus of the power system shown in Figure 6.7, the current entering bus $i$ is given by (6.24). This equation can be rewritten in terms of the bus admittance matrix as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \tag{6.48}$$

In the above equation, $j$ includes bus $i$. Expressing this equation in polar form, we have

$$I_i = \sum_{j=1}^{n} |Y_{ij}||V_j| \angle \theta_{ij} + \delta_j \tag{6.49}$$

The complex power at bus $i$ is

$$P_i - jQ_i = V_i^* I_i \tag{6.50}$$

Substituting from (6.49) for $I_i$ in (6.50),

$$P_i - jQ_i = |V_i|\angle -\delta_i \sum_{j=1}^{n} |Y_{ij}||V_j|\angle \theta_{ij} + \delta_j \tag{6.51}$$

Separating the real and imaginary parts,

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{6.52}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{6.53}$$

Equations (6.52) and (6.53) constitute a set of nonlinear algebraic equations in terms of the independent variables, voltage magnitude in per unit, and phase angle in radians. We have two equations for each load bus, given by (6.52) and (6.53), and one equation for each voltage-controlled bus, given by (6.52). Expanding (6.52) and (6.53) in Taylor's series about the initial estimate and neglecting all higher order terms results in the following set of linear equations.

$$\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \hline \Delta Q_2^{(k)} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{\partial P_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_2}{\partial \delta_n}^{(k)} & \frac{\partial P_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_2}{\partial |V_n|}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial P_n}{\partial \delta_n}^{(k)} & \frac{\partial P_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial P_n}{\partial |V_n|}^{(k)} \\ \hline \frac{\partial Q_2}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_2}{\partial \delta_n}^{(k)} & \frac{\partial Q_2}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_2}{\partial |V_n|}^{(k)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_2}^{(k)} & \cdots & \frac{\partial Q_n}{\partial \delta_n}^{(k)} & \frac{\partial Q_n}{\partial |V_2|}^{(k)} & \cdots & \frac{\partial Q_n}{\partial |V_n|}^{(k)} \end{array}\right] \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \hline \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix}$$

In the above equation, bus 1 is assumed to be the slack bus. The Jacobian matrix gives the linearized relationship between small changes in voltage angle $\Delta \delta_i^{(k)}$ and voltage magnitude $\Delta |V_i^{(k)}|$ with the small changes in real and reactive power $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$. Elements of the Jacobian matrix are the partial derivatives of (6.52) and (6.53), evaluated at $\Delta \delta_i^{(k)}$ and $\Delta |V_i^{(k)}|$. In short form, it can be written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \tag{6.54}$$

For voltage-controlled buses, the voltage magnitudes are known. Therefore, if m buses of the system are voltage-controlled, m equations involving $\Delta Q$ and $\Delta V$ and the corresponding columns of the Jacobian matrix are eliminated. Accordingly, there are $n-1$ real power constraints and $n-1-m$ reactive power constraints, and the Jacobian matrix is of order $(2n-2-m) \times (2n-2-m)$. $J_1$ is of the order $(n-1) \times (n-1)$, $J_2$ is of the order $(n-1) \times (n-1-m)$, $J_3$ is of the order $(n-1-m) \times (n-1)$, and $J_4$ is of the order $(n-1-m) \times (n-1-m)$.

The diagonal and the off-diagonal elements of $J_1$ are

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{6.55}$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.56}$$

The diagonal and the off-diagonal elements of $J_2$ are

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i||Y_{ii}| \cos\theta_{ii} + \sum_{j \neq i} |V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{6.57}$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.58}$$

The diagonal and the off-diagonal elements of $J_3$ are

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{j \neq i} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{6.59}$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.60}$$

The diagonal and the off-diagonal elements of $J_4$ are

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}| \sin\theta_{ii} - \sum_{j \neq i} |V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{6.61}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad j \neq i \tag{6.62}$$

The terms $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are the difference between the scheduled and calculated values, known as the *power residuals*, given by

$$\Delta P_i^{(k)} = P_i^{sch} - P_i^{(k)} \tag{6.63}$$

$$\Delta Q_i^{(k)} = Q_i^{sch} - Q_i^{(k)} \tag{6.64}$$

The new estimates for bus voltages are

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)} \tag{6.65}$$

$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta|V_i^{(k)}| \tag{6.66}$$

The procedure for power flow solution by the Newton-Raphson method is as follows:

1. For load buses, where $P_i^{sch}$ and $Q_i^{sch}$ are specified, voltage magnitudes and phase angles are set equal to the slack bus values, or 1.0 and 0.0, i.e., $|V_i^{(0)}| = 1.0$ and $\delta_i^{(0)} = 0.0$. For voltage-regulated buses, where $|V_i|$ and $P_i^{sch}$ are specified, phase angles are set equal to the slack bus angle, or 0, i.e., $\delta_i^{(0)} = 0$.
2. For load buses, $P_i^{(k)}$ and $Q_i^{(k)}$ are calculated from (6.52) and (6.53) and $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are calculated from (6.63) and (6.64).
3. For voltage-controlled buses, $P_i^{(k)}$ and $\Delta P_i^{(k)}$ are calculated from (6.52) and (6.63), respectively.
4. The elements of the Jacobian matrix ($\boldsymbol{J_1}$, $\boldsymbol{J_2}$, $\boldsymbol{J_3}$, and $\boldsymbol{J_4}$) are calculated from (6.55) – (6.62).
5. The linear simultaneous equation (6.54) is solved directly by optimally ordered triangular factorization and Gaussian elimination.
6. The new voltage magnitudes and phase angles are computed from (6.65) and (6.66).
7. The process is continued until the residuals $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are less than the specified accuracy, i.e.,

$$|\Delta P_i^{(k)}| \le \epsilon$$
$$|\Delta Q_i^{(k)}| \le \epsilon \tag{6.67}$$

The power flow solution by the Newton-Raphson method is demonstrated in the following example.

**Example 6.10** (chp6ex10)

Obtain the power flow solution by the Newton-Raphson method for the system of Example 6.8.



Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$, and $y_{23} = 16 - j32$. This results in the bus admittance matrix

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

Converting the bus admittance matrix to polar form with angles in radian yields

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -1.9029 & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle -1.1071 & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle -1.1737 \end{bmatrix}$$

From (6.52) and (6.53), the expressions for real power at bus 2 and 3 and the reactive power at bus 2 are

$$\begin{aligned} P_2 &= |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} + \\ &\quad |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\ P_3 &= |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \\ &\quad \delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33} \\ Q_2 &= -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} - \\ &\quad |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \end{aligned}$$

Elements of the Jacobian matrix are obtained by taking partial derivatives of the above equations with respect to $\delta_2$, $\delta_3$ and $|V_2|$.

$$\begin{aligned} \frac{\partial P_2}{\partial \delta_2} &= |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \\ &\quad \sin(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_2}{\partial \delta_3} &= -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_2}{\partial |V_2|} &= |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22} + \\ &\quad |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_3}{\partial \delta_2} &= -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) \\ \frac{\partial P_3}{\partial \delta_3} &= |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \\ &\quad \sin(\theta_{32} - \delta_3 + \delta_2) \\ \frac{\partial P_3}{\partial |V_2|} &= |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) \end{aligned}$$

The new estimates for bus voltages are

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)} \tag{6.65}$$
$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta|V_i^{(k)}| \tag{6.66}$$

The procedure for power flow solution by the Newton-Raphson method is as follows:

1. For load buses, where $P_i^{sch}$ and $Q_i^{sch}$ are specified, voltage magnitudes and phase angles are set equal to the slack bus values, or 1.0 and 0.0, i.e., $|V_i^{(0)}| = 1.0$ and $\delta_i^{(0)} = 0.0$. For voltage-regulated buses, where $|V_i|$ and $P_i^{sch}$ are specified, phase angles are set equal to the slack bus angle, or 0, i.e., $\delta_i^{(0)} = 0$.
2. For load buses, $P_i^{(k)}$ and $Q_i^{(k)}$ are calculated from (6.52) and (6.53) and $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are calculated from (6.63) and (6.64).
3. For voltage-controlled buses, $P_i^{(k)}$ and $\Delta P_i^{(k)}$ are calculated from (6.52) and (6.63), respectively.
4. The elements of the Jacobian matrix ($\boldsymbol{J_1}$, $\boldsymbol{J_2}$, $\boldsymbol{J_3}$, and $\boldsymbol{J_4}$) are calculated from (6.55) – (6.62).
5. The linear simultaneous equation (6.54) is solved directly by optimally ordered triangular factorization and Gaussian elimination.
6. The new voltage magnitudes and phase angles are computed from (6.65) and (6.66).
7. The process is continued until the residuals $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are less than the specified accuracy, i.e.,

$$|\Delta P_i^{(k)}| \le \epsilon$$
$$|\Delta Q_i^{(k)}| \le \epsilon \tag{6.67}$$

The power flow solution by the Newton-Raphson method is demonstrated in the following example.

**Example 6.10** (chp6ex10)

Obtain the power flow solution by the Newton-Raphson method for the system of Example 6.8.



Line impedances converted to admittances are $y_{12} = 10 - j20$, $y_{13} = 10 - j30$, and $y_{23} = 16 - j32$. This results in the bus admittance matrix

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

Converting the bus admittance matrix to polar form with angles in radian yields

$$Y_{bus} = \begin{bmatrix} 53.85165\angle -1.9029 & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle -1.1071 & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle -1.1737 \end{bmatrix}$$

From (6.52) and (6.53), the expressions for real power at bus 2 and 3 and the reactive power at bus 2 are

$$\begin{aligned} P_2 &= |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2^2||Y_{22}|\cos\theta_{22} + \\ &\quad |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\ P_3 &= |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \\ &\quad \delta_3 + \delta_2) + |V_3^2||Y_{33}|\cos\theta_{33} \\ Q_2 &= -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2^2||Y_{22}|\sin\theta_{22} - \\ &\quad |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \end{aligned}$$

Elements of the Jacobian matrix are obtained by taking partial derivatives of the above equations with respect to $\delta_2$, $\delta_3$ and $|V_2|$.

$$\begin{aligned} \frac{\partial P_2}{\partial \delta_2} &= |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \\ &\quad \sin(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_2}{\partial \delta_3} &= -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_2}{\partial |V_2|} &= |V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}|\cos\theta_{22} + \\ &\quad |V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3) \\ \frac{\partial P_3}{\partial \delta_2} &= -|V_3||V_2||Y_{32}|\sin(\theta_{32} - \delta_3 + \delta_2) \\ \frac{\partial P_3}{\partial \delta_3} &= |V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \\ &\quad \sin(\theta_{32} - \delta_3 + \delta_2) \\ \frac{\partial P_3}{\partial |V_2|} &= |V_3||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) \end{aligned}$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$
$$\frac{\partial Q_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$
$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}|\sin\theta_{22} - |V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400 + j250)}{100} = -4.0 - j2.5 \quad \text{pu}$$
$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

The slack bus voltage is $V_1 = 1.05\angle 0$ pu, and the bus 3 voltage magnitude is $|V_3| = 1.04$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0 - (-1.14) = -2.8600$$
$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0 - (0.5616) = 1.4384$$
$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5 - (-2.28) = -0.2200$$

Evaluating the elements of the Jacobian matrix with the initial estimate, the set of linear equations in the first iteration becomes

$$\begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.14000 & 16.64000 & 49.72000 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta|V_2^{(0)}| \end{bmatrix}$$

Obtaining the solution of the above matrix equation, the new bus voltages in the first iteration are

$$\Delta\delta_2^{(0)} = -0.045263 \qquad \delta_2^{(1)} = 0 + (-0.045263) = -0.045263$$
$$\Delta\delta_3^{(0)} = -0.007718 \qquad \delta_3^{(1)} = 0 + (-0.007718) = -0.007718$$
$$\Delta|V_2^{(0)}| = -0.026548 \qquad |V_2^{(1)}| = 1 + (-0.026548) = 0.97345$$

Voltage phase angles are in radians. For the second iteration, we have

$$\begin{bmatrix} -0.099218 \\ 0.021715 \\ -0.050914 \end{bmatrix} = \begin{bmatrix} 51.724675 & -31.765618 & 21.302567 \\ -32.981642 & 65.656383 & -15.379086 \\ -28.538577 & 17.402838 & 48.103589 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(1)} \\ \Delta\delta_3^{(1)} \\ \Delta|V_2^{(1)}| \end{bmatrix}$$



and

$$\Delta\delta_2^{(1)} = -0.001795 \qquad \delta_2^{(2)} = -0.045263 + (-0.001795) = -0.04706$$
$$\Delta\delta_3^{(1)} = -0.000985 \qquad \delta_3^{(2)} = -0.007718 + (-0.000985) = -0.00870$$
$$\Delta|V_2^{(1)}| = -0.001767 \qquad |V_2^{(2)}| = 0.973451 + (-0.001767) = 0.971684$$

For the third iteration, we have

$$\begin{bmatrix} -0.000216 \\ 0.000038 \\ -0.000143 \end{bmatrix} = \begin{bmatrix} 51.596701 & -31.693866 & 21.147447 \\ -32.933865 & 65.597585 & -15.351628 \\ -28.548205 & 17.396932 & 47.954870 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(2)} \\ \Delta\delta_3^{(2)} \\ \Delta|V_2^{(2)}| \end{bmatrix}$$

and

$$\Delta\delta_2^{(2)} = -0.000038 \qquad \delta_2^{(3)} = -0.047058 + (-0.0000038) = -0.04706$$
$$\Delta\delta_3^{(2)} = -0.0000024 \qquad \delta_3^{(3)} = -0.008703 + (-0.0000024) = 0.008705$$
$$\Delta|V_2^{(2)}| = -0.0000044 \qquad |V_2^{(3)}| = 0.971684 + (-0.0000044) = 0.97168$$

The solution converges in 3 iterations with a maximum power mismatch of $2.5 \times 10^{-4}$ with $V_2 = 0.97168\angle{-2.696^\circ}$ and $V_3 = 1.04\angle{-0.4988^\circ}$. From (6.52) and (6.53), the expressions for reactive power at bus 3 and the slack bus real and reactive powers are

$$Q_3 = -|V_3||V_1||Y_{31}|\sin(\theta_{31} - \delta_3 + \delta_1) - |V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2) - |V_3|^2|Y_{33}|\sin\theta_{33}$$
$$P_1 = |V_1|^2|Y_{11}|\cos\theta_{11} + |V_1||V_2||Y_{12}|\cos(\theta_{12} - \delta_1 + \delta_2) + |V_1||V_3| |Y_{13}|\cos(\theta_{13} - \delta_1 + \delta_3)$$
$$Q_1 = -|V_1|^2|Y_{11}|\sin\theta_{11} - |V_1||V_2||Y_{12}|\sin(\theta_{12} - \delta_1 + \delta_2) - |V_1||V_3| |Y_{13}|\sin(\theta_{13} - \delta_1 + \delta_3)$$

Upon substitution, we have

$$Q_3 = 1.4617 \quad \text{pu}$$
$$P_1 = 2.1842 \quad \text{pu}$$
$$Q_1 = 1.4085 \quad \text{pu}$$

Finally, the line flows are calculated in the same manner as the line flow calculations in the Gauss-Seidel method described in Example 6.7, and the power flow diagram is as shown in Figure 6.13.

A program named **lfnewton** is developed for power flow solution by the Newton-Raphson method for practical power systems. This program must be preceded by the **lfybus** program. **busout** and **lineflow** programs can be used to print the load flow solution and the line flow results. The format is the same as the Gauss-Seidel. The following is a brief description of the **lfnewton** program.

**lfnewton** This program obtains the power flow solution by the Newton-Raphson method and requires the **busdata** and the **linedata** files described in Section 6.9. It is designed for the direct use of load and generation in MW and Mvar, bus voltages in per unit, and angle in degrees. Loads and generation are converted to per unit quantities on the base MVA selected. A provision is made to maintain the generator reactive power of the voltage-controlled buses within their specified limits. The violation of reactive power limit may occur if the specified voltage is either too high or too low. In the second iteration, the var calculated at the generator buses are examined. If a limit is reached, the voltage magnitude is adjusted in steps of 0.5 percent up to $\pm 5$ percent to bring the var demand within the specified limits.

**Example 6.11** (chp6ex11)

Obtain the power flow solution for the IEEE-30 bus test system by the Newton-Raphson method.

The data required is the same as in Example 6.9 with the following commands

```
clear       % clears all variables from the workspace.
basemva = 100;  accuracy = 0.001;  maxiter = 12;

busdata =  [ same as in Example 6.9 ];
linedata = [ same as in Example 6.9 ];

    lfybus                % Forms the bus admittance matrix
    lfnewton   % Power flow solution by Newton-Raphson method
    busout     % Prints the power flow solution on the screen
    lineflow % Computes and displays the line flow and losses
```

The output of **lfnewton** is

```
         Power Flow Solution by Newton-Raphson Method
            Maximum Power mismatch = 7.54898e-07
                   No. of iterations = 4
```

| Bus No. | Voltage Mag. | Angle Degree | Load MW | Load Mvar | Generation MW | Generation Mvar | Injected Mvar |
|---|---|---|---|---|---|---|---|
| 1 | 1.060 | 0.000 | 0.000 | 0.000 | 260.998 | -17.021 | 0.00 |
| 2 | 1.043 | -5.497 | 21.700 | 12.700 | 40.000 | 48.822 | 0.00 |
| 3 | 1.022 | -8.004 | 2.400 | 1.200 | 0.000 | 0.000 | 0.00 |
| 4 | 1.013 | -9.661 | 7.600 | 1.600 | 0.000 | 0.000 | 0.00 |
| 5 | 1.010 | -14.381 | 94.200 | 19.000 | 0.000 | 35.975 | 0.00 |
| 6 | 1.012 | -11.398 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 7 | 1.003 | -13.150 | 22.800 | 10.900 | 0.000 | 0.000 | 0.00 |
| 8 | 1.010 | -12.115 | 30.000 | 30.000 | 0.000 | 30.826 | 0.00 |
| 9 | 1.051 | -14.434 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 10 | 1.044 | -16.024 | 5.800 | 2.000 | 0.000 | 0.000 | 19.00 |
| 11 | 1.082 | -14.434 | 0.000 | 0.000 | 0.000 | 16.119 | 0.00 |
| 12 | 1.057 | -15.302 | 11.200 | 7.500 | 0.000 | 0.000 | 0.00 |
| 13 | 1.071 | -15.302 | 0.000 | 0.000 | 0.000 | 10.423 | 0.00 |
| 14 | 1.042 | -16.191 | 6.200 | 1.600 | 0.000 | 0.000 | 0.00 |
| 15 | 1.038 | -16.278 | 8.200 | 2.500 | 0.000 | 0.000 | 0.00 |
| 16 | 1.045 | -15.880 | 3.500 | 1.800 | 0.000 | 0.000 | 0.00 |
| 17 | 1.039 | -16.188 | 9.000 | 5.800 | 0.000 | 0.000 | 0.00 |
| 18 | 1.028 | -16.884 | 3.200 | 0.900 | 0.000 | 0.000 | 0.00 |
| 19 | 1.025 | -17.052 | 9.500 | 3.400 | 0.000 | 0.000 | 0.00 |
| 20 | 1.029 | -16.852 | 2.200 | 0.700 | 0.000 | 0.000 | 0.00 |
| 21 | 1.032 | -16.468 | 17.500 | 11.200 | 0.000 | 0.000 | 0.00 |
| 22 | 1.033 | -16.455 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 23 | 1.027 | -16.662 | 3.200 | 1.600 | 0.000 | 0.000 | 0.00 |
| 24 | 1.022 | -16.830 | 8.700 | 6.700 | 0.000 | 0.000 | 4.30 |
| 25 | 1.019 | -16.424 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 26 | 1.001 | -16.842 | 3.500 | 2.300 | 0.000 | 0.000 | 0.00 |
| 27 | 1.026 | -15.912 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 28 | 1.011 | -12.057 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 29 | 1.006 | -17.136 | 2.400 | 0.900 | 0.000 | 0.000 | 0.00 |
| 30 | 0.995 | -18.015 | 10.600 | 1.900 | 0.000 | 0.000 | 0.00 |
| Total | | | 283.400 | 126.200 | 300.998 | 125.144 | 23.30 |



The output of the **lineflow** is the same as the line flow output of Example 6.9 with the power mismatch as dictated by the Newton-Raphson method.

## 6.11 FAST DECOUPLED POWER FLOW SOLUTION

Power system transmission lines have a very high $X/R$ ratio. For such a system, real power changes $\Delta P$ are less sensitive to changes in the voltage magnitude and are most sensitive to changes in phase angle $\Delta\delta$. Similarly, reactive power is less sensitive to changes in angle and are mainly dependent on changes in voltage magnitude. Therefore, it is reasonable to set elements $\boldsymbol{J_2}$ and $\boldsymbol{J_3}$ of the Jacobian matrix to zero. Thus, (6.54) becomes

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{6.68}$$

or

$$\boldsymbol{\Delta P} = \boldsymbol{J_1 \Delta \delta} = [\frac{\boldsymbol{\partial P}}{\boldsymbol{\partial \delta}}]\ \boldsymbol{\Delta \delta} \tag{6.69}$$

$$\boldsymbol{\Delta Q} = \boldsymbol{J_4 \Delta |V|} = [\frac{\boldsymbol{\partial Q}}{\boldsymbol{\partial |V|}}]\ \boldsymbol{\Delta |V|} \tag{6.70}$$

(6.69) and (6.70) show that the matrix equation is separated into two decoupled equations requiring considerably less time to solve compared to the time required for the solution of (6.54). Furthermore, considerable simplification can be made to eliminate the need for recomputing $\boldsymbol{J_1}$ and $\boldsymbol{J_4}$ during each iteration. This procedure results in the decoupled power flow equations developed by Stott and Alsac[75–76]. The diagonal elements of $\boldsymbol{J_1}$ described by (6.55) may be written as

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) - |V_i|^2 |Y_{ii}| \sin \theta_{ii}$$

Replacing the first term of the above equation with $-Q_i$, as given by (6.53), results in

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 |Y_{ii}| \sin \theta_{ii}$$
$$= -Q_i - |V_i|^2 B_{ii}$$

Where $B_{ii} = |Y_{ii}| \sin \theta_{ii}$ is the imaginary part of the diagonal elements of the bus admittance matrix. $B_{ii}$ is the sum of susceptances of all the elements incident to bus $i$. In a typical power system, the self-susceptance $B_{ii} \gg Q_i$, and we may neglect $Q_i$. Further simplification is obtained by assuming $|V_i|^2 \approx |V_i|$, which yields

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i| B_{ii} \tag{6.71}$$

Under normal operating conditions, $\delta_j - \delta_i$ is quite small. Thus, in (6.56) assuming $\theta_{ii} - \delta_i + \delta_j \approx \theta_{ii}$, the off-diagonal elements of $\boldsymbol{J_1}$ becomes

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j| B_{ij}$$

Further simplification is obtained by assuming $|V_j| \approx 1$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i| B_{ij} \tag{6.72}$$

Similarly, the diagonal elements of $\boldsymbol{J_4}$ described by (6.61) may be approximated to



$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}| \sin \theta_{ii} - \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j)$$

replacing the second term of the above equation with $-Q_i$, as given by (6.53), results in

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}| \sin \theta_{ii} + Q_i$$

Again, since $B_{ii} = Y_{ii} \sin \theta_{ii} \gg Q_i$, $Q_i$ may be neglected and (6.61) reduces to

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii} \tag{6.73}$$

Likewise in (6.62), assuming $\theta_{ij} - \delta_i + \delta_j \approx \theta_{ij}$ yields

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i| B_{ij} \tag{6.74}$$

With these assumptions, equations (6.69) and (6.70) take the following form

$$\frac{\boldsymbol{\Delta P}}{\boldsymbol{|V_i|}} = -\boldsymbol{B'}\ \boldsymbol{\Delta \delta} \tag{6.75}$$

$$\frac{\boldsymbol{\Delta Q}}{\boldsymbol{|V_i|}} = -\boldsymbol{B''}\ \boldsymbol{\Delta |V|} \tag{6.76}$$

Here, $\boldsymbol{B'}$ and $\boldsymbol{B''}$ are the imaginary part of the bus admittance matrix $\boldsymbol{Y_{bus}}$. Since the elements of this matrix are constant, they need to be triangularized and inverted only once at the beginning of the iteration. $\boldsymbol{B'}$ is of order of $(n-1)$. For voltage-controlled buses where $|V_i|$ and $P_i$ are specified and $Q_i$ is not specified, the corresponding row and column of $\boldsymbol{Y_{bus}}$ are eliminated. Thus, $\boldsymbol{B''}$ is of order of $(n-1-m)$, where $m$ is the number of voltage-regulated buses. Therefore, in the fast decoupled power flow algorithm, the successive voltage magnitude and phase angle changes are

$$\boldsymbol{\Delta \delta} = -[\boldsymbol{B'}]^{-1} \frac{\boldsymbol{\Delta P}}{\boldsymbol{|V|}} \tag{6.77}$$

$$\boldsymbol{\Delta |V|} = -[\boldsymbol{B''}]^{-1} \frac{\boldsymbol{\Delta Q}}{\boldsymbol{|V|}} \tag{6.78}$$

The fast decoupled power flow solution requires more iterations than the Newton-Raphson method, but requires considerably less time per iteration, and a power flow solution is obtained very rapidly. This technique is very useful in contingency analysis where numerous outages are to be simulated or a power flow solution is required for on-line control.

**Example 6.12** (chp6ex12)

Obtain the power flow solution by the fast decoupled method for the system of Example 6.8.

The bus admittance matrix of the system as obtained in Example 6.10 is

$$Y_{bus} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

In this system, bus 1 is the slack bus and the corresponding bus susceptance matrix for evaluation of phase angles $\Delta\delta_2$ and $\Delta\delta_3$ is

$$B' = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix}$$

The inverse of the above matrix is

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

From (6.52) and (6.53), the expressions for real power at bus 2 and 3 and the reactive power at bus 2 are

$$\begin{aligned} P_2 &= |V_2||V_1||Y_{21}|\cos(\theta_{21}-\delta_2+\delta_1)+|V_2^2||Y_{22}|\cos\theta_{22} \\ &\quad +|V_2||V_3||Y_{23}|\cos(\theta_{23}-\delta_2+\delta_3) \\ P_3 &= |V_3||V_1||Y_{31}|\cos(\theta_{31}-\delta_3+\delta_1)+|V_3||V_2||Y_{32}|\cos(\theta_{32} \\ &\quad -\delta_3+\delta_2)+|V_3^2||Y_{33}|\cos\theta_{33} \\ Q_2 &= -|V_2||V_1||Y_{21}|\sin(\theta_{21}-\delta_2+\delta_1)-|V_2^2||Y_{22}|\sin\theta_{22} \\ &\quad -|V_2||V_3||Y_{23}|\sin(\theta_{23}-\delta_2+\delta_3) \end{aligned}$$

The load and generation expressed in per units are

$$S_2^{sch} = -\frac{(400+j250)}{100} = -4.0-j2.5 \quad \text{pu}$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \quad \text{pu}$$

The slack bus voltage is $V_1 = 1.05\angle 0$ pu, and the bus 3 voltage magnitude is $|V_3| = 1.04$ pu. Starting with an initial estimate of $|V_2^{(0)}| = 1.0$, $\delta_2^{(0)} = 0.0$, and $\delta_3^{(0)} = 0.0$, the power residuals are computed from (6.63) and (6.64)

$$\Delta P_2^{(0)} = P_2^{sch} - P_2^{(0)} = -4.0-(-1.14) = -2.86$$

$$\Delta P_3^{(0)} = P_3^{sch} - P_3^{(0)} = 2.0-(0.5616) = 1.4384$$

$$\Delta Q_2^{(0)} = Q_2^{sch} - Q_2^{(0)} = -2.5-(-2.28) = -0.22$$



The fast decoupled power flow algorithm given by (6.77) becomes

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \end{bmatrix} = -\begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}\begin{bmatrix} \frac{-2.8600}{1.0} \\ \frac{1.4384}{1.04} \end{bmatrix} = \begin{bmatrix} -0.060483 \\ -0.008909 \end{bmatrix}$$

Since bus 3 is a regulated bus, the corresponding row and column of $B'$ are eliminated and we get

$$B'' = [-52]$$

From (6.78), we have

$$\Delta|V_2| = -\left[\frac{-1}{52}\right]\left[\frac{-.22}{1.0}\right] = -0.0042308$$

The new bus voltages in the first iteration are

$$\begin{aligned} \Delta\delta_2^{(0)} &= -0.060483 & \delta_2^{(1)} &= 0+(-0.060483) = -0.060483 \\ \Delta\delta_3^{(0)} &= -0.008989 & \delta_3^{(1)} &= 0+(-0.008989) = -0.008989 \\ \Delta|V_2^{(0)}| &= -0.0042308 & |V_2^{(1)}| &= 1+(-0.0042308) = 0.995769 \end{aligned}$$

The voltage phase angles are in radians. The process is continued until power residuals are within a specified accuracy. The result is tabulated in the table below.

| Iter | $\delta_2$ | $\delta_3$ | $\lvert V_2\rvert$ | $\Delta P_2$ | $\Delta P_3$ | $\Delta Q_2$ |
|---|---|---|---|---|---|---|
| 1 | -0.060482 | -0.008909 | 0.995769 | -2.860000 | 1.438400 | -0.220000 |
| 2 | -0.056496 | -0.007952 | 0.965274 | 0.175895 | -0.070951 | -1.579042 |
| 3 | -0.044194 | -0.008690 | 0.965711 | 0.640309 | -0.457039 | 0.021948 |
| 4 | -0.044802 | -0.008986 | 0.972985 | -0.021395 | 0.001195 | 0.365249 |
| 5 | -0.047665 | -0.008713 | 0.973116 | -0.153368 | 0.112899 | 0.006657 |
| 6 | -0.047614 | -0.008645 | 0.971414 | 0.000520 | 0.002610 | -0.086136 |
| 7 | -0.046936 | -0.008702 | 0.971333 | 0.035980 | -0.026190 | -0.004067 |
| 8 | -0.046928 | -0.008720 | 0.971732 | 0.000948 | -0.001411 | 0.020119 |
| 9 | -0.047087 | -0.008707 | 0.971762 | -0.008442 | 0.006133 | 0.001558 |
| 10 | -0.047094 | -0.008702 | 0.971669 | -0.000470 | 0.000510 | -0.004688 |
| 11 | -0.047057 | -0.008705 | 0.971660 | 0.001971 | -0.001427 | -0.000500 |
| 12 | -0.047054 | -0.008706 | 0.971681 | 0.000170 | -0.000163 | 0.001087 |
| 13 | -0.047063 | -0.008706 | 0.971684 | -0.000458 | 0.000330 | 0.000151 |
| 14 | -0.047064 | -0.008706 | 0.971680 | -0.000053 | 0.000048 | -0.000250 |

Converting phase angles to degrees the final solution is $V_2 = 0.97168\angle-2.696°$ and $V_3 = 1.04\angle-0.4988°$. Using (6.52) and (6.53) as in Example 6.10, the reactive power at bus 3 and the slack bus real and reactive powers are

$$Q_3 = 1.4617 \text{ pu}$$
$$P_1 = 2.1842 \text{ pu}$$
$$Q_1 = 1.4085 \text{ pu}$$

The fast decoupled power flow for this example has taken 14 iterations with the maximum power mismatch of $2.5 \times 10^{-4}$ pu compared to the Newton-Raphson method which took only three iterations. The highest $X/R$ ratio of the transmission lines in this example is 3. For systems with a higher $X/R$ ratio, the fast decoupled power flow method converges in relatively fewer iterations. However, the number of iterations is a function of system size.

Finally, the line flows are calculated in the same manner as the line flow calculations in the Gauss-Seidel method described in Example 6.7, and the power flow diagram is as shown in Figure 6.13.

A program named **decouple** is developed for power flow solution by the fast decoupled method for practical power systems. This program must be preceded by the **lfybus** program. **busout** and **lineflow** programs can be used to print the load flow solution and the line flow results. The format is the same as the Gauss-Seidel method. The following is a brief description of the **decouple** program:

**decouple** This program finds the power flow solution by the fast decouple method and requires the **busdata** and the **linedata** files described in Section 6.9. It is designed for the direct use of load and generation in MW and Mvar, bus voltages in per unit, and angle in degrees. Loads and generation are converted to per unit quantities on the base MVA selected. A provision is made to maintain the generator reactive power of the voltage-controlled buses within their specified limits. The violation of reactive power limit may occur if the specified voltage is either too high or too low. In the 10th iteration, the vars calculated at the generator buses are examined. If a limit is reached, the voltage magnitude is adjusted in steps of 0.5 percent up to ±5 percent to bring the var demand within the specified limits.

**Example 6.13** (chp6ex13)

Obtain the power flow solution for the IEEE-30 bus test system by the fast decoupled method.

Data required is the same as in Example 6.9 with the following commands



```
clear       % clears all variables from the workspace.
basemva = 100;  accuracy = 0.001;  maxiter = 20;

busdata =  [ same as in Example 6.9 ];
linedata = [ same as in Example 6.9 ];

     lfybus               % Forms the bus admittance matrix
     decouple    % Power flow solution by fast decoupled method
     busout      % Prints the power flow solution on the screen
     lineflow % Computes and displays the line flow and losses
```

The output of **decouple** is

```
          Power Flow Solution by Fast Decoupled Method
             Maximum Power mismatch = 0.000919582
                    No. of iterations = 15
```

| Bus No. | Voltage Mag. | Angle Degree | Load MW | Load Mvar | Generation MW | Generation Mvar | Injected Mvar |
|---|---|---|---|---|---|---|---|
| 1 | 1.060 | 0.000 | 0.000 | 0.000 | 260.998 | -17.021 | 0.00 |
| 2 | 1.043 | -5.497 | 21.700 | 12.700 | 40.000 | 48.822 | 0.00 |
| 3 | 1.022 | -8.004 | 2.400 | 1.200 | 0.000 | 0.000 | 0.00 |
| 4 | 1.013 | -9.662 | 7.600 | 1.600 | 0.000 | 0.000 | 0.00 |
| 5 | 1.010 | -14.381 | 94.200 | 19.000 | 0.000 | 35.975 | 0.00 |
| 6 | 1.012 | -11.398 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 7 | 1.003 | -13.149 | 22.800 | 10.900 | 0.000 | 0.000 | 0.00 |
| 8 | 1.010 | -12.115 | 30.000 | 30.000 | 0.000 | 30.828 | 0.00 |
| 9 | 1.051 | -14.434 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 10 | 1.044 | -16.024 | 5.800 | 2.000 | 0.000 | 0.000 | 19.00 |
| 11 | 1.082 | -14.434 | 0.000 | 0.000 | 0.000 | 16.120 | 0.00 |
| 12 | 1.057 | -15.303 | 11.200 | 7.500 | 0.000 | 0.000 | 0.00 |
| 13 | 1.071 | -15.303 | 0.000 | 0.000 | 0.000 | 10.421 | 0.00 |
| 14 | 1.042 | -16.198 | 6.200 | 1.600 | 0.000 | 0.000 | 0.00 |
| 15 | 1.038 | -16.276 | 8.200 | 2.500 | 0.000 | 0.000 | 0.00 |
| 16 | 1.045 | -15.881 | 3.500 | 1.800 | 0.000 | 0.000 | 0.00 |
| 17 | 1.039 | -16.188 | 9.000 | 5.800 | 0.000 | 0.000 | 0.00 |
| 18 | 1.028 | -16.882 | 3.200 | 0.900 | 0.000 | 0.000 | 0.00 |
| 19 | 1.025 | -17.051 | 9.500 | 3.400 | 0.000 | 0.000 | 0.00 |
| 20 | 1.029 | -16.852 | 2.200 | 0.700 | 0.000 | 0.000 | 0.00 |
| 21 | 1.032 | -16.468 | 17.500 | 11.200 | 0.000 | 0.000 | 0.00 |
| 22 | 1.033 | -16.454 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 23 | 1.027 | -16.661 | 3.200 | 1.600 | 0.000 | 0.000 | 0.00 |
| 24 | 1.022 | -16.829 | 8.700 | 6.700 | 0.000 | 0.000 | 4.30 |
| 25 | 1.019 | -16.423 | 0.000 | 0.000 | 0.000 | 0.000 | 0.00 |
| 26 | 1.001 | -16.840 | 3.500 | 2.300 | 0.000 | 0.000 | 0.00 |

```
27  1.026  -15.912    0.000    0.000    0.000    0.000    0.00
28  1.011  -12.057    0.000    0.000    0.000    0.000    0.00
29  1.006  -17.136    2.400    0.900    0.000    0.000    0.00
30  0.995  -18.014   10.600    1.900    0.000    0.000    0.00

Total                283.400  126.200  300.998  125.145   23.30
```

The output of the **lineflow** is the same as the line flow output of Example 6.9 with the power mismatch as dictated by the fast decoupled method.

## PROBLEMS

**6.1.** A power system network is shown in Figure 6.17. The generators at buses 1 and 2 are represented by their equivalent current sources with their reactances in per unit on a 100-MVA base. The lines are represented by $\pi$ model where series reactances and shunt reactances are also expressed in per unit on a 100 MVA base. The loads at buses 3 and 4 are expressed in MW and Mvar.
(a) Assuming a voltage magnitude of 1.0 per unit at buses 3 and 4, convert the loads to per unit impedances. Convert network impedances to admittances and obtain the bus admittance matrix by inspection.
(b) Use the function **Y = ybus(zdata)** to obtain the bus admittance matrix. The function argument **zdata** is a matrix containing the line bus numbers, resistance and reactance. (See Example 6.1.)

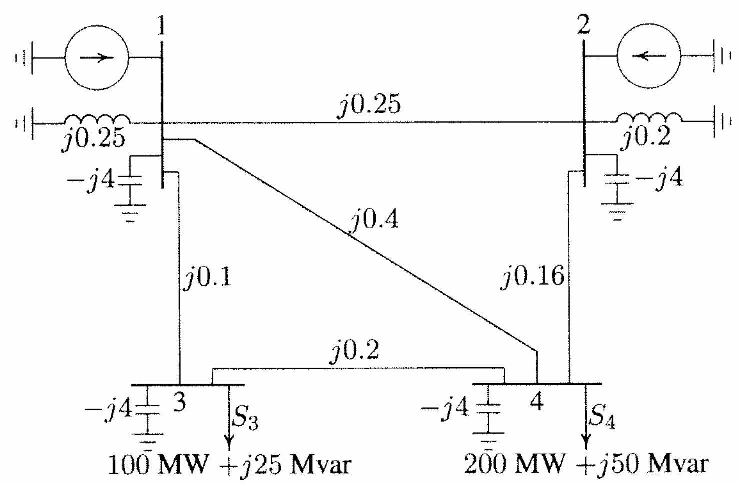



**FIGURE 6.17**
One-line diagram for Problem 6.1.



**6.2.** A power system network is shown in Figure 6.18. The values marked are impedances in per unit on a base of 100 MVA. The currents entering buses 1 and 2 are

$$I_1 = 1.38 - j2.72 \text{ pu}$$
$$I_2 = 0.69 - j1.36 \text{ pu}$$

(a) Determine the bus admittance matrix by inspection.
(b) Use the function **Y = ybus(zdata)** to obtain the bus admittance matrix. The function argument **zdata** is a matrix containing the line bus numbers, resistance and reactance. (See Example 6.1.) Write the necessary *MATLAB* commands to obtain the bus voltages.

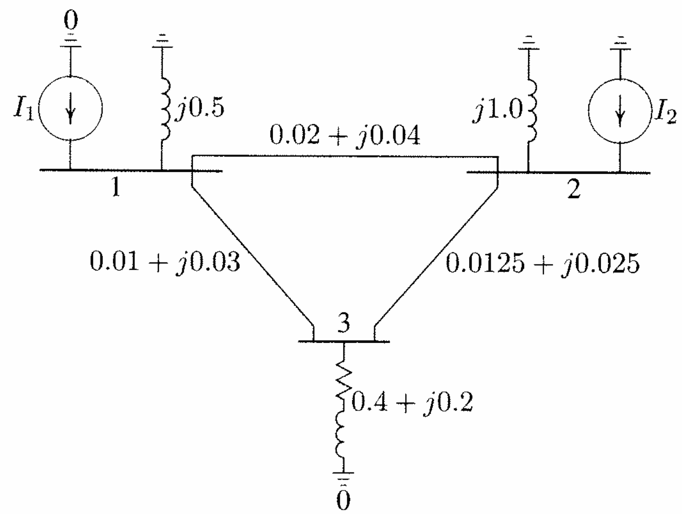



**FIGURE 6.18**
One-line diagram for Problem 6.2.

**6.3.** Use Gauss-Seidel method to find the solution of the following equations

$$x_1 + x_1 x_2 = 10$$
$$x_1 + x_2 = 6$$

with the following initial estimates
(a) $x_1^{(0)} = 1$ and $x_2^{(0)} = 1$
(b) $x_1^{(0)} = 1$ and $x_2^{(0)} = 2$
Continue the iterations until $|\Delta x_1^{(k)}|$ and $|\Delta x_2^{(k)}|$ are less than 0.001.

**6.4.** A fourth-order polynomial equation is given by

$$x^4 - 21x^3 + 147x^2 - 379x + 252 = 0$$

(a) Use Newton-Raphson method and hand calculations to find one of the roots of the polynomial equation. Start with the initial estimate of $x^{(0)} = 0$ and continue until $|\Delta x^{(k)}| < 0.001$.
(b) Write a *MATLAB* program to find the roots of the above polynomial by Newton-Raphson method. The program should prompt the user to input the initial estimate. Run using the initial estimates of 0, 3, 6, 10.
(c) Check your answers using the *MATLAB* function **r = roots(A)**, where **A** is a row vector containing the polynomial coefficients in descending powers.

**6.5.** Use Newton-Raphson method and hand calculation to find the solution of the following equations:

$$x_1^2 - 2x_1 - x_2 = 3$$
$$x_1^2 + x_2^2 = 41$$

(a) Start with the initial estimates of $x_1^{(0)} = 2$, $x_2^{(0)} = 3$. Perform three iterations.
(b) Write a *MATLAB* program to find one of the solutions of the above equations by Newton-Raphson method. The program should prompt the user to input the initial estimates. Run the program with the above initial estimates.

**6.6.** In the power system network shown in Figure 6.19, bus 1 is a slack bus with $V_1 = 1.0\angle 0°$ per unit and bus 2 is a load bus with $S_2 = 280$ MW + $j60$ Mvar. The line impedance on a base of 100 MVA is $Z = 0.02 + j0.04$ per unit.
(a) Using Gauss-Seidel method, determine $V_2$ . Use an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and perform four iterations.
(b) If after several iterations voltage at bus 2 converges to $V_2 = 0.90 - j0.10$, determine $S_1$ and the real and reactive power loss in the line.

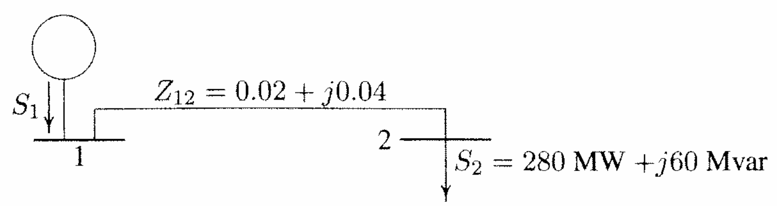



**FIGURE 6.19**
One-line diagram for Problem 6.6.



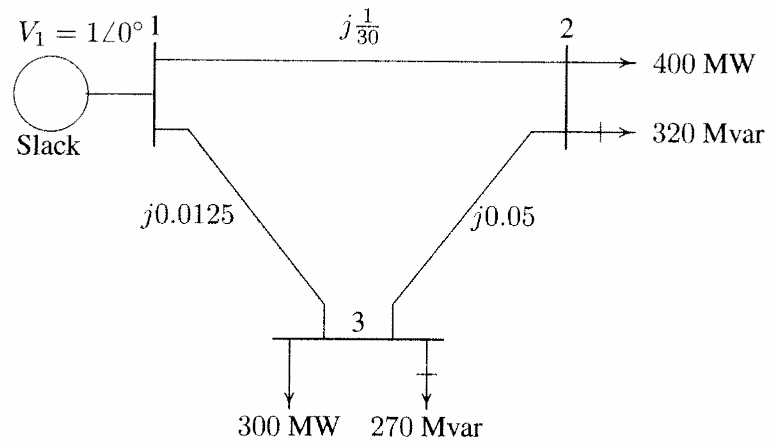



**FIGURE 6.20**
One-line diagram for Problem 6.7.

**6.7.** Figure 6.20 shows the one-line diagram of a simple three-bus power system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.0\angle 0°$ per unit. The scheduled loads on buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100-MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.
(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, determine $V_2$ and $V_3$. Perform two iterations.
(b) If after several iterations the bus voltages converge to

$$V_2 = 0.90 - j0.10 \text{ pu}$$
$$V_3 = 0.95 - j0.05 \text{ pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.
(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.) Use a power accuracy of 0.00001 and an acceleration factor of 1.0.

**6.8.** Figure 6.21 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 3. The voltage at bus 1 is $V_1 = 1.025\angle 0°$ per unit. Voltage magnitude at bus 3 is fixed at 1.03 pu with a real power generation of 300 MW. A load consisting of 400 MW and 200 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100-MVA base. For the

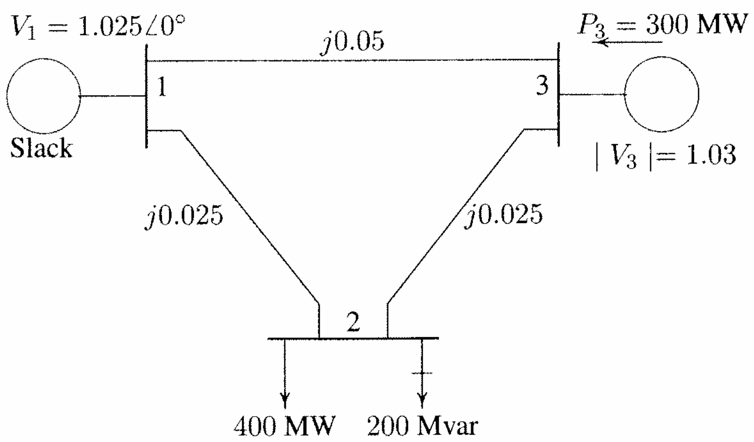



**FIGURE 6.21**
One-line diagram for Problem 6.8.

purpose of hand calculations, line resistances and line charging susceptances are neglected.

(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.03 + j0$ and keeping $|V_3| = 1.03$ pu, determine the phasor values of $V_2$ and $V_3$ . Perform two iterations.

(b) If after several iterations the bus voltages converge to

$$V_2 = 1.001243\angle{-2.1°} = 1.000571 - j0.0366898 \text{ pu}$$
$$V_3 = 1.03\angle 1.36851° = 1.029706 + j0.0246 \text{ pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.

(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.)

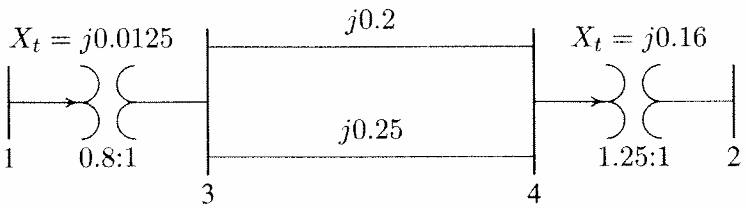



**FIGURE 6.22**
One-line diagram for Problem 6.9.



**6.9.** The one-line diagram of a four-bus power system is as shown in Figure 6.22. Reactances are given in per unit on a common MVA base. Transformers $T_1$ and $T_2$ have tap settings of 0.8:1, and 1.25:1 respectively. Obtain the bus admittance matrix.

**6.10.** In the two-bus system shown in Figure 6.23, bus 1 is a slack bus with $V_1 = 1.0\angle 0°$ pu. A load of 150 MW and 50 Mvar is taken from bus 2. The line admittance is $y_{12} = 10\angle{-73.74°}$ pu on a base of 100 MVA. The expression for real and reactive power at bus 2 is given by

$$P_2 = 10|V_2||V_1|\cos(106.26° - \delta_2 + \delta_1) + 10|V_2|^2\cos(-73.74°)$$
$$Q_2 = -10|V_2||V_1|\sin(106.26° - \delta_2 + \delta_1) - 10|V_2|^2\sin(-73.74°)$$

Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2|^{(0)} = 1.0$ pu and $\delta_2^{(0)} = 0°$. Perform two iterations.

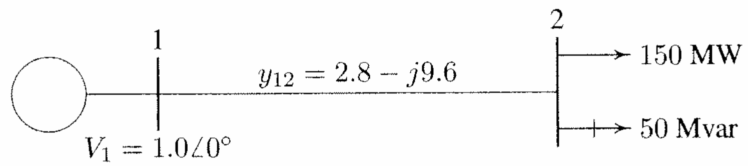



**FIGURE 6.23**
One-line diagram for Problem 6.10.

**6.11.** In the two-bus system shown in Figure 6.24, bus 1 is a slack bus with $V_1 = 1.0\angle 0°$ pu. A load of 100 MW and 50 Mvar is taken from bus 2. The line impedance is $z_{12} = 0.12 + j0.16$ pu on a base of 100 MVA. Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2|^{(0)} = 1.0$ pu and $\delta_2^{(0)} = 0°$. Perform two iterations.

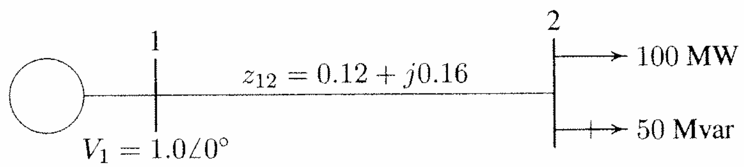



**FIGURE 6.24**
One-line diagram for Problem 6.11.

**6.12.** Figure 6.25 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.0\angle 0°$ per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

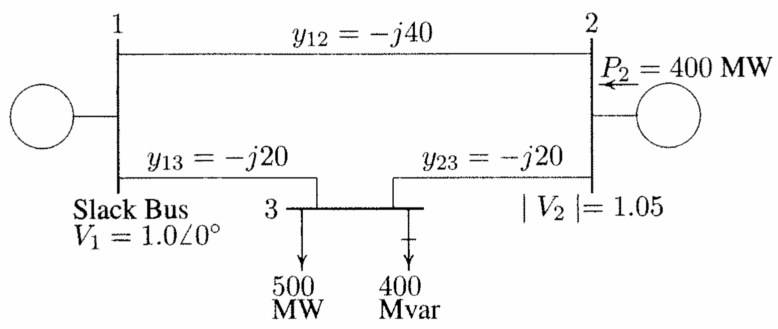



**FIGURE 6.25**
One-line diagram for Problem 6.12

(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$P_2 = 40|V_2||V_1|\cos(90° - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90° - \delta_2 + \delta_3)$$
$$P_3 = 20|V_3||V_1|\cos(90° - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90° - \delta_3 + \delta_2)$$
$$Q_3 = -20|V_3||V_1|\sin(90° - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90° - \delta_3 + \delta_2) + 40|V_3|^2$$

(b) Using Newton-Raphson method, start with the initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, and keeping $|V_2| = 1.05$ pu, determine the phasor values of $V_2$ and $V_3$. Perform two iterations.
(c) Check the power flow solution for Problem 6.12 using **lfnewton** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

**6.13.** For Problem 6.12:
(a) Obtain the power flow solution using the fast decoupled algorithm. Perform two iterations.
(b) Check the power flow solution for Problem 6.12 using **decouple** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

**6.14.** The 26-bus power system network of an electric utility company is shown in Figure 6.26 (page 256). Obtain the power flow solution by the following



methods:
(a) Gauss-Seidel power flow (see Example 6.9).
(b) Newton-Raphson power flow (see Example 6.11).
(c) Fast decoupled power flow (see Example 6.13).

The load data is as follows.

**LOAD DATA**

| Bus No. | Load MW | Load Mvar | Bus No. | Load MW | Load Mvar |
|---|---|---|---|---|---|
| 1 | 51.0 | 41.0 | 14 | 24.0 | 12.0 |
| 2 | 22.0 | 15.0 | 15 | 70.0 | 31.0 |
| 3 | 64.0 | 50.0 | 16 | 55.0 | 27.0 |
| 4 | 25.0 | 10.0 | 17 | 78.0 | 38.0 |
| 5 | 50.0 | 30.0 | 18 | 153.0 | 67.0 |
| 6 | 76.0 | 29.0 | 19 | 75.0 | 15.0 |
| 7 | 0.0 | 0.0 | 20 | 48.0 | 27.0 |
| 8 | 0.0 | 0.0 | 21 | 46.0 | 23.0 |
| 9 | 89.0 | 50.0 | 22 | 45.0 | 22.0 |
| 10 | 0.0 | 0.0 | 23 | 25.0 | 12.0 |
| 11 | 25.0 | 15.0 | 24 | 54.0 | 27.0 |
| 12 | 89.0 | 48.0 | 25 | 28.0 | 13.0 |
| 13 | 31.0 | 15.0 | 26 | 40.0 | 20.0 |

Voltage magnitude, generation schedule, and the reactive power limits for the regulated buses are tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.025\angle 0°$, is taken as the slack bus.

**GENERATION DATA**

| Bus No. | Voltage Mag. | Generation MW | Mvar Limits Min. | Mvar Limits Max. |
|---|---|---|---|---|
| 1 | 1.025 | | | |
| 2 | 1.020 | 79.0 | 40.0 | 250.0 |
| 3 | 1.025 | 20.0 | 40.0 | 150.0 |
| 4 | 1.050 | 100.0 | 40.0 | 80.0 |
| 5 | 1.045 | 300.0 | 40.0 | 160.0 |
| 26 | 1.015 | 60.0 | 15.0 | 50.0 |

The Mvar of the shunt capacitors installed at substations and the transformer tap settings are given below.

| SHUNT CAPACITORS | |
|---|---|
| Bus No. | Mvar |
| 1 | 4.0 |
| 4 | 2.0 |
| 5 | 5.0 |
| 6 | 2.0 |
| 11 | 1.5 |
| 12 | 2.0 |
| 15 | 0.5 |
| 19 | 5.0 |

| TRANSFORMER TAP | |
|---|---|
| Designation | Tap Setting |
| 2 – 3 | 0.960 |
| 2 – 13 | 0.960 |
| 3 – 13 | 1.017 |
| 4 – 8 | 1.050 |
| 4 – 12 | 1.050 |
| 6 – 19 | 0.950 |
| 7 – 9 | 0.950 |

The line and transformer series resistance, reactance and one-half the total capacitive susceptance in per unit on a 100-MVA base are tabulated below.

| LINE AND TRANSFORMER DATA | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Bus No. | Bus No. | $R$, pu | $X$, pu | $\frac{1}{2}B$, pu | Bus No. | Bus No. | $R$, pu | $X$, pu | $\frac{1}{2}B$, pu |
| 1 | 2 | 0.0005 | 0.0048 | 0.0300 | 10 | 22 | 0.0069 | 0.0298 | 0.005 |
| 1 | 18 | 0.0013 | 0.0110 | 0.0600 | 11 | 25 | 0.0960 | 0.2700 | 0.010 |
| 2 | 3 | 0.0014 | 0.0513 | 0.0500 | 11 | 26 | 0.0165 | 0.0970 | 0.004 |
| 2 | 7 | 0.0103 | 0.0586 | 0.0180 | 12 | 14 | 0.0327 | 0.0802 | 0.000 |
| 2 | 8 | 0.0074 | 0.0321 | 0.0390 | 12 | 15 | 0.0180 | 0.0598 | 0.000 |
| 2 | 13 | 0.0035 | 0.0967 | 0.0250 | 13 | 14 | 0.0046 | 0.0271 | 0.001 |
| 2 | 26 | 0.0323 | 0.1967 | 0.0000 | 13 | 15 | 0.0116 | 0.0610 | 0.000 |
| 3 | 13 | 0.0007 | 0.0054 | 0.0005 | 13 | 16 | 0.0179 | 0.0888 | 0.001 |
| 4 | 8 | 0.0008 | 0.0240 | 0.0001 | 14 | 15 | 0.0069 | 0.0382 | 0.000 |
| 4 | 12 | 0.0016 | 0.0207 | 0.0150 | 15 | 16 | 0.0209 | 0.0512 | 0.000 |
| 5 | 6 | 0.0069 | 0.0300 | 0.0990 | 16 | 17 | 0.0990 | 0.0600 | 0.000 |
| 6 | 7 | 0.0053 | 0.0306 | 0.0010 | 16 | 20 | 0.0239 | 0.0585 | 0.000 |
| 6 | 11 | 0.0097 | 0.0570 | 0.0001 | 17 | 18 | 0.0032 | 0.0600 | 0.038 |
| 6 | 18 | 0.0037 | 0.0222 | 0.0012 | 17 | 21 | 0.2290 | 0.4450 | 0.000 |
| 6 | 19 | 0.0035 | 0.0660 | 0.0450 | 19 | 23 | 0.0300 | 0.1310 | 0.000 |
| 6 | 21 | 0.0050 | 0.0900 | 0.0226 | 19 | 24 | 0.0300 | 0.1250 | 0.002 |
| 7 | 8 | 0.0012 | 0.0069 | 0.0001 | 19 | 25 | 0.1190 | 0.2249 | 0.004 |
| 7 | 9 | 0.0009 | 0.0429 | 0.0250 | 20 | 21 | 0.0657 | 0.1570 | 0.000 |
| 8 | 12 | 0.0020 | 0.0180 | 0.0200 | 20 | 22 | 0.0150 | 0.0366 | 0.000 |
| 9 | 10 | 0.0010 | 0.0493 | 0.0010 | 21 | 24 | 0.0476 | 0.1510 | 0.000 |
| 10 | 12 | 0.0024 | 0.0132 | 0.0100 | 22 | 23 | 0.0290 | 0.0990 | 0.000 |
| 10 | 19 | 0.0547 | 0.2360 | 0.0000 | 22 | 24 | 0.0310 | 0.0880 | 0.000 |
| 10 | 20 | 0.0066 | 0.0160 | 0.0010 | 23 | 25 | 0.0987 | 0.1168 | 0.000 |



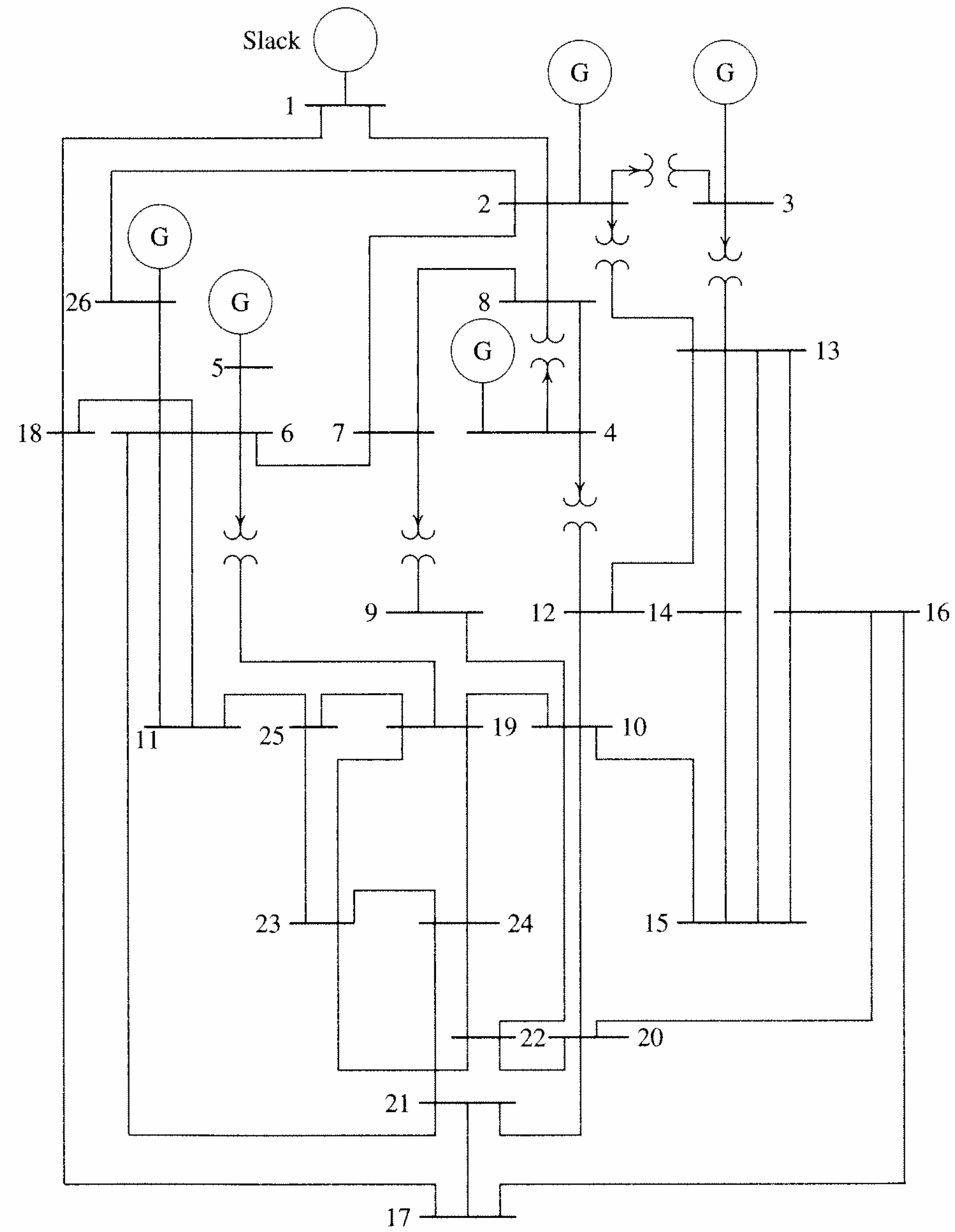



**FIGURE 6.26**
One-line diagram for Problem 6.14.