CHAPTER
9

BALANCED FAULT

9.1 INTRODUCTION

Fault studies form an important part of power system analysis. The problem consists of determining bus voltages and line currents during various types of faults. Faults on power systems are divided into *three-phase balanced faults* and *unbalanced faults*. Different types of unbalanced faults are *single line-to-ground fault*, *line-to-line fault*, and *double line-to-ground fault*, which are dealt with in Chapter 10. The information gained from fault studies are used for proper relay setting and coordination. The three-phase balanced fault information is used to select and set phase relays, while the line-to-ground fault is used for ground relays. Fault studies are also used to obtain the rating of the protective switchgears.

The magnitude of the fault currents depends on the internal impedance of the generators plus the impedance of the intervening circuit. We have seen in Chapter 8 that the reactance of a generator under short circuit condition is not constant. For the purpose of fault studies, the generator behavior can be divided into three periods: the *subtransient period*, lasting only for the first few cycles; the *transient period*, covering a relatively longer time; and, finally, the *steady state period*. In this chapter, three-phase balanced faults are discussed. The bus impedance matrix by the *building algorithm* is formulated and is employed for the systematic computation of bus voltages and line currents during the fault. Two functions are

developed for the formation of the bus impedance matrix. These function are **Zbus = zbuild(zdata)** and **Zbus = zbuildpi(linedata, gendata, yload)**. The latter one is compatible with power flow input/output files. A program named **symfault** is developed for systematic computation of three-phase balanced faults for a large interconnected power system.

9.2 BALANCED THREE-PHASE FAULT

This type of fault is defined as the simultaneous short circuit across all three phases. It occurs infrequently, and when the ratio of system zero sequence impedance to positive sequence impedance is greater than 1, it is the most sever type of fault encountered. Because the network is balanced, it is solved on a per-phase basis. The other two phases carry identical currents except for the phase shift.

In Chapter 8 it was shown that the reactance of the synchronous generator under short-circuit conditions is a time-varying quantity, and for network analysis three reactances were defined. The subtransient reactance X''_d, for the first few cycles of the short circuit current, transient reactance X'_d, for the next (say) 30 cycles, and the synchronous reactance X_d, thereafter. Since the duration of the short circuit current depends on the time of operation of the protective system, it is not always easy to decide which reactance to use. Generally, the subtransient reactance is used for determining the interrupting capacity of the circuit breakers. In fault studies required for relay setting and coordination, transient reactance is used. Also, in typical transient stability studies, transient reactance is used.

A fault represents a structural network change equivalent with that caused by the addition of an impedance at the place of fault. If the fault impedance is zero, the fault is referred to as the *bolted fault* or the *solid fault*. The faulted network can be solved conveniently by the Thévenin's method. The procedure is demonstrated in the following example.

Example 9.1 (chp9ex1)

The one-line diagram of a simple three-bus power system is shown in Figure 9.1. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common 100 MVA base, and for simplicity, resistances are neglected. The following assumptions are made.

(*i*) Shunt capacitances are neglected and the system is considered on no-load.

(*ii*) All generators are running at their rated voltage and rated frequency with their emfs in phase.

Determine the fault current, the bus voltages, and the line currents during the fault when a balanced three-phase fault with a fault impedance $Z_f = 0.16$ per unit occurs on

(a) Bus 3.
(b) Bus 2.
(c) Bus 1.

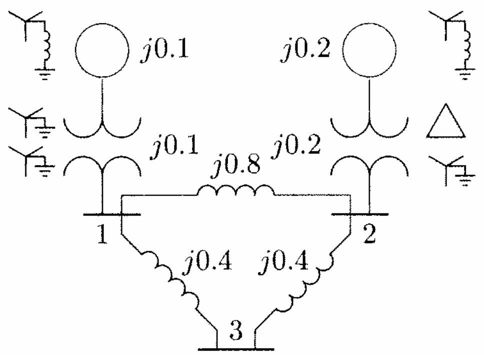


FIGURE 9.1
The impedance diagram of a simple power system.

The fault is simulated by switching on an impedance Z_f at bus 3 as shown in Figure 9.2(a). Thévenin's theorem states that the changes in the network voltage caused by the added branch (the fault impedance) shown in Figure 9.2(a) is equivalent to those caused by the added voltage $V_3(0)$ with all other sources short-circuited as shown in Figure 9.2(b).

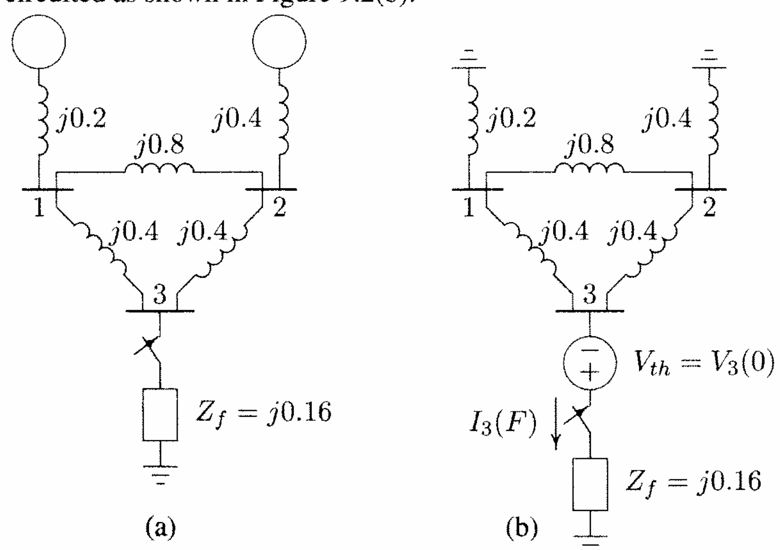


FIGURE 9.2
(a) The impedance network for fault at bus 3. (b) Thévenin's equivalent network.

(a) From 9.2(b), the fault current at bus 3 is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f}$$

where $V_3(0)$ is the Thévenin's voltage or the prefault bus voltage. The prefault bus voltage can be obtained from the results of the power flow solution. In this example, since the loads are neglected and generator's emfs are assumed equal to the rated value, all the prefault bus voltages are equal to 1.0 per unit, i.e.,

$$V_1(0) = V_2(0) = V_3(0) = 1.0 \quad \text{pu}$$

Z_{33} is the Thévenin's impedance viewed from the faulted bus.

To find the Thévenin's impedance, we convert the Δ formed by buses 123 to an equivalent Y as shown in Figure 9.3(a).

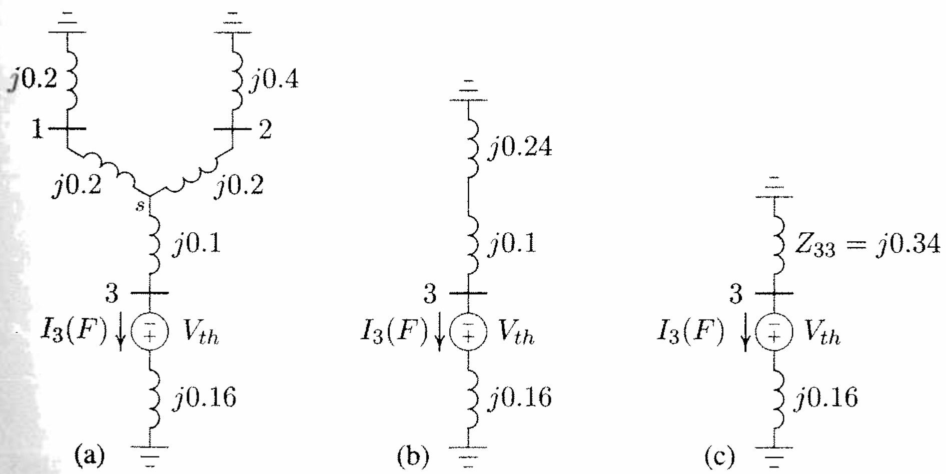


FIGURE 9.3
Reduction of Thévenin's equivalent network.

$$Z_{1s} = Z_{2s} = \frac{(j0.4)(j0.8)}{j1.6} = j0.2 \qquad Z_{3s} = \frac{(j0.4)(j0.4)}{j1.6} = j0.1$$

Combining the parallel branches, Thévenin's impedance is

$$\begin{aligned} Z_{33} &= \frac{(j0.4)(j0.6)}{j0.4 + j0.6} + j0.1 \\ &= j0.24 + j0.1 = j0.34 \end{aligned}$$

From Figure 9.3(c), the fault current is

$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \quad \text{pu}$$

With reference to Figure 9.3(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.6}{j0.4 + j0.6} I_3(F) = -j1.2 \quad \text{pu}$$
$$I_{G2} = \frac{j0.4}{j0.4 + j0.6} I_3(F) = -j0.8 \quad \text{pu}$$

For the bus voltage changes from Figure 9.3(b), we get

$$\Delta V_1 = 0 - (j0.2)(-j1.2) \;\; = -0.24 \quad \text{pu}$$
$$\Delta V_2 = 0 - (j0.4)(-j0.8) \;\; = -0.32 \quad \text{pu}$$
$$\Delta V_3 = (j0.16)(-j2) - 1.0 = -0.68 \quad \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.2(b), i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.24 = 0.76 \quad \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.32 = 0.68 \quad \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.68 = 0.32 \quad \text{pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu}$$

(b) The fault with impedance Z_f at bus 2 is depicted in Figure 9.4(a), and its Thévenin's equivalent circuit is shown in Figure 9.4(b). To find the Thévenin's impedance, we combine the parallel branches in Figure 9.4(b). Also, combining parallel branches from ground to bus 2 in Figure 9.5(a), results in

$$Z_{22} = \frac{(j0.6)(j0.4)}{j0.6 + j0.4} = j0.24$$

From Figure 9.5(b), the fault current is

$$I_2(F) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.24 + j0.16} = -j2.5 \quad \text{pu}$$

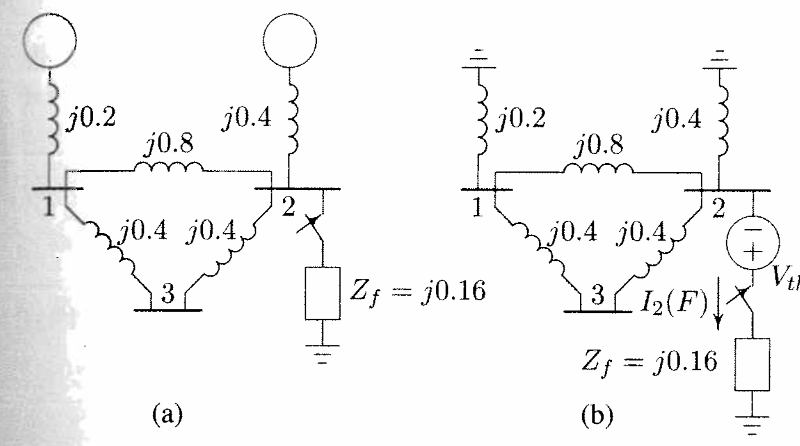


FIGURE 9.4
(a) The impedance network for fault at bus 2. (b) Thévenin's equivalent network.

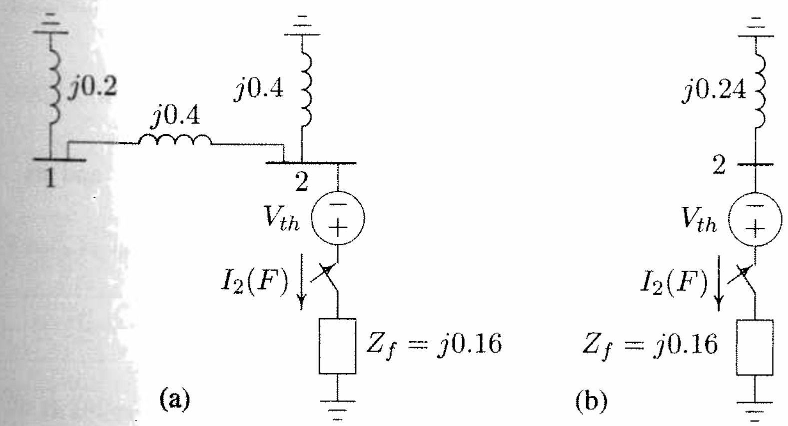


FIGURE 9.5
Reduction of Thévenin's equivalent network.

With reference to Figure 9.5(a), the current divisions between the generators are

$$I_{G1} = \frac{j0.4}{j0.4 + j0.6} I_2(F) = -j1.0 \quad \text{pu}$$
$$I_{G2} = \frac{j0.6}{j0.4 + j0.6} I_2(F) = -j1.5 \quad \text{pu}$$

For the bus voltage changes from Figure 9.4(a), we get

$$\Delta V_1 = 0 - (j0.2)(-j1.0) = -0.2 \quad \text{pu}$$
$$\Delta V_2 = 0 - (j0.4)(-j1.5) = -0.6 \quad \text{pu}$$
$$\Delta V_3 = -0.2 - (j0.4)(\frac{-j1.0}{2}) = -0.4 \quad \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.4(b), i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.2 = 0.8 \quad \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.6 = 0.4 \quad \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.4 = 0.6 \quad \text{pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.8 - 0.4}{j0.8} = -j0.5 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.8 - 0.6}{j0.4} = -j0.5 \quad \text{pu}$$
$$I_{32}(F) = \frac{V_3(F) - V_2(F)}{z_{32}} = \frac{0.6 - 0.4}{j0.4} = -j0.5 \quad \text{pu}$$

(c) The fault with impedance Z_f at bus 1 is depicted in Figure 9.6(a), and its Thévenin's equivalent circuit is shown in Figure 9.6(b).

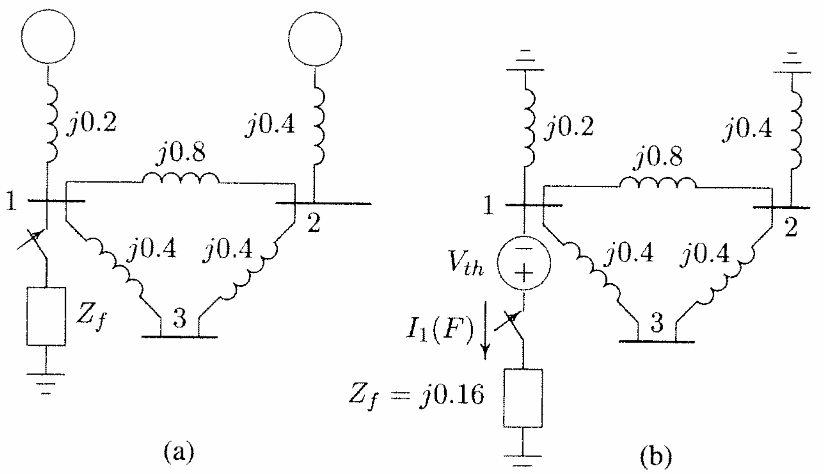


FIGURE 9.6
(a) The impedance network for fault at bus 1. (b) Thévenin's equivalent network.

To find the Thévenin's impedance, we combine the parallel branches in Figure 9.6(b). Also, combining parallel branches from ground to bus 1 in Figure 9.7(a),

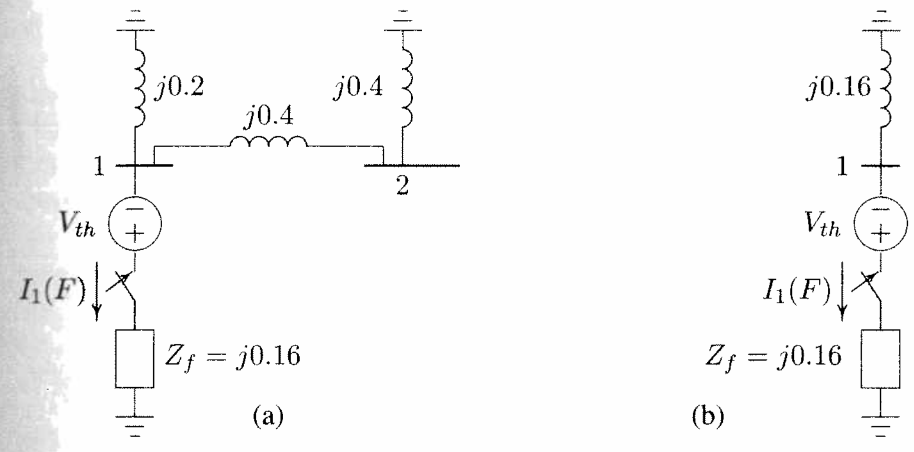


FIGURE 9.7
Reduction of Thévenin's equivalent network.

results in

$$Z_{11} = \frac{(j0.2)(j0.8)}{j0.2 + j0.8} = j0.16$$

From Figure 9.7(b), the fault current is

$$I_1(F) = \frac{V_1(0)}{Z_{11} + Z_f} = \frac{1.0}{j0.16 + j0.16} = -j3.125 \quad \text{pu}$$

With reference to Figure 9.7(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.8}{j0.2 + j0.8} I_2(F) = -j2.50 \quad \text{pu}$$
$$I_{G2} = \frac{j0.2}{j0.2 + j0.8} I_2(F) = -j0.625 \quad \text{pu}$$

For the bus voltage changes from Figure 9.6(b), we get

$$\Delta V_1 = 0 - (j0.2)(-j2.5) = -0.50 \quad \text{pu}$$
$$\Delta V_2 = 0 - (j0.4)(-j0.625) = -0.25 \quad \text{pu}$$
$$\Delta V_3 = -0.5 + (j0.4)(\frac{-j0.625}{2}) = -0.375 \quad \text{pu}$$

Bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, as shown in Figure 9.6(b), i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.50 = 0.50 \quad \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.25 = 0.75 \quad \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.375 = 0.625 \quad \text{pu}$$

The short-circuit currents in the lines are

$$I_{21}(F) = \frac{V_2(F) - V_1(F)}{z_{21}} = \frac{0.75 - 0.5}{j0.8} = -j0.3125 \quad \text{pu}$$
$$I_{31}(F) = \frac{V_3(F) - V_1(F)}{z_{31}} = \frac{0.625 - 0.5}{j0.4} = -j0.3125 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.75 - 0.625}{j0.4} = -j0.3125 \quad \text{pu}$$

In the above example the load currents were neglected and all prefault bus voltages were assumed to be equal to 1.0 per unit. For more accurate calculation, the prefault bus voltages can be obtained from the power flow solution. As we have seen in Chapter 6, in a power system, loads are specified and the load currents are unknown. One way to include the effects of load currents in the fault analysis is to express the loads by a constant impedance evaluated at the prefault bus voltages. This is a very good approximation which results in linear nodal equations. The procedure is summarized in the following steps.

- The prefault bus voltages are obtained from the results of the power flow solution.

- In order to preserve the linearity feature of the network, loads are converted to constant admittances using the prefault bus voltages.

- The faulted network is reduced into a Thévenin's equivalent circuit as viewed from the faulted bus. Applying Thévenin's theorem, changes in the bus voltages are obtained.

- Bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages computed in the previous step.

- The currents during the fault in all branches of the network are then obtained.

9.3 SHORT-CIRCUIT CAPACITY (SCC)

The short-circuit capacity at a bus is a common measure of the strength of a bus. The short-circuit capacity or the short-circuit MVA at bus k is defined as the product of the magnitudes of the rated bus voltage and the fault current. The short-circuit MVA is used for determining the dimension of a bus bar, and the *interrupting* capacity of a circuit breaker. The interrupting capacity is only one of many ratings of a circuit breaker and should not be confused with the *momentary duty* of the breaker described in (8.63).

Based on the above definition, the short-circuit capacity or the short-circuit MVA at bus k is given by

$$SCC = \sqrt{3}\, V_{Lk} I_k(F) \times 10^{-3} \quad \text{MVA} \tag{9.1}$$

where the line-to-line voltage V_{Lk} is expressed in kilovolts and $I_k(F)$ is expressed in amperes. The symmetrical three-phase fault current in per unit is given by

$$I_k(F)_{pu} = \frac{V_k(0)}{X_{kk}} \tag{9.2}$$

where $V_k(0)$ is the per unit prefault bus voltage, and X_{kk} is the per unit reactance to the point of fault. System resistance is neglected and only the inductive reactance of the system is allowed for. This gives minimum system impedance and maximum fault current and a pessimistic answer. The base current is given by

$$I_B = \frac{S_B \times 10^3}{\sqrt{3}\, V_B} \tag{9.3}$$

where S_B is the base MVA and V_B is the line-to-line base voltage in kilovolts. Thus, the fault current in amperes is

$$\begin{aligned} I_k(F) &= I_k(F)_{pu} I_B \\ &= \frac{V_k(0)}{X_{kk}} \frac{S_B \times 10^3}{\sqrt{3}\, V_B} \end{aligned} \tag{9.4}$$

Substituting for $I_k(F)$ from (9.4) into (9.1) results in

$$SCC = \frac{V_k(0) S_B}{X_{kk}} \frac{V_L}{V_B} \tag{9.5}$$

If the base voltage is equal to the rated voltage, i.e., $V_L = V_B$

$$SCC = \frac{V_k(0) S_B}{X_{kk}} \tag{9.6}$$

The prefault bus voltage is usually assumed to be 1.0 per unit, and we therefore obtain from (9.6) the following approximate formula for the short-circuit capacity or the short-circuit MVA.

$$SCC = \frac{S_B}{X_{kk}} \quad \text{MVA} \tag{9.7}$$

9.4 SYSTEMATIC FAULT ANALYSIS USING BUS IMPEDANCE MATRIX

The network reduction used in the preceding example is not efficient and is not applicable to large networks. In this section a more general fault circuit analysis using nodal method is obtained. We see that by utilizing the elements of the bus impedance matrix, the fault current as well as the bus voltages during fault are readily and easily calculated.

Consider a typical bus of an n-bus power system network as shown in Figure 9.8. The system is assumed to be operating under balanced condition and a per phase circuit model is used. Each machine is represented by a constant voltage source behind proper reactances which may be X''_d, X'_d, or X_d. Transmission lines are represented by their equivalent π model and all impedances are expressed in per unit on a common MVA base. A balanced three-phase fault is to be applied at bus k through a fault impedance Z_f. The prefault bus voltages are obtained from the power flow solution and are represented by the column vector

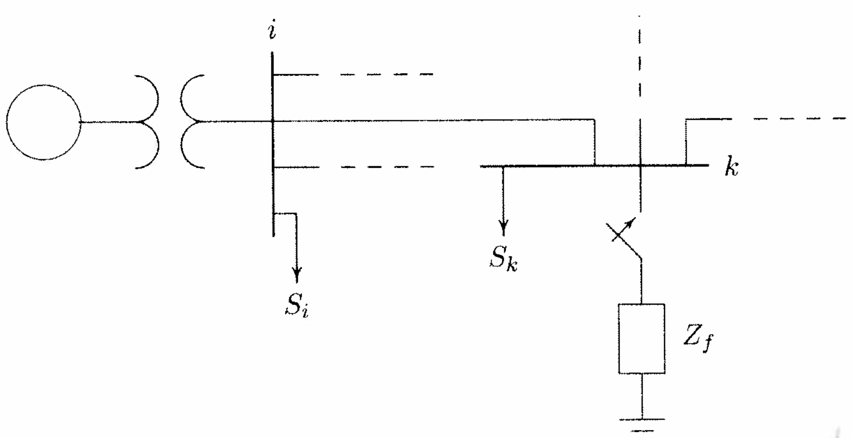


FIGURE 9.8
A typical bus of a power system.

$$\mathbf{V}_{bus}(0) = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix} \tag{9.8}$$

As already mentioned, short circuit currents are so much larger than the steady-state values that we may neglect the latter. However, a good approximation is to represent the bus load by a constant impedance evaluated at the prefault bus voltage, i.e.,

$$Z_{iL} = \frac{|V_i(0)|^2}{S_L^*} \tag{9.9}$$

The changes in the network voltage caused by the fault with impedance Z_f is equivalent to those caused by the added voltage $V_k(0)$ with all other sources short-circuited. Zeroing all voltage sources and representing all components and loads by their appropriate impedances, we obtain the Thévenin's circuit shown in Figure 9.9. The bus voltage changes caused by the fault in this circuit are represented by the column vector

$$\mathbf{\Delta V}_{bus} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} \tag{9.10}$$

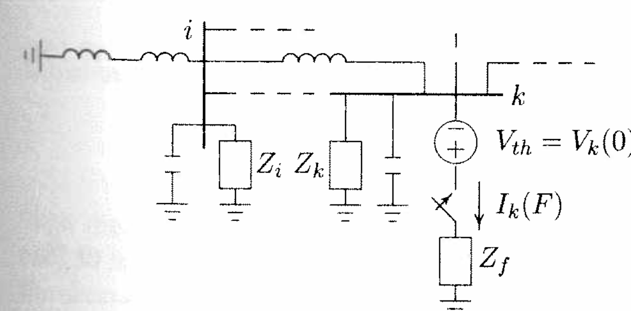


FIGURE 9.9
A typical bus of a power system.

From Thévenin's theorem bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages given by

$$\mathbf{V}_{bus}(F) = \mathbf{V}_{bus}(0) + \mathbf{\Delta V}_{bus} \tag{9.11}$$

In Section 6.2, we obtained the node-voltage equation for an n-bus network. The injected bus currents are expressed in terms of the bus voltages (with bus 0 as reference), i.e.,

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\mathbf{V}_{bus} \tag{9.12}$$

where $\mathbf{I}_{bus}$ is the bus current vector entering the bus and $\mathbf{Y}_{bus}$ is the bus admittance matrix. The diagonal element of each bus is the sum of admittances connected to it, i.e.,

$$Y_{ii} = \sum_{j=0}^{m} y_{ij} \qquad j \neq i \tag{9.13}$$

The off-diagonal element is equal to the negative of the admittance between the buses, i.e.,

$$Y_{ij} = Y_{ji} = -y_{ij} \tag{9.14}$$

where y_{ij} (lower case) is the actual admittance of the line i-j. For more details refer to Section 6.2.

In the Thévenin's circuit of Figure 9.9, current entering every bus is zero except at the faulted bus. Since the current at faulted bus is leaving the bus, it is taken as a negative current entering bus k. Thus the nodal equation applied to the Thévenin's circuit in Figure 9.9 becomes

$$\begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{1k} & \cdots & Y_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{k1} & \cdots & Y_{kk} & \cdots & Y_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & \cdots & Y_{nk} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} \tag{9.15}$$

or

$$\mathbf{I}_{bus}(F) = \mathbf{Y}_{bus}\mathbf{\Delta V}_{bus} \tag{9.16}$$

Solving for $\mathbf{\Delta V}_{bus}$, we have

$$\mathbf{\Delta V}_{bus} = \mathbf{Z}_{bus}\mathbf{I}_{bus}(F) \tag{9.17}$$

where $\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1}$ is known as the *bus impedance matrix*. Substituting (9.17) into (9.11), the bus voltage vector during the fault becomes

$$\mathbf{V}_{bus}(F) = \mathbf{V}_{bus}(0) + \mathbf{Z}_{bus}\mathbf{I}_{bus}(F) \tag{9.18}$$

Writing the above matrix equation in terms of its elements, we have

$$\begin{bmatrix} V_1(F) \\ \vdots \\ V_k(F) \\ \vdots \\ V_n(F) \end{bmatrix} = \begin{bmatrix} V_1(0) \\ \vdots \\ V_k(0) \\ \vdots \\ V_n(0) \end{bmatrix} + \begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{n1} & \cdots & Z_{nk} & \cdots & Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_k(F) \\ \vdots \\ 0 \end{bmatrix} \tag{9.19}$$

Since we have only one single nonzero element in the current vector, the kth equation in (9.19) becomes

$$V_k(F) = V_k(0) - Z_{kk}I_k(F) \tag{9.20}$$

Also from the Thévenin's circuit shown in Figure 9.9, we have

$$V_k(F) = Z_f I_k(F) \tag{9.21}$$

For bolted fault, $Z_f = 0$ and $V_k(F) = 0$. Substituting for $V_k(F)$ from (9.21) into (9.20) and solving for the fault current, we get

$$I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f} \tag{9.22}$$

Thus for a fault at bus k we need only the Z_{kk} element of the bus impedance matrix. This element is indeed the Thévenin's impedance as viewed from the faulted bus. Also, writing the ith equation in (9.19) in terms of its element, we have

$$V_i(F) = V_i(0) - Z_{ik}I_k(F) \tag{9.23}$$

Substituting for $I_k(F)$, bus voltage during the fault at bus i becomes

$$V_i(F) = V_i(0) - \frac{Z_{ik}}{Z_{kk} + Z_f}V_k(0) \tag{9.24}$$

With the knowledge of bus voltages during the fault, we can calculate the fault current in all the lines. For the line connecting buses i and j with impedance z_{ij}, the short circuit current in this line (defined positive in the direction $i \rightarrow j$) is

$$I_{ij}(F) = \frac{V_i(F) - V_j(F)}{z_{ij}} \tag{9.25}$$

We note that with the knowledge of the bus impedance matrix, the fault current and bus voltages during the fault are readily obtained for any faulted bus in the network. This method is very simple and practical. Thus, all fault calculations are formulated in the bus frame of reference using bus impedance matrix Z_{bus}.

One way to find $\mathbf{Z}_{bus}$ is to formulate Y_{bus} matrix for the system and then find its inverse. The matrix inversion for a large power system with a large number of buses is not feasible. A computationally attractive and efficient method for finding Z_{bus} matrix is "building" or "assembling" the impedance matrix by adding one network element at a time. In effect, this is an indirect matrix inversion of the bus admittance matrix. The algorithm for building the bus impedance matrix is described in the next section.

Example 9.2 (chp9ex2)

A three-phase fault with a fault impedance $Z_f = j0.16$ per unit occurs at bus 3 in the network of Example 9.1. Using the bus impedance matrix method, compute the fault current, the bus voltages, and the line currents during the fault.

In this example the bus impedance matrix is obtained by finding the inverse of the bus admittance matrix. In the next section, we describe an efficient method of finding the bus impedance matrix by the method of building algorithm.

To find the bus admittance matrix, the Thévenin's circuit in Figure 9.2(b) is redrawn with impedances converted to admittances as shown in Figure 9.10. The

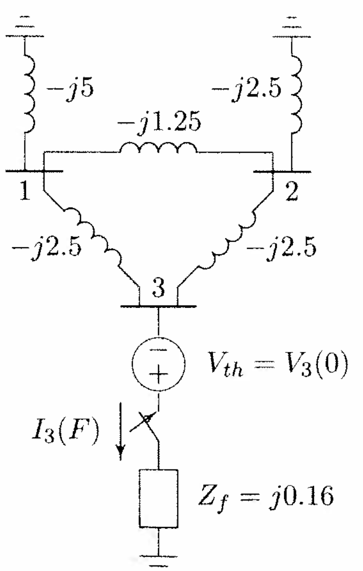


FIGURE 9.10
The admittance diagram for system of Figure 9.2 (b).

ith diagonal element of the bus admittance matrix is the sum of all admittances connected to bus i, and the ijth off-diagonal element is the negative of the admittance between buses i and j. Referring to Figure 9.10, the bus admittance matrix by inspection is

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.5 \\ j1.25 & -j6.25 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix}$$

Using *MATLAB* inverse function **inv**, the bus impedance matrix is obtained

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

From (9.22), for a fault at bus 3 with fault impedance $Z_f = j0.16$ per unit, the fault current is

$$I_3(F) = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$V_1(F) = V_1(0) - Z_{13}I_3(F) = 1.0 - (j0.12)(-j2.0) = 0.76 \quad \text{pu}$$
$$V_2(F) = V_2(0) - Z_{23}I_3(F) = 1.0 - (j0.16)(-j2.0) = 0.68 \quad \text{pu}$$
$$V_3(F) = V_3(0) - Z_{33}I_3(F) = 1.0 - (j0.34)(-j2.0) = 0.32 \quad \text{pu}$$

From (9.25), the short circuit currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76 - 0.68}{j0.8} = -j0.1 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76 - 0.32}{j0.4} = -j1.1 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68 - 0.32}{j0.4} = -j0.9 \quad \text{pu}$$

The results are exactly the same as the values found in Example 9.1(a). The reader is encouraged to repeat the above calculations for fault at buses 2 and 1, and compare the results with those obtained from parts (b) and (c) in Example 9.1.

Note that the values of the diagonal elements in the bus impedance matrix are the same as the Thévenin's impedances found in Example 9.1, thus eliminating the repeated need for network reduction for each fault location. Furthermore, the off-diagonal elements are utilized in (9.24) to obtain bus voltages during the fault. Therefore, the bus impedance matrix method is an indispensable tool for fault studies.

9.5 ALGORITHM FOR FORMATION OF THE BUS IMPEDANCE MATRIX

Before we present the building algorithm for the bus impedance matrix, a few definitions from the discipline of the graph theory are introduced. The *graph* of a network describes the geometrical structure of the network. The graph consists of redrawing the network, with a line representing each element of the network. The graph of the network for Figure 9.2(a) before the fault application is shown in Figure 9.11(a). The buses are represented by *nodes* or *vertices* and impedances by

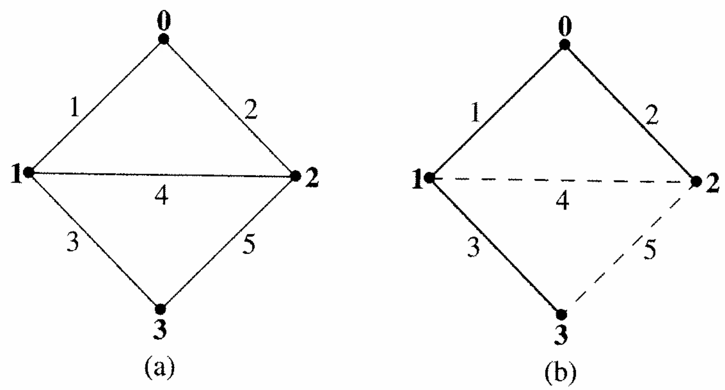


FIGURE 9.11
Graph, a selected tree, and a cotree for the network of Figure 9.2(b).

line segments called *elements* or *edges*. A *tree* of a connected graph is a connected subgraph connecting all the nodes without forming a loop. The elements of a tree are called *branches*. In general, a graph contains multiple trees. The number of branches in any selected tree denoted by b is always one less than the nodes, i.e.,

$$b = n - 1 \tag{9.26}$$

where n is the number of nodes including the reference node 0. Once a tree for a graph has been defined, the remaining elements are referred to as *links*. The collection of links is called a *cotree*. If e is the total number of elements in a graph, the number of links in a cotree is

$$l = e - b = e - n + 1 \tag{9.27}$$

A loop that contains one link is called a *basic loop*. The number of basic loops is unique; it equals the number of links and is the number of independent loop equations. A *cut set* is a minimal set of branches that, when cut, divides the graph into two connected subgraphs. A *fundamental cut set* is a cut set that contains only one branch. The number of fundamental cut sets is unique; it equals the number of branches and is the number of independent node equations. Figure 9.11(b) shows a tree of a graph with the tree branches highlighted by heavy lines and the cotree links by dashed lines.

The bus impedance matrix can be built up starting with a single element and the process is continued until all nodes and elements are included. Let us assume that $\mathbf{Z}_{bus}$ matrix exists for a partial network having m buses and a reference bus 0 as shown in Figure 9.12.

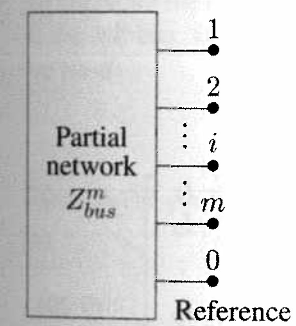


FIGURE 9.12
Partial network.

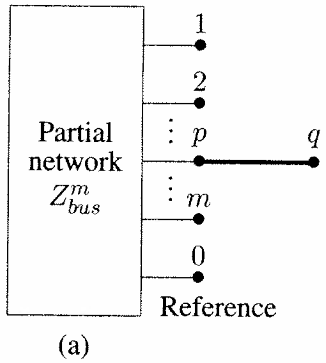
The corresponding network equation for this partial network is

$$\mathbf{V}_{bus} = \mathbf{Z}_{bus}\mathbf{I}_{bus} \tag{9.28}$$

For an n-bus system, m buses are included in the network and $\mathbf{Z}_{bus}$ is of order $m \times m$. We shall add one element at a time from the remaining portion of the network until all elements are included. The added element may be a branch or a link described as follows.

ADDITION OF A BRANCH

When the added element is a branch, a new bus is added to the partial network creating a new row and a column, and the new bus impedance matrix is of order $(m+1) \times (m+1)$. Let us add a branch with impedance z_{pq} from an existing bus p to a new bus q as shown in Figure 9.13(a). The network equation becomes



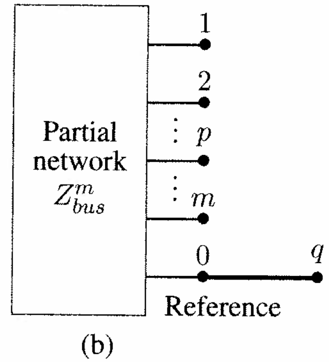


FIGURE 9.13
Addition of a branch p-q.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_p \\ \vdots \\ V_m \\ V_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \tag{9.29}$$

The addition of branch does not affect the original matrix, but requires the calculation of the elements in the q row and column. Since the elements of the power system network are linear and bilateral, $Z_{qi} = Z_{iq}$, for $q = 1, \ldots, m$.

First, let us compute the elements Z_{qi} for $i = 1, \ldots, m$ and $i \neq q$ (i.e., excluding diagonal element Z_{qq}). To calculate these elements we will apply a current source of 1 per unit at the ith bus, i.e., $I_i = 1$ pu, and keep remaining buses open-circuited, i.e., $I_k = 0$, $k = 1, \ldots, m$ and $k \neq i$. From (9.29), we get

$$\begin{aligned} V_1 &= Z_{1i} \\ V_2 &= Z_{2i} \\ &\vdots \\ V_p &= Z_{pi} \\ &\vdots \\ V_m &= Z_{mi} \\ V_q &= Z_{qi} \end{aligned} \tag{9.30}$$

From Figure 9.13(a)

$$V_q = V_p - v_{pq} \tag{9.31}$$

where v_{pq} is the voltage across the added branch with impedance z_{pq}, and is given by

$$v_{pq} = z_{pq} i_{pq} \tag{9.32}$$

Since added element p-q is a branch, $i_{pq} = 0$, thus $v_{pq} = 0$ and (9.31) reduces to

$$Z_{qi} = Z_{pi} \qquad i = 1, \ldots, m \qquad i \neq q \tag{9.33}$$

To calculate the diagonal element Z_{qq}, we will inject a current source of 1 per unit at the qth bus, i.e., $I_q = 1$ pu, and keep other buses open-circuited. From (9.29), we have

$$V_q = Z_{qq} \tag{9.34}$$

Since at the qth bus, the injected current flows from the bus q towards the bus p, $i_{pq} = -I_q = -1$. Hence, (9.32) reduces to

$$v_{pq} = -z_{pq} \tag{9.35}$$

Substituting for v_{pq} in (9.31), we get

$$V_q = V_p + z_{pq} \tag{9.36}$$

Now, since from (9.30) for $i = q$, $V_q = Z_{qq}$ and $V_p = Z_{pq}$, (9.36) becomes

$$Z_{qq} = Z_{pq} + z_{pq} \tag{9.37}$$

If node p is the reference node as shown in Figure 9.13(b), $V_p = 0$ and we obtain

$$Z_{qi} = Z_{pi} = V_p = 0 \qquad i = 1, \ldots, m \qquad i \neq q \tag{9.38}$$

From (9.37), the diagonal element becomes

$$Z_{qq} = z_{pq} \tag{9.39}$$

ADDITION OF A LINK

When the added element is a cotree link between the bus p and q, no new bus is created. The dimension of the $\mathbf{Z}_{bus}$ matrix remains the same but all the elements are required to be calculated. Let us add a link with impedance z_{pq} between two existing buses p and q as shown in Figure 9.14(a). If I_ℓ is the current through the added link in the direction shown in Figure 9.14(a), we have

$$z_{pq} I_\ell = V_p - V_q \tag{9.40}$$

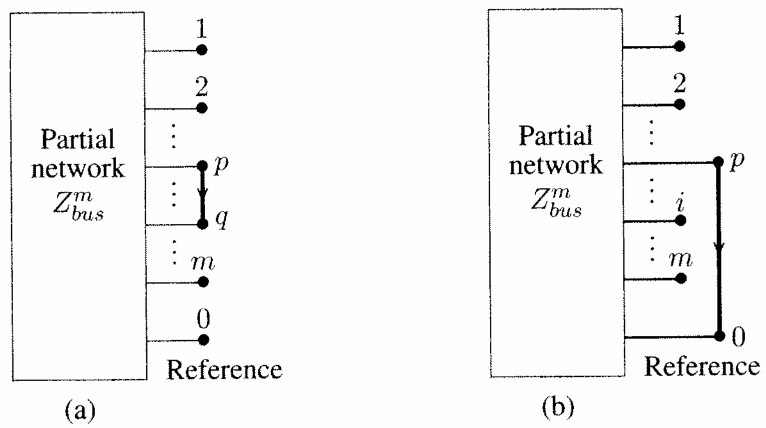


FIGURE 9.14
Addition of a link p-q.

or

$$V_q - V_p + z_{pq} I_\ell = 0 \tag{9.41}$$

The added link modifies the old current I_p to $(I_p - I_\ell)$ and the old current I_q to $(I_q + I_\ell)$ as shown in Figure 9.14(a), and the network equation becomes

$$\begin{aligned}
V_1 &= Z_{11}I_1 + \cdots + Z_{1p}(I_p - I_\ell) + Z_{1q}(I_q + I_\ell) + \cdots + Z_{1m}I_m \\
&\vdots \\
V_p &= Z_{p1}I_1 + \cdots + Z_{pp}(I_p - I_\ell) + Z_{pq}(I_q + I_\ell) + \cdots + Z_{pm}I_m \\
V_q &= Z_{q1}I_1 + \cdots + Z_{qp}(I_p - I_\ell) + Z_{qq}(I_q + I_\ell) + \cdots + Z_{qm}I_m \\
&\vdots \\
V_m &= Z_{m1}I_1 + \cdots + Z_{mp}(I_p - I_\ell) + Z_{mq}(I_q + I_\ell) + \cdots + Z_{mm}I_m
\end{aligned} \tag{9.42}$$

Substituting for V_p and V_q from (9.42) into (9.41) results in

$$\begin{aligned}
&(Z_{q1} - Z_{p1})I_1 + \cdots + (Z_{qp} - Z_{pp})I_p + \cdots + (Z_{qq} - Z_{pq})I_q + \cdots + \\
&(Z_{qm} - Z_{pm})I_m + (z_{pq} + Z_{pp} + Z_{qq} - 2Z_{pq})I_\ell = 0
\end{aligned} \tag{9.43}$$

Equations in (9.42) plus (9.43) result in $m + 1$ simultaneous equations, which is written in matrix form as

$$\begin{bmatrix} V_1 \\ \vdots \\ V_p \\ V_q \\ \vdots \\ V_m \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1p} & Z_{1q} & \cdots & Z_{1m} & Z_{1\ell} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pp} & Z_{pq} & \cdots & Z_{pm} & Z_{p\ell} \\ Z_{q1} & \cdots & Z_{qp} & Z_{qq} & \cdots & Z_{qm} & Z_{q\ell} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{m1} & \cdots & Z_{mp} & Z_{mq} & \cdots & Z_{mm} & Z_{m\ell} \\ Z_{\ell 1} & \cdots & Z_{\ell p} & Z_{\ell q} & \cdots & Z_{\ell m} & Z_{\ell\ell} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_p \\ I_q \\ \vdots \\ I_m \\ I_\ell \end{bmatrix} \tag{9.44}$$

where

$$Z_{\ell i} = Z_{i\ell} = Z_{iq} - Z_{ip} \tag{9.45}$$

and

$$Z_{\ell\ell} = z_{pq} + Z_{pp} + Z_{qq} - 2Z_{pq} \tag{9.46}$$

Now the link current I_ℓ can be eliminated. Equation (9.44) can be partitioned and rewritten in compact form as

$$\begin{bmatrix} \mathbf{V}_{bus} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{bus}^{old} & \mathbf{\Delta Z} \\ \mathbf{\Delta Z}^T & Z_{\ell\ell} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{bus} \\ I_\ell \end{bmatrix} \tag{9.47}$$

where

$$\mathbf{\Delta Z} = \begin{bmatrix} Z_{1\ell} & \cdots & Z_{p\ell} & Z_{q\ell} & \cdots & Z_{m\ell} \end{bmatrix}^T \tag{9.48}$$

Expanding (9.47), we get

$$\mathbf{V}_{bus} = \mathbf{Z}_{bus}^{old}\mathbf{I}_{bus} + \mathbf{\Delta Z} I_\ell \tag{9.49}$$

and

$$0 = \mathbf{\Delta Z}^T \mathbf{I}_{bus} + Z_{\ell\ell} I_\ell \tag{9.50}$$

or

$$I_\ell = -\frac{\mathbf{\Delta Z}^T}{Z_{\ell\ell}} \mathbf{I}_{bus} \tag{9.51}$$

Substituting from (9.51) for I_ℓ in (9.49), we have

$$\mathbf{V}_{bus} = \left[\mathbf{Z}_{bus}^{old} - \frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{\ell\ell}} \right] \mathbf{I}_{bus} \tag{9.52}$$

or

$$\mathbf{V}_{bus} = \mathbf{Z}_{bus}^{new}\mathbf{I}_{bus} \tag{9.53}$$

where

$$\mathbf{Z}_{bus}^{new} = \mathbf{Z}_{bus}^{old} - \frac{\Delta\mathbf{Z}\,\Delta\mathbf{Z}^T}{Z_{\ell\ell}} \tag{9.54}$$

Note that (9.54) reduces the matrix to its original size. The reason for this is that we have not added a new node but only linked two existing nodes.

The bus impedance matrix can be constructed with addition of branches and links in any sequence. However, it is best to select a tree that contains the elements connected to the reference node. If more than one element is connected between a given node and the reference node, only one element can be selected as a branch placing other elements in the cotree. The step-by-step procedure for building the bus impedance matrix which takes us from a given bus impedance matrix $\mathbf{Z}_{bus}^{old}$ to a new $\mathbf{Z}_{bus}^{new}$ is summarized below.

Rule 1: Addition of a Tree Branch to the Reference

Start with the branches connected to the reference node. Addition of a branch z_{q0} between a new node q and the reference node 0 to the given $\mathbf{Z}_{bus}^{old}$ matrix of order $(m \times m)$, results in the $\mathbf{Z}_{bus}^{new}$ matrix of order $(m+1) \times (m+1)$. From the results of (9.38) and (9.39), we have

$$\mathbf{Z}_{bus}^{new} = \begin{bmatrix} Z_{11} & \cdots & Z_{1m} & 0 \\ \vdots & \ddots & 0 & 0 \\ Z_{m1} & \cdots & Z_{mm} & 0 \\ 0 & \cdots & 0 & z_{q0} \end{bmatrix} \tag{9.55}$$

This matrix is diagonal with the impedance values of the branches on the diagonal.

Rule 2: Addition of a Tree Branch from a New Bus to an Old Bus

Continue with the remaining branches of the tree connecting a new node to the existing node. Addition of a branch z_{pq} between a new node q and the existing node p to the given $\mathbf{Z}_{bus}^{old}$ matrix of order $(m \times m)$, results in the $\mathbf{Z}_{bus}^{new}$ matrix of order $(m+1) \times (m+1)$. From the results of (9.33) and (9.37), we have

$$\mathbf{Z}_{bus}^{new} = \begin{bmatrix} Z_{11} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pp} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{m1} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mp} \\ Z_{p1} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pp} + z_{pq} \end{bmatrix} \tag{9.56}$$

Rule 3: Addition of a Cotree Link between two existing Buses

When a link with impedance z_{pq} is added between two existing nodes p and q, we augment the $\mathbf{Z}_{bus}^{old}$ matrix with a new row and a new column, and from (9.44) and (9.45) we have

$$\mathbf{Z}_{bus}^{new} = \begin{bmatrix} Z_{11} & Z_{1p} & Z_{1q} & \cdots & Z_{1m} & Z_{1q}-Z_{1p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{p1} & Z_{pp} & Z_{pq} & \cdots & Z_{pm} & Z_{pq}-Z_{pp} \\ Z_{q1} & Z_{qp} & Z_{qq} & \cdots & Z_{qm} & Z_{qq}-Z_{qp} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{m1} & Z_{mp} & Z_{mq} & \cdots & Z_{mm} & Z_{mq}-Z_{mp} \\ Z_{q1}-Z_{p1} & Z_{qp}-Z_{pp} & Z_{qq}-Z_{pq} & \cdots & Z_{qm}-Z_{pm} & Z_{\ell\ell} \end{bmatrix} \tag{9.57}$$

where

$$Z_{\ell\ell} = z_{pq} + Z_{pp} + Z_{qq} - 2Z_{pq} \tag{9.58}$$

The new row and column is eliminated using the relation in (9.54), which is repeated here

$$\mathbf{Z}_{bus}^{new} = \mathbf{Z}_{bus}^{old} - \frac{\Delta\mathbf{Z}\,\Delta\mathbf{Z}^T}{Z_{\ell\ell}} \tag{9.59}$$

and $\Delta\mathbf{Z}$ is defined as

$$\Delta\mathbf{Z} = \begin{bmatrix} Z_{1q} - Z_{1p} \\ \vdots \\ Z_{pq} - Z_{pp} \\ Z_{qq} - Z_{qp} \\ \vdots \\ Z_{mq} - Z_{mp} \end{bmatrix} \tag{9.60}$$

When bus q is the reference bus, $Z_{qi} = Z_{iq} = 0$ (for $i = 1, m$), and (9.57) reduces to

$$\mathbf{Z}_{bus}^{new} = \begin{bmatrix} Z_{11} & \cdots & Z_{1p} & \cdots & Z_{1m} & -Z_{1p} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{p1} & \cdots & Z_{pp} & \cdots & Z_{pm} & -Z_{pp} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{m1} & \cdots & Z_{mp} & \cdots & Z_{mm} & -Z_{mp} \\ -Z_{p1} & \cdots & -Z_{pp} & \cdots & -Z_{pm} & Z_{\ell\ell} \end{bmatrix} \tag{9.61}$$

where $Z_{\ell\ell} = z_{pq} + Z_{pp}$, and

$$\Delta \mathbf{Z} = \begin{bmatrix} -Z_{1p} \\ \vdots \\ -Z_{pp} \\ \vdots \\ -Z_{mp} \end{bmatrix} \tag{9.62}$$

The algorithm to construct the $\mathbf{Z}_{bus}$ matrix by adding one element at a time can be used to remove lines or generators from the network. The procedure is identical to that of adding elements, except that the removed element is considered as negative impedance, in order to cancel the effect of the element.

Based on the above algorithm, two functions named **Zbus = zbuild(zdata)** and **Zbus = zbuildpi(linedata, gendata, yload)** are developed for the formation of the bus impedance matrix. These functions are described in Section 9.6. Before demonstrating this program, for the sake of better understanding the building algorithm, we shall demonstrate the hand calculation procedure for the simple three-bus network of Example 9.1.

Example 9.3 (chp9ex3)

Construct the bus impedance matrix for the network in Example 9.1. The one-line impedance diagram is shown in Figure 9.15(a).

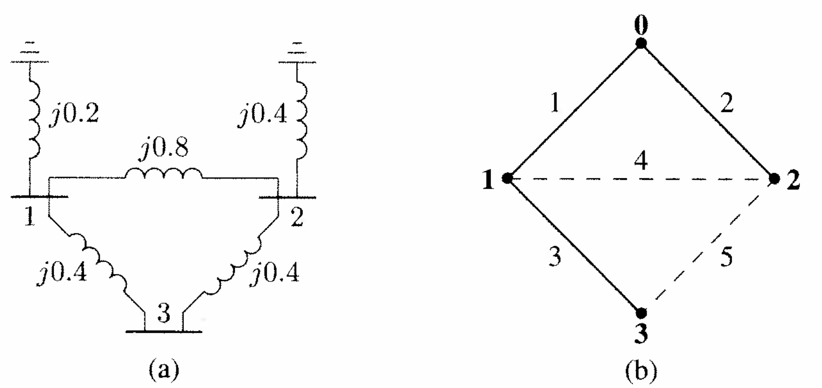


FIGURE 9.15
Impedance diagram of Example 9.1 and a proper tree.

The elements connected to the reference node are included in the proper tree as shown in Figure 9.15(b). We start with those branches of the tree connected to the reference node. Add branch 1, $z_{10} = j0.2$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.20$$

Next, add branch 2, $z_{20} = j0.4$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 \\ 0 & j0.4 \end{bmatrix}$$

Note that the off-diagonal elements of the bus impedance matrix are zero. This is because there is no connection between these buses other than to the reference. In this example, there are no more branches from a new bus to the reference. We continue with the remaining branches of the tree. Add branch 3, $z_{13} = j0.4$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.4 & 0 \\ j0.2 & 0 & j0.6 \end{bmatrix}$$

All tree branches are in place. We now proceed with the links. Add link 4, $z_{12} = j0.8$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{21} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

$$= \begin{bmatrix} j0.2 & 0 & j0.2 & -j0.2 \\ 0 & j0.4 & 0 & j0.4 \\ j0.2 & 0 & j0.6 & -j0.2 \\ -j0.2 & j0.4 & -j0.2 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.8 + j0.2 + j0.4 - 2(j0) = j1.4$$

and

$$\frac{\Delta \mathbf{Z}\, \Delta \mathbf{Z}^T}{Z_{44}} = \frac{1}{j1.4} \begin{bmatrix} -j0.2 \\ j0.4 \\ -j0.2 \end{bmatrix} \begin{bmatrix} -j0.2 & j0.4 & -j0.2 \end{bmatrix}$$

$$= \begin{bmatrix} j0.02857 & -j0.05714 & j0.02857 \\ -j0.05714 & j0.11428 & -j0.05714 \\ j0.02857 & -j0.05714 & j0.02857 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.4 & 0 \\ j0.2 & 0 & j0.6 \end{bmatrix} - \begin{bmatrix} j0.02857 & -j0.05714 & j0.02857 \\ -j0.05714 & j0.11428 & -j0.05714 \\ j0.02857 & -j0.05714 & j0.02857 \end{bmatrix}$$

$$= \begin{bmatrix} j0.17143 & j0.05714 & j0.17143 \\ j0.05714 & j0.28571 & j0.05714 \\ j0.17143 & j0.05714 & j0.57143 \end{bmatrix}$$

Finally, we add link 5, $z_{23} = j0.4$ between node $q = 3$ and node $p = 2$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(5)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13}-Z_{12} \\ Z_{21} & Z_{22} & Z_{23} & Z_{23}-Z_{22} \\ Z_{31} & Z_{32} & Z_{33} & Z_{33}-Z_{32} \\ Z_{31}-Z_{21} & Z_{32}-Z_{22} & Z_{33}-Z_{23} & Z_{44} \end{bmatrix}$$

$$= \begin{bmatrix} j0.17143 & j0.05714 & j0.17143 & j0.11429 \\ j0.05714 & j0.28571 & j0.05714 & -j0.22857 \\ j0.17143 & j0.05714 & j0.57143 & j0.51429 \\ j0.11429 & -j0.22857 & j0.51429 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{23} + Z_{22} + Z_{33} - 2Z_{23} = j0.4 + j0.28571 + j0.57143 - 2(j0.05714) = j1.14$$

and

$$\frac{\Delta\mathbf{Z}\,\Delta\mathbf{Z}^T}{Z_{44}} = \frac{1}{j1.4}\begin{bmatrix} j0.11429 \\ -j0.22857 \\ j0.51429 \end{bmatrix}\begin{bmatrix} j0.11429 & -j0.22857 & j0.51429 \end{bmatrix}$$

$$= \begin{bmatrix} j0.01143 & -j0.02286 & j0.05143 \\ -j0.02286 & j0.04571 & -j0.10286 \\ j0.05143 & -j0.10286 & j0.23143 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.17143 & j0.05714 & j0.17143 \\ j0.05714 & j0.28571 & j0.05714 \\ j0.17143 & j0.05714 & j0.57143 \end{bmatrix} - \begin{bmatrix} j0.01143 & -j0.02286 & j0.05143 \\ -j0.02286 & j0.04571 & -j0.10286 \\ j0.05143 & -j0.10286 & j0.23143 \end{bmatrix}$$

$$= \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

This is the desired bus impedance matrix $\mathbf{Z}_{bus}$, which is the same as the one obtained by inverting the $\mathbf{Y}_{bus}$ matrix in Example 9.2.

Example 9.4 (chp9ex4)

The bus impedance matrix for the network shown in Figure 9.16 is found to be

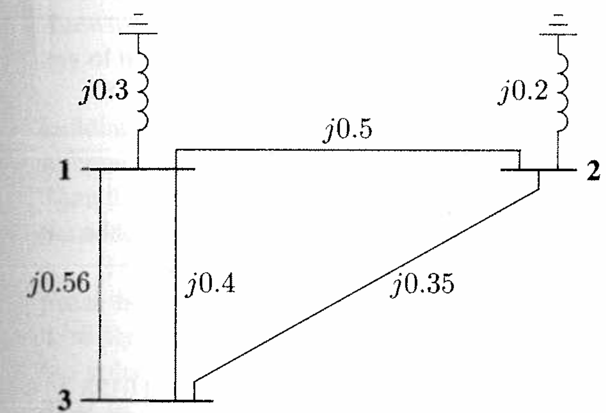


FIGURE 9.16
Impedance diagram for Example 9.4.

$$Z_{bus} = \begin{bmatrix} j0.183 & j0.078 & j0.141 \\ j0.078 & j0.148 & j0.106 \\ j0.141 & j0.106 & j0.267 \end{bmatrix}$$

The line between buses 1 and 3 with impedance $Z_{13} = j0.56$ is removed by the simultaneous opening of breakers at both ends of the line. Determine the new bus impedance matrix.

The removal of an element is equivalent to connecting a link having an impedance equal to the negated value of the original impedance. Therefore, we add link $z_{13} = -j0.56$ between node $q = 3$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13}-Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{23}-Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{33}-Z_{31} \\ Z_{31}-Z_{11} & Z_{32}-Z_{12} & Z_{33}-Z_{13} & Z_{44} \end{bmatrix}$$

Thus, we get

$$Z_{bus} = \begin{bmatrix} j0.183 & j0.078 & j0.141 & -j0.042 \\ j0.078 & j0.148 & j0.106 & j0.028 \\ j0.141 & j0.106 & j0.267 & j0.126 \\ -j0.042 & j0.028 & j0.126 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{13} + Z_{11} + Z_{33} - 2Z_{13} = -j0.56 + j0.183 + j0.267 - 2(j0.141) = -j0.392$$

and

$$\frac{\Delta \mathbf{Z}\, \Delta \mathbf{Z}^T}{Z_{44}} = \frac{1}{-j0.392}\begin{bmatrix} -j0.042 \\ j0.028 \\ j0.126 \end{bmatrix}\begin{bmatrix} -j0.042 & j0.028 & j0.126 \end{bmatrix}$$

$$= \begin{bmatrix} -j0.0045 & j0.0030 & j0.0135 \\ j0.0030 & -j0.0020 & -j0.0090 \\ j0.0135 & -j0.0090 & -j0.0405 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.183 & j0.078 & j0.141 \\ j0.078 & j0.148 & j0.106 \\ j0.141 & j0.106 & j0.267 \end{bmatrix} - \begin{bmatrix} -j0.0045 & j0.0030 & j0.0135 \\ j0.0030 & -j0.0020 & -j0.0090 \\ j0.0135 & -j0.0090 & -j0.0405 \end{bmatrix}$$

$$= \begin{bmatrix} j0.1875 & j0.0750 & j0.1275 \\ j0.0750 & j0.1500 & j0.1150 \\ j0.1275 & j0.1150 & j0.3075 \end{bmatrix}$$

9.6 ZBUILD AND SYMFAULT PROGRAMS

Two functions are developed for the formation of the bus impedance matrix. One function is named **Zbus = zbuild(zdata)**, where the argument **zdata** is an $e \times 4$ matrix containing the impedance data of an e-element network. Columns 1 and 2 are the element bus numbers and columns 3 and 4 contain the element resistance and reactance, respectively, in per unit. Bus number 0 to generator buses contain generator impedances. These may be the subtransient, transient, or synchronous reactances. Also, any other shunt impedances such as capacitors and load impedance to ground (bus 0) may be included in this matrix.

The other function for the formation of the bus impedance matrix is **zbus = zbuildpi(linedata, gendata, yload)**, which is compatible with the power flow programs. The first argument **linedata** is consistent with the data required for the power flow solution. Columns 1 and 2 are the line bus numbers. Columns 3 through 5 contain line resistance, reactance, and one-half of the total line charging susceptance in per unit on the specified MVA base. The last column is for the transformer tap setting; for lines, 1 must be entered in this column. The lines may be entered in any sequence or order. The generator reactances are not included in the **linedata** of the power flow program and must be specified separately as required by the **gendata** in the second argument. **gendata** is an $n_g \times 4$ matrix, where each row contains bus 0, generator bus number, resistance and reactance. The last argument, **yload** is optional. This is a two-column matrix containing bus number and the complex load admittance. This data is provided by any of the power flow programs **lfgauss**, **lfnewton** or **decouple**. **yload** is automatically generated following the execution of any of the above power flow programs.

The **zbuild** and **zbuildpi** functions obtain the bus impedance matrix by the building algorithm method. These functions select a tree containing elements to the reference node. First, all branches connected to the reference node are processed. Then the remaining branches of the tree are connected, and finally the cotree links are added.

The program **symfault(zdata, Zbus, V)** is developed for the balanced three-phase fault studies. The function requires the **zdata** and the **Zbus** matrices. The third argument **V** is optional. If it is not included, the program sets all the prefault bus voltages to 1.0 per unit. If the variable **V** is included, the prefault bus voltages must be specified by the array **V** containing bus numbers and the complex bus voltage. The voltage vector **V** is automatically generated following the execution of any of the power flow programs. The use of the above functions are demonstrated in the following examples. When **symfault** is executed, it prompts the user to enter the faulted bus number and the fault impedance. The program computes the total fault current and tabulates the magnitude of the bus voltages and line currents during the fault.

Example 9.5 (chp9ex5)

Use the function **zbus = zbuild(zdata)** to obtain the bus impedance matrix for the network in Example 9.3.

The network configuration containing resistances and reactances are specified and the **zbuild** function is used as follows.

```
zdata = [ 0   1   0    0.2
          0   2   0    0.4
          1   2   0    0.8
          1   3   0    0.4
          2   3   0    0.4];

Zbus  = zbuild(zdata)
```

The result is

```
Zbus =
         0 + 0.16i     0 + 0.08i     0 + 0.12i
         0 + 0.08i     0 + 0.24i     0 + 0.16i
         0 + 0.12i     0 + 0.16i     0 + 0.34i
```

Example 9.6 (chp9ex6)

A three-phase fault with a fault impedance $Z_f = j0.16$ per unit occurs at bus 3 in the network of Example 9.1. Use the **symfault** function to compute the fault current, the bus voltages and line currents during the fault.

In this example all shunt capacitances and loads are neglected and all the prefault bus voltages are assumed to be unity. The impedance diagram in Figure 9.2(b) is described by the variable **zdata** and the following commands are used.

```
zdata =    [ 0   1   0    0.2
             0   2   0    0.4
             1   2   0    0.8
             1   3   0    0.4
             2   3   0    0.4];

Zbus = zbuild(zdata)
symfault(zdata, Zbus)
```

The result is

```
Zbus =
         0 + 0.1600i      0 + 0.0800i      0 + 0.1200i
         0 + 0.0800i      0 + 0.2400i      0 + 0.1600i
         0 + 0.1200i      0 + 0.1600i      0 + 0.3400i

Enter Faulted Bus No. -> 3
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*0.16

Balanced three-phase fault at bus No. 3
Total fault current =    2.0000 Per unit

Bus Voltages during the fault in per unit
      Bus       Voltage        Angle
      No.       Magnitude      Degree
       1         0.7600        0.0000
       2         0.6800        0.0000
       3         0.3200        0.0000
```



```
Line currents for fault at bus No. 3
     From       To     Current        Angle
     Bus        Bus    Magnitude      Degree
      G          1      1.2000      -90.0000
      1          2      0.1000      -90.0000
      1          3      1.1000      -90.0000
      G          2      0.8000      -90.0000
      2          3      0.9000      -90.0000
      3          F      2.0000      -90.0000
```

Example 9.7 (chp9ex7)

The 11-bus power system network of an electric utility company is shown in Figure 9.17.

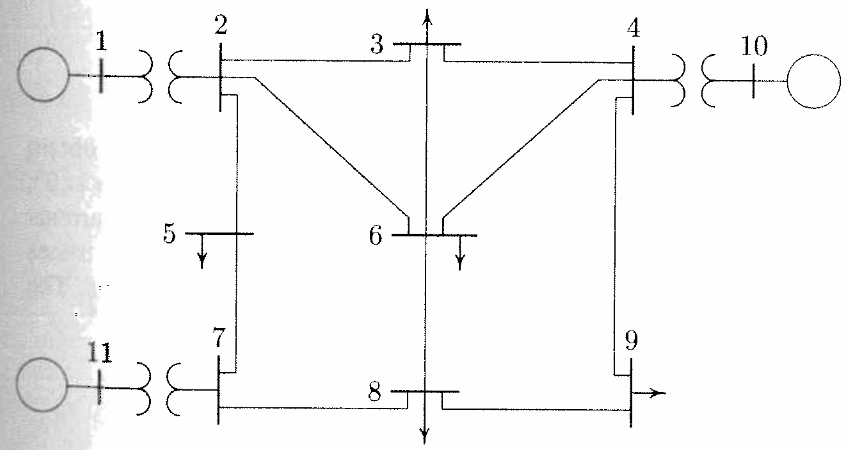


FIGURE 9.17
One-line diagram for Example 9.7

The transient impedance of the generators on a 100-MVA base are given below.

GEN. TRANSIENT IMPEDANCE PU		
Gen. No.	R_a	X'_d
1	0	0.20
10	0	0.15
11	0	0.25

The line and transformer data containing the series resistance and reactance in per unit, and one-half of the total capacitance in per unit susceptance on a 100-MVA base is tabulated below.

LINE AND TRANSFORMER DATA

Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}$ B, PU
1	2	0.00	0.06	0.0000
2	3	0.08	0.30	0.0004
2	5	0.04	0.15	0.0002
2	6	0.12	0.45	0.0005
3	4	0.10	0.40	0.0005
3	6	0.04	0.40	0.0005
4	6	0.15	0.60	0.0008
4	9	0.18	0.70	0.0009
4	10	0.00	0.08	0.0000
5	7	0.05	0.43	0.0003
6	8	0.06	0.48	0.0000
7	8	0.06	0.35	0.0004
7	11	0.00	0.10	0.0000
8	9	0.052	0.48	0.0000

Neglecting the shunt capacitors and the loads, use **zbuild(zdata)** function to obtain the bus impedance matrix. Assuming all the prefault bus voltages are equal to $1\angle 0°$, use **symfault** function to compute the fault current, bus voltages, and line currents for a bolted fault at bus 8. When using **zbuild** function, the generator reactances must be included in the impedance data with bus zero as the reference bus. The impedance data and the required commands are as follows.

```
%        Bus   Bus     R        X
%        No.   No.     pu       pu
zdata = [0      1     0.00     0.20
         0     10     0.00     0.15
         0     11     0.00     0.25
         1      2     0.00     0.06
         2      3     0.08     0.30
         2      5     0.04     0.15
         2      6     0.12     0.45
         3      4     0.10     0.40
         3      6     0.04     0.40
         4      6     0.15     0.60
         4      9     0.18     0.70
         4     10     0.00     0.08
         5      7     0.05     0.43
         6      8     0.06     0.48
         7      8     0.06     0.35
         7     11     0.00     0.10
         8      9     0.052    0.48
```

```
Zbus = zbuild(zdata)
symfault(zdata, Zbus)
```

The bus impedance matrix is displayed on the screen, and the three-phase short circuit result is

```
Enter Faulted Bus No. -> 8
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0
Balanced three-phase fault at bus No. 8
Total fault current =   3.3319 per unit

Bus Voltages during the fault in per unit
     Bus      Voltage        Angle
     No.      Magnitude      Degree
      1        0.8082       -1.8180
      2        0.7508       -2.5443
      3        0.6882       -1.5987
      4        0.7491       -2.4902
      5        0.7007       -2.3762
      6        0.5454       -1.0194
      7        0.5618       -3.8128
      8        0.0000        0.0000
      9        0.3008        2.4499
     10        0.8362       -1.4547
     11        0.6866       -2.2272
Line currents for fault at bus No. 8
     From       To      Current       Angle
     Bus        Bus     Magnitude     Degree
      G          1       0.9697      -82.4034
      1          2       0.9697      -82.4034
      2          3       0.2053      -87.8751
      2          5       0.3230      -79.9626
      2          6       0.4427      -81.6497
      3          6       0.3556      -88.0987
      4          3       0.1503      -88.4042
      4          6       0.3305      -82.3804
      4          9       0.6229      -81.3672
      5          7       0.3230      -79.9626
      6          8       1.1274      -83.8944
      7          8       1.5820      -84.0852
      8          F       3.3319      -83.5126
      9          8       0.6229      -81.3672
      G         10       1.1029      -82.6275
     10          4       1.1029      -82.6275
      G         11       1.2601      -85.1410
     11          7       1.2601      -85.1410
```

Example 9.8 (chp9ex8)

In Example 9.7 consider the shunt capacitors and neglect the loads. Use **zbuildpi** function to obtain the bus impedance matrix. Assuming all the prefault bus voltages are equal to $1\angle 0°$, use **symfault** function to compute the fault current, bus voltages, and line currents for a bolted fault at bus 8.

The **zbuildpi(linedata, gendata, yload)** is designed to be compatible with the power flow programs. The first argument **linedata** is consistent with the data required for the power flow program. The generator reactances are not included in the **linedata** and must be specified separately by the **gendata**. The optional argument **yload** contains the load admittance which is generated from the power flow solution. The loads are neglected in this example, therefore, the argument **yload** is omitted. The impedance data and the required commands are as follows:

```
%         Bus   Bus    R       X       1/2B
%         No.   No.    pu      pu      pu
linedata=[1      2    0.00    0.06    0.0000
          2      3    0.08    0.30    0.0004
          2      5    0.04    0.15    0.0002
          2      6    0.12    0.45    0.0005
          3      4    0.10    0.40    0.0005
          3      6    0.04    0.40    0.0005
          4      6    0.15    0.60    0.0008
          4      9    0.18    0.70    0.0009
          4     10    0.00    0.08    0.0000
          5      7    0.05    0.43    0.0003
          6      8    0.06    0.48    0.0000
          7      8    0.06    0.35    0.0004
          7     11    0.00    0.10    0.0000
          8      9    0.052   0.48    0.0000];
%         Gen.  Ra     Xd'
gendata=[ 1      0    0.20
          10     0    0.15
          11     0    0.25];
Zbus=zbuildpi(linedata, gendata)
symfault(linedata, Zbus)
```

The bus impedance matrix is displayed on the screen, and the three-phase short circuit result is

```
Enter Faulted Bus No. -> 8
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0
Balanced three-phase fault at bus No. 8
```



```
Total fault current =   3.3301 per unit

Bus Voltages during the fault in per unit

  Bus     Voltage        Angle
  No.     Magnitude      Degree
   1       0.8080       -1.8188
   2       0.7506       -2.5456
   3       0.6879       -1.5986
   4       0.7489       -2.4915
   5       0.7006       -2.3774
   6       0.5451       -1.0185
   7       0.5617       -3.8137
   8       0.0000        0.0000
   9       0.3005        2.4564
  10       0.8361       -1.4553
  11       0.6866       -2.2276

Line currents for fault at bus No. 8

  From     To     Current      Angle
  Bus      Bus    Magnitude    Degree
   1        2      0.9704     -82.4068
   2        3      0.2056     -87.7898
   2        5      0.3230     -79.9386
   2        6      0.4429     -81.6055
   3        6      0.3556     -88.0454
   4        3      0.1505     -88.2647
   4        6      0.3308     -82.2823
   4        9      0.6232     -81.3096
   5        7      0.3228     -79.9261
   6        8      1.1269     -83.8935
   7        8      1.5818     -84.0781
   8        F      3.3301     -83.5110
   9        8      0.6224     -81.3606
  10        4      1.1038     -82.6316
  11        7      1.2604     -85.1416
```

Example 9.9 (chp9ex9)

Repeat the symmetrical three-phase short circuit analysis for Example 9.7 considering the prefault bus voltages and the effect of load currents. The load data is as follows:

LOAD DATA					
Bus	Load		Bus	Load	
No.	MW	Mvar	No.	MW	Mvar
1	0.0	0.0	7	0.0	0.0
2	0.0	0.0	8	110.0	90.0
3	150.0	120.0	9	80.0	50.0
4	0.0	0.0	10	0.0	0.0
5	120.0	60.0	11	0.0	0.0
6	140.0	90.0			

Voltage magnitude, generation schedule and the reactive power limits for the regulated buses are tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.04\angle 0°$, is taken as the slack bus.

GENERATION DATA				
Bus	Voltage	Generation,	Mvar Limits	
No.	Mag.	MW	Min.	Max.
1	1.040			
10	1.035	200.0	0.0	180.0
11	1.030	160.0	0.0	120.0

Anyone of the power flow programs can be used to obtain the prefault bus voltages and the load admittance. The **lfnewton** program is used which returns the prefault bus voltage array **V** and the bus load admittance array **yload**. The required commands are as follows.

```
clear       % clears all variables from workspace.
basemva = 100;  accuracy = 0.0001;  maxiter = 10;
%      Bus Bus Voltage Angle --Load--  ---Generator---Injected
%       No code Mag. Degree MW  Mvar   MW  Mvar Qmin Qmax Mvar
busdata=[1  1  1.06   0     0.0  0.0   0.0  0.0   0    0    0
         2  0  1.0    0     0.0  0.0   0.0  0     0    0    0
         3  0  1.0    0   150.0 120.0  0.0  0     0    0    0
         4  0  1.0    0     0.0  0.0   0.0  0     0    0    0
         5  0  1.0    0   120.0  60.0  0.0  0     0    0    0
         6  0  1.0    0   140.0  90.0  0.0  0     0    0    0
         7  0  1.0    0     0.0  0.0   0.0  0     0    0    0
         8  0  1.0    0   110.0  90.0  0.0  0     0    0    0
         9  0  1.0    0    80.0  50.0  0.0  0     0    0    0
        10  2  1.035  0     0.0  0.0 200.0  0     0  180    0
        11  2  1.03   0     0.0  0.0 160.0  0     0  120    0];

%         Bus    Bus     R        X        1/2B
%         No.    No.     pu       pu       pu
```



```
linedata=[1    2    0.00    0.06    0.0000   1
          2    3    0.08    0.30    0.0004   1
          2    6    0.12    0.45    0.0005   1
          3    4    0.10    0.40    0.0005   1
          3    6    0.04    0.40    0.0005   1
          4    6    0.15    0.60    0.0008   1
          4    9    0.18    0.70    0.0009   1
          4   10    0.00    0.08    0.0000   1
          5    7    0.05    0.43    0.0003   1
          6    8    0.06    0.48    0.0000   1
          7    8    0.06    0.35    0.0004   1
          7   11    0.00    0.10    0.0000   1
          8    9    0.052   0.48    0.0000   1];
%         Gen.  Ra    Xd'
gendata=[ 1     0     0.20
          10    0     0.15
          11    0     0.25];
lfybus                 % Forms the bus admittance matrix
lfnewton         % Power flow solution by Newton-Raphson method
busout           % Prints the power flow solution on the screen
Zbus=zbuildpi(linedata, gendata, yload) %  Zbus including load
symfault(linedata,Zbus,V)%3-phase fault including load current
```

The result is

```
          Power Flow Solution by Newton-Raphson Method
              Maximum Power Mismatch = 0.0000533178
                     No. of Iterations = 3

Bus  Voltage  Angle      ----Load----      --Generation-- Injected
No.  Mag.     Degree     MW      Mvar       MW      Mvar      Mvar

1   1.040    0.000     0.000    0.000    248.622  149.163    0.0
2   1.031   -0.797     0.000    0.000      0.000    0.000    0.0
3   0.997   -2.619   150.000  120.000      0.000    0.000    0.0
4   1.024   -1.737     0.000    0.000      0.000    0.000    0.0
5   0.981   -7.414   120.000   60.000      0.000    0.000    0.0
6   0.992   -3.336   140.000   90.000      0.000    0.000    0.0
7   1.014   -4.614     0.000    0.000      0.000    0.000    0.0
8   0.981   -5.093   110.000   90.000      0.000    0.000    0.0
9   0.977   -4.842    80.000   50.000      0.000    0.000    0.0
10  1.035   -0.872     0.000    0.000    200.000  144.994    0.0
11  1.020   -3.737     0.000    0.000    160.000  161.121    0.0

Total                600.000  410.000    608.622  455.278    0.0
```

```
Enter Faulted Bus No. -> 8
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0
Balanced three-phase fault at bus No. 8
Total fault current =   3.3571 per unit

Bus Voltages during the in per unit
     Bus      Voltage          Angle
     No.      Magnitude       Degree
      1        0.8876        -0.9467
      2        0.8350        -2.0943
      3        0.7321        -2.5619
      4        0.7866        -3.1798
      5        0.5148        -8.3043
      6        0.5792        -2.4214
      7        0.5179        -8.2563
      8        0.0000         0.0000
      9        0.3156         0.9877
     10        0.8785        -1.7237
     11        0.6631        -5.7789

Line currents for fault at bus No. 8
     From         To     Current         Angle
     Bus         Bus     Magnitude      Degree
      1           2       0.9219      -73.3472
      2           3       0.3321      -73.7856
      2           6       0.5494      -76.3804
      3           6       0.3804      -87.3283
      4           3       0.1336      -87.2217
      4           6       0.3357      -81.1554
      4           9       0.6537      -81.4818
      6           8       1.1974      -85.2964
      7           5       0.0073      -82.5471
      7           8       1.4585      -88.5207
      8           F       3.3571      -85.4214
      9           8       0.6538      -82.8293
     10           4       1.1787      -79.4854
     11           7       1.4733      -87.0395
```

PROBLEMS

9.1. The system shown in Figure 9.18 is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is $j160\ \Omega$. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the short-circuit current and the short-circuit MVA.

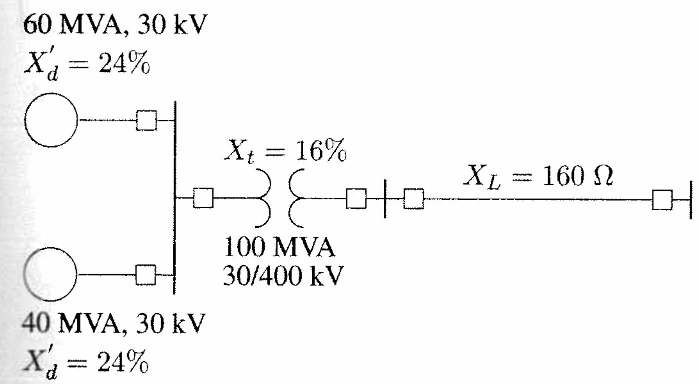


FIGURE 9.18
One-line diagram for Problem 9.1.

9.2. The system shown in Figure 9.19 shows an existing plant consisting of a generator of 100 MVA, 30 kV, with 20 percent subtransient reactance and a generator of 50 MVA, 30 kV with 15 percent subtransient reactance, connected in parallel to a 30-kV bus bar. The 30-kV bus bar feeds a transmission line via the circuit breaker C which is rated at 1250 MVA. A grid supply is connected to the station bus bar through a 500-MVA, 400/30-kV transformer with 20 percent reactance. Determine the reactance of a current limiting reactor in ohm to be connected between the grid system and the existing bus bar such that the short-circuit MVA of the breaker C does not exceed.

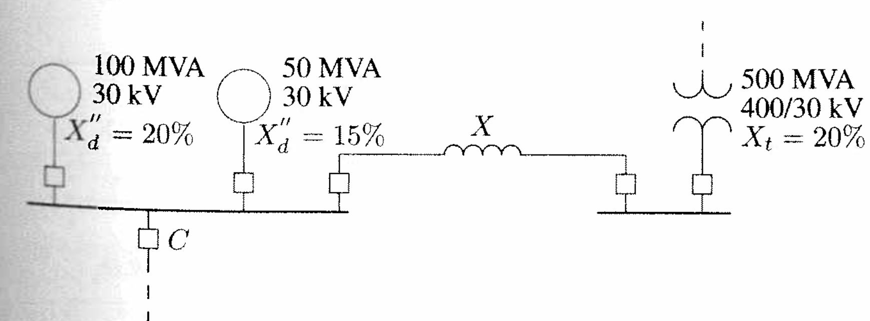


FIGURE 9.19
One-line diagram for Problem 9.2.

9.3. The one-line diagram of a simple power system is shown in Figure 9.20. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 1 through a fault impedance of $Z_f = j0.08$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

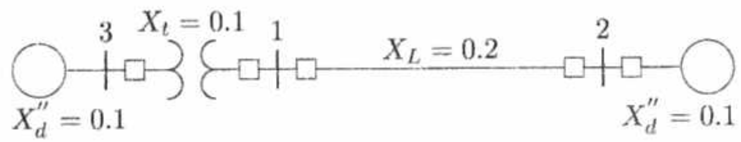


FIGURE 9.20
One-line diagram for Problem 9.3.

9.4. The one-line diagram of a simple three-bus power system is shown in Figure 9.21 Each generator is represented by an emf behind the subtransient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

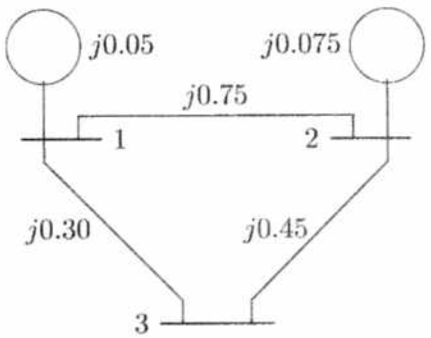


FIGURE 9.21
One-line diagram for Problem 9.4.

9.5. The one-line diagram of a simple four-bus power system is shown in Figure 9.22 Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A bolted three-phase fault occurs at bus 4.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.
(c) Repeat (a) and (b) for a fault at bus 2 with a fault impedance of $Z_f = j0.0225$.

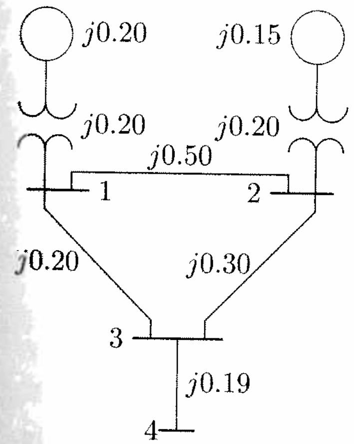


FIGURE 9.22
One-line diagram for Problem 9.5.

9.6. Using the method of building algorithm find the bus impedance matrix for the network shown in Figure 9.23.

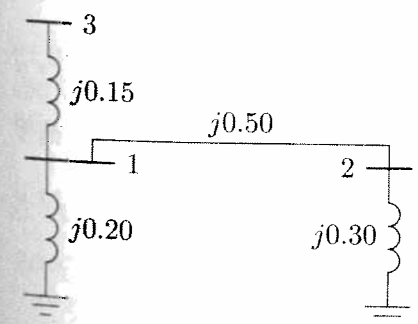


FIGURE 9.23
One-line diagram for Problem 9.6.

9.7. Obtain the bus impedance matrix for the network of Problem 9.3.

9.8. Obtain the bus impedance matrix for the network of Problem 9.4.

9.9. The bus impedance matrix for the network shown in Figure 9.24 is given by

$$Z_{bus} = j \begin{bmatrix} 0.300 & 0.200 & 0.275 \\ 0.200 & 0.400 & 0.250 \\ 0.275 & 0.250 & 0.41875 \end{bmatrix}$$

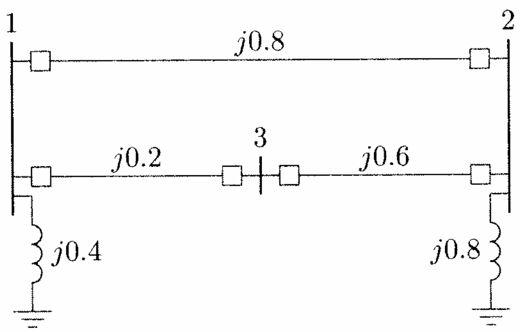


FIGURE 9.24
One-line diagram for Problem 9.9.

There is a line outage and the line from bus 1 to 2 is removed. Using the method of building algorithm determine the new bus impedance matrix.

9.10. The per unit bus impedance matrix for the power system of Problem 9.4 is given by

$$Z_{bus} = j \begin{bmatrix} 0.0450 & 0.0075 & 0.0300 \\ 0.0075 & 0.06375 & 0.0300 \\ 0.0300 & 0.0300 & 0.2100 \end{bmatrix}$$

A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

9.11. The per unit bus impedance matrix for the power system of Problem 9.5 is given by

$$Z_{bus} = j \begin{bmatrix} 0.240 & 0.140 & 0.200 & 0.200 \\ 0.140 & 0.2275 & 0.175 & 0.175 \\ 0.200 & 0.175 & 0.310 & 0.310 \\ 0.200 & 0.1750 & 0.310 & 0.500 \end{bmatrix}$$

(a) A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault.
(b) Repeat (a) for a three-phase fault at bus 2 with a fault impedance of $Z_f = j0.0225$.
(c) Check your result using the **Zbuild** and **symfault** programs.

9.12. The per unit bus impedance matrix for the power system shown in Figure 9.25 is given by

$$Z_{bus} = j \begin{bmatrix} 0.150 & 0.075 & 0.140 & 0.135 \\ 0.075 & 0.1875 & 0.090 & 0.0975 \\ 0.140 & 0.090 & 0.2533 & 0.210 \\ 0.135 & 0.0975 & 0.210 & 0.2475 \end{bmatrix}$$

A three-phase fault occurs at bus4 through a fault impedance of $Z_f = j0.0025$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

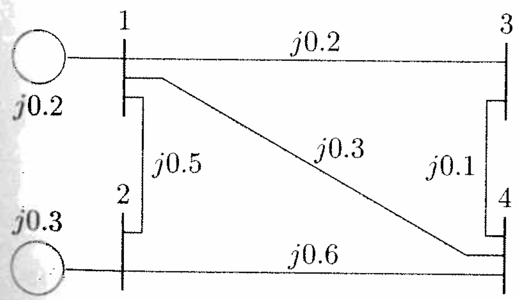


FIGURE 9.25
One-line diagram for Problem 9.12.

9.13. Repeat Example 9.7 for a bolted three-phase fault at bus 9.

9.14. Repeat Example 9.8 for a bolted three-phase fault at bus 9.

9.15. Repeat Example 9.9 for a bolted three-phase fault at bus 9.

9.16. The 6-bus power system network of an electric utility company is shown in Figure 9.26. The line and transformer data containing the series resistance and reactance in per unit, and one-half of the total capacitance in per unit susceptance on a 100-MVA base, is tabulated below.

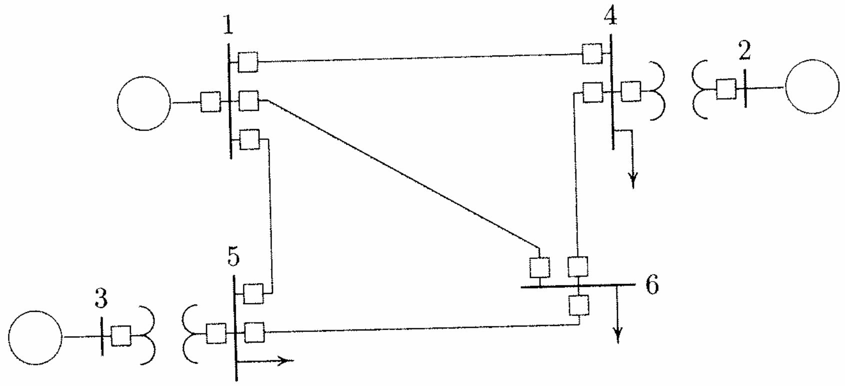


FIGURE 9.26
One-line diagram for Problem 9.16.

LINE AND TRANSFORMER DATA				
Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}$ B, PU
1	4	0.035	0.225	0.0065
1	5	0.025	0.105	0.0045
1	6	0.040	0.215	0.0055
2	4	0.000	0.035	0.0000
3	5	0.000	0.042	0.0000
4	6	0.028	0.125	0.0035
5	6	0.026	0.175	0.0300

The transient impedance of the generators on a 100-MVA base are given below.

GEN. TRANSIENT IMPEDANCE, PU		
Gen. No.	R_a	X'_d
1	0	0.20
2	0	0.15
3	0	0.25

Neglecting the shunt capacitors and the loads, use **Zbus = zbuild(zdata)** function to obtain the bus impedance matrix. Assuming all the prefault bus voltages are equal to $1\angle 0°$, use **symfault(zdata, Zbus)** function to compute the fault current, bus voltages, and line currents for a bolted fault at bus 6. When using **Zbus = zbuild(zdata)** function, the generator reactances must be included in the **zdata** array with bus zero as the reference bus.

9.17. In Problem 9.16 consider the shunt capacitors and neglect the loads. use **zbuildpi(linedata, gendata, yload)** function to obtain the bus impedance matrix. Assuming all the prefault bus voltages are equal to $1\angle 0°$, use **symfault(linedata, Zbus)** function to compute the fault current, bus voltages, and line currents for a bolted fault at bus 6.

9.18. Repeat the symmetrical three-phase short circuit analysis for Problem 9.16 considering the prefault bus voltages and the effect of load currents. The load data is as follows.

LOAD DATA		
Bus No.	Load MW	Load Mvar
1	0	0
2	0	0
3	0	0
4	100	70
5	90	30
6	160	110

Voltage magnitude, generation schedule, and the reactive power limits for the regulated buses are tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.06\angle 0°$, is taken as the slack bus.

GENERATION DATA				
Bus No.	Voltage Mag.	Generation, MW	Mvar Limits Min.	Mvar Limits Max.
1	1.060			
2	1.040	150.0	0.0	140.0
3	1.030	100.0	0.0	90.0

Use anyone of the power flow programs to obtain the prefault bus voltages and the load admittance. The power flow program returns the prefault bus voltage array **V** and the bus load admittance array **yload**.

CHAPTER

10

SYMMETRICAL COMPONENTS AND UNBALANCED FAULT

10.1 INTRODUCTION

Different types of unbalanced faults are the *single line-to-ground fault*, *line-to-line fault*, and *double line-to-ground fault*.

The fault study that was presented in Chapter 9 has considered only three-phase balanced faults, which lends itself to a simple per phase approach. Various methods have been devised for the solution of unbalanced faults. However, since the one-line diagram simplifies the solution of the balanced three-phase problems, the method of symmetrical components that resolves the solution of unbalanced circuit into a solution of a number of balanced circuits is used. In this chapter, the symmetrical components method is discussed. It is then applied to the unbalanced faults, which allows once again the treatment of the problem on a simple per phase basis. Two functions are developed for the symmetrical components transformations. These are **abc2sc**, which provides transformation from phase quantities to symmetrical components, and **sc2abc** for the inverse transformation. In addition, these functions produce plots of unbalanced phasors and their symmetrical components. Finally, unbalanced faults are computed using the concept of symmetrical components. Three functions named **lgfault(zdata0, zbus0, zdata1, zbus1, zdata2, zbus2, V)**, **llfault(zdata1, zbus1, zdata2, zbus2, V)**, and **dlgfault(zdata0, zbus0, zdata1, zbus1, zdata2, zbus2, V)** are developed for the line-to-ground, line-to-line, and the double line-to-ground fault studies.

10.2 FUNDAMENTALS OF SYMMETRICAL COMPONENTS

Symmetrical components allow unbalanced phase quantities such as currents and voltages to be replaced by three separate balanced symmetrical components.

In three-phase system the phase sequence is defined as the order in which they pass through a positive maximum. Consider the phasor representation of a three-phase balanced current shown in Figure 10.1(a).

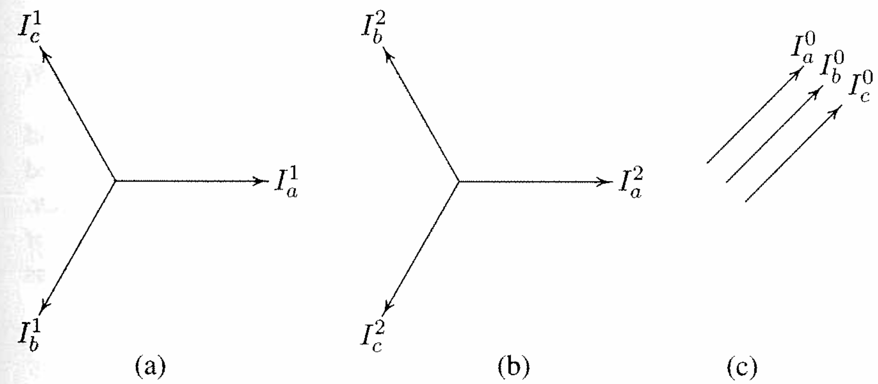


FIGURE 10.1
Representation of symmetrical components.

By convention, the direction of rotation of the phasors is taken to be counterclockwise. The three phasors are written as

$$\begin{aligned} I_a^1 &= I_a^1\angle 0^\circ = I_a^1 \\ I_b^1 &= I_a^1\angle 240^\circ = a^2 I_a^1 \\ I_c^1 &= I_a^1\angle 120^\circ = a I_a^1 \end{aligned} \tag{10.1}$$

where we have defined an operator a that causes a counterclockwise rotation of 120°, such that

$$\begin{aligned} a &= 1\angle 120^\circ = -0.5 + j0.866 \\ a^2 &= 1\angle 240^\circ = -0.5 - j0.866 \\ a^3 &= 1\angle 360^\circ = 1 + j0 \end{aligned} \tag{10.2}$$

From above, it is clear that

$$1 + a + a^2 = 0 \tag{10.3}$$

The order of the phasors is abc. This is designated the *positive phase sequence*. When the order is acb as in Figure 10.1(b), it is designated the *negative phase* *sequence*. The negative phase sequence quantities are represented as

$$\begin{aligned} I_a^2 &= I_a^2\angle 0^\circ &&= I_a^2 \\ I_b^2 &= I_a^2\angle 120^\circ &&= aI_a^2 \\ I_c^2 &= I_a^2\angle 240^\circ &&= a^2 I_a^2 \end{aligned} \tag{10.4}$$

When analyzing certain types of unbalanced faults, it will be found that a third set of balanced phasors must be introduced. These phasors, known as the *zero phase sequence*, are found to be in phase with each other. Zero phase sequence currents, as in Figure 10.1(c), would be designated

$$I_a^0 = I_b^0 = I_c^0 \tag{10.5}$$

The superscripts 1, 2, and 0 are being used to represent positive, negative, and zero-sequence quantities, respectively. In some texts the notation 0, +, − is used instead of 0, 1, 2. The symmetrical components method was introduced by Dr. C. L. Fortescue in 1918. Based on his theory, three-phase unbalanced phasors of a three-phase system can be resolved into three balanced systems of phasors as follows.

1. Positive-sequence components consisting of a set of balanced three-phase components with a phase sequence *abc*.
2. Negative-sequence components consisting of a set of balanced three-phase components with a phase sequence *acb*.
3. Zero-sequence components consisting of three single-phase components, all equal in magnitude but with the same phase angles.

Consider the three-phase unbalanced currents I_a, I_b, and I_c shown in Figure 10.2 (page 405). We are seeking to find the three symmetrical components of the current such that

$$\begin{aligned} I_a &= I_a^0 + I_a^1 + I_a^2 \\ I_b &= I_b^0 + I_b^1 + I_b^2 \\ I_c &= I_c^0 + I_c^1 + I_c^2 \end{aligned} \tag{10.6}$$

According to the definition of the symmetrical components as given by (10.1), (10.4), and (10.5), we can rewrite (10.6) all in terms of phase a components.

$$\begin{aligned} I_a &= I_a^0 + I_a^1 + I_a^2 \\ I_b &= I_a^0 + a^2 I_a^1 + aI_a^2 \\ I_c &= I_a^0 + aI_a^1 + a^2 I_a^2 \end{aligned} \tag{10.7}$$

or

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} \tag{10.8}$$

We rewrite the above equation in matrix notation as

$$\mathbf{I}^{abc} = \mathbf{A}\ \mathbf{I}_a^{012} \tag{10.9}$$

where $\mathbf{A}$ is known as *symmetrical components transformation matrix* (SCTM) which transforms phasor currents $\mathbf{I}^{abc}$ into component currents $\mathbf{I}_a^{012}$, and is

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{10.10}$$

Solving (10.9) for the symmetrical components of currents, we have

$$\mathbf{I}_a^{012} = \mathbf{A}^{-1}\ \mathbf{I}^{abc} \tag{10.11}$$

The inverse of $\mathbf{A}$ is given by

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{10.12}$$

From (10.10) and (10.12), we conclude that

$$\mathbf{A}^{-1} = \frac{1}{3}\mathbf{A}^* \tag{10.13}$$

Substituting for $\mathbf{A}^{-1}$ in (10.11), we have

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{10.14}$$

or in component form, the symmetrical components are

$$\begin{aligned} I_a^0 &= \tfrac{1}{3}(I_a + I_b + I_c) \\ I_a^1 &= \tfrac{1}{3}(I_a + aI_b + a^2 I_c) \\ I_a^2 &= \tfrac{1}{3}(I_a + a^2 I_b + aI_c) \end{aligned} \tag{10.15}$$

From (10.15), we note that the zero-sequence component of current is equal to one-third of the sum of the phase currents. Therefore, when the phase currents sum to zero, e.g., in a three-phase system with ungrounded neutral, the zero-sequence current cannot exist. If the neutral of the power system is grounded, zero-sequence current flows between the neutral and the ground.

Similar expressions exist for voltages. Thus the unbalanced phase voltages in terms of the symmetrical components voltages are

$$\begin{aligned} V_a &= V_a^0 + V_a^1 + V_a^2 \\ V_b &= V_a^0 + a^2V_a^1 + aV_a^2 \\ V_c &= V_a^0 + aV_a^1 + a^2V_a^2 \end{aligned} \qquad (10.16)$$

or in matrix notation

$$\mathbf{V}^{abc} = \mathbf{A}\ \mathbf{V}_a^{012} \qquad (10.17)$$

The symmetrical components in terms of the unbalanced voltages are

$$\begin{aligned} V_a^0 &= \tfrac{1}{3}(V_a + V_b + V_c) \\ V_a^1 &= \tfrac{1}{3}(V_a + aV_b + a^2V_c) \\ V_a^2 &= \tfrac{1}{3}(V_a + a^2V_b + aV_c) \end{aligned} \qquad (10.18)$$

or in matrix notation

$$\mathbf{V}_a^{012} = \mathbf{A}^{-1}\ \mathbf{V}^{abc} \qquad (10.19)$$

The apparent power may also be expressed in terms of the symmetrical components. The three-phase complex power is

$$S_{(3\phi)} = \mathbf{V}^{abc^T}\mathbf{I}^{abc^*} \qquad (10.20)$$

Substituting (10.9) and (10.17) in (10.20), we obtain

$$\begin{aligned} S_{(3\phi)} &= \left(\mathbf{A}\mathbf{V}_a^{012}\right)^T\left(\mathbf{A}\mathbf{I}_a^{012}\right)^* \\ &= \mathbf{V}_a^{012^T}\mathbf{A}^T\mathbf{A}^*\mathbf{I}_a^{012^*} \end{aligned} \qquad (10.21)$$

Since $\mathbf{A}^T = \mathbf{A}$, then from (10.13), $\mathbf{A}^T\mathbf{A}^* = 3$, and the complex power becomes

$$\begin{aligned} S_{(3\phi)} &= 3\left(\mathbf{V}^{012^T}\mathbf{I}^{012^*}\right) \\ &= 3V_a^0I_a^{0^*} + 3V_a^1I_a^{1^*} + 3V_a^2I_a^{2^*} \end{aligned} \qquad (10.22)$$

Equation (10.22) shows that the total unbalanced power can be obtained from the sum of the symmetrical component powers. Often the subscript a of the symmetrical components are omitted, e.g., I^0, I^1, and I^2 are understood to refer to phase a.

Transformation from phase quantities to symmetrical components in *MATLAB* is very easy. Once the symmetrical components transformation matrix **A** is defined, its inverse is found using the *MATLAB* function **inv**. However, for quick calculations and graphical demonstration, the following functions are developed for symmetrical components analysis.

sctm The symmetrical components transformation matrix **A** is defined in this script file. Typing **sctm** defines **A**.

phasor(F) This function makes plots of phasors. The variable F may be expressed in an $n \times 1$ array in rectangular complex form or as an $n \times 2$ matrix. In the latter case, the first column is the phasor magnitude and the second column is its phase angle in degree.

F_{012} = **abc2sc**(F_{abc}) This function returns the symmetrical components of a set of unbalanced phasors in rectangular form. F_{abc} may be expressed in a 3×1 array in rectangular complex form or as a 3×2 matrix. In the latter case, the first column is the phasor magnitude and the second column is its phase angle in degree for a, b, and c phases. In addition, the function produces a plot of the unbalanced phasors and its symmetrical components.

F_{abc} = **sc2abc**(F_{012}) This function returns the unbalanced phasor in rectangular form when symmetrical components are specified. F_{012} may be expressed in a 3×1 array in rectangular complex form or as a 3×2 matrix. In the latter case, the first column is the phasor magnitude and the second column is its phase angle in degree for the zero-, positive-, and negative-sequence components, respectively. In addition, the function produces a plot of the unbalanced phasors and its symmetrical components.

Z_{012} = **zabc2sc**(Z_{abc}) This function transforms the phase impedance matrix to the sequence impedance matrix, given by (10.30).

F_p = **rec2pol**(F_r) This function converts the rectangular phasor F_r into polar form F_p.

F_r = **pol2rec**(F_p) This function converts the polar phasor F_p into rectangular form F_r.

Example 10.1 (chp10ex1)

Obtain the symmetrical components of a set of unbalanced currents $I_a = 1.6\angle 25°$, $I_b = 1.0\angle 180°$, and $I_c = 0.9\angle 132°$.

The commands

```
Iabc = [1.6    25
        1.0   180
        0.9   132];
I012 = abc2sc(Iabc);  % Symmetrical components of phase a
I012p= rec2pol(I012)          % Rectangular to polar form
```

result in

```
I012P =
        0.4512    96.4529
        0.9435    -0.0550
        0.6024    22.3157
```

and the plots of the phasors are shown in Figure 10.2.

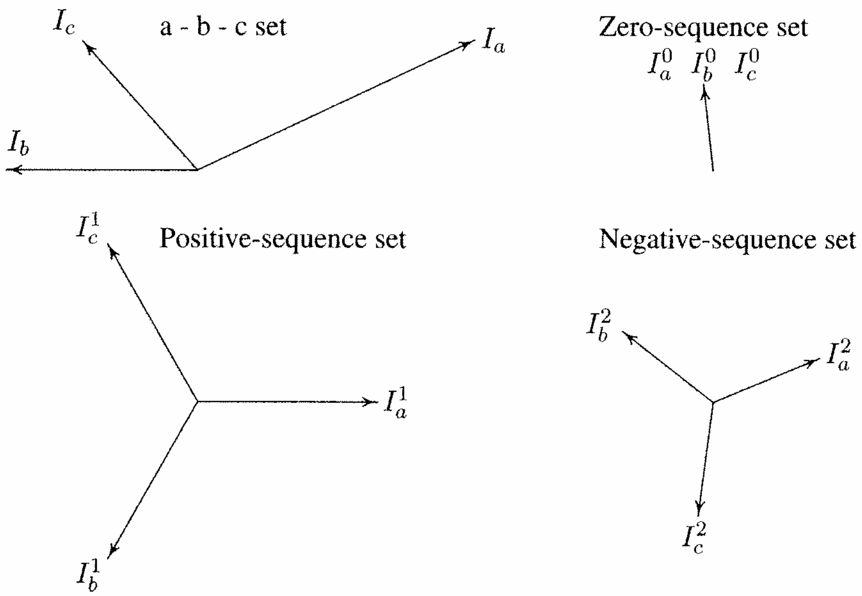


FIGURE 10.2
Resolution of unbalanced phasors into symmetrical components.

Example 10.2 (chp10ex2)

The symmetrical components of a set of unbalanced three-phase voltages are $V_a^0 = 0.6\angle 90°$, $V_a^1 = 1.0\angle 30°$, and $V_a^2 = 0.8\angle -30°$. Obtain the original unbalanced phasors.

The commands


```
V012 = [0.6    90
        1.0    30
        0.8   -30];
Vabc = sc2abc(V012);%Unbalanced phasor to symmetrical comp.
Vabcp= rec2pol(Vabc)               % Rectangular to polar form
```

result in

```
Vabcp =
        1.7088     24.1825
        0.400      90.0000
        1.7088    155.8175
```

and the plots of the phasors are shown in Figure 10.3.

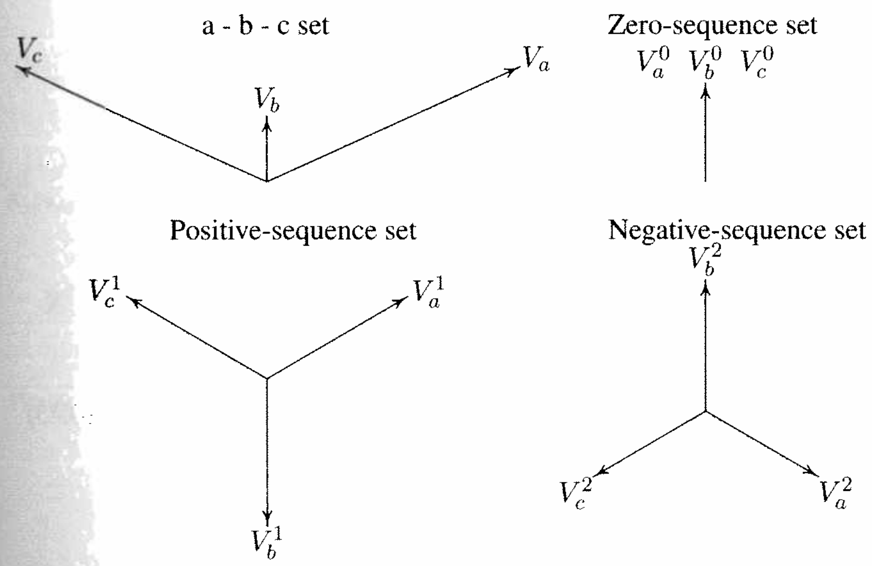


FIGURE 10.3
Transformation of the symmetrical components into phasor components.

10.3 SEQUENCE IMPEDANCES

This is the impedance of an equipment or component to the current of different sequences. The impedance offered to the flow of positive-sequence currents is known as the *positive-sequence impedance* and is denoted by Z^1. The impedance offered to the flow of negative-sequence currents is known as the *negative-sequence impedance*, shown by Z^2. When zero-sequence currents flow, the impedance is called the *zero-sequence impedance*, shown by Z^0. The sequence impedances of transmission lines, generators, and transformers are considered briefly here.

10.3.1 SEQUENCE IMPEDANCES OF Y-CONNECTED LOADS

A three-phase balanced load with self and mutual elements is shown in Figure 10.4. The load neutral is grounded through an impedance Z_n.

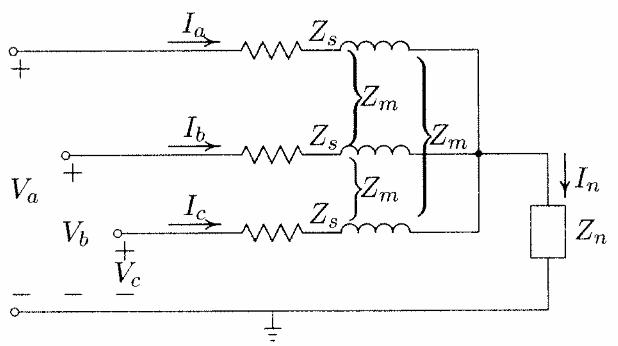


FIGURE 10.4
Balanced Y-connected load.

The line-to-ground voltages are

$$\begin{aligned} V_a &= Z_s I_a + Z_m I_b + Z_m I_c + Z_n I_n \\ V_b &= Z_m I_a + Z_s I_b + Z_m I_c + Z_n I_n \\ V_c &= Z_m I_a + Z_m I_b + Z_s I_c + Z_n I_n \end{aligned} \tag{10.23}$$

From Kirchhoff's current law, we have

$$I_n = I_a + I_b + I_c \tag{10.24}$$

Substituting for I_n from (10.24) into (10.23) and rewriting this equation in matrix form, yields

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Zm + Z_n \\ Z_m + Z_n & Z_s + Z_n & Zm + Z_n \\ Z_m + Z_n & Z_m + Z_n & Zs + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{10.25}$$

or in compact form

$$\mathbf{V}^{abc} = \mathbf{Z}^{abc}\mathbf{I}^{abc} \tag{10.26}$$

where

$$\mathbf{Z}^{abc} = \begin{bmatrix} Z_s + Z_n & Z_m + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_s + Z_n & Z_m + Z_n \\ Z_m + Z_n & Z_m + Z_n & Z_s + Z_n \end{bmatrix} \tag{10.27}$$

Writing $\mathbf{V}^{abc}$ and $\mathbf{I}^{abc}$ in terms of their symmetrical components, we get

$$\mathbf{A}\mathbf{V}_a^{012} = \mathbf{Z}^{abc}\mathbf{A}\mathbf{I}_a^{012} \tag{10.28}$$

Multiplying (10.28) by $\mathbf{A}^{-1}$, we get

$$\begin{aligned} \mathbf{V}_a^{012} &= \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}\mathbf{I}_a^{012} \\ &= \mathbf{Z}^{012}\mathbf{I}_a^{012} \end{aligned} \tag{10.29}$$

where

$$\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A} \tag{10.30}$$

Substituting for $\mathbf{Z}^{abc}$, $\mathbf{A}$, and $\mathbf{A}^{-1}$ from (10.27), (10.10), and (10.12), we have

$$\mathbf{Z}^{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_s{+}Z_n & Z_m{+}Z_n & Z_m{+}Z_n \\ Z_m{+}Z_n & Z_s{+}Z_n & Z_m{+}Z_n \\ Z_m{+}Z_n & Z_m{+}Z_n & Z_s{+}Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{10.31}$$

Performing the above multiplications, we get

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 3Z_n + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix} \tag{10.32}$$

When there is no mutual coupling, we set $Z_m = 0$, and the impedance matrix becomes

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} \tag{10.33}$$

The impedance matrix has nonzero elements appearing only on the principal diagonal, and it is a diagonal matrix. Therefore, for a balanced load, the three sequences are independent. That is, currents of each phase sequence will produce voltage drops of the same phase sequence only. This is a very important property, as it permits the analysis of each sequence network on a per phase basis.

10.3.2 SEQUENCE IMPEDANCES OF TRANSMISSION LINES

Transmission line parameters were derived in Chapter 4. For static devices such as transmission lines, the phase sequence has no effect on the impedance, because the voltages and currents encounter the same geometry of the line, irrespective of the sequence. Thus, positive- and negative-sequence impedances are equal, i.e., $Z^1 = Z^2$.

In deriving the line parameters, the effect of ground and shielding conductors were neglected. Zero-sequence currents are in phase and flow through the a,b,c conductors to return through the grounded neutral. The ground or any shielding wire are effectively in the path of zero sequence. Thus, Z^0, which includes the effect of the return path through the ground, is generally different from Z^1 and Z^2. The determination of the zero sequence impedance with the presence of earth neutral wires is quite involved and the interested reader is referred to the Carson's formula [14]. To get an idea of the order of Z^0 we will consider the following simplified configuration. Consider 1-m length of a three-phase line with equilaterally spaced conductors as shown in Figure 10.5. The phase conductors carry zero-sequence (single-phase) currents with return paths through a grounded neutral. The ground surface is approximated to an equivalent fictitious conductor located at the average distance D_n from each of the three phases. Since conductor n carries the return current in opposite direction, we have

$$I_a^0 + I_b^0 + I_c^0 + I_n = 0 \qquad (10.34)$$

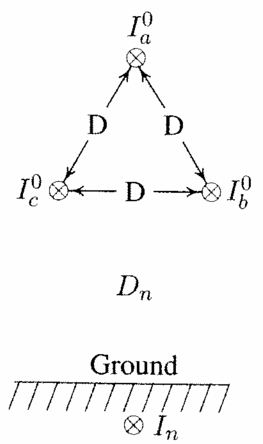


FIGURE 10.5
Zero-sequence current flow with earth return.

Since $I_a^0 = I_b^0 = I_c^0$, we have

$$I_n = -3I_a^0 \qquad (10.35)$$

Utilizing the relation for the flux linkages of a conductor in a group expressed by (4.29), the total flux linkage of phase a conductor is

$$\lambda_{a0} = 2 \times 10^{-7}\left(I_a^0 \ln \frac{1}{r'} + I_b^0 \ln \frac{1}{D} + I_c^0 \ln \frac{1}{D} + I_n \ln \frac{1}{D_n}\right) \qquad (10.36)$$

Substituting for I_b^0, I_c^0, and I_n in terms of I_a^0, we get

$$\lambda_{a0} = 2 \times 10^{-7} I_a^0 \left(\ln \frac{1}{r'} + \ln \frac{1}{D} + \ln \frac{1}{D} - 3 \ln \frac{1}{D_n}\right)$$
$$= 2 \times 10^{-7} I_a^0 \ln \frac{D_n^3}{r'D^2} \quad \text{Wb/m} \qquad (10.37)$$

Since $L_0 = \lambda_{a0}/I_a^0$, the zero sequence inductance per phase in mH per kilometer length is

$$L_0 = 0.2 \ln \frac{D_n^3}{r'D^2}$$
$$= 0.2 \ln \frac{DD_n^3}{r'D^3}$$
$$= 0.2 \ln \frac{D}{r'} + 3\left(0.2 \ln \frac{D_n}{D}\right) \quad \text{mH/Km} \qquad (10.38)$$

The first term above is the same as the positive-sequence inductance given by (4.33). Thus the zero sequence reactance can be expressed as

$$X^0 = X^1 + 3X_n \qquad (10.39)$$

where

$$X_n = 2\pi f \left(0.2 \ln \frac{D_n}{D}\right) \quad \text{m}\Omega/\text{km} \qquad (10.40)$$

The zero-sequence impedance of the transmission line is more than three times larger than the positive- or negative-sequence impedance.

10.3.3 SEQUENCE IMPEDANCES OF SYNCHRONOUS MACHINE

The inductances of a synchronous machine depend upon the phase order of the sequence current relative to the direction of rotation of the rotor. The positive-sequence generator impedance is the value found when positive-sequence current flows from the action of an imposed positive-sequence set of voltages. We have seen that the generator positive-sequence reactance varies, and in Section 9.2 one of the reactances X''_d, X'_d, or X_d was used for the balanced three-phase fault studies.

When negative-sequence currents are impressed in the stator, the net flux in the air gap rotates at opposite direction to that of the rotor. That is, the net flux rotates at twice synchronous speed relative to the rotor. Since the field voltage is associated with the positive-sequence variables, the field winding has no influence. Consequently, only the damper winding produces an effect in the quadrature axis. Hence, there is no distinction between the transient and subtransient reactances in the quadrature axis as there is in the direct axis. The negative-sequence reactance is close to the positive-sequence subtransient reactance, i.e.,

$$X^2 \simeq X''_d \tag{10.41}$$

Zero-sequence impedance is the impedance offered by the machine to the flow of the zero-sequence current. We recall that a set of zero sequence currents are all identical. Therefore, if the spatial distribution of mmf is assumed sinusoidal, the resultant air-gap flux would be zero, and there is no reactance due to armature reaction. The machine offers a very small reactance due to the leakage flux. Therefore, the zero-sequence reactance is approximated to the leakage reactance, i.e.,

$$X^0 \simeq X_\ell \tag{10.42}$$

10.3.4 SEQUENCE IMPEDANCES OF TRANSFORMER

In Chapter 3 we obtained the per phase equivalent circuit for a three-phase transformer. In power transformers, the core losses and the magnetization current are on the order of 1 percent of the rated value; therefore, the magnetizing branch is neglected. The transformer is modeled with the equivalent series leakage impedance. Since the transformer is a static device, the leakage impedance will not change if the phase sequence is changed. Therefore, the positive- and negative-sequence impedances are the same. Also, if the transformer permits zero-sequence current flow at all, the phase impedance to zero-sequence is equal to the leakage impedance, and we have

$$Z^0 = Z^1 = Z^2 = Z_\ell \tag{10.43}$$

From Section 3.9.1, we recall that in a Y-Δ, or a Δ-Y transformer, the positive-sequence line voltage on HV side leads the corresponding line voltage on the

LV side by 30°. For the negative-sequence voltage the corresponding phase shift is −30°. The equivalent circuit for the zero-sequence impedance depends on the winding connections and also upon whether or not the neutrals are grounded. Figure 10.6 shows some of the more common transformer configurations and their zero-sequence equivalent circuits. We recall that in a transformer, when the core reluctance is neglected, there is an exact mmf balance between the primary and secondary. This means that current can flow in the primary only if there is a current in the secondary. Based on this observation we can check the validity of the zero-sequence circuits by applying a set of zero-sequence voltage to the primary and calculating the resulting currents.

(a) Y-Y connections with both neutrals grounded – We know that the zero sequence current equals the sum of phase currents. Since both neutrals are grounded, there is a path for the zero sequence current to flow in the primary and secondary, and the transformer exhibits the equivalent leakage impedance per phase as shown in Figure 10.6(a).

(b) Y-Y connection with the primary neutral grounded – The primary neutral is grounded, but since the secondary neutral is isolated, the secondary phase current must sum up to zero. This means that the zero-sequence current in the secondary is zero. Consequently, the zero sequence current in the primary is zero, reflecting infinite impedance or an open circuit as shown in Figure 10.6(b).

(c) Y-Δ with grounded neutral – In this configuration the primary currents can flow because there is zero-sequence circulating current in the Δ-connected secondary and a ground return path for the Y-connected primary. Note that no zero-sequence current can leave the Δ terminals, thus there is an isolation between the primary and secondary sides as shown in Figure 10.6(c).

(d) Y-Δ connection with isolated neutral – In this configuration, because the neutral is isolated, zero sequence current cannot flow and the equivalent circuit reflects an infinite impedance or an open as shown in Figure 10.6(d).

(e) Δ-Δ connection – In this configuration zero-sequence currents circulate in the Δ-connected windings, but no currents can leave the Δ terminals, and the equivalent circuit is as shown in Figure 10.6(e).

Notice that the neutral impedance plays an important part in the equivalent circuit. When the neutral is grounded through an impedance Z_n, because $I_n = 3I_0$, in the equivalent circuit the neutral impedance appears as $3Z_n$ in the path of I_0.

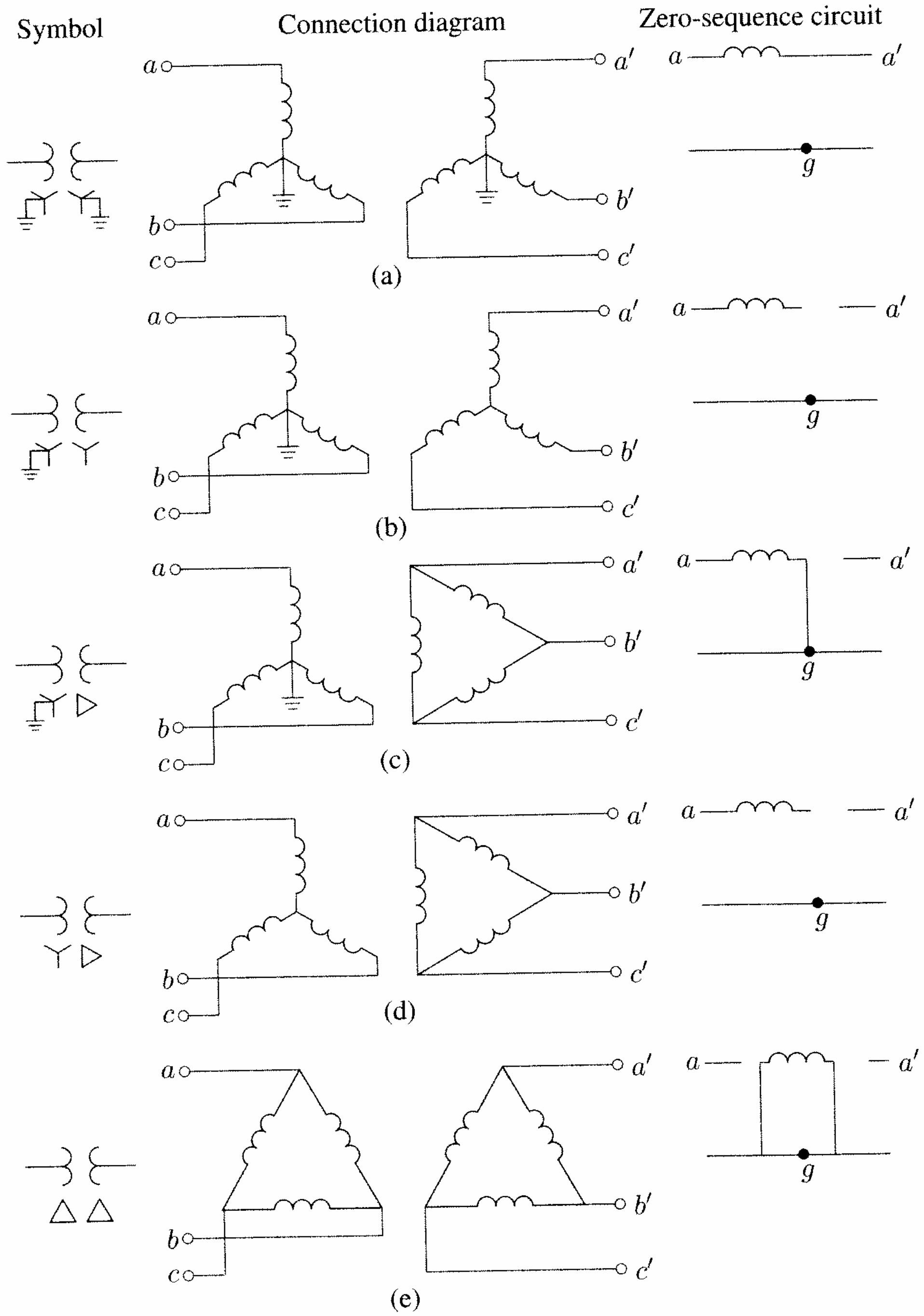


FIGURE 10.6
Transformer zero-sequence equivalent circuits.

Example 10.3 (chp10ex3)

A balanced three-phase voltage of 100-V line-to-neutral is applied to a balanced Y-connected load with ungrounded neutral as shown in Figure 10.7. The three-phase load consists of three mutually-coupled reactances. Each phase has a series reactance of $Z_s = j12\ \Omega$, and the mutual coupling between phases is $Z_m = j4\ \Omega$.

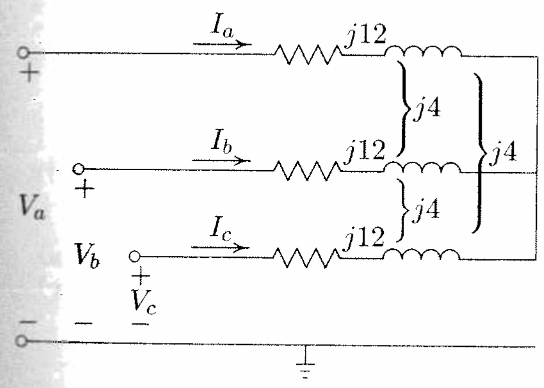


FIGURE 10.7
Circuit for Example 10.3.

(a) Determine the line currents by mesh analysis without using symmetrical components.
(b) Determine the line currents using symmetrical components.

(a) Applying KVL to the two independent mesh equations yields

$$Z_sI_a + Z_mI_b - Z_sI_b - Z_mI_a = V_a - V_b = |V_L|\angle\pi/6$$
$$Z_sI_b + Z_mI_c - Z_sI_c - Z_mI_b = V_b - V_c = |V_L|\angle-\pi/2$$

Also from KCL, we have

$$I_a + I_b + I_c = 0$$

Writing above equations in matrix form, results in

$$\begin{bmatrix} (Z_s - Z_m) & -(Z_s - Z_m) & 0 \\ 0 & (Z_s - Z_m) & -(Z_s - Z_m) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} |V_L|\angle\pi/6 \\ |V_L|\angle-\pi/2 \\ 0 \end{bmatrix}$$

or in compact form

$$\mathbf{Z}_{mesh}\mathbf{I}^{abc} = \mathbf{V}_{mesh}$$

Solving the above equations results in the line currents

$$\mathbf{I}^{abc} = \mathbf{Z}_{mesh}{}^{-1}\mathbf{V}_{mesh}$$

The following commands

```
% (a) Solution by mesh analysis
Zs=j*12; Zm=j*4; Va = 100; VL=Va*sqrt(3);
Z= [(Zs-Zm)  -(Zs-Zm)      0
       0      (Zs-Zm)  -(Zs-Zm)
       1         1         1   ];
V=[VL*cos(pi/6)+j*VL*sin(pi/6)
   VL*cos(-pi/2)+j*VL*sin(-pi/2)
             0                   ];
Y=inv(Z)
Iabc=Y*V;                    % Line currents (Rectangular form)
Iabcp=[abs(Iabc), angle(Iabc)*180/pi] %  Line currents (Polar)
```

result in

```
     Iabcp =
             12.5    -90.0
             12.5    150.0
             12.5     30.0
```

(b) Using the symmetrical components method, we have

$$\mathbf{V}^{012} = \mathbf{Z}^{012}\mathbf{I}^{012}$$

where

$$\mathbf{V}^{012} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

and from (10.32)

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

for the sequence components of currents, we get

$$\mathbf{I}^{012} = [\mathbf{Z}^{012}]^{-1}\mathbf{V}^{012}$$

We write the following commands


```
% (b) Solution by symmetrical components method
Z012=[Zs+2*Zm   0      0        % Symmetrical components matrix
        0     Zs-Zm    0
        0       0    Zs-Zm];
V012=[0; Va ; 0];    %Symmetrical components of phase voltages
I012=inv(Z012)*V012; %Symmetrical components of line  currents
a=cos(2*pi/3)+j*sin(2*pi/3);
A=[ 1  1  1; 1 a^2  a; 1 a a^2];       % Transformation matrix
Iabc=A*I012;                  % Line currents (Rectangular form)
Iabcp=[abs(Iabc), angle(Iabc)*180/pi]  % Line currents (Polar)
```

which result in

```
     Iabcp =
             12.5    -90.0
             12.5    150.0
             12.5     30.0
```

This is the same result as in part (a).

Example 10.4 (chp10ex4)

A three-phase unbalanced source with the following phase-to-neutral voltages

$$\mathbf{V}^{abc} = \begin{bmatrix} 200 & \angle 25^\circ \\ 100 & \angle -155^\circ \\ 80 & \angle 100^\circ \end{bmatrix}$$

is applied to the circuit in Figure 10.4 (page 407). The load series impedance per phase is $Z_s = 8 + j24$ and the mutual impedance between phases is $Z_m = j4$. The load and source neutrals are solidly grounded. Determine

(a) The load sequence impedance matrix $\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}$.
(b) The symmetrical components of voltage.
(c) The symmetrical components of current.
(d) The load phase currents.
(e) The complex power delivered to the load in terms of symmetrical components, $S_{3\phi} = 3(V_a^0 I_a^{0*} + V_a^1 I_a^{1*} + V_a^2 I_a^{2*})$.
(f) The complex power delivered to the load by summing up the power in each phase, $S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$.

We write the following commands

```
Vabc = [200    25
        100  -155
         80   100];
Zabc = [8+j*24     j*4     j*4
           j*4  8+j*24     j*4
          j*4      j*4  8+j*24];
Z012 = zabc2sc(Zabc)      % Symmetrical components of impedance
V012 = abc2sc(Vabc);        % Symmetrical components of voltage
V012p= rec2pol(V012)                % Rectangular to polar form
I012 = inv(Z012)*V012;      % Symmetrical components of current
I012p= rec2pol(I012)                % Rectangular to polar form
Iabc = sc2abc(I012);                           % Phase currents
Iabcp= rec2pol(Iabc)                % Rectangular to polar form
S3ph =3*(V012.')*conj(I012)%Power using symmetrical components
Vabcr = Vabc(:, 1).*(cos(pi/180*Vabc(:, 2)) +...
j*sin(pi/180*Vabc(:, 2)));
S3ph=(Vabcr.')*conj(Iabc)
                     % Power using phase currents and voltages
```

The result is

```
Z012 =
       8.00 + 32.00i   0.00 +  0.00i   0.00 +  0.00i
       0.00 +  0.00i   8.00 + 20.00i   0.00 +  0.00i
       0.00 -  0.00i   0.00 -  0.00i   8.00 + 20.00i

V012p =
        47.7739     57.6268
       112.7841     -0.0331
        61.6231     45.8825

I012p =
         1.4484    -18.3369
         5.2359    -68.2317
         2.8608    -22.3161

Iabcp =
         8.7507    -47.0439
         5.2292    143.2451
         3.0280     39.0675

S3ph =
      9.0471e+002+ 2.3373e+003i

S3ph =
      9.0471e+002+ 2.3373e+003i
```


10.4 SEQUENCE NETWORKS OF A LOADED GENERATOR

Figure 10.8 represents a three-phase synchronous generator with neutral grounded through an impedance Z_n. The generator is supplying a three-phase balanced load.

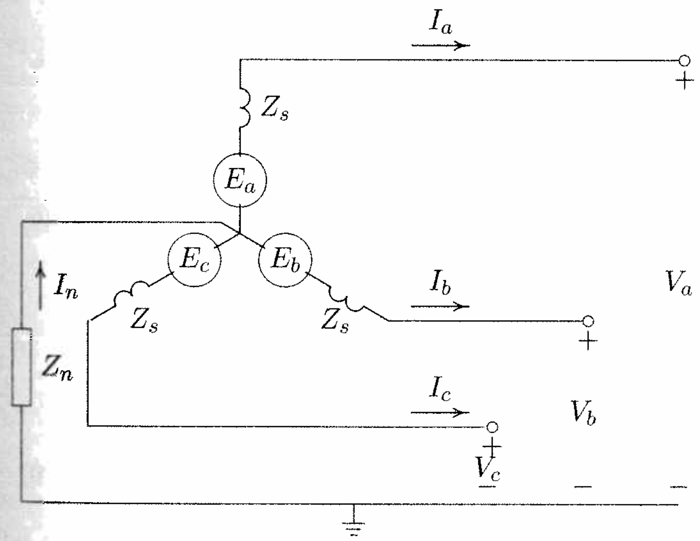


FIGURE 10.8
Three-phase balanced source and impedance.

The synchronous machine generates balanced three-phase internal voltages and is represented as a positive-sequence set of phasors

$$\mathbf{E}^{abc} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} E_a \tag{10.44}$$

The machine is supplying a three-phase balanced load. Applying Kirchhoff's voltage law to each phase we obtain

$$\begin{aligned} V_a &= E_a - Z_sI_a - Z_nI_n \\ V_b &= E_b - Z_sI_b - Z_nI_n \\ V_c &= E_c - Z_cI_c - Z_nI_n \end{aligned} \tag{10.45}$$

Substituting for $I_n = I_a + I_b + I_c$, and writing (10.45) in matrix form, we get

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} - \begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{10.46}$$

or in compact form, we have

$$\mathbf{V}^{abc} = \mathbf{E}^{abc} - \mathbf{Z}^{abc}\mathbf{I}^{abc} \tag{10.47}$$

where $\mathbf{V}^{abc}$ is the phase terminal voltage vector and $\mathbf{I}^{abc}$ is the phase current vector. Transforming the terminal voltages and current phasors into their symmetrical components results in

$$\mathbf{A}\mathbf{V}_a^{012} = \mathbf{A}\mathbf{E}_a^{012} - \mathbf{Z}^{abc}\mathbf{A}\mathbf{I}_a^{012} \tag{10.48}$$

Multiplying (10.48) by $\mathbf{A}^{-1}$, we get

$$\begin{aligned}\mathbf{V}_a^{012} &= \mathbf{E}_a^{012} - \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}\mathbf{I}_a^{012}\\ &= \mathbf{E}_a^{012} - \mathbf{Z}^{012}\mathbf{I}_a^{012}\end{aligned} \tag{10.49}$$

where

$$\mathbf{Z}^{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{10.50}$$

Performing the above multiplications, we get

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 3Z_n & 0 & 0 \\ 0 & Z_s & 0 \\ 0 & 0 & Z_s \end{bmatrix} = \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^1 & 0 \\ 0 & 0 & Z^2 \end{bmatrix} \tag{10.51}$$

Since the generated emf is balanced, there is only positive-sequence voltage, i.e.,

$$\mathbf{E}_a^{012} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} \tag{10.52}$$

Substituting for $\mathbf{E}_a^{012}$ and $\mathbf{Z}^{012}$ in (10.49), we get

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^1 & 0 \\ 0 & 0 & Z^2 \end{bmatrix}\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} \tag{10.53}$$

Since the above equation is very important, we write it in component form, and we get

$$\begin{aligned} V_a^0 &= 0 - Z^0 I_a^0 \\ V_a^1 &= E_a - Z^1 I_a^1 \\ V_a^2 &= 0 - Z^2 I_a^2 \end{aligned} \tag{10.54}$$

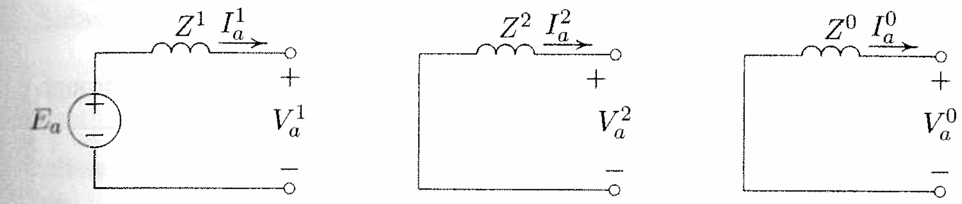


FIGURE 10.9
Sequence networks: (a) Positive-sequence; (b) negative-sequence; (c) zero-sequence.

The three equations given by (10.54) can be represented by the three equivalent sequence networks shown in Figure 10.9.

We make the following important observations.

- The three sequences are independent.
- The positive-sequence network is the same as the one-line diagram used in studying balanced three-phase currents and voltages.
- Only the positive-sequence network has a voltage source. Therefore, the positive-sequence current causes only positive-sequence voltage drops.
- There is no voltage source in the negative- or zero-sequence networks.
- Negative- and zero-sequence currents cause negative- and zero-sequence voltage drops only.
- The neutral of the system is the reference for positive-and negative-sequence networks, but ground is the reference for the zero-sequence networks. Therefore, the zero-sequence current can flow only if the circuit from the system neutrals to ground is complete.
- The grounding impedance is reflected in the zero sequence network as $3Z_n$.
- The three-sequence systems can be solved separately on a per phase basis. The phase currents and voltages can then be determined by superposing their symmetrical components of current and voltage respectively.

We are now ready with mathematical tools to analyze various types of unbalanced faults. First, the fault current is obtained using Thévenin's method and algebraic manipulation of sequence networks. The analysis will then be extended to find the bus voltages and fault current during fault, for different types of faults using the bus impedance matrix.

10.5 SINGLE LINE-TO-GROUND FAULT

Figure 10.10 illustrates a three-phase generator with neutral grounded through impedance Z_n.

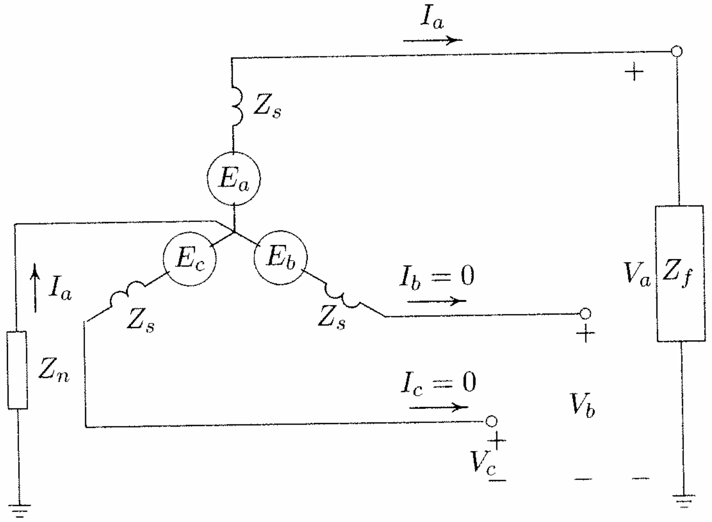


FIGURE 10.10
Line-to-ground fault on phase a.

Suppose a line-to-ground fault occurs on phase a through impedance Z_f. Assuming the generator is initially on no-load, the boundary conditions at the fault point are

$$V_a = Z_f I_a \tag{10.55}$$

$$I_b = I_c = 0 \tag{10.56}$$

Substituting for $I_b = I_c = 0$, the symmetrical components of currents from (10.14) are

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \tag{10.57}$$

From the above equation, we find that

$$I_a^0 = I_a^1 = I_a^2 = \frac{1}{3} I_a \tag{10.58}$$

Phase a voltage in terms of symmetrical components is

$$V_a = V_a^0 + V_a^1 + V_a^2 \tag{10.59}$$

Substituting for V_a^0, V_a^1, and V_a^2 from (10.54) and noting $I_a^0 = I_a^1 = I_a^2$, we get

$$V_a = E_a - (Z^1 + Z^2 + Z^0) I_a^0 \tag{10.60}$$

where $Z^0 = Z_s + 3Z_n$. Substituting for V_a from (10.55), and noting $I_a = 3I_a^0$, we get

$$3Z_f I_a^0 = E_a - (Z^1 + Z^2 + Z^0) I_a^0 \tag{10.61}$$

or

$$I_a^0 = \frac{E_a}{Z^1 + Z^2 + Z^0 + 3Z_f} \tag{10.62}$$

The fault current is

$$I_a = 3I_a^0 = \frac{3E_a}{Z^1 + Z^2 + Z^0 + 3Z_f} \tag{10.63}$$

Substituting for the symmetrical components of currents in (10.54), the symmetrical components of voltage and phase voltages at the point of fault are obtained.

Equations (10.58) and (10.62) can be represented by connecting the sequence networks in series as shown in the equivalent circuit of Figure 10.11. Thus, for line-to-ground faults, the Thévenin impedance to the point of fault is obtained for each sequence network, and the three sequence networks are placed in series. In many practical applications, the positive- and negative-sequence impedances are found to be equal. If the generator neutral is solidly grounded, $Z_n = 0$ and for bolted faults $Z_f = 0$.

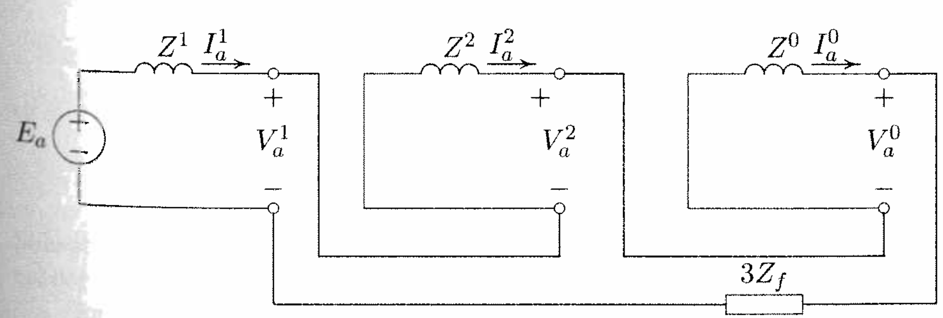


FIGURE 10.11
Sequence network connection for line-to-ground fault.

10.6 LINE-TO-LINE FAULT

Figure 10.12 shows a three-phase generator with a fault through an impedance Z_f between phases b and c. Assuming the generator is initially on no-load, the boundary conditions at the fault point are

$$V_b - V_c = Z_f I_b \tag{10.64}$$

$$I_b + I_c = 0 \tag{10.65}$$

$$I_a = 0 \tag{10.66}$$

Substituting for $I_a = 0$, and $I_c = -I_b$, the symmetrical components of currents from (10.14) are

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \tag{10.67}$$

From the above equation, we find that

$$I_a^0 = 0 \tag{10.68}$$

$$I_a^1 = \frac{1}{3}(a - a^2)I_b \tag{10.69}$$

$$I_a^2 = \frac{1}{3}(a^2 - a)I_b \tag{10.70}$$

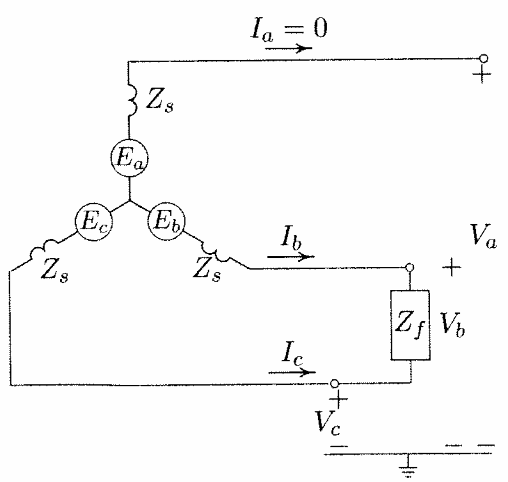


FIGURE 10.12
Line-to-line fault between phase b and c.

Also, from (10.69) and (10.70), we note that

$$I_a^1 = -I_a^2 \tag{10.71}$$

From (10.16), we have

$$\begin{aligned} V_b - V_c &= (a^2 - a)(V_a^1 - V_a^2) \\ &= Z_f I_b \end{aligned} \tag{10.72}$$

Substituting for V_a^1 and V_a^2 from (10.54) and noting $I_a^2 = -I_a^1$, we get

$$(a^2 - a)[E_a - (Z^1 + Z^2)I_a^1] = Z_f I_b \tag{10.73}$$

Substituting for I_b from (10.69), we get

$$E_a - (Z^1 + Z^2)I_a^1 = Z_f \frac{3I_a^1}{(a - a^2)(a^2 - a)} \tag{10.74}$$

Since $(a - a^2)(a^2 - a) = 3$, solving for I_a^1 results in

$$I_a^1 = \frac{E_a}{Z^1 + Z^2 + Z_f} \tag{10.75}$$

The phase currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0 \\ I_a^1 \\ -I_a^1 \end{bmatrix} \tag{10.76}$$

The fault current is

$$I_b = -I_c = (a^2 - a)I_a^1 \tag{10.77}$$

or

$$I_b = -j\sqrt{3}\, I_a^1 \tag{10.78}$$

Substituting for the symmetrical components of currents in (10.54), the symmetrical components of voltage and phase voltages at the point of fault are obtained.

Equations (10.71) and (10.75) can be represented by connecting the positive- and negative-sequence networks in opposition as shown in the equivalent circuit of Figure 10.13. In many practical applications, the positive- and negative-sequence impedances are found to be equal. For a bolted fault, $Z_f = 0$.

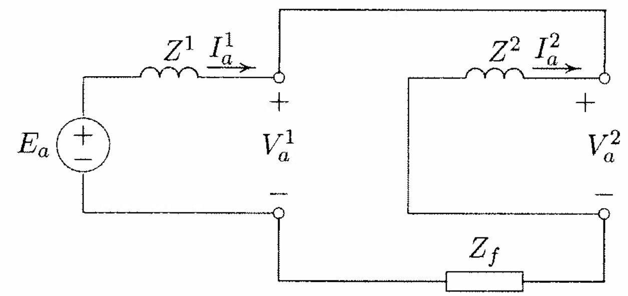


FIGURE 10.13
Sequence network connection for line-to-line fault.

10.7 DOUBLE LINE-TO-GROUND FAULT

Figure 10.14 shows a three-phase generator with a fault on phases b and c through an impedance Z_f to ground. Assuming the generator is initially on no-load, the boundary conditions at the fault point are

$$V_b = V_c = Z_f(I_b + I_c) \tag{10.79}$$

$$I_a = I_a^0 + I_a^1 + I_a^2 = 0 \tag{10.80}$$

From (10.16), the phase voltages V_b and V_c are

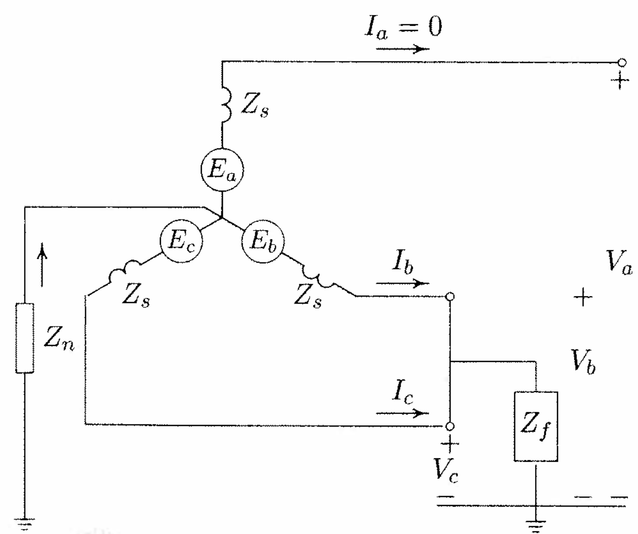


FIGURE 10.14
Double line-to-ground fault.

$$V_b = V_a^0 + a^2V_a^1 + aV_a^2 \tag{10.81}$$

$$V_c = V_a^0 + aV_a^1 + a^2V_a^2 \tag{10.82}$$

Since $V_b = V_c$, from above we note that

$$V_a^1 = V_a^2 \tag{10.83}$$

Substituting for the symmetrical components of currents in (10.79), we get

$$\begin{aligned} V_b &= Z_f(I_a^0 + a^2I_a^1 + aI_a^2 + I_a^0 + aI_a^1 + a^2I_a^2) \\ &= Z_f(2I_a^0 - I_a^1 - I_a^2) \\ &= 3Z_fI_a^0 \end{aligned} \tag{10.84}$$

Substituting for V_b from (10.84) and for V_a^2 from (10.83) into (10.81), we have

$$\begin{aligned} 3Z_fI_a^0 &= V_a^0 + (a^2 + a)V_a^1 \\ &= V_a^0 - V_a^1 \end{aligned} \tag{10.85}$$

Substituting for the symmetrical components of voltage from (10.54) into (10.85) and solving for I_a^0, we get

$$I_a^0 = -\frac{E_a - Z^1I_a^1}{Z^0 + 3Z_f} \tag{10.86}$$

Also, substituting for the symmetrical components of voltage in (10.83), we obtain

$$I_a^2 = -\frac{E_a - Z^1I_a^1}{Z^2} \tag{10.87}$$

Substituting for I_a^0 and I_a^2 into (10.80) and solving for I_a^1, we get

$$I_a^1 = \frac{E_a}{Z^1 + \frac{Z^2(Z^0+3Z_f)}{Z^2+Z^0+3Z_f}} \tag{10.88}$$

Equations (10.86)–(10.88) can be represented by connecting the positive-sequence impedance in series with the parallel combination of the negative-sequence and zero-sequence networks as shown in the equivalent circuit of Figure 10.15. The value of I_a^1 found from (10.88) is substituted in (10.86) and (10.87), and I_a^0 and I_a^2 are found. The phase currents are then found from (10.8). Finally, the fault current is obtained from

$$I_f = I_b + I_c = 3I_a^0 \tag{10.89}$$

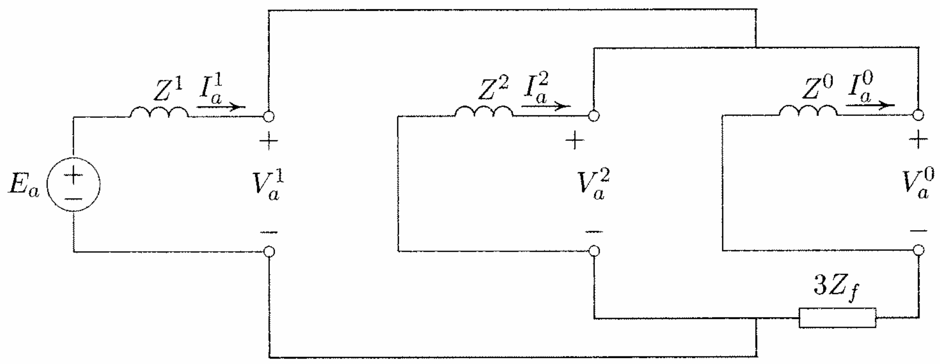


FIGURE 10.15
Sequence network connection for double line-to-ground fault.

Example 10.5 (chp10ex5)

The one-line diagram of a simple power system is shown in Figure 10.16. The neutral of each generator is grounded through a current-limiting reactor of 0.25/3 per unit on a 100-MVA base. The system data expressed in per unit on a common 100-MVA base is tabulated below. The generators are running on no-load at their rated voltage and rated frequency with their emfs in phase.

Determine the fault current for the following faults.

(a) A balanced three-phase fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

(b) A single line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.10$ per unit.

(c) A line-to-line fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

(d) A double line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

Item	Base MVA	Voltage Rating	X^1	X^2	X^0
G_1	100	20 kV	0.15	0.15	0.05
G_2	100	20 kV	0.15	0.15	0.05
T_1	100	20/220 kV	0.10	0.10	0.10
T_2	100	20/220 kV	0.10	0.10	0.10
L_{12}	100	220 kV	0.125	0.125	0.30
L_{13}	100	220 kV	0.15	0.15	0.35
L_{23}	100	220 kV	0.25	0.25	0.7125

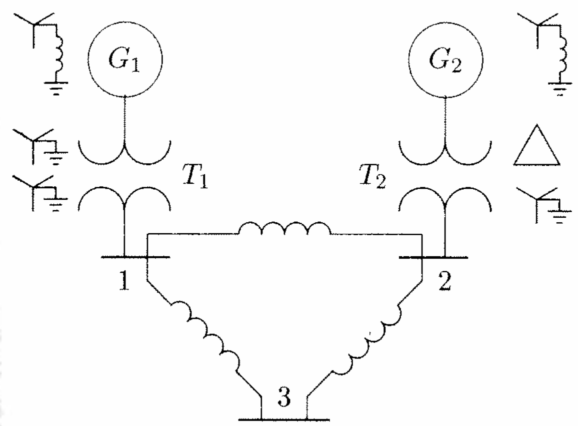


FIGURE 10.16
The one-line diagram for Example 10.5.

The positive-sequence impedance network is shown in Figure 10.17.

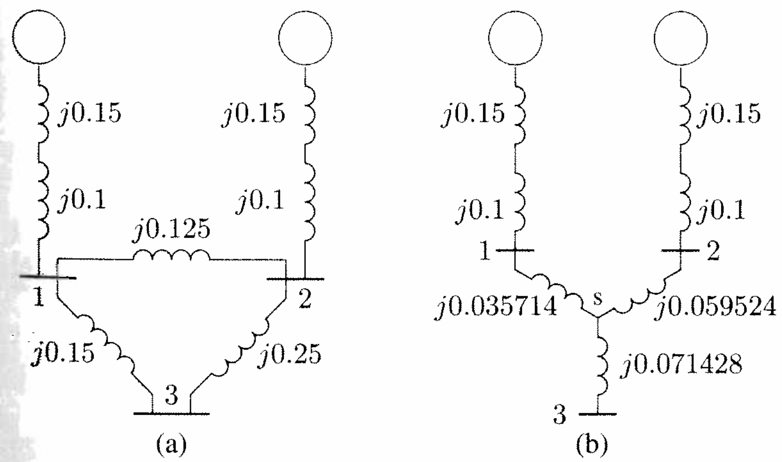


FIGURE 10.17
Positive-sequence impedance diagram for Example 10.5.

To find Thévenin impedance viewed from the faulted bus (bus 3), we convert the delta formed by buses 123 to an equivalent Y as shown in Figure 10.17(b).

$$Z_{1s} = \frac{(j0.125)(j0.15)}{j0.525} = j0.0357143$$

$$Z_{2s} = \frac{(j0.125)(j0.25)}{j0.525} = j0.0595238$$

$$Z_{3s} = \frac{(j0.15)(j0.25)}{j0.525} = j0.0714286$$

Combining the parallel branches, the positive-sequence Thévenin impedance is

$$Z_{33}^{1} = \frac{(j0.2857143)(j0.3095238)}{j0.5952381} + j0.0714286$$
$$= j0.1485714 + j0.0714286 = j0.22$$

This is shown in Figure 10.18(a).

(a) Positive-sequence network (b) Negative-sequence network

FIGURE 10.18
Reduction of the positive-sequence Thévenin equivalent network.

Since the negative-sequence impedance of each element is the same as the positive-sequence impedance, we have

$$Z_{33}^{2} = Z_{33}^{1} = j0.22$$

and the negative-sequence network is as shown in Figure 10.18(b). The equivalent circuit for the zero-sequence network is constructed according to the transformer winding connections of Figure 10.6 and is shown in Figure 10.19.

To find Thévenin impedance viewed from the faulted bus (bus 3), we convert the delta formed by buses 123 to an equivalent Y as shown in Figure 10.19(b).

$$Z_{1s} = \frac{(j0.30)(j0.35)}{j1.3625} = j0.0770642$$
$$Z_{2s} = \frac{(j0.30)(j0.7125)}{j1.3625} = j0.1568807$$
$$Z_{3s} = \frac{(j0.35)(j0.7125)}{j1.3625} = j0.1830257$$

Combining the parallel branches, the zero-sequence Thévenin impedance is

$$Z_{33}^{0} = \frac{(j0.4770642)(j0.2568807)}{j0.7339449} + j0.1830275$$
$$= j0.1669725 + j0.1830275 = j0.35$$

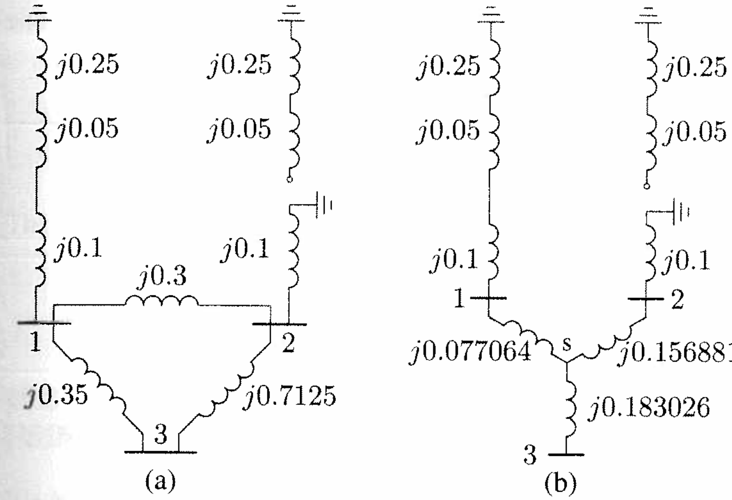


FIGURE 10.19
Zero-sequence impedance diagram for Example 10.5.

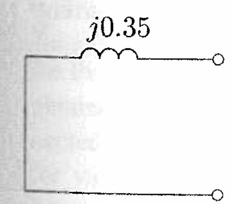


FIGURE 10.20
Zero-sequence network for Example 10.5.

The zero-sequence impedance diagram is shown in Figure 10.20.

(a) Balanced three-phase fault at bus 3.

Assuming the no-load generated emfs are equal to 1.0 per unit, the fault current is

$$I_3^a(F) = \frac{V_{3(0)}^a}{Z_{33}^1 + Z_f} = \frac{1.0}{j0.22 + j0.1} = -j3.125 \quad \text{pu}$$
$$= 820.1\angle{-90^\circ} \quad \text{A}$$

(b) Single line-to-ground fault at bus 3.

From (10.62), the sequence components of the fault current are

$$I_3^0 = I_3^1 = I_3^2 = \frac{V_3^a(0)}{Z_{33}^1 + Z_{33}^2 + Z_{33}^0 + 3Z_f}$$
$$= \frac{1.0}{j0.22 + j0.22 + j0.35 + 3(j0.1)}$$
$$= -j0.9174 \text{ pu}$$

The fault current is

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_3^0 \\ I_3^0 \\ I_3^0 \end{bmatrix} = \begin{bmatrix} 3I_3^0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j2.7523 \\ 0 \\ 0 \end{bmatrix} \text{ pu}$$

(c) Line-to line fault at bus 3.

The zero-sequence component of current is zero, i.e.,

$$I_3^0 = 0$$

From (10.75), the positive- and negative-sequence components of the fault current are

$$I_3^1 = -I_3^2 = \frac{V_{3(0)}^a}{Z_{33}^1 + Z_{33}^2 + Z_f} = \frac{1}{j0.22 + j0.22 + j0.1} = -j1.8519 \text{ pu}$$

The fault current is

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.8519 \\ j1.8519 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.2075 \\ 3.2075 \end{bmatrix}$$

(d) Double line-to line-fault at bus 3.

From (10.88), the positive-sequence component of the fault current is

$$I_3^1 = \frac{V_{3(0)}^a}{Z_{33}^1 + \frac{Z_{33}^2(Z_{33}^0+3Z_f)}{Z_{33}^2+Z_{33}^0+3Z_f}} = \frac{1}{j0.22 + \frac{j0.22(j0.35+j0.3)}{j0.22+j0.35+j.3}} = -j2.6017 \text{ pu}$$

The negative-sequence component of current from (10.87) is

$$I_3^2 = -\frac{V_{3(0)}^a - Z_{33}^1 I_3^1}{Z_{33}^2} = -\frac{1-(j0.22)(-j2.6017)}{j0.22} = j1.9438 \text{ pu}$$

The zero-sequence component of current from (10.86) is

$$I_3^0 = -\frac{V_{3(0)}^a - Z_{33}^1 I_3^1}{Z_{33}^0 + 3Z_f} = -\frac{1-(j0.22)(-j2.6017)}{j0.35 + j0.3} = j0.6579 \text{ pu}$$

and the phase currents are

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j0.6579 \\ -j2.6017 \\ j1.9438 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.058\angle 165.93^\circ \\ 4.058\angle 14.07^\circ \end{bmatrix}$$

The fault current is

$$I_3(F) = I_3^b + I_3^c = 1.9732\angle 90^\circ$$

10.8 UNBALANCED FAULT ANALYSIS USING BUS IMPEDANCE MATRIX

We have seen that when the network is balanced, the symmetrical components impedances are diagonal, so that it is possible to calculate $\mathbf{Z}_{bus}$ separately for zero-, positive-, and negative-sequence networks. Also, we have observed that for a fault at bus k, the diagonal element in the k axis of the bus impedance matrix $\mathbf{Z}_{bus}$ is the Thévenin impedance to the point of fault. In order to obtain a solution for the unbalanced faults, the bus impedance matrix for each sequence network is obtained separately, then the sequence impedances $Z^0{}_{kk}$, $Z^1{}_{kk}$, and $Z^2{}_{kk}$ are connected together as described in Figures 10.11, 10.13, and 10.15. The fault formulas for various unbalanced faults is summarized below. In writing the symmetrical components of voltage and currents, the subscript a is left out and the symmetrical components are understood to refer to phase a.

10.8.1 SINGLE LINE-TO-GROUND FAULT USING $\mathbf{Z}_{bus}$

Consider a fault between phase a and ground through an impedance Z_f at bus k as shown in Figure 10.21. The line-to-ground fault requires that positive-, negative-, and zero-sequence networks for phase a be placed in series in order to compute the zero-sequence fault current as given by (10.62). Thus, in general, for a fault at bus k, the symmetrical components of fault current is

$$I_k^0 = I_k^1 = I_k^2 = \frac{V_k(0)}{Z_{kk}^1 + Z_{kk}^2 + Z_{kk}^0 + 3Z_f} \quad (10.90)$$

where Z_{kk}^1, Z_{kk}^2, and Z_{kk}^0 are the diagonal elements in the k axis of the corresponding bus impedance matrix and $V_k(0)$ is the prefault voltage at bus k. The fault phase current is

$$I_k^{abc} = \mathbf{A} I_k^{012} \quad (10.91)$$

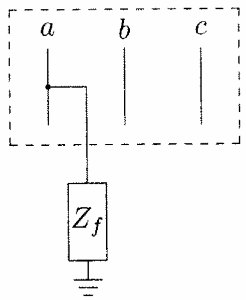


FIGURE 10.21
Line-to-ground fault at bus k.

10.8.2 LINE-TO-LINE FAULT USING $\mathbf{Z}_{bus}$

Consider a fault between phases b and c through an impedance Z_f at bus k as shown in Figure 10.22.

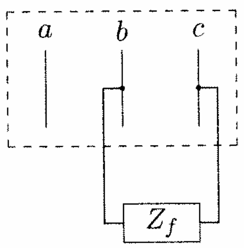


FIGURE 10.22
Line-to-line fault at bus k.

The phase a sequence network of Figure 10.13 is applicable here, where the positive- and negative-sequence networks are placed in opposition. The symmetrical components of the fault current as given from (10.68), (10.71), and (10.75) are

$$I_k^0 = 0 \tag{10.92}$$

$$I_k^1 = -I_k^2 = \frac{V_k(0)}{Z_{kk}^1 + Z_{kk}^2 + Z_f} \tag{10.93}$$

where Z_{kk}^1, and Z_{kk}^2 are the diagonal elements in the k axis of the corresponding bus impedance matrix. The fault phase current is then obtained from (10.91).

10.8.3 DOUBLE LINE-TO-GROUND FAULT USING $\mathbf{Z}_{bus}$

Consider a fault between phases b and c through an impedance Z_f to ground at bus k as shown in Figure 10.23.

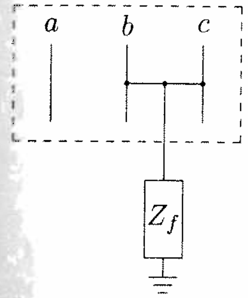


FIGURE 10.23
Double line-to-ground fault at bus k.

The phase a sequence network of Figure 10.15 is applicable here, where the positive-sequence impedance is placed in series with the parallel combination of the negative- and zero-sequence networks. The symmetrical components of the fault current as given from (10.86)–(10.88) are

$$I_k^1 = \frac{V_k(0)}{Z_{kk}^1 + \frac{Z_{kk}^2(Z_{kk}^0 + 3Z_f)}{Z_{kk}^2 + Z_{kk}^0 + 3Z_f}} \tag{10.94}$$

$$I_k^2 = -\frac{V_k(0) - Z_{kk}^1 I_k^1}{Z_{kk}^2} \tag{10.95}$$

$$I_k^0 = -\frac{V_k(0) - Z_{kk}^1 I_k^1}{Z_{kk}^0 + 3Z_f} \tag{10.96}$$

where Z_{kk}^1, and Z_{kk}^2, and Z_{kk}^0 are the diagonal elements in the k axis of the corresponding bus impedance matrix. The phase currents are obtained from (10.91), and the fault current is

$$I_k(F) = I_k^b + I_k^c \tag{10.97}$$

10.8.4 BUS VOLTAGES AND LINE CURRENTS DURING FAULT

Using the sequence components of the fault current given by the formulas in (10.54), the symmetrical components of the ith bus voltages during fault are obtained

$$V_i^0(F) = 0 - Z_{ik}^0 I_k^0$$

$$V_i^1(F) = V_i^1(0) - Z_{ik}^1 I_k^1 \quad (10.98)$$
$$V_i^2(F) = 0 - Z_{ik}^2 I_k^2$$

where $V_i^1(0) = V_i(0)$ is the prefault phase voltage at bus i. The phase voltages during fault are

$$V_i^{abc} = \mathbf{A} V_i^{012} \quad (10.99)$$

The symmetrical components of fault current in line i to j is given by

$$I_{ij}^0 = \frac{V_i^0(F) - V_j^0(F)}{z_{ij}^0}$$
$$I_{ij}^1 = \frac{V_i^1(F) - V_j^1(F)}{z_{ij}^1} \quad (10.100)$$
$$I_{ij}^2 = \frac{V_i^2(F) - V_j^2(F)}{z_{ij}^2}$$

where z_{ij}^0, z_{ij}^1, and z_{ij}^2, are the zero-, positive-, and negative-sequence components of the actual line impedance between buses i and j. Having obtained the symmetrical components of line current, the phase fault current in line i to j is

$$I_{ij}^{abc} = \mathbf{A} I_{ij}^{012} \quad (10.101)$$

Example 10.6 (chp10ex6)

Solve Example 10.5 using the bus impedance matrix. In addition, for each type of fault determine the bus voltages and line currents during fault.

Using the function **Zbus = zbuild(zdata)**, $\mathbf{Z}_{bus}^1$ and $\mathbf{Z}_{bus}^0$ are found for the positive-sequence network of Figure 10.17 and the zero-sequence network of Figure 10.19. The positive-sequence bus impedance matrix is

$$\mathbf{Z}_{bus}^1 = \begin{bmatrix} j0.1450 & j0.1050 & j0.1300 \\ j0.1050 & j0.1450 & j0.1200 \\ j0.1300 & j0.1200 & j0.2200 \end{bmatrix}$$

and the zero-sequence bus impedance matrix is

$$\mathbf{Z}_{bus}^0 = \begin{bmatrix} j0.1820 & j0.0545 & j0.1400 \\ j0.0545 & j0.0864 & j0.0650 \\ j0.1400 & j0.0650 & j0.3500 \end{bmatrix}$$

Since positive- and negative-sequence reactances for the system in Example 10.5 are identical, $\mathbf{Z}_{bus}^1 = \mathbf{Z}_{bus}^2$.

(a) Balanced three-phase fault at bus 3 through a fault impedance $Z_f = j0.1$.

The symmetrical components of fault current is given by

$$I_3^{012}(F) = \begin{bmatrix} 0 \\ \frac{1}{Z_{33}^1 + Z_f} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{j0.22+j0.1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -j3.125 \\ 0 \end{bmatrix}$$

The fault current is

$$I_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j3.125 \\ 0 \end{bmatrix} = \begin{bmatrix} 3.125\angle{-90^\circ} \\ 3.125\angle 150^\circ \\ 3.125\angle 30^\circ \end{bmatrix}$$

For balanced fault we only have the positive-sequence component of voltage. Thus, from (10.98), bus voltages during fault for phase a are

$$V_1(F) = 1 - Z_{13}^1 I_3(F) = 1 - j0.13(-j3.125) = 0.59375$$
$$V_2(F) = 1 - Z_{23}^1 I_3(F) = 1 - j0.12(-j3.125) = 0.62500$$
$$V_3(F) = 1 - Z_{33}^1 I_3(F) = 1 - j0.22(-j3.125) = 0.31250$$

Fault currents in lines for phase a are

$$I_{21}(F) = \frac{V_2(F) - V_1(F)}{z_{12}^1} = \frac{0.62500 - 0.59375}{j0.125} = 0.2500\angle{-90^\circ}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}^1} = \frac{0.59375 - 0.31250}{j0.15} = 0.1875\angle{-90^\circ}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}^1} = \frac{0.62500 - 0.31250}{j0.25} = 0.125\angle{-90^\circ}$$

(b) Single line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.1$.

From (10.90), the symmetrical components of fault current is given by

$$I_3^0(F) = I_3^1(F) = I_3^2(F) = \frac{1.0}{Z_{33}^1 + Z_{33}^2 + Z_{33}^0 + 3Z_f}$$
$$= \frac{1.0}{j0.22 + j0.22 + j0.35 + j3(0.1)} = -j0.9174$$

The fault current is

$$I_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j0.9174 \\ -j0.9174 \\ -j0.9174 \end{bmatrix} = \begin{bmatrix} 2.7523\angle{-90^\circ} \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{13}^0 I_3^0 \\ V_1^1(0) - Z_{13}^1 I_3^1 \\ 0 - Z_{13}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.140(-j0.9174) \\ 1 - j0.130(-j0.9174) \\ 0 - j0.130(-j0.9174) \end{bmatrix} = \begin{bmatrix} -0.1284 \\ 0.8807 \\ -0.1193 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{23}^0 I_3^0 \\ V_2^1(0) - Z_{23}^1 I_3^1 \\ 0 - Z_{23}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.065(-j0.9174) \\ 1 - j0.120(-j0.9174) \\ 0 - j0.120(-j0.9174) \end{bmatrix} = \begin{bmatrix} -0.0596 \\ 0.8899 \\ -0.1101 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{33}^0 I_3^0 \\ V_3^1(0) - Z_{33}^1 I_3^1 \\ 0 - Z_{33}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.350(-j0.9174) \\ 1 - j0.220(-j0.9174) \\ 0 - j0.220(-j0.9174) \end{bmatrix} = \begin{bmatrix} -0.3211 \\ 0.7982 \\ -0.2018 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.1284 \\ 0.8807 \\ -0.1193 \end{bmatrix} = \begin{bmatrix} 0.633\angle 0^\circ \\ 1.0046\angle -120.45^\circ \\ 1.0046\angle +120.45^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.0596 \\ 0.8899 \\ -0.1101 \end{bmatrix} = \begin{bmatrix} 0.7207\angle 0^\circ \\ 0.9757\angle -117.43^\circ \\ 0.9757\angle +117.43^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.3211 \\ 0.7982 \\ -0.2018 \end{bmatrix} = \begin{bmatrix} 0.2752\angle 0^\circ \\ 1.0647\angle -125.56^\circ \\ 1.0647\angle +125.56^\circ \end{bmatrix}$$

The symmetrical components of fault currents in lines for phase a are

$$I_{21}^{012} = \begin{bmatrix} \frac{V_2^0(F) - V_1^0(F)}{z_{12}^0} \\ \frac{V_2^1(F) - V_1^1(F)}{z_{12}^1} \\ \frac{V_2^2(F) - V_1^2(F)}{z_{12}^2} \end{bmatrix} = \begin{bmatrix} \frac{-0.0596-(-0.1284)}{j0.3} \\ \frac{0.8899-0.8807)}{j0.125} \\ \frac{-0.1101-(-0.1193)}{j0.125} \end{bmatrix} = \begin{bmatrix} 0.2294\angle -90^\circ \\ 0.0734\angle -90^\circ \\ 0.0734\angle -90^\circ \end{bmatrix}$$

$$I_{13}^{012} = \begin{bmatrix} \frac{V_1^0(F) - V_3^0(F)}{z_{13}^0} \\ \frac{V_1^1(F) - V_3^1(F)}{z_{13}^1} \\ \frac{V_1^2(F) - V_3^2(F)}{z_{13}^2} \end{bmatrix} = \begin{bmatrix} \frac{-0.1284-(-0.3211)}{j0.35} \\ \frac{0.8807-0.7982)}{j0.15} \\ \frac{-0.1193-(-0.2018)}{j0.15} \end{bmatrix} = \begin{bmatrix} 0.5505\angle -90^\circ \\ 0.5505\angle -90^\circ \\ 0.5505\angle -90^\circ \end{bmatrix}$$

$$I_{23}^{012} = \begin{bmatrix} \frac{V_2^0(F) - V_3^0(F)}{z_{23}^0} \\ \frac{V_2^1(F) - V_3^1(F)}{z_{23}^1} \\ \frac{V_2^2(F) - V_3^2(F)}{z_{23}^2} \end{bmatrix} = \begin{bmatrix} \frac{-0.0596-(-0.3211)}{j0.7125} \\ \frac{0.8899-0.7982)}{j0.25} \\ \frac{-0.1101-(-0.2018)}{j0.25} \end{bmatrix} = \begin{bmatrix} 0.3670\angle -90^\circ \\ 0.3670\angle -90^\circ \\ 0.3670\angle -90^\circ \end{bmatrix}$$

The line fault currents are

$$I_{21}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.2294\angle -90^\circ \\ 0.0734\angle -90^\circ \\ 0.0734\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 0.3761\angle -90^\circ \\ 0.1560\angle -90^\circ \\ 0.1560\angle -90^\circ \end{bmatrix}$$

$$I_{13}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.5505\angle -90^\circ \\ 0.5505\angle -90^\circ \\ 0.5505\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 1.6514\angle -90^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$I_{23}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.3670\angle -90^\circ \\ 0.3670\angle -90^\circ \\ 0.3670\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 1.1009\angle -90^\circ \\ 0 \\ 0 \end{bmatrix}$$

(c) Line-to-line fault at bus 3 through a fault impedance $Z_f = j0.1$.

From (10.92) and (10.93), the symmetrical components of fault current are

$$I_3^0 = 0$$

$$I_3^1 = -I_3^2 = \frac{V_3(0)}{Z_{33}^1 + Z_{33}^2 + Z_f} = \frac{1}{j0.22 + j0.22 + j0.1} = -j1.8519$$

The fault current is

$$I_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j1.8519 \\ j1.8519 \end{bmatrix} = \begin{bmatrix} 0 \\ -3.2075 \\ 3.2075 \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 \\ V_1^1(0) - Z_{13}^1 I_3^1 \\ 0 - Z_{13}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.130(-j1.8519) \\ 0 - j0.130(j1.8519) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7593 \\ 0.2407 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 \\ V_2^1(0) - Z_{23}^1 I_3^1 \\ 0 - Z_{23}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.120(-j1.8519) \\ 0 - j0.120(j1.8519) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7778 \\ 0.2222 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 \\ V_3^1(0) - Z_{33}^1 I_3^1 \\ 0 - Z_{33}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.220(-j1.8519) \\ 0 - j0.220(j1.8519) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5926 \\ 0.4074 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.7593 \\ 0.2407 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.672\angle -138.07^\circ \\ 0.672\angle +138.07^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.7778 \\ 0.2222 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6939\angle -136.10^\circ \\ 0.6939\angle +136.10^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5926 \\ 0.4074 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.5251\angle -162.21^\circ \\ 0.5251\angle +162.21^\circ \end{bmatrix}$$

The symmetrical components of fault currents in lines for phase a are

$$I_{21}^{012} = \begin{bmatrix} 0 \\ \frac{V_2^1(F) - V_1^1(F)}{z_{12}^1} \\ \frac{V_2^2(F) - V_1^2(F)}{z_{12}^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{0.7778 - 0.7593)}{j0.125} \\ \frac{0.2222 - 0.2407}{j0.125} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.148\angle -90^\circ \\ 0.148\angle +90^\circ \end{bmatrix}$$

$$I_{13}^{012} = \begin{bmatrix} 0 \\ \frac{V_1^1(F) - V_3^1(F)}{z_{13}^1} \\ \frac{V_1^2(F) - V_3^2(F)}{z_{13}^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{0.7593 - 0.5926}{j0.15} \\ \frac{0.2407 - 0.4074}{j0.15} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.1111\angle -90^\circ \\ 1.1111\angle +90^\circ \end{bmatrix}$$

$$I_{23}^{012} = \begin{bmatrix} 0 \\ \frac{V_2^1(F) - V_3^1(F)}{z_{23}^1} \\ \frac{V_2^2(F) - V_3^2(F)}{z_{23}^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{0.7778 - 0.5926}{j0.25} \\ \frac{0.2222 - 0.4074}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7407\angle -90^\circ \\ 0.7407\angle +90^\circ \end{bmatrix}$$

The line fault currents are

$$I_{21}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.148\angle -90^\circ \\ 0.148 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.2566 \\ 0.2566 \end{bmatrix}$$

$$I_{13}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.1111\angle -90^\circ \\ 1.1111\angle +90^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ -1.9245 \\ 1.9245 \end{bmatrix}$$

$$I_{23}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.7407\angle -90^\circ \\ 0.7407\angle +90^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ -1.283 \\ 1.283 \end{bmatrix}$$

(d) Double line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.1$.

From (10.94)–(10.96), the symmetrical components of fault current is given by

$$I_3^1 = \frac{V_3(0)}{Z_{33}^1 + \frac{Z_{33}^2(Z_{33}^0 + 3Z_f)}{Z_{33}^2 + Z_{33}^0 + 3Z_f}} = -\frac{1}{j0.22 + \frac{j0.22(j0.35 + j0.3)}{j0.22 + j0.35 + j0.3}} = -j2.6017$$

$$I_3^2 = -\frac{V_3(0) - Z_{33}^1 I_3^1}{Z_{33}^2} - \frac{1 - j0.22(-j2.6017)}{j0.22} = j1.9438$$

$$I_3^0 = -\frac{V_3(0) - Z_{33}^1 I_3^1}{Z_{33}^0 + 3Z_f} = -\frac{1 - j0.22(-j2.6017)}{j0.35 + j0.3} = j0.6579$$

The phase currents at the faulted bus are

$$I_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j0.6579 \\ -j2.6017 \\ j1.9438 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.0583\angle 165.93^\circ \\ 4.0583\angle 14.07^\circ \end{bmatrix}$$

and the total fault current is

$$I_3^b + I_3^c = 4.0583\angle 165.93^\circ + 4.0583\angle 14.07^\circ = 1.9732\angle 90^\circ$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{13}^0 I_3^0 \\ V_1^1(0) - Z_{13}^1 I_3^1 \\ 0 - Z_{13}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.140(j0.6579) \\ 1 - j0.130(-j2.6017) \\ 0 - j0.130(j1.9438) \end{bmatrix} = \begin{bmatrix} 0.0921 \\ 0.6618 \\ 0.2527 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{23}^0 I_3^0 \\ V_2^1(0) - Z_{23}^1 I_3^1 \\ 0 - Z_{23}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.065(j0.6579) \\ 1 - j0.120(-j2.6017 \\ 0 - j0.120(j1.9438) \end{bmatrix} = \begin{bmatrix} 0.0428 \\ 0.6878 \\ 0.2333 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{33}^0 I_3^0 \\ V_3^1(0) - Z_{33}^1 I_3^1 \\ 0 - Z_{33}^2 I_3^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.350(j0.6579) \\ 1 - j0.220(-j2.6017) \\ 0 - j0.220(j1.9438) \end{bmatrix} = \begin{bmatrix} 0.2303 \\ 0.4276 \\ 0.4276 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.0921 \\ 0.6618 \\ 0.2527 \end{bmatrix} = \begin{bmatrix} 1.0066\angle 0^\circ \\ 0.5088\angle -135.86^\circ \\ 0.5088\angle +135.86^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.0428 \\ 0.6878 \\ 0.2333 \end{bmatrix} = \begin{bmatrix} 0.9638\angle 0^\circ \\ 0.5740\angle -136.70^\circ \\ 0.5740\angle +136.70^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.2303 \\ 0.4276 \\ 0.4276 \end{bmatrix} = \begin{bmatrix} 1.0855\angle 0^\circ \\ 0.1974\angle 180^\circ \\ 0.1974\angle +180^\circ \end{bmatrix}$$

The symmetrical components of fault currents in lines for phase a are

$$I_{12}^{012} = \begin{bmatrix} \frac{V_1^0(F)-V_2^0(F)}{z_{12}^0} \\ \frac{V_1^1(F)-V_2^1(F)}{z_{12}^1} \\ \frac{V_1^2(F)-V_2^2(F)}{z_{12}^2} \end{bmatrix} = \begin{bmatrix} \frac{0.0921-0.0428}{j0.3} \\ \frac{0.6618-0.687)}{j0.125} \\ \frac{0.2527-0.2333}{j0.125} \end{bmatrix} = \begin{bmatrix} 0.1645\angle -90^\circ \\ 0.2081\angle +90^\circ \\ 0.1555\angle -90^\circ \end{bmatrix}$$

$$I_{13}^{012} = \begin{bmatrix} \frac{V_1^0(F)-V_3^0(F)}{z_{13}^0} \\ \frac{V_1^1(F)-V_3^1(F)}{z_{13}^1} \\ \frac{V_1^2(F)-V_3^2(F)}{z_{13}^2} \end{bmatrix} = \begin{bmatrix} \frac{0.0921-0.2303}{j0.35} \\ \frac{0.6618-0.4276}{j0.15} \\ \frac{0.2527-0.4276}{j0.15} \end{bmatrix} = \begin{bmatrix} 0.3947\angle +90^\circ \\ 1.5610\angle -90^\circ \\ 1.1663\angle +90^\circ \end{bmatrix}$$

$$I_{23}^{012} = \begin{bmatrix} \frac{V_2^0(F)-V_3^0(F)}{z_{23}^0} \\ \frac{V_2^1(F)-V_3^1(F)}{z_{23}^1} \\ \frac{V_2^2(F)-V_3^2(F)}{z_{23}^2} \end{bmatrix} = \begin{bmatrix} \frac{0.0428-0.2303}{j0.7125} \\ \frac{0.6878-0.4276}{j0.25} \\ \frac{0.2333-0.4276}{j0.25} \end{bmatrix} = \begin{bmatrix} 0.2632\angle +90^\circ \\ 1.0407\angle -90^\circ \\ 0.7775\angle +90^\circ \end{bmatrix}$$

The line fault currents are

$$I_{12}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.1645\angle -90^\circ \\ 0.2081\angle +90^\circ \\ 0.1555\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 0.1118\angle -90^\circ \\ 0.3682\angle -31.21 \\ 0.3682\angle -148.79^\circ \end{bmatrix}$$

$$I_{13}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.3947\angle +90^\circ \\ 1.5610\angle -90^\circ \\ 1.1663\angle +90^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 2.435\angle 165.93^\circ \\ 2.435\angle 14.07^\circ \end{bmatrix}$$

$$I_{23}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.2632\angle +90^\circ \\ 1.0407\angle -90^\circ \\ 0.7775\angle +90^\circ \end{bmatrix} = \begin{bmatrix} 0 \\ 1.6233\angle 165.93^\circ \\ 1.6233\angle 14.07^\circ \end{bmatrix}$$

10.9 UNBALANCED FAULT PROGRAMS

Three functions are developed for the unbalanced fault analysis. These functions are are **lgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2, V)**, **llfault(zdata1, Zbus1,zdata2, Zbus2, V)**, and **dlgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2, V)**. **lgfault** is designed for the single line-to-ground fault analysis, **llfault** for the line-to-line fault analysis, and **dlgfault** for the double line-to-ground fault analysis of a power system network. **lgfault** and **dlgfault** require the positive-, negative-, and zero-sequence bus impedance matrices **Zbus0**, **Zbus1**, and **Zbus2**, and **llfault** requires the positive- and negative-sequence bus impedance matrices **Zbus1**, and **Zbus2**. The last argument **V** is optional. If it is not included, the program sets all the prefault bus voltages to 1.0 per unit. If the variable **V** is included, the prefault bus voltages must be specified by the array **V** containing bus numbers and the complex bus voltage. The voltage vector **V** is automatically generated following the execution of any of the power flow programs.

The bus impedance matrices may be obtained from **Zbus0 = zbuild(zdata0)**, and **Zbus1 = zbuild(zdata1)**. The argument **zdata1** contains the positive-sequence network impedances. **zdata0** contains the zero-sequence network impedances. Arguments **zdata0**, **zdata1** and **zdata2** are an $e \times 4$ matrices containing the impedance data of an e-element network. Columns 1 and 2 are the element bus numbers and columns 3 and 4 contain the element resistance and reactance, respectively, in per unit. Bus number 0 to generator buses contain generator impedances. These may be the subtransient, transient, or synchronous reactances. Also, any other shunt impedances such as capacitors and load impedances to ground (bus 0) may be included in this matrix.

The negative-sequence network has the same topology as the positive-sequence network. The line and transformer negative-sequence impedances are the same as the positive-sequence impedances, however, the generator negative-sequence reactances are different from the positive-sequence values. In the fault analysis of large power system usually the negative-sequence network impedances are assumed to be identical to the positive-sequence impedances. The zero-sequence network topology is different from the positive-sequence network. The zero-sequence network must be constructed according to the transformer winding connections of Figure 10.6. All transformer connections except Y-Y with both neutral grounded result in isolation between the primary and secondary in the zero-sequence network. For these connections the corresponding resistance and reactance columns in the zero-sequence data must be filled with **inf**. For grounded Y-Δ connections, additional entries must be included to represent the transformer impedance from bus 0 to the grounded Y-side. In case the neutral is grounded through an impedance Z_n, an impedance of $3Z_n$ must be added to the transformer reactance. The reader is reminded of the 30° phase shift in a Y-Δ or Δ-Y transformer. According to the ASA convention, the positive-sequence voltage is advanced by 30° when stepping up from the low-voltage side to the high-voltage side. Similarly, the negative-sequence voltage is retarded by 30° when stepping up from low-voltage to the high-voltage side. The phase shifts due to Δ-Y transformers have no effect on the bus voltages and line currents in that part of the system where the fault occurs. However, on the other side of the Δ-Y transformers, the sequence voltages, and currents must be shifted in phase before transforming to the phase quantities. The unbalanced fault programs presently ignores the 30° phase shift in the Δ-Y transformers.

The other function for the formation of the bus impedance matrix is **Zbus = zbuildpi(linedata, gendata, yload)**, which is compatible with the power flow programs. The first argument **linedata** is consistent with the data required for the power flow solution. Columns 1 and 2 are the line bus numbers. Columns 3 through 5 contain the line resistance, reactance, and one-half of the total line charging susceptance in per unit on the specified MVA base. The last column is for the transformer tap setting; for lines, 1 must be entered in this column. The generator reactances are not included in the **linedata** for the power flow program and must be specified separately as required by the **gendata** in the second argument. **gendata** is an $e_g \times 4$ matrix, where each row contains bus 0, generator bus number, resistance and reactance. The last argument **yload** is optional. This is a two-column matrix containing bus number and the complex load admittance. This data is provided by any of the power flow programs **lfgauss**, **lfnewton** or **decouple**. **yload** is automatically generated following the execution of the above power flow programs.

The program prompts the user to enter the faulted bus number and the fault impedance **Zf**. The program obtains the total fault current, bus voltages and line currents during the fault. The use of the above functions are demonstrated in the following examples.

Example 10.7 (chp10ex7)

Use the **lgfault**, **llfault**, and **dlgfault** functions to compute the fault current, bus voltages and line currents in the circuit given in Example 10.5 for the following fault.

(a) A balanced three-phase fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.
(b) A single-line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.
(c) A line-to-line fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.
(d) A double line-to-ground fault at bus 3 through a fault impedance $Z_f = j0.1$ per unit.

In this example all shunt capacitances and loads are neglected and all the prefault bus voltages are assumed to be unity. The positive-sequence impedance diagram in Figure 10.17 is described by the variable **zdata1** and the zero-sequence impedance diagram in Figure 10.19 is described by the variable **zdata0**. The negative-sequence data is assumed to be the same as the positive-sequence data. We use the following commands.

```
zdata1 = [0   1   0   0.25
          0   2   0   0.25
          1   2   0   0.125
          1   3   0   0.15
          2   3   0   0.25];

zdata0 = [0   1   0   0.40
          0   2   0   0.10
          1   2   0   0.30
          1   3   0   0.35
          2   3   0   0.7125];

zdata2 = zdata1;
Zbus1 = zbuild(zdata1)
Zbus0 = zbuild(zdata0)
Zbus2 = Zbus1;
symfault(zdata1, Zbus1)
lgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
llfault(zdata1, Zbus1, zdata2, Zbus2)
dlgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
```

The result is

```
Three-phase balanced fault analysis
Enter Faulted Bus No. -> 3
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*0.1
Balanced three-phase fault at bus No. 3
Total fault current =   3.1250 per unit

Bus Voltages during fault in per unit
     Bus      Voltage        Angle
     No.      Magnitude      Degree
      1        0.5938        0.0000
      2        0.6250        0.0000
      3        0.3125        0.0000

Line currents for fault at bus No. 3
```

```
    From      To     Current      Angle
    Bus       Bus    Magnitude    Degree
     G         1      1.6250     -90.0000
     1         3      1.8750     -90.0000
     G         2      1.5000     -90.0000
     2         1      0.2500     -90.0000
     2         3      1.2500     -90.0000
     3         F      3.1250     -90.0000

Another fault location?
Enter 'y' or 'n' within single quote -> 'n'

Line-to-ground fault analysis
Enter Faulted Bus No. -> 3
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*0.1
Single line to-ground fault at bus No. 3
Total fault current =    2.7523 per unit

Bus Voltages during the fault in per unit
     Bus    -------Voltage Magnitude-------
     No.    Phase a      Phase b      Phase c
     1      0.6330       1.0046       1.0046
     2      0.7202       0.9757       0.9757
     3      0.2752       1.0647       1.0647

Line currents for fault at bus No. 3
     From      To       -----Line Current Magnitude----
     Bus       Bus      Phase a      Phase b      Phase c
      1         3       1.6514       0.0000       0.0000
      2         1       0.3761       0.1560       0.1560
      2         3       1.1009       0.0000       0.0000
      3         F       2.7523       0.0000       0.0000

Another fault location?
Enter 'y' or 'n' within single quote -> 'n'

Line-to-line fault analysis
Enter Faulted Bus No. -> 3
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*0.1
Line-to-line fault at bus No. 3
Total fault current =    3.2075 per unit

Bus Voltages during the fault in per unit
```



```
     Bus    -------Voltage Magnitude-------
     No.    Phase a      Phase b      Phase c
     1      1.0000       0.6720       0.6720
     2      1.0000       0.6939       0.6939
     3      1.0000       0.5251       0.5251

Line currents for fault at bus No. 3
     From      To       -----Line Current Magnitude----
     Bus       Bus      Phase a      Phase b      Phase c
      1         3       0.0000       1.9245       1.9245
      2         1       0.0000       0.2566       0.2566
      2         3       0.0000       1.2830       1.2830
      3         F       0.0000       3.2075       3.2075

Another fault location?
Enter 'y' or 'n' within single quote -> 'n'

Double line-to-ground fault analysis
Enter Faulted Bus No. -> 3
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*0.1
Double line-to-ground fault at bus No. 3
Total fault current =    1.9737 per unit

Bus Voltages during the fault in per unit
     Bus    -------Voltage Magnitude-------
     No.    Phase a      Phase b      Phase c
     1      1.0066       0.5088       0.5088
     2      0.9638       0.5740       0.5740
     3      1.0855       0.1974       0.1974

Line currents for fault at bus No. 3
     From      To       -----Line Current Magnitude----
     Bus       Bus      Phase a      Phase b      Phase c
      1         3       0.0000       2.4350       2.4350
      2         1       0.1118       0.3682       0.3682
      2         3       0.0000       1.6233       1.6233
      3         F       0.0000       4.0583       4.0583

Another fault location?
Enter 'y' or 'n' within single quote -> 'n'
```

Example 10.8 (chp10ex8)

The 11-bus power system network of an electric utility company is shown in Figure 10.24. The positive- and zero-sequence reactances of the lines and transformers in per unit on a 100-MVA base is tabulated below. The transformer connections are shown in Figure 10.24. The Δ-Y transformer between buses 11 and 7 is grounded through a reactor of reactance 0.08 per unit. The generators positive-, and zero-sequence reactances including the reactance of grounding neutrals on a 100-MVA base is also tabulated below. Resistances, shunt reactances, and loads are neglected, and all negative-sequence reactances are assumed equal to the positive-sequence reactances. Use **zbuild** function to obtain the positive- and zero-sequence bus impedance matrices. Assuming all the prefault bus voltages are equal to $1\angle 0°$, use **lgfault**, **llfault**, and **dlgfault** to compute the fault current, bus voltages, and line currents for the following unbalanced faults.

(a) A bolted single line-to-ground fault at bus 8.
(b) A bolted line-to-line fault at bus 8.
(c) A bolted double line-to-ground fault at bus 8.

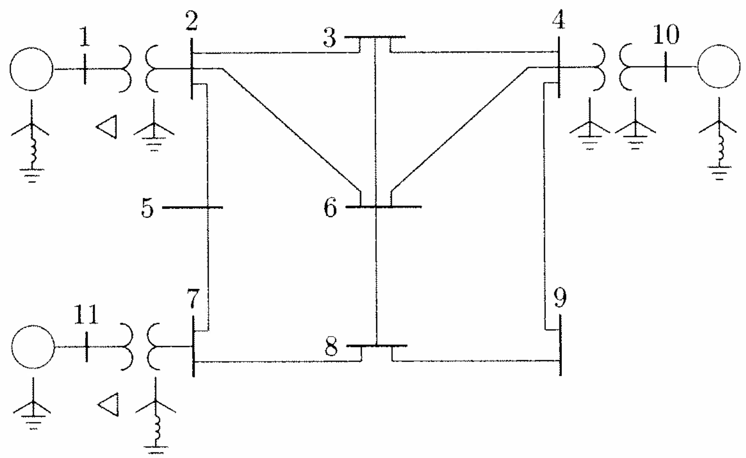


FIGURE 10.24
One-line diagram for Example 10.8.

GENERATOR TRANSIENT IMPEDANCE, PU			
Gen. No.	X^1	X^0	X_n
1	0.20	0.06	0.05
10	0.15	0.04	0.05
11	0.25	0.08	0.00

LINE AND TRANSFORMER DATA			
Bus No.	Bus No.	X^1, PU	X^0, PU
1	2	0.06	0.06
2	3	0.30	0.60
2	5	0.15	0.30
2	6	0.45	0.90
3	4	0.40	0.80
3	6	0.40	0.80
4	6	0.60	1.00
4	9	0.70	1.10
4	10	0.08	0.08
5	7	0.43	0.80
6	8	0.48	0.95
7	8	0.35	0.70
7	11	0.10	0.10
8	9	0.48	0.90

The equivalent circuit for the zero-sequence network is constructed according to the transformer winding connections of Figure 10.6 and is shown in Figure 10.25.

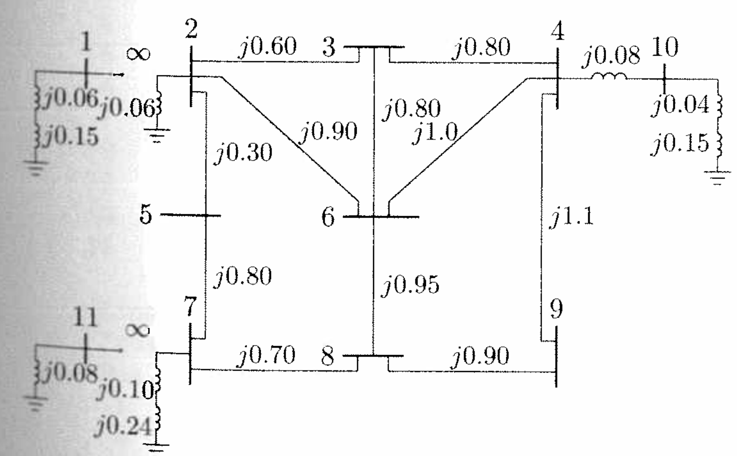


FIGURE 10.25
Zero-sequence network for Example 10.8.

When using **zbuild** function, the generator reactances must be included in the impedance data with bus zero as the reference bus.

The Δ-Y transformers result in isolation between the primary and secondary in the zero-sequence network. For these connections **inf** is entered in the corresponding resistance and reactance columns in the zero-sequence data. For grounded Y-Δconnections, additional entries are included to represent the transformer impedance from bus 0 to the grounded Y-side. The generators and transformers neutral reactor are included in the zero-sequence circuit each with a reactance of $3X_n$.

The positive- and zero-sequence impedance data and the required commands are as follows.

```
zdata1 = [ 0    1    0.00   0.20
           0   10    0.00   0.15
           0   11    0.00   0.25
           1    2    0.00   0.06
           2    3    0.00   0.30
           2    5    0.00   0.15
           2    6    0.00   0.45
           3    4    0.00   0.40
           3    6    0.00   0.40
           4    6    0.00   0.60
           4    9    0.00   0.70
           4   10    0.00   0.08
           5    7    0.00   0.43
           6    8    0.00   0.48
           7    8    0.00   0.35
           7   11    0.00   0.10
           8    9    0.00   0.48];

zdata0 = [ 0    1    0.00   0.06+3*0.05
           0   10    0.00   0.04+3*0.05
           0   11    0.00   0.08
           0    2    0.00   0.06
           0    7    0.00   0.10+3*.08
           1    2    inf    inf
           2    3    0.00   0.60
           2    5    0.00   0.30
           2    6    0.00   0.90
           3    4    0.00   0.80
           3    6    0.00   0.80
           4    6    0.00   1.00
           4    9    0.00   1.10
           4   10    0.00   0.08
           5    7    0.00   0.80
           6    8    0.00   0.95
           7    8    0.00   0.70
           7   11    inf    inf
           8    9    0.00   0.90];
```

```
zdata2=zdata1;
Zbus0 = zbuild(zdata0)
Zbus1 = zbuild(zdata1)
Zbus2 = Zbus1;
lgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
llfault(zdata1, Zbus1,zdata2, Zbus2)
dlgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
```

The result is

```
Line-to-ground fault analysis
Enter Faulted Bus No. -> 8
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0
Single line to-ground fault at bus No. 8
Total fault current =     2.8135 per unit

Bus Voltages during the fault in per unit
Bus    -------Voltage Magnitude-------
No.     Phase a     Phase b     Phase c
1       0.8907      0.9738      0.9738
2       0.8377      0.9756      0.9756
3       0.7451      0.9954      0.9954
4       0.7731      1.0063      1.0063
5       0.7824      0.9823      0.9823
6       0.5936      1.0123      1.0123
7       0.6295      0.9995      0.9995
8       0.0000      1.0898      1.0898
9       0.3299      1.0453      1.0453
10      0.8612      0.9995      0.9995
11      0.8231      0.9588      0.9588

Line currents for fault at bus No. 8
From     To      -----Line Current Magnitude----
Bus      Bus     Phase a     Phase b     Phase c
 1        2      0.5464      0.2732      0.2732
 2        3      0.2113      0.0407      0.0407
 2        6      0.3966      0.0207      0.0207
 3        6      0.2877      0.0073      0.0073
 4        3      0.0764      0.0479      0.0479
 4        6      0.2540      0.0255      0.0255
 4        9      0.5311      0.0023      0.0023
 5        2      0.2753      0.0023      0.0023
 6        8      0.9383      0.0121      0.0121
 7        5      0.2753      0.0023      0.0023
```

```
 7        8      1.3441     0.0098     0.0098
 8        F      2.8135     0.0000     0.0000
 9        8      0.5311     0.0023     0.0023
10        4      0.8615     0.0711     0.0711
11        7      0.7075     0.3538     0.3538

Another fault location?
Enter 'y' or 'n' within single quote -> 'n'

Line-to-line fault analysis
Enter Faulted Bus No. -> 8
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0
Line-to-line fault at bus No. 8
Total fault current =    2.9060 per unit

Bus Voltages during the fault in per unit
Bus     -------Voltage Magnitude-------
No.     Phase a     Phase b     Phase c
1       1.0000      0.8576      0.8576
2       1.0000      0.8168      0.8168
3       1.0000      0.7757      0.7757
4       1.0000      0.8157      0.8157
5       1.0000      0.7838      0.7838
6       1.0000      0.6871      0.6871
7       1.0000      0.6947      0.6947
8       1.0000      0.5000      0.5000
9       1.0000      0.5646      0.5646
10      1.0000      0.8778      0.8778
11      1.0000      0.7749      0.7749

Line currents for fault at bus No. 8
From      To      -----Line Current Magnitude----
Bus       Bus     Phase a     Phase b     Phase c
 1        2       0.0000      0.8465      0.8465
 2        3       0.0000      0.1762      0.1762
 2        5       0.0000      0.2820      0.2820
 2        6       0.0000      0.3883      0.3883
 3        6       0.0000      0.3047      0.3047
 4        3       0.0000      0.1285      0.1285
 4        6       0.0000      0.2887      0.2887
 4        9       0.0000      0.5461      0.5461
 5        7       0.0000      0.2820      0.2820
 6        8       0.0000      0.9817      0.9817
 7        8       0.0000      1.3782      1.3782
```



```
 8        F       0.0000      2.9060      2.9060
 9        8       0.0000      0.5461      0.5461
10        4       0.0000      0.9633      0.9633
11        7       0.0000      1.0962      1.0962

Another fault location?
Enter 'y' or 'n' within single quote -> 'n'

Double line-to-ground fault analysis
Enter Faulted Bus No. -> 8
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0
Double line-to-ground fault at bus No. 8
Total fault current =    2.4222 per unit

Bus Voltages during the fault in per unit
Bus     -------Voltage Magnitude-------
No.     Phase a     Phase b     Phase c
1       0.9530      0.8441      0.8441
2       0.9562      0.7884      0.7884
3       0.9919      0.7122      0.7122
4       1.0107      0.7569      0.7569
5       0.9686      0.7365      0.7365
6       1.0208      0.5666      0.5666
7       0.9992      0.5907      0.5907
8       1.1391      0.0000      0.0000
9       1.0736      0.3151      0.3151
10      0.9991      0.8455      0.8455
11      0.9239      0.7509      0.7509

Line currents for fault at bus No. 8
From      To      -----Line Current Magnitude----
Bus       Bus     Phase a     Phase b     Phase c
 1        2       0.2352      0.8546      0.8546
 2        3       0.0350      0.2069      0.2069
 2        5       0.0020      0.3063      0.3063
 2        6       0.0178      0.4278      0.4278
 3        6       0.0063      0.3277      0.3277
 4        3       0.0413      0.1290      0.1290
 4        6       0.0220      0.3050      0.3050
 4        9       0.0020      0.5924      0.5924
 5        7       0.0020      0.3063      0.3063
 6        8       0.0104      1.0596      1.0596
 7        8       0.0084      1.4963      1.4963
 8        F       0.0000      3.1483      3.1483
```

```
  9        8      0.0020      0.5924      0.5924
 10        4      0.0612      1.0217      1.0217
 11        7      0.3046      1.1067      1.1067

 Another fault location?
Enter 'y' or 'n' within single quote -> 'n'
```

PROBLEMS

10.1. Obtain the symmetrical components for the set of unbalanced voltages $V_a = 300\angle-120°$, $V_b = 200\angle 90°$, and $V_c = 100\angle-30°$.

10.2. The symmetrical components of a set of unbalanced three-phase currents are $I_a^0 = 3\angle-30°$, $I_a^1 = 5\angle 90°$, and $I_a^2 = 4\angle 30°$. Obtain the original unbalanced phasors.

10.3. The operator a is defined as $a = 1\angle 120°$; show that
(a) $\frac{(1+a)}{(1+a^2)} = 1\angle 120°$
(b) $\frac{(1-a)^2}{(1+a)^2} = 3\angle-180°$
(c) $(a - a^2)(a^2 - a) = 3\angle 0°$
(d) $V_{an}^1 = \frac{1}{\sqrt{3}}V_{bc}^1\angle 90°$
(e) $V_{an}^2 = \frac{1}{\sqrt{3}}V_{bc}^2\angle-90°$

10.4. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 1000\angle 0°$, $V_{bc} = 866.0254\angle-150°$, and $V_{ca} = 500\angle 120°$. Determine the symmetrical components for line and phase voltages, then find the phase voltages V_{an}, V_{bn}, and V_{cn}.

10.5. In the three-phase system shown in Figure 10.26, phase a is on no load and phases b and c are short-circuited to ground.

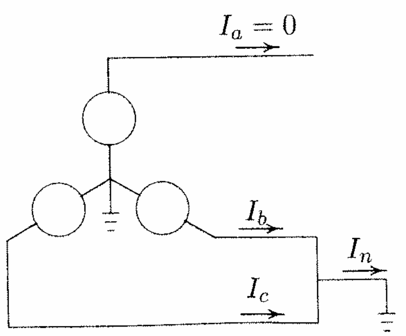


FIGURE 10.26
Circuit for Problem 10.5.

The following currents are given:
$I_b = 91.65\angle 160.9°$
$I_n = 60.00\angle 90°$
Find the symmetrical components of current I_a^0, I_a^1, and I_a^2.

10.6. A balanced three-phase voltage of 360-V line-to-neutral is applied to a balanced Y-connected load with ungrounded neutral, as shown in Figure 10.27. The three-phase load consists of three mutually-coupled reactances. Each phase has a series reactance of $Z_s = j24\ \Omega$, and the mutual coupling between phases is $Z_m = j6\ \Omega$.
(a) Determine the line currents by mesh analysis without using symmetrical components.
(b) Determine the line currents using symmetrical components.

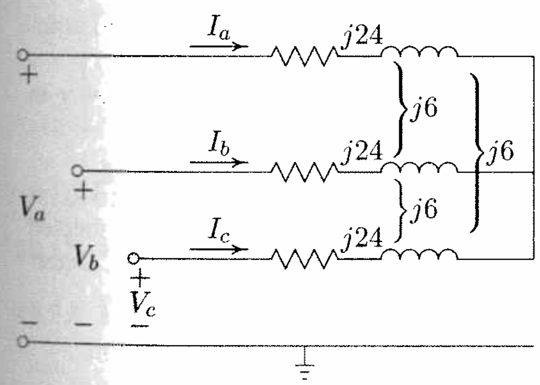


FIGURE 10.27
Circuit for Problem 10.6.

10.7. A three-phase unbalanced source with the following phase-to-neutral voltages

$$\mathbf{V}^{abc} = \begin{bmatrix} 300 & \angle-120° \\ 200 & \angle 90° \\ 100 & \angle-30° \end{bmatrix}$$

is applied to the circuit in Figure 10.28. The load series impedance per phase is $Z_s = 10 + j40$ and the mutual impedance between phases is $Z_m = j5$. The load and source neutrals are solidly grounded. Determine
(a) The load sequence impedance matrix, $\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}$.
(b) The symmetrical components of voltage.
(c) The symmetrical components of current.
(d) The load phase currents.

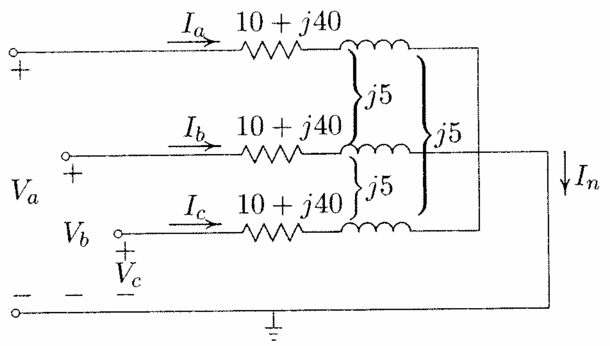


FIGURE 10.28
Circuit for Problem 10.7.

(e) The complex power delivered to the load in terms of symmetrical components, $S_{3\phi} = 3(V_a^0 I_a^{0*} + V_a^1 I_a^{1*} + V_a^2 I_a^{2*})$.
(f) The complex power delivered to the load by summing up the power in each phase, $S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$.

10.8. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 600\angle 36.87°$, $V_{bc} = 800\angle 126.87°$, and $V_{ca} = 1000\angle -90°$. A Y-connected load with a resistance of 37 Ω per phase is connected to the supply. Determine
(a) The symmetrical components of voltage.
(b) The phase voltages.
(c) The line currents.

10.9. A generator having a solidly grounded neutral and rated 50-MVA, 30-kV has positive-, negative-, and zero-sequence reactances of 25, 15, and 5 percent, respectively. What reactance must be placed in the generator neutral to limit the fault current for a bolted line-to-ground fault to that for a bolted three-phase fault?

10.10. What reactance must be placed in the neutral of the generator of Problem 9 to limit the magnitude of the fault current for a bolted double line-to-ground fault to that for a bolted three-phase fault?

10.11. Three 15-MVA, 30-kV synchronous generators A, B, and C are connected via three reactors to a common bus bar, as shown in Figure 10.29. The neutrals of generators A and B are solidly grounded, and the neutral of generator C is grounded through a reactor of 2.0 Ω. The generator data and the reactance of the reactors are tabulated below. A line-to-ground fault occurs on phase a of the common bus bar. Neglect prefault currents and assume generators are operating at their rated voltage. Determine the fault current in phase a.

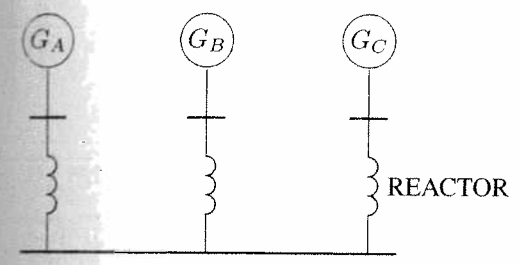


FIGURE 10.29
Circuit for Problem 10.11.

Item	X^1	X^2	X^0
G_A	0.25 pu	0.155 pu	0.056 pu
G_B	0.20 pu	0.155 pu	0.056 pu
G_C	0.20 pu	0.155 pu	0.060 pu
Reactor	6.0 Ω	6.0 Ω	6.0 Ω

10.12. Repeat Problem 10.11 for a bolted line-to-line fault between phases b and c.

10.13. Repeat Problem 10.11 for a bolted double line-to-ground fault on phases b and c.

10.14. The zero-, positive-, and negative-sequence bus impedance matrices for a three-bus power system are

$$\mathbf{Z}_{bus}^0 = j \begin{bmatrix} 0.20 & 0.05 & 0.12 \\ 0.05 & 0.10 & 0.08 \\ 0.12 & 0.08 & 0.30 \end{bmatrix} \quad \text{pu}$$

$$\mathbf{Z}_{bus}^1 = \mathbf{Z}_{bus}^2 = j \begin{bmatrix} 0.16 & 0.10 & 0.15 \\ 0.10 & 0.20 & 0.12 \\ 0.15 & 0.12 & 0.25 \end{bmatrix} \quad \text{pu}$$

Determine the per unit fault current and the bus voltages during fault for
(a) A bolted three-phase fault at bus 2.
(b) A bolted single line-to-ground fault at bus 2.
(c) A bolted line-to-line fault at bus 2.
(d) A bolted double line-to-ground fault at bus 2.

10.15. The reactance data for the power system shown in Figure 10.30 in per unit on a common base is as follows:

Item	X^1	X^2	X^0
G_1	0.10	0.10	0.05
G_2	0.10	0.10	0.05
T_1	0.25	0.25	0.25
T_2	0.25	0.25	0.25
Line 1–2	0.30	0.30	0.50

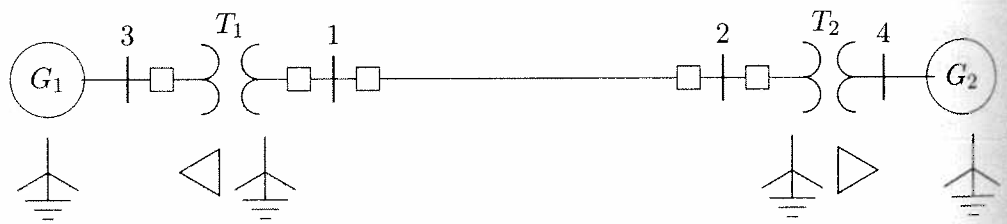


FIGURE 10.30
Circuit for Problem 10.15.

Obtain the Thévenin sequence impedances for the fault at bus 1 and compute the fault current in per unit for the following faults:

(a) A bolted three-phase fault at bus 1.
(b) A bolted single line-to-ground fault at bus 1.
(c) A bolted line-to-line fault at bus 1.
(d) A bolted double line-to-ground fault at bus 1.

10.16. For Problem 10.15, obtain the bus impedance matrices for the sequence networks. A bolted single line-to-ground fault occurs at bus 1. Find the fault current, the three-phase bus voltages during fault, and the line currents in each phase. Check your results using the **zbuild** and **lgfault** programs.

10.17. Repeat Problem 10.16 for a bolted line-to-line fault. Check your results using the **zbuild** and **llfault** programs.

10.18. Repeat Problem 10.16 for a bolted double line-to-ground fault. Check your results using the **zbuild** and **dlgfault** programs.

10.19. The positive-sequence reactances for the power system shown in Figure 10.31 are in per unit on a common MVA base. Resistances are neglected and the negative-sequence impedances are assumed to be the same as the positive-sequence impedances. A bolted line-to-line fault occurs between phases b and c at bus 2. Before the fault occurrence, all bus voltages are 1.0 per unit. Obtain the positive-sequence bus impedance matrix. Find the fault current, the three-phase bus voltages during fault, and the line currents in each phase. Check your results using the **zbuild** and **llfault** programs.

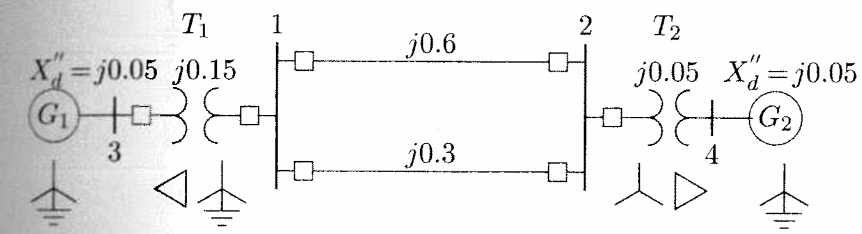


FIGURE 10.31
Circuit for Problem 10.19.

10.20. Use the **lgfault**, **llfault**, and **dlgfault** functions to compute the fault current, bus voltages, and line currents in the circuit given in Example 10.8 for the following unbalanced fault.

(a) A bolted single line-to-ground fault at bus 9.
(b) A bolted line-to-line fault at bus 9.
(c) A bolted double line-to-ground fault at bus 9.

All shunt capacitances and loads are neglected and the negative-sequence data is assumed to be the same as the positive-sequence data. All the prefault bus voltages are assumed to be unity.

10.21. The six-bus power system network of an electric utility company is shown in Figure 10.32. The positive- and zero-sequence reactances of the lines and transformers in per unit on a 100-MVA base is tabulated below.

LINE AND TRANSFORMER DATA

Bus No.	Bus No.	X^1, PU	X^0, PU
1	4	0.225	0.400
1	5	0.105	0.200
1	6	0.215	0.390
2	4	0.035	0.035
3	5	0.042	0.042
4	6	0.125	0.250
5	6	0.175	0.350

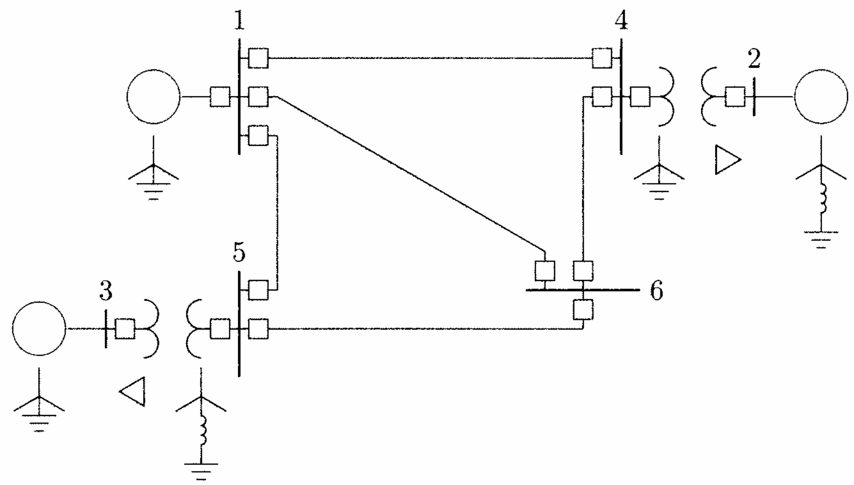


FIGURE 10.32
One-line diagram for Problem 10.21.

The transformer connections are shown in Figure 10.32. The Δ-Y transformer between buses 3 and 5 is grounded through a reactor of reactance 0.10 per unit. The generator's positive- and zero-sequence reactances including the reactance of grounding neutrals on a 100-MVA base is tabulated below.

GENERATOR TRANSIENT IMPEDANCE, PU			
Gen. No.	X^1	X^0	X_n
1	0.20	0.06	0.00
2	0.15	0.04	0.05
3	0.25	0.08	0.00

Resistances, shunt reactances, and loads are neglected, and all negative-sequence reactances are assumed equal to the positive-sequence reactances. Use **zbuild** function to obtain the positive- and zero-sequence bus impedance matrices. Assume all the prefault bus voltages are equal to $1\angle 0°$, use **lgfault**, **llfault**, and **dlgfault** to compute the fault current, bus voltages, and line currents for the following unbalanced faults.

(a) A bolted single line-to-ground fault at bus 6.
(b) A bolted line-to-line fault at bus 6.
(c) A bolted double line-to-ground fault at bus 6.

CHAPTER 11

STABILITY

11.1 INTRODUCTION

The tendency of a power system to develop restoring forces equal to or greater than the disturbing forces to maintain the state of equilibrium is known as *stability*. If the forces tending to hold machines in synchronism with one another are sufficient to overcome the disturbing forces, the system is said to remain stable (to stay in synchronism).

The stability problem is concerned with the behavior of the synchronous machines after a disturbance. For convenience of analysis, stability problems are generally divided into two major categories — *steady-state stability* and *transient stability*. Steady-state stability refers to the ability of the power system to regain synchronism after small and slow disturbances, such as gradual power changes. An extension of the steady-state stability is known as the *dynamic stability*. The dynamic stability is concerned with small disturbances lasting for a long time with the inclusion of automatic control devices. Transient stability studies deal with the effects of large, sudden disturbances such as the occurrence of a fault, the sudden outage of a line or the sudden application or removal of loads. Transient stability studies are needed to ensure that the system can withstand the transient condition following a major disturbance. Frequently, such studies are conducted when new generating and transmitting facilities are planned. The studies are helpful in determining such things as the nature of the relaying system needed, critical clearing time of circuit breakers, voltage level of, and transfer capability between systems.