In the analysis of power systems, it is frequently convenient to use the per-unit system. The advantage of this method is the elimination of transformers by simple impedances. The per-unit system is presented, followed by the impedance diagram of the network, expressed to a common MVA base.

3.2 SYNCHRONOUS GENERATORS

Large-scale power is generated by three-phase synchronous generators, known as *alternators*, driven either by steam turbines, hydroturbines, or gas turbines. The armature windings are placed on the stationary part called *stator*. The armature windings are designed for generation of balanced three-phase voltages and are arranged to develop the same number of magnetic poles as the field winding that is on the rotor. The field which requires a relatively small power (0.2–3 percent of the machine rating) for its excitation is placed on the rotor. The rotor is also equipped with one or more short-circuited windings known as *damper windings*. The rotor is driven by a prime mover at constant speed and its field circuit is excited by direct current. The excitation may be provided through slip rings and brushes by means of dc generators (referred to as *exciters*) mounted on the same shaft as the rotor of the synchronous machine. However, modern excitation systems usually use ac generators with rotating rectifiers, and are known as *brushless excitation*. The generator excitation system maintains generator voltage and controls the reactive power flow.

The rotor of the synchronous machine may be of cylindrical or salient construction. The cylindrical type of rotor, also called *round rotor*, has one distributed winding and a uniform air gap. These generators are driven by steam turbines and are designed for high speed 3600 or 1800 rpm (two- and four-pole machines, respectively) operation. The rotor of these generators has a relatively large axial length and small diameter to limit the centrifugal forces. Roughly 70 percent of large synchronous generators are cylindrical rotor type ranging from about 150 to 1500 MVA. The salient type of rotor has concentrated windings on the poles and nonuniform air gaps. It has a relatively large number of poles, short axial length, and large diameter. The generators in hydroelectric power stations are driven by hydraulic turbines, and they have salient-pole rotor construction.

3.2.1 GENERATOR MODEL

An elementary two-pole three-phase generator is illustrated in Figure 3.1. The stator contains three coils, aa', bb', and cc', displaced from each other by 120 electrical degrees. The concentrated full-pitch coils shown here may be considered to represent distributed windings producing sinusoidal mmf waves concentrated on the magnetic axes of the respective phases. When the rotor is excited to produce

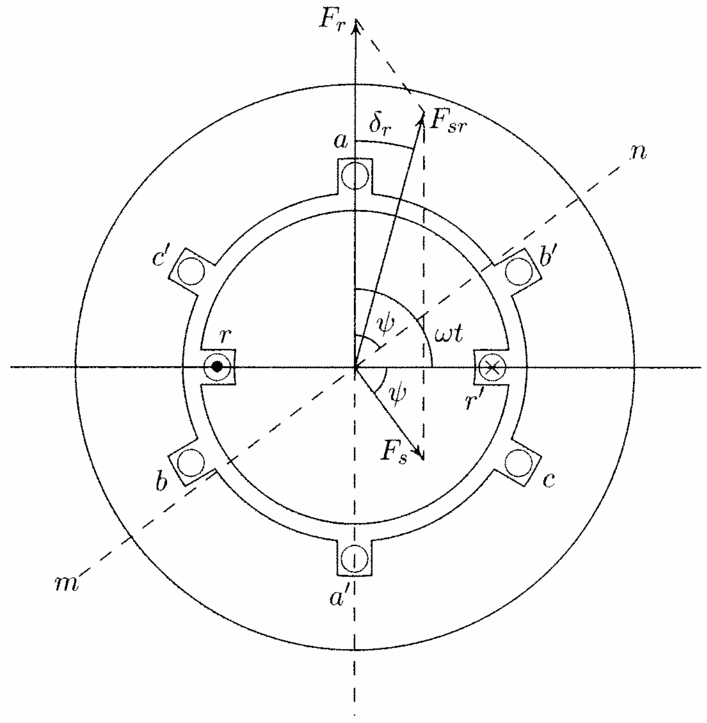


FIGURE 3.1
Elementary two-pole three-phase synchronous generator.

an air gap flux of ϕ per pole and is revolving at constant angular velocity ω, the flux linkage of the coil varies with the position of the rotor mmf axis ωt, where ωt is measured in electrical radians from coil aa' magnetic axis. The flux linkage for an N-turn concentrated coil aa' will be maximum $(N\phi)$ at $\omega t = 0$ and zero at $\omega t = \pi/2$. Assuming distributed winding, the flux linkage λ_a will vary as the cosine of the angle ωt. Thus, the flux linkage with coil a is

$$\lambda_a = N\phi \cos \omega t \tag{3.1}$$

The voltage induced in coil aa' is obtained from Faraday's law as

$$\begin{aligned} e_a &= -\frac{d\lambda}{dt} = \omega N\phi \sin \omega t \\ &= E_{max} \sin \omega t \\ &= E_{max} \cos(\omega t - \frac{\pi}{2}) \end{aligned} \tag{3.2}$$

where

$$E_{max} = \omega N\phi = 2\pi f N\phi$$

Therefore, the rms value of the generated voltage is

$$E = 4.44 f N \phi \tag{3.3}$$

where f is the frequency in hertz. In actual ac machine windings, the armature coil of each phase is distributed in a number of slots. Since the emfs induced in different slots are not in phase, their phasor sum is less than their numerical sum. Thus, a reduction factor K_w, called the *winding factor*, must be applied. For most three-phase windings K_w is about 0.85 to 0.95. Therefore, for a distributed phase winding, the rms value of the generated voltage is

$$E = 4.44 K_w f N \phi \tag{3.4}$$

The magnetic field of the rotor revolving at constant speed induces three-phase sinusoidal voltages in the armature, displaced by $2\pi/3$ radians. The frequency of the induced armature voltages depends on the speed at which the rotor runs and on the number of poles for which the machine is wound. The frequency of the armature voltage is given by

$$f = \frac{P}{2}\frac{n}{60} \tag{3.5}$$

where n is the rotor speed in rpm, referred to as *synchronous speed*. During normal conditions, the generator operates synchronously with the power grid. This results in three-phase balanced currents in the armature. Assuming current in phase a is lagging the generated emf e_a by an angle ψ, which is indicated by line mn in Figure 3.1, the instantaneous armature currents are

$$\begin{aligned} i_a &= I_{max}\sin(\omega t - \psi) \\ i_b &= I_{max}\sin(\omega t - \psi - \frac{2\pi}{3}) \\ i_c &= I_{max}\sin(\omega t - \psi - \frac{4\pi}{3}) \end{aligned} \tag{3.6}$$

According to (3.2) the generated emf e_a is maximum when the rotor magnetic axis is under phase a. Since i_a is lagging e_a by an angle ψ, when line mn reaches the axis of coil aa', current in phase a reaches its maximum value. At any instant of time, each phase winding produces a sinusoidally distributed mmf wave with its peak along the axis of the phase winding. These sinusoidally distributed fields can be represented by vectors referred to as *space phasors*. The amplitude of the sinusoidally distributed mmf $f_a(\theta)$ is represented by the vector F_a along the axis of phase a. Similarly, the amplitude of the mmfs $f_b(\theta)$ and $f_c(\theta)$ are shown by vectors F_b and F_c along their respective axis. The mmf amplitudes are proportional to the instantaneous value of the phase current, i.e.,

$$\begin{aligned} F_a &= Ki_a = KI_{max}\sin(\omega t - \psi) = F_m \sin(\omega t - \psi) \\ F_b &= Ki_b = KI_{max}\sin(\omega t - \psi - \frac{2\pi}{3}) = F_m \sin(\omega t - \psi - \frac{2\pi}{3}) \\ F_c &= Ki_c = KI_{max}\sin(\omega t - \psi - \frac{4\pi}{3}) = F_m \sin(\omega t - \psi - \frac{4\pi}{3}) \end{aligned} \tag{3.7}$$

where K is proportional to the number of armature turns per phase and is a function of the winding type. The resultant armature mmf is the vector sum of the above mmfs. A suitable method for finding the resultant mmf is to project these mmfs on line mn and obtain the resultant in-phase and quadrature-phase components. The resultant in-phase components are

$$\begin{aligned} F_1 = F_m \sin(\omega t - \psi)\cos(\omega t - \psi) + F_m \sin(\omega t - \psi - \frac{2\pi}{3}) \\ \cos(\omega t - \psi - \frac{2\pi}{3}) + F_m \sin(\omega t - \psi - \frac{4\pi}{3})\cos(\omega t - \psi - \frac{4\pi}{3}) \end{aligned}$$

Using the trigonometric identity $\sin\alpha\cos\alpha = (1/2)\sin 2\alpha$, the above expression becomes

$$\begin{aligned} F_1 = \frac{F_m}{2}[\sin 2(\omega t - \psi) + \sin 2(\omega t - \psi - \frac{2\pi}{3}) \\ + \sin 2(\omega t - \psi - \frac{4\pi}{3})] \end{aligned}$$

The above expression is the sum of three sinusoidal functions displaced from each other by $2\pi/3$ radians, which adds up to zero, i.e., $F_1 = 0$.

The sum of quadrature components results in

$$\begin{aligned} F_2 = F_m \sin(\omega t - \psi)\sin(\omega t - \psi) + F_m \sin(\omega t - \psi - \frac{2\pi}{3})\sin(\omega t - \psi - \frac{2\pi}{3}) \\ + F_m \sin(\omega t - \psi - \frac{4\pi}{3})\sin(\omega t - \psi - \frac{4\pi}{3}) \end{aligned}$$

Using the trigonometric identity $\sin^2\alpha = (1/2)(1 - \cos 2\alpha)$, the above expression becomes

$$\begin{aligned} F_2 = \frac{F_m}{2}[3 - \cos 2(\omega t - \psi) + \cos 2(\omega t - \psi - \frac{2\pi}{3}) \\ + \cos 2(\omega t - \psi - \frac{4\pi}{3})] \end{aligned}$$

The sinusoidal terms of the above expression are displaced from each other by $2\pi/3$ radians and add up to zero, with $F_2 = 3/2F_m$. Thus, the amplitude of the resultant armature mmf or stator mmf becomes

$$F_s = \frac{3}{2}F_m \tag{3.8}$$

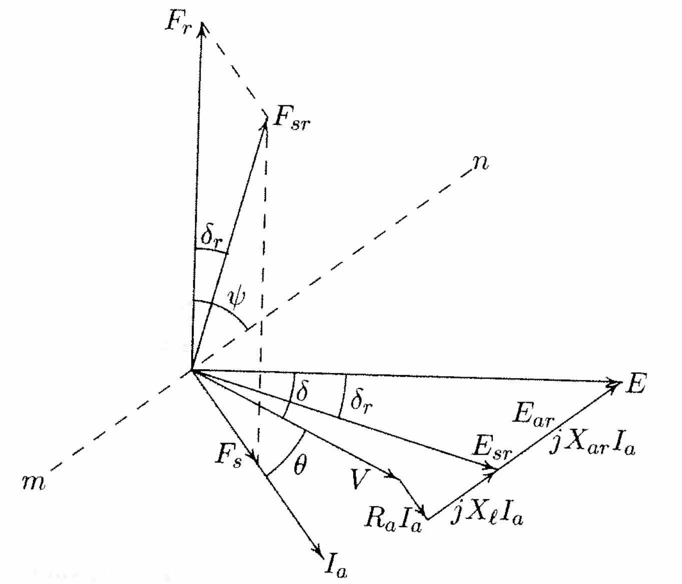


FIGURE 3.2
Combined phasor/vector diagram for one phase of a cylindrical rotor generator.

We thus conclude that the resultant armature mmf has a constant amplitude perpendicular to line mn, and rotates at a constant speed and in synchronism with the field mmf F_r. To see a demonstration of the rotating magnetic field, type **rotfield** at the *MATLAB* prompt.

A typical synchronous machine field alignment for operation as a generator is shown in Figure 3.2, using space vectors to represent the various fields. When the rotor is revolving at synchronous speed and the armature current is zero, the field mmf F_r produces the no-load generated emf E in each phase. The no-load generated voltage which is proportional to the field current is known as the *excitation voltage*. The phasor voltage for phase a, which is lagging F_r by 90°, is combined with the mmf vector diagram as shown in Figure 3.2. This combined phasor/vector diagram leads to a circuit model for the synchronous machine. It must be emphasized that in Figure 3.2 mmfs are space vectors, whereas the emfs are time phasors. When the armature is carrying balanced three-phase currents, F_s is produced perpendicular to line mn. The interaction of armature mmf and the field mmf, known as *armature reaction*, gives rise to the resultant air gap mmf F_{sr}. The resultant mmf F_{sr} is the vector sum of the field mmf F_r and the armature mmf F_s. The resultant mmf is responsible for the resultant air gap flux ϕ_{sr} that induces the generated emf on-load, shown by E_{sr}. The armature mmf F_s induces the emf E_{ar}, known as the *armature reaction voltage*, which is perpendicular to F_s. The voltage E_{ar} leads I_a by 90° and thus can be represented by a voltage drop across a reactance X_{ar} due to the current I_a. X_{ar} is known as the *reactance of the armature reaction*. The phasor sum of E and E_{ar} is shown by E_{sr} perpendicular to F_{sr}, which represents the on-load generated emf.

$$E = E_{sr} + jX_{ar}I_a \tag{3.9}$$

The terminal voltage V is less than E_{sr} by the amount of resistive voltage drop R_aI_a and leakage reactance voltage drop $X_\ell I_a$. Thus

$$E = V + [R_a + j(X_\ell + X_{ar})]I_a \tag{3.10}$$

or

$$E = V + [R_a + jX_s]I_a \tag{3.11}$$

where $X_s = (X_\ell + X_{ar})$ is known as the *synchronous reactance*. The cosine of the angle between I and V, i.e., $\cos\theta$ represents the power factor at the generator terminals. The angle between E and E_{sr} is equal to the angle between the rotor mmf F_r and the air gap mmf F_{sr}, shown by δ_r. The power developed by the machine is proportional to the product of F_r, F_{sr} and $\sin\delta_r$. The relative positions of these mmfs dictates the action of the synchronous machine. When F_r is ahead of F_{sr} by an angle δ_r, the machine is operating as a generator and when F_r falls behind F_{sr}, the machine will act as a motor. Since E and E_{sr} are proportional to F_r and F_{sr}, respectively, the power developed by the machine is proportional to the products of E, E_{sr}, and $\sin\delta_r$. The angle δ_r is thus known as the *power angle*. This is a very important result because it relates the time angle between the phasor emfs with the space angle between the magnetic fields in the machine. Usually the developed power is expressed in terms of the excitation voltage E, the terminal voltage V, and $\sin\delta$. The angle δ is approximately equal to δ_r because the leakage impedance is very small compared to the magnetization reactance.

Due to the nonlinearity of the machine magnetization curve, the synchronous reactance is not constant. The unsaturated synchronous reactance can be found from the open- and short-circuit data. For operation at or near rated terminal voltage, it is usually assumed that the machine is equivalent to an unsaturated one whose magnetization curve is a straight line through the origin and the rated voltage point on the open-circuit characteristic. For steady-state analysis, a constant value known as the *saturated value of the synchronous reactance* corresponding to the rated voltage is used. A simple per-phase model for a cylindrical rotor generator based on (3.11) is obtained as shown in Figure 3.3. The armature resistance is generally much smaller than the synchronous reactance and is often neglected. The equivalent circuit connected to an infinite bus becomes that shown in Figure 3.4, and (3.11) reduces to

$$E = V + jX_sI_a \tag{3.12}$$

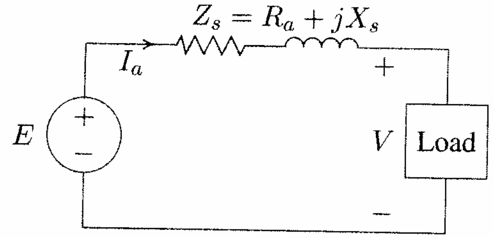


FIGURE 3.3
Synchronous machine equivalent circuit.

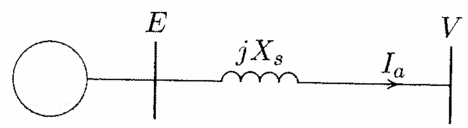
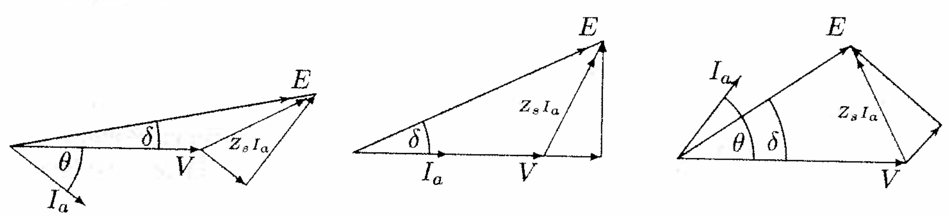


FIGURE 3.4
Synchronous machine connected to an infinite bus.

Figure 3.5 shows the phasor diagram of the generator with terminal voltage as reference for excitations corresponding to lagging, unity, and leading power factors. The voltage regulation of an alternator is a figure of merit used for comparison with other machines. It is defined as the percentage change in terminal voltage from no-load to rated load. This gives an indication of the change in field current required to maintain system voltage when going from no-load to rated load at some specific power factor.



(a) Lagging pf load (b) Upf load (c) Leading pf load

FIGURE 3.5
Synchronous generator phasor diagram.

$$\text{VR} = \frac{|V_{nl}| - |V_{rated}|}{|V_{rated}|} \times 100 = \frac{|E| - |V_{rated}|}{|V_{rated}|} \times 100 \qquad (3.13)$$

The no-load voltage for a specific power factor may be determined by operating the machine at rated load conditions and then removing the load and observing the no-load voltage. Since this is not a practical method for very large machines, an accurate analytical method recommended by IEEE as given in reference [43] may be used. An approximate method that provides reasonable results is to consider a hypothetical linearized magnetization curve drawn to intersect the actual magnetization curve at rated voltage. The value of E calculated from (3.12) is then used to find the field current from the linearized curve. Finally, the no-load voltage corresponding to this field current is found from the actual magnetization curve.

3.3 STEADY-STATE CHARACTERISTICS—CYLINDRICAL ROTOR

3.3.1 POWER FACTOR CONTROL

Most synchronous machines are connected to large interconnected electric power networks. These networks have the important characteristic that the system voltage at the point of connection is constant in magnitude, phase angle, and frequency. Such a point in a power system is referred to as an *infinite bus*. That is, the voltage at the generator bus will not be altered by changes in the generator's operating condition.

The ability to vary the rotor excitation is an important feature of the synchronous machine, and we now consider the effect of such a variation when the machine operates as a generator with constant mechanical input power. The per-phase equivalent circuit of a synchronous generator connected to an infinite bus is shown in Figure 3.4. Neglecting the armature resistance, the output power is equal to the power developed, which is assumed to remain constant given by

$$P_{3\phi} = \Re[3VI_a^*] = 3|V||I_a|\cos\theta \qquad (3.14)$$

where V is the phase-to-neutral terminal voltage assumed to remain constant. From (3.14) we see that for constant developed power at a fixed terminal voltage V, $I_a \cos\theta$ must be constant. Thus, the tip of the armature current phasor must fall on a vertical line as the power factor is varied by varying the field current as shown in Figure 3.6. From this diagram we have

$$cd = E_1 \sin\delta_1 = X_s I_{a1} \cos\theta_1 \qquad (3.15)$$

Thus $E_1 \sin\delta_1$ is a constant, and the locus of E_1 is on the line ef. In Figure 3.6, phasor diagrams are drawn for three armature currents. Application of (3.12) for a lagging power factor armature current I_{a1} results in E_1. If θ is zero, the generator operates at unity power factor and armature current has a minimum value, shown by I_{a2}, which results in E_2. Similarly, E_3 is obtained corresponding to I_{a3} at a leading power factor. Figure 3.6 shows that the generation of reactive power can

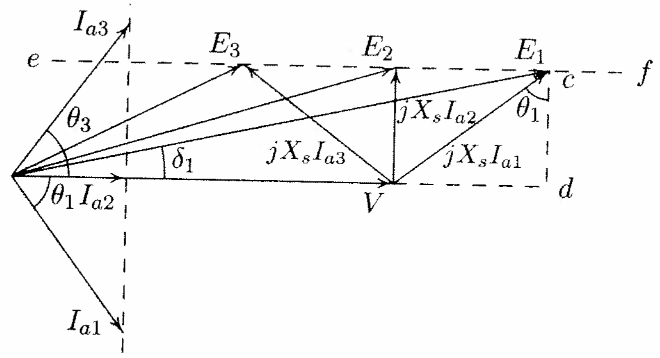


FIGURE 3.6
Variation of field current at constant power.

be controlled by means of the rotor excitation while maintaining a constant real power output. The variation in the magnitude of armature current as the excitation voltage is varied is best shown by a curve. Usually the field current is used as the abscissa instead of excitation voltage because the field current is readily measured. The curve of the armature current as the function of the field current resembles the letter V and is often referred to as the *V curve* of synchronous machines. These curves constitute one of the generator's most important characteristics. There is, of course, a limit beyond which the excitation cannot be reduced. This limit is reached when $\delta = 90°$. Any reduction in excitation below the stability limit for a particular load will cause the rotor to pull out of synchronism. The V curve is illustrated in Figure 3.7 (page 62) for the machine in Example 3.3.

3.3.2 POWER ANGLE CHARACTERISTICS

Consider the per-phase equivalent circuit shown in Figure 3.4. The three-phase complex power at the generator terminal is

$$S_{3\phi} = 3VI_a^* \tag{3.16}$$

Expressing the phasor voltages in polar form, the armature current is

$$I_a = \frac{|E|\angle\delta \;-|V|\angle 0}{|Z_s|\angle\gamma} \tag{3.17}$$

Substituting for I_a^* in (3.16) results in

$$S_{3\phi} = 3\frac{|E||V|}{|Z_s|}\angle\gamma-\delta \;-3\frac{|V|^2}{|Z_s|}\angle\gamma \tag{3.18}$$

Thus, the real power $P_{3\phi}$ and reactive power $Q_{3\phi}$ are

$$P_{3\phi} = 3\frac{|E||V|}{|Z_s|}\cos(\gamma-\delta) - 3\frac{|V|^2}{|Z_s|}\cos\gamma \tag{3.19}$$

$$Q_{3\phi} = 3\frac{|E||V|}{|Z_s|}\sin(\gamma-\delta) - 3\frac{|V|^2}{|Z_s|}\sin\gamma \tag{3.20}$$

If R_a is neglected, then $Z_s = jX_s$ and $\gamma = 90°$. Equations (3.19) and (3.20) reduce to

$$P_{3\phi} = 3\frac{|E||V|}{X_s}\sin\delta \tag{3.21}$$

$$Q_{3\phi} = 3\frac{|V|}{X_s}(|E|\cos\delta - |V|) \tag{3.22}$$

Equation (3.21) shows that if $|E|$ and $|V|$ are held fixed and the power angle δ is changed by varying the mechanical driving torque, the power transfer varies sinusoidally with the angle δ. From (3.21), the theoretical maximum power occurs when $\delta = 90°$

$$P_{max(3\phi)} = 3\frac{|E||V|}{X_s} \tag{3.23}$$

The behavior of the synchronous machine can be described as follows. If we start with $\delta = 0°$ and increase the driving torque, the machine accelerates, and the rotor mmf F_r advances with respect to the resultant mmf F_{sr}. This results in an increase in δ, causing the machine to deliver electric power. At some value of δ the machine reaches equilibrium where the electric power output balances the increased mechanical power owing to the increased driving torque. It is clear that if an attempt were made to advance δ further than 90° by increasing the driving torque, the electric power output would decrease from the P_{max} point. Therefore, the excess driving torque continues to accelerate the machine, and the mmfs will no longer be magnetically coupled. The machine loses synchronism and automatic equipment disconnects it from the system. The value P_{max} is called the *steady-state stability limit* or *static stability limit*. In general, stability considerations dictate that a synchronous machine achieve steady-state operation for a power angle at considerably less than 90°. The control of real power flow is maintained by the generator governor through the frequency-power control channel.

Equation (3.22) shows that for small δ, $\cos\delta$ is nearly unity and the reactive power can be approximated to

$$Q_{3\phi} \simeq 3\frac{|V|}{X_s}(|E| - |V|) \tag{3.24}$$

From (3.24) we see that when $|E| > |V|$ the generator delivers reactive power to the bus, and the generator is said to be overexcited. If $|E| < |V|$, the reactive power delivered to the bus is negative; that is, the bus is supplying positive reactive power to the generator. Generators are normally operated in the overexcited mode since the generators are the main source of reactive power for inductive load throughout the system. Therefore, we conclude that the flow of reactive power is governed mainly by the difference in the excitation voltage $|E|$ and the bus bar voltage $|V|$. The adjustment in the excitation voltage for the control of reactive power is achieved by the generator excitation system.

Example 3.1 (chp3ex1)

A 50-MVA, 30-kV, three-phase, 60-Hz synchronous generator has a synchronous reactance of 9 Ω per phase and a negligible resistance. The generator is delivering rated power at a 0.8 power factor lagging at the rated terminal voltage to an infinite bus.
(a) Determine the excitation voltage per phase E and the power angle δ.
(b) With the excitation held constant at the value found in (a), the driving torque is reduced until the generator is delivering 25 MW. Determine the armature current and the power factor.
(c) If the generator is operating at the excitation voltage of part (a), what is the steady-state maximum power the machine can deliver before losing synchronism? Also, find the armature current corresponding to this maximum power.

(a) The three-phase apparent power is

$$\begin{aligned} S_{3\phi} = 50\angle\cos^{-1} 0.8 &= 50\angle 36.87^\circ \quad \text{MVA} \\ &= 40 \;\; \text{MW} + j30 \;\; \text{Mvar} \end{aligned}$$

The rated voltage per phase is

$$V = \frac{30}{\sqrt{3}} = 17.32\angle 0^\circ \quad \text{kV}$$

The rated current is

$$I_a = \frac{S^*_{3\phi}}{3V^*} = \frac{(50\angle{-36.87})10^3}{3(17.32\angle 0^\circ)} = 962.25\angle{-36.87^\circ} \quad \text{A}$$

The excitation voltage per phase from (3.12) is

$$E = 17320.5 + (j9)(962.25\angle{-36.87}) = 23558\angle 17.1^\circ \quad \text{V}$$

The excitation voltage per phase (line to neutral) is 23.56 kV and the power angle is 17.1°.

(b) When the generator is delivering 25 MW from (3.21) the power angle is

$$\delta = \sin^{-1}\left[\frac{(25)(9)}{(3)(23.56)(17.32)}\right] = 10.591^\circ$$

The armature current is

$$I_a = \frac{(23,558\angle 10.591^\circ - 17,320\angle 0^\circ)}{j9} = 807.485\angle{-53.43^\circ} \quad \text{A}$$

The power factor is given by $\cos(53.43) = 0.596$ lagging.

(c) The maximum power occurs at $\delta = 90^\circ$

$$P_{max(3\phi)} = 3\frac{|E||V|}{X_s} = 3\frac{(23.56)(17.32)}{9} = 136 \quad \text{MW}$$

The armature current is

$$I_a = \frac{(23,558\angle 90^\circ - 17,320\angle 0^\circ)}{j9} = 3248.85\angle 36.32^\circ \quad \text{A}$$

The power factor is given by $\cos(36.32) = 0.8057$ leading.

Example 3.2 (chp3ex2)

The generator of Example 3.1 is delivering 40 MW at a terminal voltage of 30 kV. Compute the power angle, armature current, and power factor when the field current is adjusted for the following excitations.
(a) The excitation voltage is decreased to 79.2 percent of the value found in Example 3.1.
(b) The excitation voltage is decreased to 59.27 percent of the value found in Example 3.1.
(c) Find the minimum excitation below which the generator will lose synchronism.

(a) The new excitation voltage is

$$E = 0.792 \times 23,558 = 18,657 \quad \text{V}$$

From (3.21) the power angle is

$$\delta = \sin^{-1}\left[\frac{(40)(9)}{(3)(18.657)(17.32)}\right] = 21.8^\circ$$

The armature current is

$$I_a = \frac{(18657\angle 21.8^\circ - 17320\angle 0^\circ)}{j9} = 769.8\angle 0^\circ \quad \text{A}$$

The power factor is given by $\cos(0) = 1$.

(b) The new excitation voltage is

$$E = 0.5927 \times 23,558 = 13,963 \quad \text{V}$$

From (3.21) the power angle is

$$\delta = \sin^{-1}\left[\frac{(40)(9)}{(3)(13.963)(17.32)}\right] = 29.748^\circ$$

The armature current is

$$I_a = \frac{(13,963\angle 29.748^\circ - 17,320\angle 0^\circ)}{j9} = 962.3\angle 36.87^\circ \quad \text{A}$$

From current phase angle, the power factor is $\cos 36.87 = 0.8$ leading. The generator is underexcited and is actually receiving reactive power.

(c) From (3.23), the minimum excitation corresponding to $\delta = 90^\circ$ is

$$E = \frac{(40)(9)}{(3)(17.32)(1)} = 6.928 \quad \text{kV}$$

The armature current is

$$I_a = \frac{(6,928\angle 90^\circ - 17,320\angle 0^\circ)}{j9} = 2073\angle 68.2^\circ \quad \text{A}$$

The current phase angle shows that the power factor is $\cos 68.2 = 0.37$ leading. The generator is underexcited and is receiving reactive power.

Example 3.3 (chp3ex3)

For the generator of Example 3.1, construct the V curve for the rated power of 40 MW with varying field excitation from 0.4 power factor leading to 0.4 power factor lagging. Assume the open-circuit characteristic in the operating region is given by $E = 2000 I_f$ V.

The following *MATLAB* command results in the V curve shown in Figure 3.7.

```
P = 40;                                             % real power, MW
V = 30/sqrt(3)+ j*0;                                % phase voltage, kV
Zs = j*9;                                           % synchronous impedance
ang = acos(0.4);
theta=ang:-0.01:-ang;%Angle 0.4 leading to 0.4 lagging pf
P = P*ones(1,length(theta));%generates array of same size
Iam = P./(3*abs(V)*cos(theta));     % current magnitude kA
Ia = Iam.*(cos(theta) + j*sin(theta));   % current phasor
E = V + Zs.*Ia;                       % excitation voltage phasor
Em = abs(E);                % excitation voltage magnitude, kV
If = Em*1000/2000;                            % field current, A
plot(If, Iam), grid, xlabel('If - A')
ylabel('Ia - kA'), text(3.4, 1, 'Leading pf')
text(13, 1, 'Lagging pf'), text(9, .71, 'Upf')
```

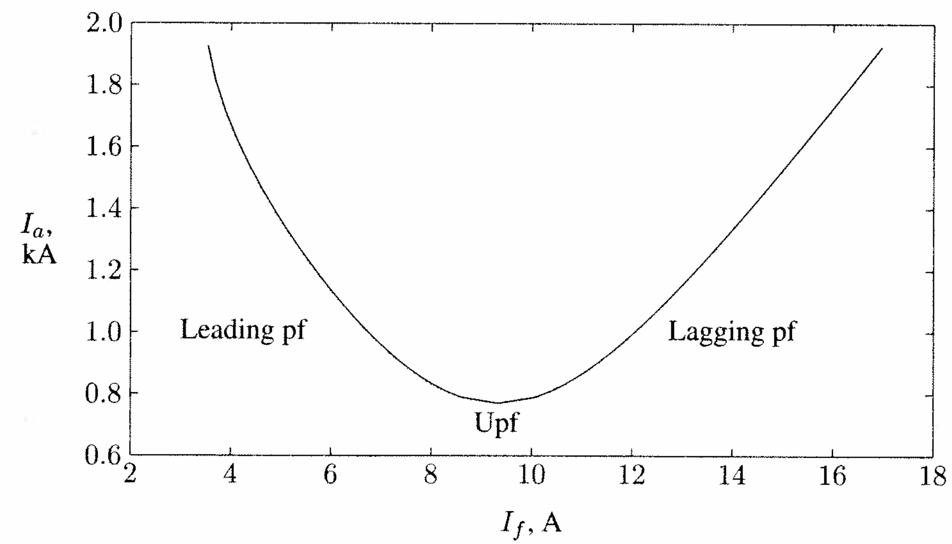


FIGURE 3.7
V curve for generator of Example 3.3.

3.4 SALIENT-POLE SYNCHRONOUS GENERATORS

The model developed in Section 3.2 is only valid for cylindrical rotor generators with uniform air gaps. The salient-pole rotor results in nonuniformity of the magnetic reluctance of the air gap. The reluctance along the polar axis, commonly referred to as the rotor *direct axis*, is appreciably less than that along the interpolar axis, commonly referred to as the *quadrature axis*. Therefore, the reactance has a high value X_d along the direct axis, and a low value X_q along the quadrature axis. These reactances produce voltage drop in the armature and can be taken into account by resolving the armature current I_a into two components I_q, in phase, and I_d in time quadrature, with the excitation voltage. The phasor diagram with the armature resistance neglected is shown in Figure 3.8. It is no longer possible to represent the machine by a simple equivalent circuit. The excitation voltage magnitude is

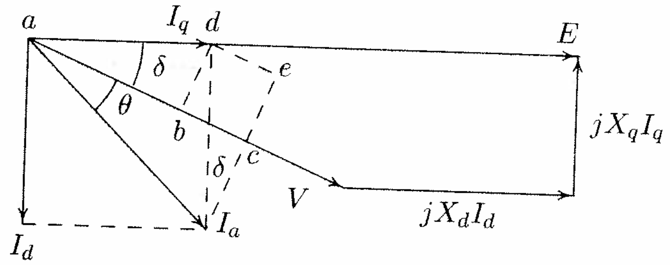


FIGURE 3.8
Phasor diagram for a salient-pole generator.

$$|E| = |V|\cos\delta + X_d I_d \tag{3.25}$$

The three-phase real power at the generator terminal is

$$P = 3|V||I_a|\cos\theta \tag{3.26}$$

The power component of the armature current can be expressed in terms of I_d and I_q as follows.

$$\begin{aligned} |I_a|\cos\theta &= ab + de \\ &= I_q\cos\delta + I_d\sin\delta \end{aligned} \tag{3.27}$$

Substituting from (3.27) into (3.26), we have

$$P = 3|V|(I_q\cos\delta + I_d\sin\delta) \tag{3.28}$$

Now from the phasor diagram given in Figure 3.8,

$$|V|\sin\delta = X_q I_q \tag{3.29}$$

or

$$I_q = \frac{|V|\sin\delta}{X_q} \tag{3.30}$$

Also, from (3.25), I_d is given by

$$I_d = \frac{|E| - |V|\cos\delta}{X_d} \tag{3.31}$$

Substituting for I_d and I_q from (3.31) and (3.30) into (3.28), the real power with armature resistance neglected becomes

$$P_{3\phi} = 3\frac{|E||V|}{X_d}\sin\delta + 3|V|^2\frac{X_d - X_q}{2X_dX_q}\sin 2\delta \tag{3.32}$$

The power equation contains an additional term known as the *reluctance power*. Equations (3.25) and (3.32) can be utilized for steady-state analysis. For short-circuit analysis, assuming a high X/R ratio, the power factor approaches zero and the quadrature component of current can often be neglected. In such a case, X_d merely replaces the X_s used for the cylindrical rotor machine. Generators are thus modeled by their direct axis reactance in series with a constant-voltage power source. Later in the text it will be shown that X_d takes on different values, depending upon the transient time following the short circuit. These reactances are usually expressed in per-unit and are available from the manufacturer's data.

3.5 POWER TRANSFORMER

Transformers are essential elements in any power system. They allow the relatively low voltages from generators to be raised to a very high level for efficient power transmission. At the user end of the system, transformers reduce the voltage to values most suitable for utilization. In modern utility systems, the energy may undergo four or five transformations between generator and ultimate user. As a result, a given system is likely to have about five times more kVA of installed capacity of transformers than of generators.

3.6 EQUIVALENT CIRCUIT OF A TRANSFORMER

The equivalent circuit model of a single-phase transformer is shown in Figure 3.9. The equivalent circuit consists of an ideal transformer of ratio $N_1{:}N_2$ together with elements which represent the imperfections of the real transformer. An ideal transformer would have windings with zero resistance and a lossless, infinite permeability core. The voltage E_1 across the primary of the ideal transformer represents the rms voltage induced in the primary winding by the mutual flux ϕ. This is the portion of the core flux which links both primary and secondary coils. Assuming

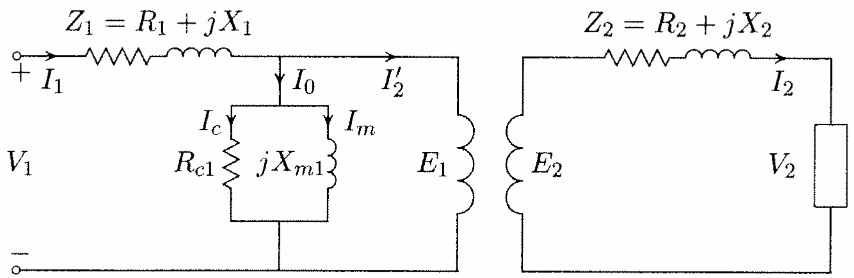


FIGURE 3.9
Equivalent circuit of a transformer.

sinusoidal flux $\phi = \Phi_{max} \cos \omega t$, the instantaneous voltage e_1 is

$$\begin{aligned} e_1 &= N_1 \frac{d\phi}{dt} \\ &= -\omega N_1 \Phi_{max} \sin \omega t \\ &= E_{1max} \cos(\omega t + 90^\circ) \end{aligned} \tag{3.33}$$

where

$$E_{1max} = 2\pi f N_1 \Phi_{max} \tag{3.34}$$

or the rms voltage magnitude E_1 is

$$E_1 = 4.44 f N_1 \Phi_{max} \tag{3.35}$$

It is important to note that the phasor flux is lagging the induced voltage E_1 by 90°. Similarly the rms voltage E_2 across the secondary of the ideal transformer represents the voltage induced in the secondary winding by the mutual flux ϕ, given by

$$E_2 = 4.44 f N_2 \Phi_{max} \tag{3.36}$$

In the ideal transformer, the core is assumed to have a zero reluctance and there is an exact mmf balanced between the primary and secondary. If I_2' represents the component of current to neutralize the secondary mmf, then

$$I_2' N_1 = I_2 N_2 \tag{3.37}$$

Therefore, for an ideal transformer, from (3.35) through (3.37) we have

$$\frac{E_1}{E_2} = \frac{I_2}{I_2'} = \frac{N_1}{N_2} \tag{3.38}$$

In a real transformer, the reluctance of the core is finite, and when the secondary current I_2 is zero, the primary current has a finite value. Since at no-load, induced voltage E_1 is almost equal to the supply voltage V_1, the induced voltage and the flux are sinusoidal. However, because of the nonlinear characteristics of the ferromagnetic core, the no-load current is not sinusoidal and contains odd harmonics. The third harmonic is particularly troublesome in certain three-phase connections of transformers. For the purpose of modeling, we assume a sinusoidal no-load current with the rms value of I_0, known as the *no-load current*. This current has a component I_m, in phase with flux, known as the *magnetizing current*, to set up the core flux. Since flux is lagging the induced voltage E_1 by 90°, I_m is also lagging the induced voltage E_1 by 90°. Thus, this component can be represented in the circuit by the magnetizing reactance jX_{m1}. The other component of I_0 is I_c, which supplies the eddy-current and hysteresis losses in the core. Since this is a power component, it is in phase with E_1 and is represented by the resistance R_{c1} as shown in Figure 3.9.

In a real transformer with finite reluctance, all of the flux is not common to both primary and secondary windings. The flux has three components: mutual flux, primary leakage flux, and secondary leakage flux. The leakage flux associated with one winding does not link the other, and the voltage drops caused by the leakage flux are expressed in terms of leakage reactances X_1 and X_2. Finally, R_1 and R_2 are included to represent the primary and secondary winding resistances.

To obtain the performance characteristics of a transformer, it is convenient to use an equivalent circuit model referred to one side of the transformer. From Kirchhoff's voltage law (KVL), the voltage equation of the secondary side is

$$E_2 = V_2 + Z_2 I_2 \tag{3.39}$$

From the relationship (3.38) developed for the ideal transformer, the secondary induced voltage and current are $E_2 = (N_2/N_1)E_1$ and $I_2 = (N_1/N_2)I_2'$, respectively. Upon substitution, (3.39) reduces to

$$\begin{aligned} E_1 &= \frac{N_1}{N_2} V_2 + \left(\frac{N_1}{N_2}\right)^2 Z_2 I_2' \\ &= V_2' + Z_2' I_2' \end{aligned} \tag{3.40}$$

where

$$Z_2' = R_2' + jX_2' = \left(\frac{N_1}{N_2}\right)^2 R_2 + j\left(\frac{N_1}{N_2}\right)^2 X_2$$

Relation (3.40) is the KVL equation of the secondary side referred to the primary, and the equivalent circuit of Figure 3.9 can be redrawn as shown in Figure 3.10, so the same effects are produced in the primary as would be produced in the secondary.

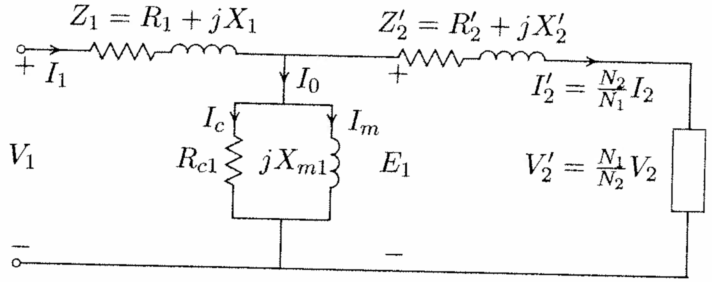


FIGURE 3.10
Exact equivalent circuit referred to the primary side.

On no-load, the primary voltage drop is very small, and V_1 can be used in place of E_1 for computing the no-load current I_0. Thus, the shunt branch can be moved to the left of the primary series impedance with very little loss of accuracy. In this manner, the primary quantities R_1 and X_1 can be combined with the referred secondary quantities R_2' and X_2' to obtain the equivalent primary quantities R_{e1} and X_{e1}. The equivalent circuit is shown in Figure 3.11 where we have dispensed with the coils of the ideal transformer. From Figure 3.11

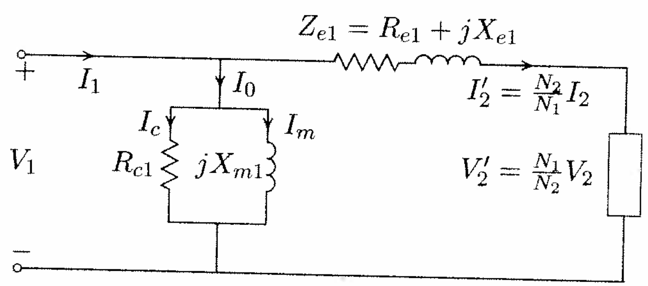


FIGURE 3.11
Approximate equivalent circuit referred to the primary.

$$V_1 = V_2' + (R_{e1} + jX_{e1})I_2' \tag{3.41}$$

where

$$R_{e1} = R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 \quad X_{e1} = X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2 \quad \text{and} \quad I_2' = \frac{S_L^*}{3V_2'^*}$$

The equivalent circuit referred to the secondary is also shown in Figure 3.12. From Figure 3.12 the referred primary voltage V_1' is given by

$$V_1' = V_2 + (R_{e2} + jX_{e2})I_2 \tag{3.42}$$

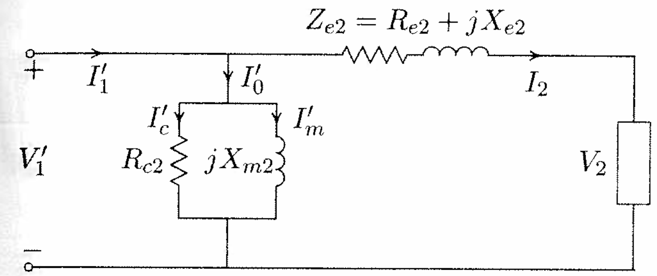


FIGURE 3.12
Approximate equivalent circuit referred to the secondary.

Power transformers are generally designed with very high permeability core and very small core loss. Consequently, a further approximation of the equivalent circuit can be made by omitting the shunt branch, as shown in Figure 3.13. The equivalent circuit referred to the secondary is also shown in Figure 3.13.

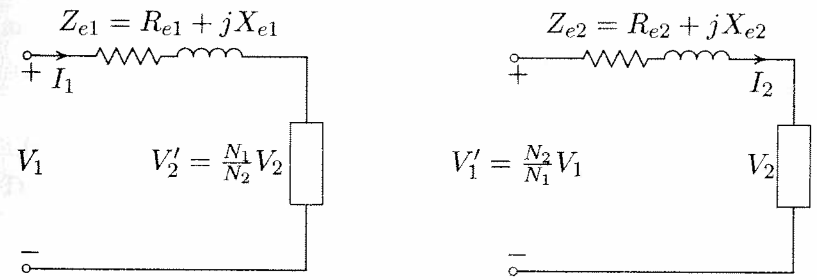


FIGURE 3.13
Simplified circuits referred to one side.

3.7 DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS

The parameters of the approximate equivalent circuit are readily obtained from open-circuit and short-circuit tests. In the open-circuit test, rated voltage is applied at the terminals of one winding while the other winding terminals are open-circuited. Instruments are connected to measure the input voltage V_1, the no-load input current I_0, and the input power P_0. If the secondary is open-circuited, the referred secondary current I_2' will be zero, and only a small no-load current will be drawn from the supply. Also, the primary voltage drop $(R_1 + jX_1)I_0$ can be neglected, and the equivalent circuit reduces to the form shown in Figure 3.14.

Since the secondary winding copper loss (resistive power loss) is zero and the

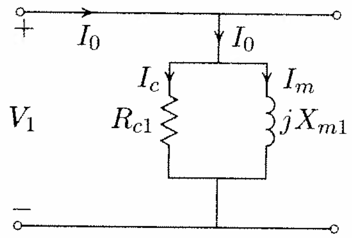


FIGURE 3.14
Equivalent circuit for the open-circuit test.

primary copper loss $R_1 I_0{}^2$ is negligible, the no-load input power P_0 represents the transformer core loss commonly referred to as *iron loss*. The shunt elements R_c and X_m may then be determined from the relations

$$R_{c1} = \frac{V_1{}^2}{P_0} \tag{3.43}$$

The two components of the no-load current are

$$I_c = \frac{V_1}{R_{c1}} \tag{3.44}$$

and

$$I_m = \sqrt{I_0{}^2 - I_c{}^2} \tag{3.45}$$

Therefore, the magnetizing reactance is

$$X_{m1} = \frac{V_1}{I_m} \tag{3.46}$$

In the short-circuit test, a reduced voltage V_{sc} is applied at the terminals of one winding while the other winding terminals are short-circuited. Instruments are connected to measure the input voltage V_{sc}, the input current I_{sc}, and the input power P_{sc}. The applied voltage is adjusted until rated currents are flowing in the windings. The primary voltage required to produce rated current is only a few percent of the rated voltage. At the correspondingly low value of core flux, the exciting current and core losses are entirely negligible, and the shunt branch can be omitted. Thus, the power input can be taken to represent the winding copper loss. The transformer appears as a short when viewed from the primary with the equivalent leakage impedance Z_{e1} consisting of the primary leakage impedance and the referred secondary leakage impedance as shown in Figure 3.15. The series elements

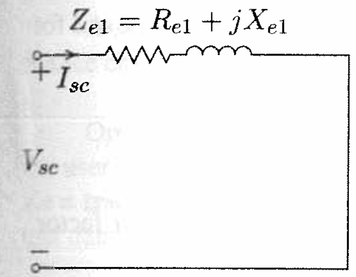


FIGURE 3.15
Equivalent circuit for the short-circuit test.

R_{e1} and X_{e1} may then be determined from the relations

$$Z_{e1} = \frac{V_{sc}}{I_{sc}}$$

and

$$R_{e1} = \frac{P_{sc}}{(I_{sc})^2} \tag{3.47}$$

Therefore, the equivalent leakage reactance is

$$X_{e1} = \sqrt{Z_{e1}{}^2 - R_{e1}{}^2} \tag{3.48}$$

3.8 TRANSFORMER PERFORMANCE

The equivalent circuit can now be used to predict the performance characteristics of the transformer. An important aspect is the transformer efficiency. Power transformer efficiencies very from 95 percent to 99 percent, the higher efficiencies being obtained from transformers with the greater ratings. The actual efficiency of a transformer in percent is given by

$$\eta = \frac{\text{output power}}{\text{input power}} \tag{3.49}$$

and the conventional efficiency of a transformer at n fraction of the full-load power is given by

$$\eta = \frac{n \times S \times PF}{(n \times S \times PF) + n^2 \times P_{cu} + P_c} \tag{3.50}$$

where S is the full-load rated volt-ampere, P_{cu} is the full-load copper loss, and for a three-phase transformer, they are given by

$$\begin{aligned} S &= 3|V_2||I_2| \\ P_{cu} &= 3R_{e2}|I_2|^2 \end{aligned}$$

and P_c is the iron loss at rated voltage. For varying I_2 at constant power factor, maximum efficiency occurs when

$$\frac{d\eta}{d|I_2|} = 0$$

For the above condition, it can be easily shown that maximum efficiency occurs when copper loss equals core loss at n per-unit loading given by

$$n = \sqrt{\frac{P_c}{P_{cu}}} \tag{3.51}$$

Another important performance characteristic of a transformer is change in the secondary voltage from no-load to full-load. A figure of merit used to compare the relative performance of different transformers is the voltage regulation. Voltage regulation is defined as the change in the magnitude of the secondary terminal voltage from no-load to full-load expressed as a percentage of the full-load value.

$$\text{Regulation} = \frac{|V_{2nl}| - |V_2|}{|V_2|} \times 100 \tag{3.52}$$

where V_2 is the full-load rated voltage. V_{2nl} in (3.52) can be calculated by using equivalent circuits referred to either primary or secondary. When the equivalent circuit is referred to the primary side, the primary no-load voltage is found from (3.41), and the voltage regulation becomes

$$\text{Regulation} = \frac{|V_1| - |V_2'|}{|V_2'|} \times 100 \tag{3.53}$$

When the equivalent circuit is referred to the secondary side, the secondary no-load voltage is found from (3.42), and the voltage regulation becomes

$$\text{Regulation} = \frac{|V_1'| - |V_2|}{|V_2|} \times 100 \tag{3.54}$$

An interesting feature arises with a capacitive load. Because partial resonance is set up between the capacitance and the reactance, the secondary voltage may actually tend to rise as the capacitive load value increases.

A program called **trans** is developed for obtaining the transformer performance characteristics. The command **trans** displays a menu with three options:

Option 1 calls upon the function **[Rc, Xm] = troct(Vo, Io, Po)** which prompts the user to enter the no-load test data and returns the shunt branch parameters. Then **Ze = trsct(Vsc, Isc, Psc)** is loaded which prompts the user to enter the short-circuit test data and returns the equivalent leakage impedance.

Option 2 calls upon the function **[Zelv, Zehv] = wz2eqz(Elv, Ehv, Zlv, Zhv)** which prompts the user to enter the individual winding impedances and the shunt branch. This function returns the referred equivalent circuit for both sides.

Option 3 prompts the user to enter the parameters of the equivalent circuit.

The above functions can be used independently when the arguments of the functions are defined in the *MATLAB* environment. If the above functions are typed without the parenthesis and the arguments, the user will be prompted to enter the required data. After the selection of any of the above options, the program prompts the user to enter the load specifications and proceeds to obtain the transformer performance characteristics including an efficiency curve from 25 to 125 percent of full-load.

A new GUI program named **transformer** is developed for the transformer tests and analysis. This program obtains the transformer equivalent circuit from open-circuit and short-circuit tests. It also finds the transformer performance characteristics using the transformer parameters.

Example 3.4 (chp3ex4)

Data obtained from short-circuit and open-circuit tests of a 240-kVA, 4800/240-V, 60-Hz transformer are:

Open-circuit test, low-side data	Short-circuit test, high-side data
$V_1 = 240$ V	$V_{sc} = 187.5$ V
$I_0 = 10$ A	$I_{sc} = 50$ A
$P_0 = 1440$ W	$P_{sc} = 2625$ W

Determine the parameters of the equivalent circuit

The command

```
trans
```

display the following menu

```
Type of parameters for input                          Select
To obtain equivalent circuit from tests                  1
To input individual winding impedances                   2
To input transformer equivalent impedance                3
To quit                                                  0

Select number of menu → 1
Enter Transformer rated power in kVA, S = 240
Enter rated low voltage in volts = 240
Enter rated high voltage in volts = 4800

Open circuit test data
Enter 'lv' within quotes for data ref. to low side or
enter 'hv' within quotes for data ref. to high side → 'lv'
Enter input voltage, in volts, Vo = 240
Enter no-load current in Amp, Io = 10
Enter no-load input power in Watt, Po = 1440

Short circuit test data
Enter 'lv' within quotes for data ref. to low side or
enter 'hv' within quotes for data ref. to high side → 'hv'
Enter reduced input voltage in volts, Vsc = 187.5
Enter input current in Amp, Isc = 50
Enter input power in Watt, Psc = 2625

Shunt branch ref. to LV side       Shunt branch ref. to HV side
Rc = 40.000 ohm                    Rc = 16000.000 ohm
Xm = 30.000 ohm                    Xm = 12000.000 ohm

Series branch ref. to LV side      Series branch ref. to HV side
Ze = 0.002625 + j 0.0090 ohm       Ze = 1.0500 + j 3.6000 ohm

Hit return to continue
```

At this point the user is prompted to enter the load apparent power, power factor, and voltage. The program then obtains the performance characteristics of the transformer including the efficiency curve from 25 to 125 percent of full load as shown in Figure 3.16.

```
Enter load kVA, S2 = 240
Enter load power factor, pf = 0.8
Enter 'lg' within quotes for lagging pf
or 'ld' within quotes for leading pf -> 'lg'
Enter load terminal voltage in volt, V2 = 240
```

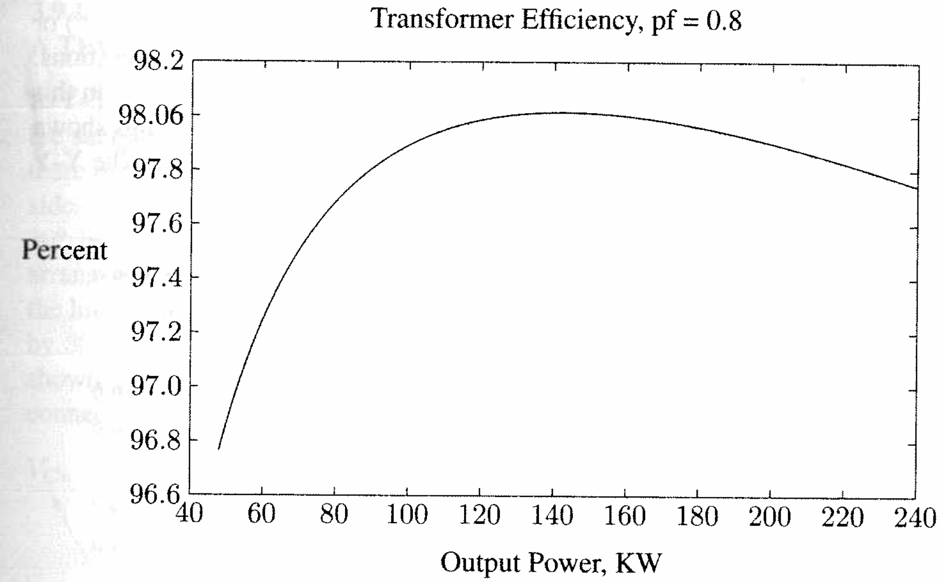


FIGURE 3.16
Efficiency curve of Example 3.4.

```
Secondary load voltage    =    240.000 V
Secondary load current    =   1000.000 A  at  -36.87 degrees
Current ref. to primary   =     50.000 A  at  -36.87 degrees
Primary no-load current   =      0.516 A  at  -53.13 degrees
Primary input current     =     50.495 A  at  -37.03 degrees
Primary input voltage     =   4951.278 V  at    1.30 degrees
Voltage regulation        =      3.152 %
Transformer efficiency    =     97.927 %

Maximum efficiency is 98.015 percent, occurs at 177.757 kVA
with 0.80 pf.
```

At the end of this analysis the program menu is displayed.

3.9 THREE-PHASE TRANSFORMER CONNECTIONS

Three-phase power is transformed by use of three-phase units. However, in large extra high voltage (EHV) units, the insulation clearances and shipping limitations may require a bank of three single-phase transformers connected in three-phase arrangements.

The primary and secondary windings can be connected in either wye (Y) or delta (Δ) configurations. This results in four possible combinations of connections: Y–Y, Δ–Δ, Y–Δ and Δ–Y shown by the simple schematic in Figure 3.17. In this diagram, transformer windings are indicated by heavy lines. The windings shown in parallel are located on the same core and their voltages are in phase. The Y–Y

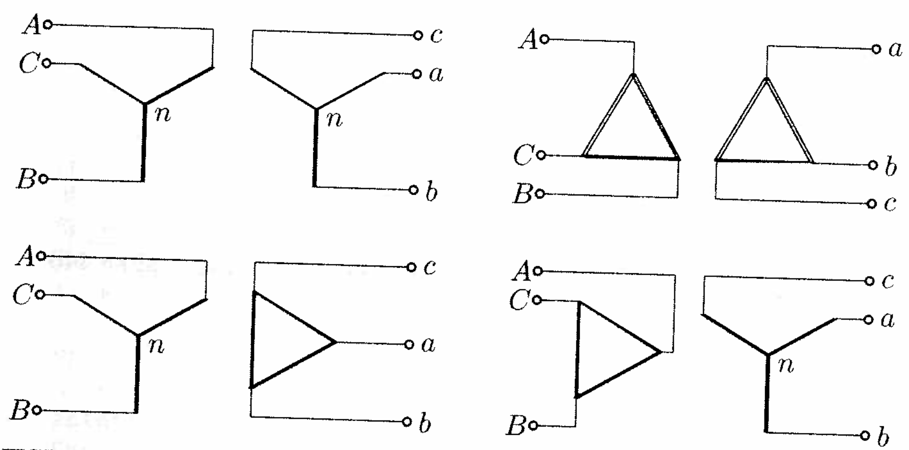


FIGURE 3.17
Three-phase transformer connections.

connection offers advantages of decreased insulation costs and the availability of the neutral for grounding purposes. However, because of problems associated with third harmonics and unbalanced operation, this connection is rarely used. To eliminate the harmonics, a third set of windings, called a *tertiary* winding, connected in Δ is normally fitted on the core to provide a path for the third harmonic currents. This is known as the *three-winding* transformer. The tertiary winding can be loaded with switched reactors or capacitors for reactive power compensation. The Δ–Δ provides no neutral connection and each transformer must withstand full line-to-line voltage. The Δ connection does, however, provide a path for third harmonic currents to flow. This connection has the advantage that one transformer can be removed for repair and the remaining two can continue to deliver three-phase power at a reduced rating of 58 percent of the original bank. This is known as the V connection. The most common connection is the Y–Δ or Δ–Y. This connection is more stable with respect to unbalanced loads, and if the Y connection is used on the high voltage side, insulation costs are reduced. The Y–Δ connection is commonly used to step down a high voltage to a lower voltage. The neutral point on the high voltage side can be grounded. This is desirable in most cases. The Δ–Y connection is commonly used for stepping up to a high voltage.

3.9.1 THE PER-PHASE MODEL OF A THREE-PHASE TRANSFORMER

In Y–Y and Δ–Δ connections, the ratio of the line voltages on HV and LV sides are the same as the ratio of the phase voltages on the HV and LV sides. Furthermore, there is no phase shift between the corresponding line voltages on the HV and LV sides. However, the Y–Δ and the Δ–Y connections will result in a phase shift of 30° between the primary and secondary line-to-line voltages. The windings are arranged in accordance to the ASA (American Standards Association) such that the line voltage on the HV side leads the corresponding line voltage on the LV side by 30° regardless of which side is Y or Δ. Consider the Y–Δ schematic diagram shown in Figure 3.17. The positive phase sequence voltage phasor diagram for this connection is shown in Figure 3.18, where V_{An} is taken as reference. Let the Y

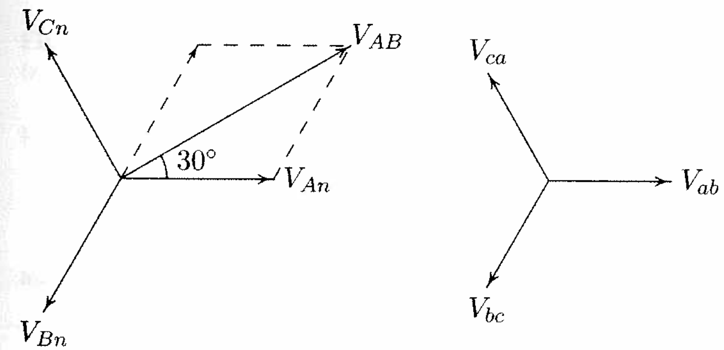


FIGURE 3.18
30° phase shift in line-to-line voltages of Y–Δ connection.

connection be the high voltage side shown by letter H and the Δ connection the low voltage side shown by X. We consider phase a only and use subscript L for line and P for phase quantities. If N_H is the number of turns on one phase of the high voltage winding and N_X is the number of turns on one phase of the low voltage winding, the transformer turns ratio is $a = N_H/N_X = V_{HP}/V_{XP}$. The relationship between the line voltage and phase voltage magnitudes is

$$V_{HL} = \sqrt{3}\, V_{HP}$$
$$V_{XL} = V_{XP}$$

Therefore, the ratio of the line voltage magnitudes for Y–Δ transformer is

$$\frac{V_{HL}}{V_{XL}} = \sqrt{3}\, a \tag{3.55}$$

Because the core losses and magnetization current for power transformers are on the order of 1 percent of the maximum ratings, the shunt impedance is neglected and only the winding resistance and leakage reactance are used to model the transformer. In dealing with Y–Δ or Δ–Y banks, it is convenient to replace the Δ connection by an equivalent Y connection and then work with only one phase. Since for balanced operations, the Y neutral and the neutral of the equivalent Y of the Δ connection are at the same potential, they can be connected together and represented by a neutral conductor. When the equivalent series impedance of one transformer is referred to the delta side, the Δ connected impedances of the transformer are replaced by equivalent Y-connected impedances, given by $Z_Y = Z_\Delta/3$. The per phase equivalent model with the shunt branch neglected is shown in Figure 3.19. Z_{e1} and Z_{e2} are the equivalent impedances based on the line-to-neutral connections, and the voltages are the line-to-neutral values.

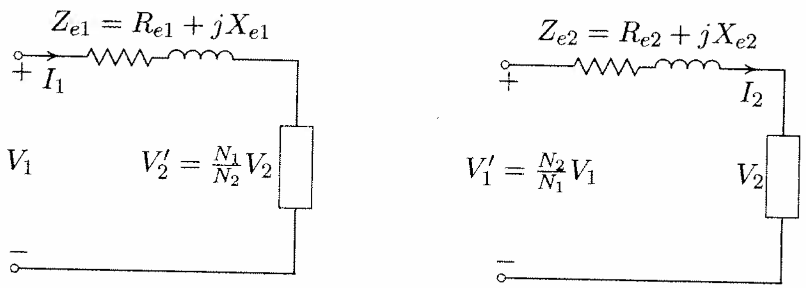


FIGURE 3.19
The per phase equivalent circuit.

3.10 AUTOTRANSFORMERS

Transformers can be constructed so that the primary and secondary coils are electrically connected. This type of transformer is called an autotransformer. A conventional two-winding transformer can be changed into an autotransformer by connecting the primary and secondary windings in series. Consider the two-winding transformer shown in Figure 3.20(a). The two-winding transformer is converted to an autotransformer arrangement as shown in Figure 3.20(b) by connecting the two windings electrically in series so that the polarities are additive. The winding from X_1 to X_2 is called the series winding, and the winding from H_1 to H_2 is called the common winding. From an inspection of this figure it follows that an autotransformer can operate as a step-up as well as a step-down transformer. In both cases, winding part H_1H_2 is common to the primary as well as the secondary side of the transformer. The performance of an autotransformer is governed by the fundamental considerations already discussed for transformers having two separate windings. For determining the power rating as an autotransformer, the ideal transformer relations are ordinarily used, which provides an adequate approximation to the actual transformer values.

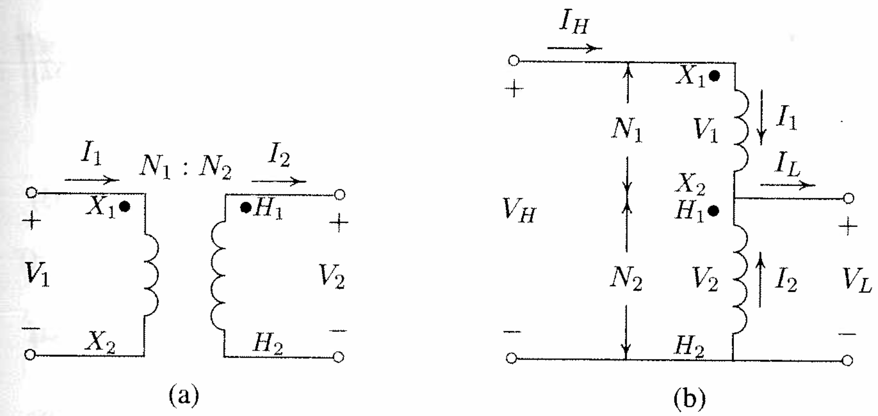


FIGURE 3.20
(a) Two-winding transformer, (b) reconnected as an autotransformer.

From Figure 3.20(a), the two-winding voltages and currents are related by

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a \tag{3.56}$$

and

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = a \tag{3.57}$$

where a is the turns ratio of the two-winding transformer. From Figure 3.20(b), we have

$$V_H = V_2 + V_1 \tag{3.58}$$

Substituting for V_1 from (3.56) into (3.58) yields

$$V_H = V_2 + \frac{N_1}{N_2}V_2 \tag{3.59}$$

Since $V_2 = V_L$, the voltage relationship between the two sides of an autotransformer becomes

$$\begin{aligned} V_H &= V_L + \frac{N_1}{N_2}V_L \\ &= (1+a)V_L \end{aligned} \tag{3.60}$$

or

$$\frac{V_H}{V_L} = 1 + a \tag{3.61}$$

Since the transformer is ideal, the mmf due to I_1 must be equal and opposite to the mmf produced by I_2. As a result, we have

$$N_2 I_2 = N_1 I_1 \tag{3.62}$$

From Kirchhoff's law, $I_2 = I_L - I_1$, and the above equation becomes

$$N_2(I_L - I_1) = N_1 I_1 \tag{3.63}$$

or

$$I_L = \frac{N_1 + N_2}{N_2} I_1 \tag{3.64}$$

Since $I_1 = I_H$, the current relationship between the two sides of an autotransformer becomes

$$\frac{I_L}{I_H} = 1 + a \tag{3.65}$$

The ratio of the apparent power rating of an autotransformer to a two-winding transformer, known as the *power rating advantage*, is found from

$$\frac{S_{auto}}{S_{2-w}} = \frac{(V_1 + V_2)I_1}{V_1 I_1} = 1 + \frac{N_2}{N_1} = 1 + \frac{1}{a} \tag{3.66}$$

From (3.66), we can see that a higher rating is obtained as an autotransformer with a higher number of turns of the common winding (N_2). The higher rating as an autotransformer is a consequence of the fact that only S_{2-w} is transformed by the electromagnetic induction. The rest passes from the primary to secondary without being coupled through the transformer's windings. This is known as the *conducted power*. Compared with a two-winding transformer of the same rating, autotransformers are smaller, more efficient, and have lower internal impedance. Three-phase autotransformers are used extensively in power systems where the voltages of the two systems coupled by the transformers do not differ by a factor greater than about three.

Example 3.5 (chp3ex5)

A two-winding transformer is rated at 60 kVA, 240/1200 V, 60 Hz. When operated as a conventional two-winding transformer at rated load, 0.8 power factor, its efficiency is 0.96. This transformer is to be used as a 1440/1200-V step-down autotransformer in a power distribution system.
(a) Assuming ideal transformer, find the transformer kVA rating when used as an autotransformer.
(b) Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

The two-winding transformer rated currents are:

$$I_1 = \frac{60,000}{240} = 250 \text{ A}$$
$$I_2 = \frac{60,000}{1200} = 50 \text{ A}$$

The autotransformer connection is as shown in Figure 3.21.

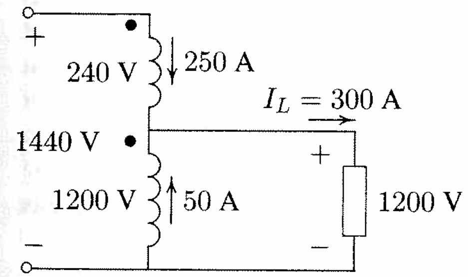


FIGURE 3.21
Auto transformer connection for Example 3.5.

(a) The autotransformer secondary current is

$$I_L = 250 + 50 = 300 \text{ A}$$

With windings carrying rated currents, the autotransformer rating is

$$S = (1200)(300)(10^{-3}) = 360 \text{ kVA}$$

Therefore, the power advantage of the autotransformer is

$$\frac{S_{auto}}{S_{2-w}} = \frac{360}{60} = 6$$

(b) When operated as a two-winding transformer at full-load, 0.8 power factor, the losses are found from the efficiency formula

$$\frac{(60)(0.8)}{(60)(0.8) + P_{loss}} = 0.96$$

Solving the above equation, the total transformer loss is

$$P_{loss} = \frac{48(1 - 0.96)}{0.96} = 2.0 \text{ kW}$$

Since the windings are subjected to the same rated voltages and currents as the two-winding transformer, the autotransformer copper loss and the core loss at the rated values are the same as the two-winding transformer. Therefore, the autotransformer efficiency at rated load, 0.8 power factor, is

$$\eta = \frac{(360)(0.8)}{(360)(0.8)+2} \times 100 = 99.31 \text{ percent}$$

3.10.1 AUTOTRANSFORMER MODEL

When a two-winding transformer is connected as an autotransformer, its equivalent impedance expressed in per-unit is much smaller compared to the equivalent value of the two-winding connection. It can be shown that the effective per-unit impedance of an autotransformer is smaller by a factor equal to the reciprocal of the power advantage of the autotransformer connection. It is common practice to consider an autotransformer as a two-winding transformer with its two winding connected in series as shown in Figure 3.22, where the equivalent impedance is referred to the N_1-turn side.

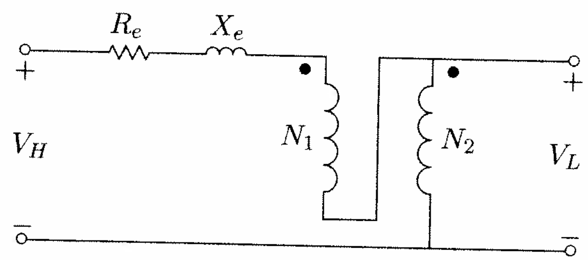


FIGURE 3.22
Autotransformer equivalent circuit.

3.11 THREE-WINDING TRANSFORMERS

Transformers having three windings are often used to interconnect three circuits which may have different voltages. These windings are called primary, secondary, and tertiary windings. Typical applications of three-winding transformers in power systems are for the supply of two independent loads at different voltages from the same source and interconnection of two transmission systems of different voltages. Usually the tertiary windings are used to provide voltage for auxiliary power purposes in the substation or to supply a local distribution system. In addition, the switched reactor or capacitors are connected to the tertiary bus for the purpose of reactive power compensation. Sometimes three-phase Y-Y transformers and Y–connected autotransformers are provided with Δ–connected tertiary windings for harmonic suppression.

3.11.1 THREE-WINDING TRANSFORMER MODEL

If the exciting current of a three-winding transformer is neglected, it is possible to draw a simple single-phase equivalent T-circuit as shown in Figure 3.23.

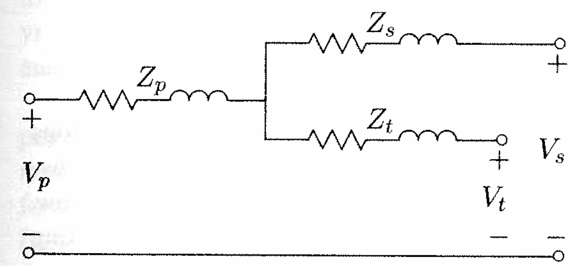


FIGURE 3.23
Equivalent circuit of three-winding transformer.

Three short-circuit tests are carried out on a three-winding transformer with N_p, N_s, and N_t turns per phase on the three windings, respectively. The three tests are similar in that in each case one winding is open, one shorted, and reduced voltage is applied to the remaining winding. The following impedances are measured on the side to which the voltage is applied.

Z_{ps} = impedance measured in the primary circuit with the secondary short-circuited and the tertiary open.

Z_{pt} = impedance measured in the primary circuit with the tertiary short-circuited and the secondary open.

Z'_{st} = impedance measured in the secondary circuit with the tertiary short-circuited and the primary open.

Referring Z'_{st} to the primary side, we obtain

$$Z_{st} = \left(\frac{N_p}{N_s}\right)^2 Z'_{st} \tag{3.67}$$

If Z_p, Z_s, and Z_t are the impedances of the three separate windings referred to the primary side, then

$$\begin{aligned} Z_{ps} &= Z_p + Z_s \\ Z_{pt} &= Z_p + Z_t \\ Z_{st} &= Z_s + Z_t \end{aligned} \tag{3.68}$$

Solving the above equations, we have

$$Z_p = \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st})$$
$$Z_s = \frac{1}{2}(Z_{ps} + Z_{st} - Z_{pt}) \qquad (3.69)$$
$$Z_t = \frac{1}{2}(Z_{pt} + Z_{st} - Z_{ps})$$

3.12 VOLTAGE CONTROL OF TRANSFORMERS

Voltage control in transformers is required to compensate for varying voltage drops in the system and to control reactive power flow over transmission lines. Transformers may also be used to control phase angle and, therefore, active power flow. The two commonly used methods are tap changing transformers and regulating transformers.

3.12.1 TAP CHANGING TRANSFORMERS

Practically all power transformers and many distribution transformers have taps in one or more windings for changing the turns ratio. This method is the most popular since it can be used for controlling voltages at all levels. Tap changing, by altering the voltage magnitude, affects the distribution of vars and may therefore be used to control the flow of reactive power. There are two types of tap changing transformers

(i) Off-load tap changing transformers.
(ii) Tap changing under load (TCUL) transformers.

The off-load tap changing transformer requires the disconnection of the transformer when the tap setting is to be changed. Off-load tap changers are used when it is expected that the ratio will need to be changed only infrequently, because of load growth or some seasonal change. A typical transformer might have four taps in addition to the nominal setting, with spacing of 2.5 percent of full-load voltage between them. Such an arrangement provides for adjustments of up to 5 percent above or below the nominal voltage rating of the transformer.

Tap changing under load (TCUL) is used when changes in ratio may be frequent or when it is undesirable to de-energize the transformer to change a tap. A large number of units are now being built with load tap changing equipment. It is used on transformers and autotransformers for transmission tie, for bulk distribution units, and at other points of load service. Basically, a TCUL transformer is a transformer with the ability to change taps while power is connected. A TCUL transformer may have built-in voltage sensing circuitry that automatically changes taps to keep the system voltage constant. Such special transformers are very common in modern power systems. Special tap changing gear are required for TCUL transformers, and the position of taps depends on a number of factors and requires special consideration to arrive at an optimum location for the TCUL equipment. Step-down units usually have TCUL in the low voltage winding and de-energized taps in the high voltage winding. For example, the high voltage winding might be equipped with a nominal voltage turns ratio plus four 2.5 percent fixed tap settings to yield ± 5 percent buck or boost voltage. In addition to this, there could be provision, on the low voltage windings, for 32 incremental steps of $\frac{5}{8}$ each, giving an automatic range of ± 10 percent.

Tapping on both ends of a radial transmission line can be adjusted to compensate for the voltage drop in the line. Consider one phase of a three-phase transmission line with a step-up transformer at the sending end and a step-down transformer at the receiving end of the line. A single-line representation is shown in Figure 3.24, where t_S and t_R are the tap setting in per-unit. In this diagram, V_1' is the supply phase voltage referred to the high voltage side, and V_2' is the load phase voltage, also referred to the high voltage side. The impedance shown includes the

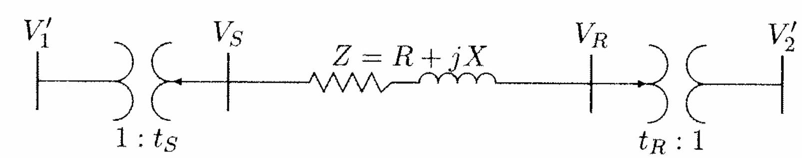


FIGURE 3.24
A radial line with tap changing transformers at both ends.

line impedance plus the referred impedances of the sending end and the receiving end transformers to the high voltage side. If V_S and V_R are the phase voltages at both ends of the line, we have

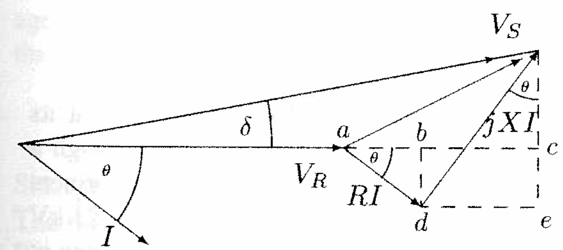


FIGURE 3.25
Voltage phasor diagram.

$$V_R = V_S + (R + jX)I \tag{3.70}$$

The phasor diagram for the above equation is shown in Figure 3.25.

The phase shift δ between the two ends of the line is usually small, and we can neglect the vertical component of V_S. Approximating V_S by its horizontal component results in

$$\begin{aligned} |V_S| &= |V_R| + ab + de \\ &= |V_R| + |I|R\cos\theta + |I|X\sin\theta \end{aligned} \tag{3.71}$$

Substituting for $|I|$ from $P_\phi = |V_R||I|\cos\theta$ and $Q_\phi = |V_R||I|\sin\theta$ will result in

$$|V_S| = |V_R| + \frac{RP_\phi + XQ_\phi}{|V_R|} \tag{3.72}$$

Since $V_S = t_S V_1'$ and $V_R = t_R V_2'$, the above relation in terms of V_1', and V_2' becomes

$$t_S|V_1'| = t_R|V_2'| + \frac{RP_\phi + XQ_\phi}{t_R|V_2'|} \tag{3.73}$$

or

$$t_S = \frac{1}{|V_1'|}\left(t_R|V_2'| + \frac{RP_\phi + XQ_\phi}{t_R|V_2'|}\right) \tag{3.74}$$

Assuming the product of t_S and t_R is unity, i.e., $t_S t_R = 1$, and substituting for t_R in (3.74), the following expression is found for t_S.

$$t_S = \sqrt{\frac{\frac{|V_2'|}{|V_1'|}}{1 - \frac{RP_\phi + XQ_\phi}{|V_1'||V_2'|}}} \tag{3.75}$$

Example 3.6 (chp3ex6)

A three-phase transmission line is feeding from a 23/230-kV transformer at its sending end. The line is supplying a 150-MVA, 0.8 power factor load through a step-down transformer of 230/23 kV. The impedance of the line and transformers at 230 kV is $18 + j60\ \Omega$. The sending end transformer is energized from a 23-kV supply. Determine the tap setting for each transformer to maintain the voltage at the load at 23 kV.

The load real and reactive power per phase are

$$P_\phi = \frac{1}{3}(150)(0.8) = 40 \text{ MW}$$
$$Q_\phi = \frac{1}{3}(150)(0.6) = 30 \text{ Mvar}$$

The source and the load phase voltages referred to the high voltage side are

$$|V_1'| = |V_2'| = \left(\frac{230}{23}\right)\left(\frac{23}{\sqrt{3}}\right) = \frac{230}{\sqrt{3}}$$

From (3.75), we have

$$t_S = \sqrt{\frac{1}{1 - \frac{(18)(40)+(60)(30)}{(230/\sqrt{3})^2}}} = 1.08 \text{ pu}$$

and

$$t_R = \frac{1}{1.08} = 0.926 \text{ pu}$$

3.12.2 REGULATING TRANSFORMERS OR BOOSTERS

Regulating transformers, also known as *boosters*, are used to change the voltage magnitude and phase angle at a certain point in the system by a small amount. A booster consists of an exciting transformer and a series transformer.

VOLTAGE MAGNITUDE CONTROL

Figure 3.26 shows the connection of a regulating transformer for phase a of a three-phase system for voltage magnitude control. Other phases have identical arrangement. The secondary of the exciting transformer is tapped, and the voltage obtained from it is applied to the primary of the series transformer. The corresponding voltage on the secondary of the series transformer is added to the input voltage. Thus, the output voltage is

$$V_{an}' = V_{an} + \Delta V_{an} \tag{3.76}$$

Since the voltages are in phase, a booster of this type is called an *in-phase booster*. The output voltage can be adjusted by changing the excitation transformer taps. By changing the switch from position 1 to 2, the polarity of the voltage across the series transformer is reversed, so that the output voltage is now less than the input voltage.

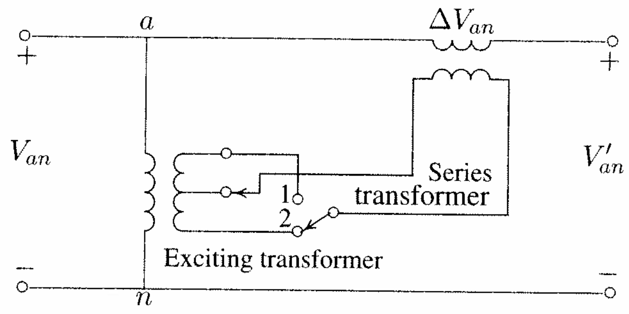


FIGURE 3.26
Regulating transformer for voltage magnitude control.

PHASE ANGLE CONTROL

Regulating transformers are also used to control the voltage phase angle. If the injected voltage is out of phase with the input voltage, the resultant voltage will have a phase shift with respect to the input voltage. Phase shifting is used to control active power flow at major intertie buses. A typical arrangement for phase a of a three-phase system is shown in Figure 3.27.

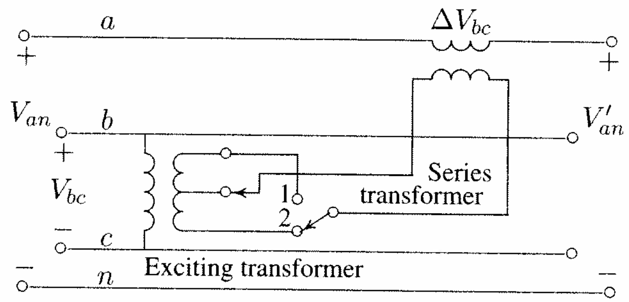


FIGURE 3.27
Regulating transformer for voltage phase angle control.

The series transformer of phase a is supplied from the secondary of the exciting transformer bc. The injected voltage ΔV_{bc} is in quadrature with the voltage V_{an}, thus the resultant voltage V'_{an} goes through a phase shift α, as shown in Figure 3.28. The output voltage is

$$V'_{an} = V_{an} + \Delta V_{bc} \tag{3.77}$$

Similar connections are made for the remaining phases, resulting in a balanced three phase output voltage. The amount of phase shift can be adjusted by changing the excitation transformer taps. By changing the switch from position 1 to 2, the

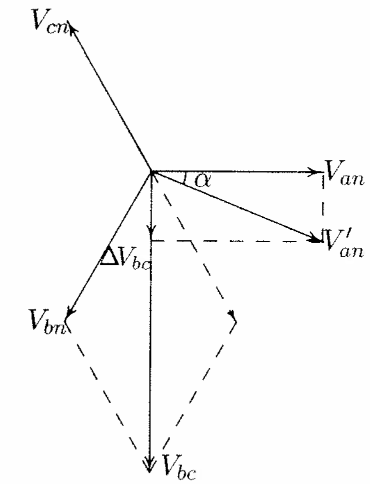


FIGURE 3.28
Voltage phasor diagram showing phase shifting effect for phase a.

output voltage can be made to lag or lead the input voltage. The advantages of the regulating transformers are

1. The main transformers are free from tappings.
2. The regulating transformers can be used at any intermediate point in the system.
3. The regulating transformers and the tap changing gears can be taken out of service for maintenance without affecting the system.

3.13 THE PER-UNIT SYSTEM

The solution of an interconnected power system having several different voltage levels requires the cumbersome transformation of all impedances to a single voltage level. However, power system engineers have devised the *per-unit system* such that the various physical quantities such as power, voltage, current and impedance are expressed as a decimal fraction or multiples of base quantities. In this system, the different voltage levels disappear, and a power network involving generators, transformers, and lines (of different voltage levels) reduces to a system of simple impedances. The per-unit value of any quantity is defined as

$$\text{Quantity in per-unit} = \frac{\text{actual quantity}}{\text{base value of quantity}} \tag{3.78}$$

For example,

$$S_{pu} = \frac{S}{S_B} \quad V_{pu} = \frac{V}{V_B} \quad I_{pu} = \frac{I}{I_B} \quad \text{and} \quad Z_{pu} = \frac{Z}{Z_B}$$

where the numerators (actual values) are phasor quantities or complex values and the denominators (base values) are always real numbers. A minimum of four base quantities are required to completely define a per-unit system: volt-ampere, voltage, current, and impedance. Usually, the three-phase base volt-ampere S_B or MVA_B and the line-to-line base voltage V_B or kV_B are selected. Base current and base impedance are then dependent on S_B and V_B and must obey the circuit laws. These are given by

$$I_B = \frac{S_B}{\sqrt{3}\,V_B} \tag{3.79}$$

and

$$Z_B = \frac{V_B/\sqrt{3}}{I_B} \tag{3.80}$$

Substituting for I_B from (3.79), the base impedance becomes

$$Z_B = \frac{(V_B)^2}{S_B}$$
$$Z_B = \frac{(kV_B)^2}{MVA_B} \tag{3.81}$$

The phase and line quantities expressed in per-unit are the same, and the circuit laws are valid, i.e.,

$$S_{pu} = V_{pu} I_{pu}^* \tag{3.82}$$

and

$$V_{pu} = Z_{pu} I_{pu} \tag{3.83}$$

The load power at its rated voltage can also be expressed by a per-unit impedance. If $S_{L(3\phi)}$ is the complex load power, the load current per phase at the phase voltage V_P is given by

$$S_{L(3\phi)} = 3 V_P I_P^* \tag{3.84}$$

The phase current in terms of the ohmic load impedance is

$$I_P = \frac{V_P}{Z_P} \tag{3.85}$$

Substituting for I_P from (3.85) into (3.84) results in the ohmic value of the load impedance

$$Z_P = \frac{3|V_P|^2}{S_{L(3\phi)}^*} = \frac{|V_{L-L}|^2}{S_{L(3\phi)}^*} \tag{3.86}$$

From (3.81) the load impedance in per-unit is

$$Z_{pu} = \frac{Z_P}{Z_B} = \left|\frac{V_{L-L}}{V_B}\right|^2 \frac{S_B}{S_{L(3\phi)}^*} \tag{3.87}$$

or

$$Z_{pu} = \frac{|V_{pu}|^2}{S_{L(pu)}^*} \tag{3.88}$$

3.14 CHANGE OF BASE

The impedance of individual generators and transformers, as supplied by the manufacturer, are generally in terms of percent or per-unit quantities based on their own ratings. The impedance of transmission lines are usually expressed by their ohmic values. For power system analysis, all impedances must be expressed in per unit on a common system base. To accomplish this, an arbitrary base for apparent power is selected; for example, 100 MVA. Then, the voltage bases must be selected. Once a voltage base has been selected for a point in a system, the remaining voltage bases are no longer independent; they are determined by the various transformer turns ratios. For example, if on a low-voltage side of a 34.5/115-kV transformer the base voltage of 36 kV is selected, the base voltage on the high-voltage side must be $36(115/34.5) = 120$ kV. Normally, we try to select the voltage bases that are the same as the nominal values.

Let Z_{pu}^{old} be the per-unit impedance on the power base S_B^{old} and the voltage base V_B^{old}, which is expressed by

$$Z_{pu}^{old} = \frac{Z_\Omega}{Z_B^{old}} = Z_\Omega \frac{S_B^{old}}{(V_B^{old})^2} \tag{3.89}$$

Expressing Z_Ω to a new power base and a new voltage base, results in the new per-unit impedance

$$Z_{pu}^{new} = \frac{Z_\Omega}{Z_B^{new}} = Z_\Omega \frac{S_B^{new}}{(V_B^{new})^2} \tag{3.90}$$

From (3.89) and (3.90), the relationship between the old and the new per-unit values is

$$Z_{pu}^{new} = Z_{pu}^{old} \frac{S_B^{new}}{S_B^{old}} \left(\frac{V_B^{old}}{V_B^{new}} \right)^2 \quad (3.91)$$

If the voltage bases are the same, (3.91) reduces to

$$Z_{pu}^{new} = Z_{pu}^{old} \frac{S_B^{new}}{S_B^{old}} \quad (3.92)$$

The advantages of the per-unit system for analysis are described below.

- The per-unit system gives us a clear idea of relative magnitudes of various quantities, such as voltage, current, power and impedance.
- The per-unit impedance of equipment of the same general type based on their own ratings fall in a narrow range regardless of the rating of the equipment. Whereas their impedance in ohms vary greatly with the rating.
- The per-unit values of impedance, voltage and current of a transformer are the same regardless of whether they are referred to the primary or the secondary side. This is a great advantage since the different voltage levels disappear and the entire system reduces to a system of simple impedance.
- The per-unit systems are ideal for the computerized analysis and simulation of complex power system problems.
- The circuit laws are valid in per-unit systems, and the power and voltage equations as given by (3.82) and (3.83) are simplified since the factors of $\sqrt{3}$ and 3 are eliminated in the per-unit system.

Example 3.7 demonstrates how a per-unit impedance diagram is obtained for a simple power system network.

Example 3.7 (chp3ex7)

The one-line diagram of a three-phase power system is shown in Figure 3.29. Select a common base of 100 MVA and 22 kV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in per-unit. The manufacturer's data for each device is given as follow:

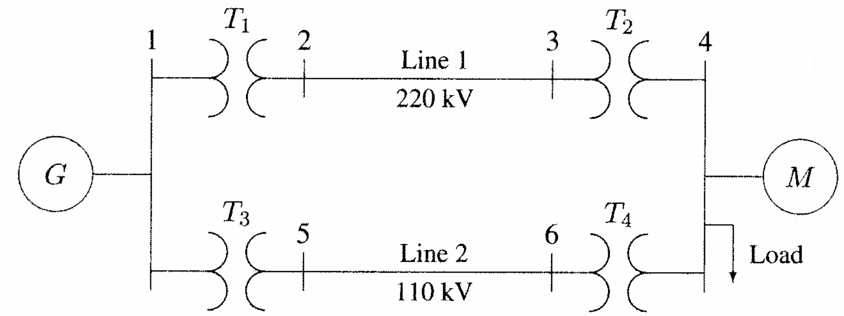


FIGURE 3.29
One-line diagram for Example 3.7.

G:	90 MVA	22 kV	$X = 18$ %
T_1:	50 MVA	22/220 kV	$X = 10$ %
T_2:	40 MVA	220/11 kV	$X = 6.0$ %
T_3:	40 MVA	22/110 kV	$X = 6.4$ %
T_4:	40 MVA	110/11 kV	$X = 8.0$ %
M:	66.5 MVA	10.45 kV	$X = 18.5$ %

The three-phase load at bus 4 absorbs 57 MVA, 0.6 power factor lagging at 10.45 kV. Line 1 and line 2 have reactances of 48.4 and 65.43 Ω, respectively.

First, the voltage bases must be determined for all sections of the network. The generator rated voltage is given as the base voltage at bus 1. This fixes the voltage bases for the remaining buses in accordance to the transformer turns ratios. The base voltage V_{B1} on the LV side of T_1 is 22 kV. Hence the base on its HV side is

$$V_{B2} = 22(\frac{220}{22}) = 220 \quad \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B3} = 220$ kV, and on its LV side at

$$V_{B4} = 220(\frac{11}{220}) = 11 \quad \text{kV}$$

Similarly, the voltage base at buses 5 and 6 are

$$V_{B5} = V_{B6} = 22(\frac{110}{22}) = 110 \quad \text{kV}$$

Since generator and transformer voltage bases are the same as their rated values, their per-unit reactances on a 100 MVA base, from (3.92) are

$$G:\ X = 0.18\left(\frac{100}{90}\right) = 0.20 \quad \text{pu}$$

$$T_1:\ X = 0.10\left(\frac{100}{50}\right) = 0.20 \quad \text{pu}$$

$$T_2:\ X = 0.06\left(\frac{100}{40}\right) = 0.15 \quad \text{pu}$$

$$T_3:\ X = 0.064\left(\frac{100}{40}\right) = 0.16 \quad \text{pu}$$

$$T_4:\ X = 0.08\left(\frac{100}{40}\right) = 0.2 \quad \text{pu}$$

The motor reactance is expressed on its nameplate rating of 66.5 MVA and 10.45 kV. However, the base voltage at bus 4 for the motor is 11 kV. From (3.91) the motor reactance on a 100 MVA, 11-kV base is

$$M:\ X = 0.185\left(\frac{100}{66.5}\right)\left(\frac{10.45}{11}\right)^2 = 0.25 \quad \text{pu}$$

Impedance bases for lines 1 and 2, from (3.81) are

$$Z_{B2} = \frac{(220)^2}{100} = 484 \quad \Omega$$

$$Z_{B5} = \frac{(110)^2}{100} = 121 \quad \Omega$$

Line 1 and 2 per-unit reactances are

$$\text{Line 1:}\ X = \left(\frac{48.4}{484}\right) = 0.10 \quad \text{pu}$$

$$\text{Line 2:}\ X = \left(\frac{65.43}{121}\right) = 0.54 \quad \text{pu}$$

The load apparent power at 0.6 power factor lagging is given by

$$S_{L(3\phi)} = 57\angle 53.13^\circ \quad \text{MVA}$$

Hence, the load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(10.45)^2}{57\angle -53.13^\circ} = 1.1495 + j1.53267 \quad \Omega$$

The base impedance for the load is

$$Z_{B4} = \frac{(11)^2}{100} = 1.21 \quad \Omega$$

Therefore, the load impedance in per-unit is

$$Z_{L(pu)} = \frac{1.1495 + j1.53267}{1.21} = 0.95 + j1.2667 \quad \text{pu}$$

The per-unit equivalent circuit is shown in Figure 3.30.

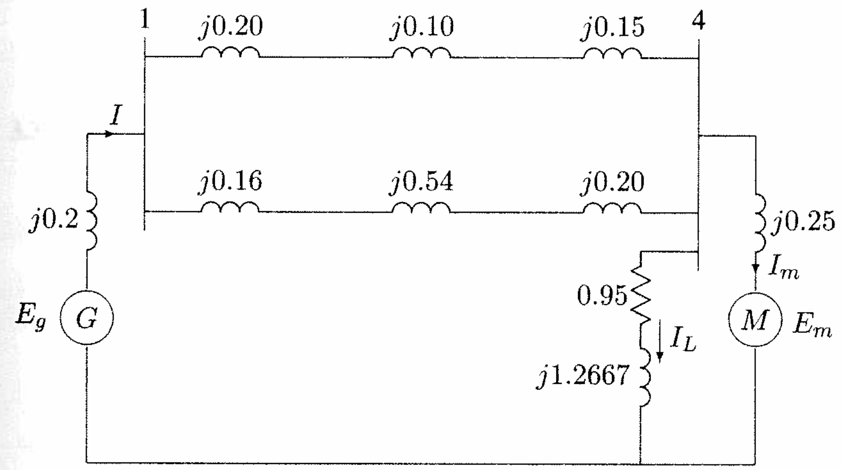


FIGURE 3.30
Per-unit impedance diagram for Example 3.7.

Example 3.8 (chp3ex8)

The motor of Example 3.7 operates at full-load 0.8 power factor leading at a terminal voltage of 10.45 kV.

(a) Determine the voltage at the generator bus bar (bus 1).
(b) Determine the generator and the motor internal emfs.

(a) The per-unit voltage at bus 4, taken as reference is

$$V_4 = \frac{10.45}{11} = 0.95\angle 0^\circ \quad \text{pu}$$

The motor apparent power at 0.8 power factor leading is given by

$$S_m = \frac{66.5}{100}\angle -36.87^\circ \quad \text{pu}$$

Therefore, current drawn by the motor is

$$I_m = \frac{S_m^*}{V_4^*} = \frac{0.665\angle 36.87}{0.95\angle 0^\circ} = 0.56 + j0.42 \quad \text{pu}$$

and current drawn by the load is

$$I_L = \frac{V_4}{Z_L} = \frac{0.95\angle 0^\circ}{0.95 + j1.2667} = 0.36 - j0.48 \quad \text{pu}$$

Total current drawn from bus 4 is

$$I = I_m + I_L = (0.56 + j0.42) + (0.36 - j0.48) = 0.92 - j0.06 \quad \text{pu}$$

The equivalent reactance of the parallel branches is

$$X_{\parallel} = \frac{0.45 \times 0.9}{0.45 + 0.9} = 0.3 \quad \text{pu}$$

The generator terminal voltage is

$$\begin{aligned} V_1 = V_4 + Z_{\parallel} I = 0.95\angle 0^\circ + j0.3(0.92 - j0.06) &= 0.968 + j0.276 \\ &= 1.0\angle 15.91^\circ \quad \text{pu} \\ &= 22\angle 15.91^\circ \quad \text{kV} \end{aligned}$$

(b) The generator internal emf is

$$\begin{aligned} E_g = V_1 + Z_g I = 0.968 + j0.276 + j0.20(0.92 - j0.06) &= 1.0826\angle 25.14^\circ \quad \text{pu} \\ &= 23.82\angle 25.14^\circ \quad \text{kV} \end{aligned}$$

and the motor internal emf is

$$\begin{aligned} E_m = V_4 - Z_m I_m = 0.95 + j0 - j0.25(0.56 + j0.42) &= 1.064\angle{-7.56^\circ} \quad \text{pu} \\ &= 11.71\angle{-7.56^\circ} \quad \text{kV} \end{aligned}$$

PROBLEMS

3.1. A three-phase, 318.75-kVA, 2300-V alternator has an armature resistance of 0.35 Ω/phase and a synchronous reactance of 1.2 Ω/phase. Determine the no-load line-to-line generated voltage and the voltage regulation at

(a) Full-load kVA, 0.8 power factor lagging, and rated voltage.

(b) Full-load kVA, 0.6 power factor leading, and rated voltage.

3.2. A 60-MVA, 69.3-kV, three-phase synchronous generator has a synchronous reactance of 15 Ω/phase and negligible armature resistance.

(a) The generator is delivering rated power at 0.8 power factor lagging at the rated terminal voltage to an infinite bus bar. Determine the magnitude of the generated emf per phase and the power angle δ.

(b) If the generated emf is 36 kV per phase, what is the maximum three-phase power that the generator can deliver before losing its synchronism?

(c) The generator is delivering 48 MW to the bus bar at the rated voltage with its field current adjusted for a generated emf of 46 kV per phase. Determine the armature current and the power factor. State whether power factor is lagging or leading?

3.3. A 24,000-kVA, 17.32-kV, 60-Hz, three-phase synchronous generator has a synchronous reactance of 5 Ω/phase and negligible armature resistance.

(a) At a certain excitation, the generator delivers rated load, 0.8 power factor lagging to an infinite bus bar at a line-to-line voltage of 17.32 kV. Determine the excitation voltage per phase.

(b) The excitation voltage is maintained at 13.4 kV/phase and the terminal voltage at 10 kV/phase. What is the maximum three-phase real power that the generator can develop before pulling out of synchronism?

(c) Determine the armature current for the condition of part (b).

3.4. A 34.64-kV, 60-MVA, three-phase salient-pole synchronous generator has a direct axis reactance of 13.5 Ω and a quadrature-axis reactance of 9.333 Ω. The armature resistance is negligible.

(a) Referring to the phasor diagram of a salient-pole generator shown in Figure 3.8, show that the power angle δ is given by

$$\delta = \tan^{-1}\left(\frac{X_q|I_a|\cos\theta}{|V| + X_q|I_a|\sin\theta}\right)$$

(b) Compute the load angle δ and the per phase excitation voltage E when the generator delivers rated MVA, 0.8 power factor lagging to an infinite bus bar of 34.64-kV line-to-line voltage.

(c) The generator excitation voltage is kept constant at the value found in part (b). Use *MATLAB* to obtain a plot of the power angle curve, i.e., equation (3.32) over a range of δ = 0:0.05:180°. Use the command **[Pmax, k] = max(P)**; **dmax = d(k)**, to obtain the steady-state maximum power **Pmax** and the corresponding power angle dmax.

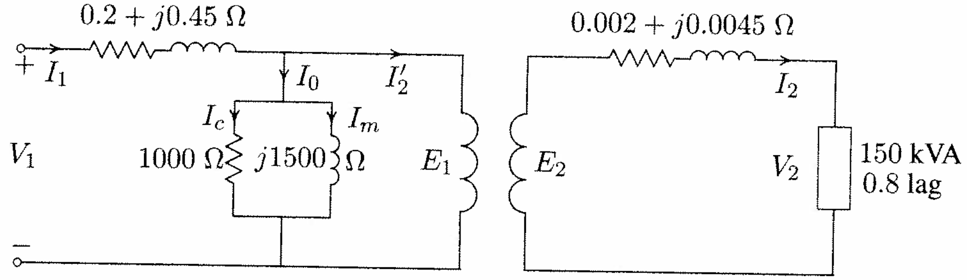


FIGURE 3.31
Transformer circuit for Problem 3.5

3.5. A 150-kVA, 2400/240-V single-phase transformer has the parameters as shown in Figure 3.31.

(a) Determine the equivalent circuit referred to the high-voltage side.

(b) Find the primary voltage and voltage regulation when transformer is operating at full load 0.8 power factor lagging and 240 V.

(c) Find the primary voltage and voltage regulation when the transformer is operating at full-load 0.8 power factor leading.

(d) Verify your answers by running the **trans** program in *MATLAB* and obtain the transformer efficiency curve.

3.6. A 60-kVA, 4800/2400-V single-phase transformer gave the following test results:

1. Rated voltage is applied to the low voltage winding and the high voltage winding is open-circuited. Under this condition, the current into the low voltage winding is 2.4 A and the power taken from the 2400 V source is 3456 W.

2. A reduced voltage of 1250 V is applied to the high voltage winding and the low voltage winding is short-circuited. Under this condition, the current flowing into the high voltage winding is 12.5 A and the power taken from the 1250 V source is 4375 W.

(a) Determine parameters of the equivalent circuit referred to the high voltage side.

(b) Determine voltage regulation and efficiency when transformer is operating at full-load, 0.8 power factor lagging, and a terminal voltage of 2400 V.

(c) What is the load kVA for maximum efficiency and the maximum efficiency at 0.8 power factor?

(d) Determine the efficiency when transformer is operating at 3/4 full-load, 0.8 power factor lagging, and a terminal voltage of 2400 V.

(e) Verify your answers by running the **trans** program in *MATLAB* and obtain the transformer efficiency curve.

3.7. A two-winding transformer rated at 9-kVA, 120/90-V, 60-HZ has a core loss of 200 W and a full-load copper loss of 500 W.

(a) The above transformer is to be connected as an auto transformer to supply a load at 120 V from a 210-V source. What kVA load can be supplied without exceeding the current rating of the windings? (For this part assume an ideal transformer.)

(b) Find the efficiency with the kVA loading of part (a) and 0.8 power factor.

3.8. Three identical 9-MVA, 7.2-kV/4.16-kV, single-phase transformers are connected in wye on the high-voltage side and delta on the low voltage side. The equivalent series impedance of each transformer referred to the high-voltage side is $0.12 + j0.82\ \Omega$ per phase. The transformer supplies a balanced three-phase load of 18 MVA, 0.8 power factor lagging at 4.16 kV. Determine the line-to-line voltage at the high-voltage terminals of the transformer.

3.9. A 400-MVA, 240-kV/24-kV, three-phase Y-Δ transformer has an equivalent series impedance of $1.2 + j6\ \Omega$ per phase referred to the high-voltage side. The transformer is supplying a three-phase load of 400-MVA, 0.8 power factor lagging at a terminal voltage of 24 kV (line to line) on its low-voltage side. The primary is supplied from a feeder with an impedance of $0.6 + j1.2\ \Omega$ per phase. Determine the line-to-line voltage at the high-voltage terminals of the transformer and the sending-end of the feeder.

3.10. In Problem 3.9, with transformer rated values as base quantities, express all impedances in per-unit. Working with per-unit values, determine the line-to-line voltage at the high-voltage terminals of the transformer and the sending-end of the feeder.

3.11. A three-phase, Y-connected, 75-MVA, 27-kV synchronous generator has a synchronous reactance of 9.0 Ω per phase. Using rated MVA and voltage as base values, determine the per-unit reactance. Then refer this per-unit value to a 100-MVA, 30-kV base.

3.12. A 40-MVA, 20-kV/400-kV, single-phase transformer has the following series impedances:

$Z_1 = 0.9 + j1.8\ \Omega$ and $Z_2 = 128 + j288\ \Omega$

Using the transformer rating as base, determine the per-unit impedance of the transformer from the ohmic value referred to the low-voltage side. Compute the per-unit impedance using the ohmic value referred to the high-voltage side.

3.13. Draw an impedance diagram for the electric power system shown in Figure 3.32 showing all impedances in per unit on a 100-MVA base. Choose 20-kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

G_1 :	90 MVA	20 kV	$X = 9\%$
T_1 :	80 MVA	20/200 kV	$X = 16\%$
T_2 :	80 MVA	200/20 kV	$X = 20\%$
G_2 :	90 MVA	18 kV	$X = 9\%$
Line:		200 kV	$X = 120\ \Omega$
Load:		200 kV	$S = 48$ MW $+j64$ Mvar

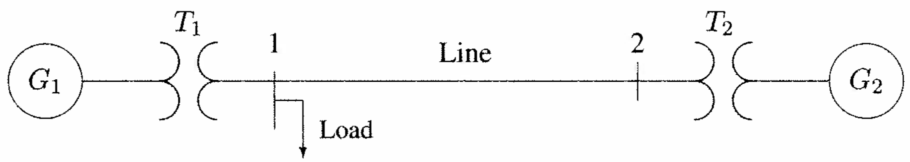


FIGURE 3.32
One-line diagram for Problem 3.13

3.14. The one-line diagram of a power system is shown in Figure 3.33.

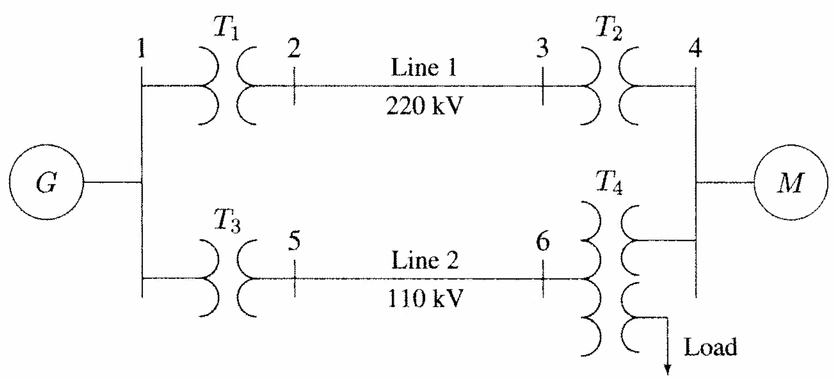


FIGURE 3.33
One-line diagram for Problem 3.14

The three-phase power and line-line ratings are given below.

G:	80 MVA	22 kV	$X = 24\%$
T_1:	50 MVA	22/220 kV	$X = 10\%$
T_2:	40 MVA	220/22 kV	$X = 6.0\%$
T_3:	40 MVA	22/110 kV	$X = 6.4\%$
Line 1:		220 kV	$X = 121\ \Omega$
Line 2:		110 kV	$X = 42.35\ \Omega$
M:	68.85 MVA	20 kV	$X = 22.5\%$
Load:	10 Mvar	4 kV	Δ-connected capacitors

The three-phase ratings of the three-winding transformer are

Primary:	Y-connected	40MVA, 110 kV
Secondary:	Y-connected	40 MVA, 22 kV
Tertiary:	Δ-connected	15 MVA, 4 kV

The per phase measured reactances at the terminal of a winding with the second one short-circuited and the third open-circuited are

$Z_{ps} = 9.6\%$ 40 MVA, 110 kV/22 kV
$Z_{pt} = 7.2\%$ 40 MVA, 110 kV/4 kV
$Z_{st} = 12\%$ 40 MVA, 22kV/4 kV

Obtain the T-circuit equivalent impedances of the three-winding transformer to the common 100-MVA base. Draw an impedance diagram showing all impedances in per-unit on a 100-MVA base. Choose 22 kV as the voltage base for generator.

3.15. The three-phase power and line-line ratings of the electric power system shown in Figure 3.34 are given below.

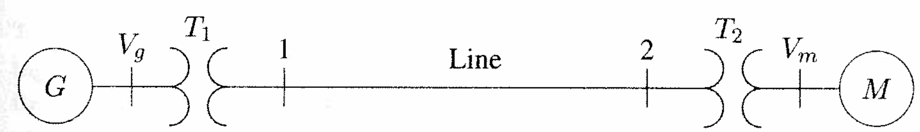


FIGURE 3.34
One-line diagram for Problem 3.15

G_1 :	60 MVA	20 kV	$X = 9\%$
T_1 :	50 MVA	20/200 kV	$X = 10\%$
T_2 :	50 MVA	200/20 kV	$X = 10\%$
M :	43.2 MVA	18 kV	$X = 8\%$
Line:		200 kV	$Z = 120 + j200\ \Omega$

(a) Draw an impedance diagram showing all impedances in per-unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

(b) The motor is drawing 45 MVA, 0.80 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per-unit and in kV.

3.16. The one-line diagram of a three-phase power system is as shown in Figure 3.35. Impedances are marked in per-unit on a 100-MVA, 400-kV base. The load at bus 2 is $S_2 = 15.93$ MW $-j33.4$ Mvar, and at bus 3 is $S_3 = 77$ MW $+j14$ Mvar. It is required to hold the voltage at bus 3 at $400\angle 0°$ kV. Working in per-unit, determine the voltage at buses 2 and 1.

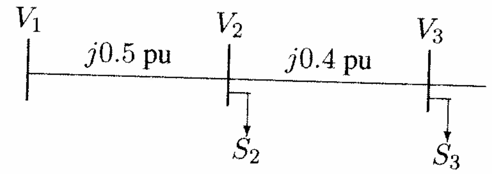


FIGURE 3.35
One-line diagram for Problem 3.16

3.17. The one-line diagram of a three-phase power system is as shown in Figure 3.36. The transformer reactance is 20 percent on a base of 100 MVA, 23/115 kV and the line impedance is $Z = j66.125\Omega$. The load at bus 2 is $S_2 = 184.8$ MW $+j6.6$ Mvar, and at bus 3 is $S_3 = 0$ MW $+j20$ Mvar. It is required to hold the voltage at bus 3 at $115\angle 0°$ kV. Working in per-unit, determine the voltage at buses 2 and 1.

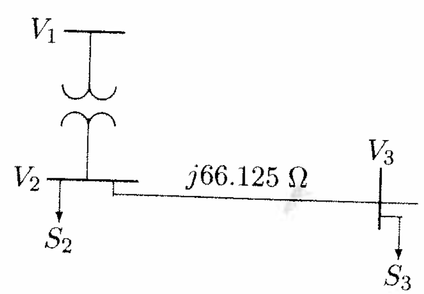


FIGURE 3.36
One-line diagram for Problem 3.17

CHAPTER
4
TRANSMISSION LINE PARAMETERS

4.1 INTRODUCTION

The purpose of a transmission network is to transfer electric energy from generating units at various locations to the distribution system which ultimately supplies the load. Transmission lines also interconnect neighboring utilities which permits not only economic dispatch of power within regions during normal conditions, but also transfer of power between regions during emergencies.

All transmission lines in a power system exhibit the electrical properties of resistance, inductance, capacitance, and conductance. The inductance and capacitance are due to the effects of magnetic and electric fields around the conductor. These parameters are essential for the development of the transmission line models used in power system analysis. The shunt conductance accounts for leakage currents flowing across insulators and ionized pathways in the air. The leakage currents are negligible compared to the current flowing in the transmission lines and may be neglected.

The first part of this chapter deals with the determination of inductance and capacitance of overhead lines. The concept of *geometric mean radius*, GMR and *geometric mean distance* GMD are discussed, and the function **[GMD, GMRL,**

GMRC] = gmd is developed for the evaluation of GMR and GMD. This function is very useful for computing the inductance and capacitance of single-circuit or double-circuit transmission lines with bundled conductors. Alternatively, the function **[L, C] = gmd2LC** returns the line inductance in mH per km and the shunt capacitance in μF per km. Finally the effects of electromagnetic and electrostatic induction are discussed. A new GUI program named **lcgui** is developed for the computation of transmission line parameters. This is a user-friendly program, which makes the data entry for various configurations very easy.

4.2 OVERHEAD TRANSMISSION LINES

A transmission circuit consists of conductors, insulators, and usually shield wires, as shown in Figure 4.1. Transmission lines are hung overhead from a tower usually made of steel, wood or reinforced concrete with its own right-of-way. Steel towers may be single-circuit or double-circuit designs. Multicircuit steel towers have been built, where the tower supports three to ten 69-kV lines over a given width of right-of-way. Less than 1 percent of the nation's total transmission lines are placed underground. Although underground ac transmission would present a solution to some of the environmental and aesthetic problems involved with overhead transmission lines, there are technical and economic reasons that make the use of underground ac transmission prohibitive.

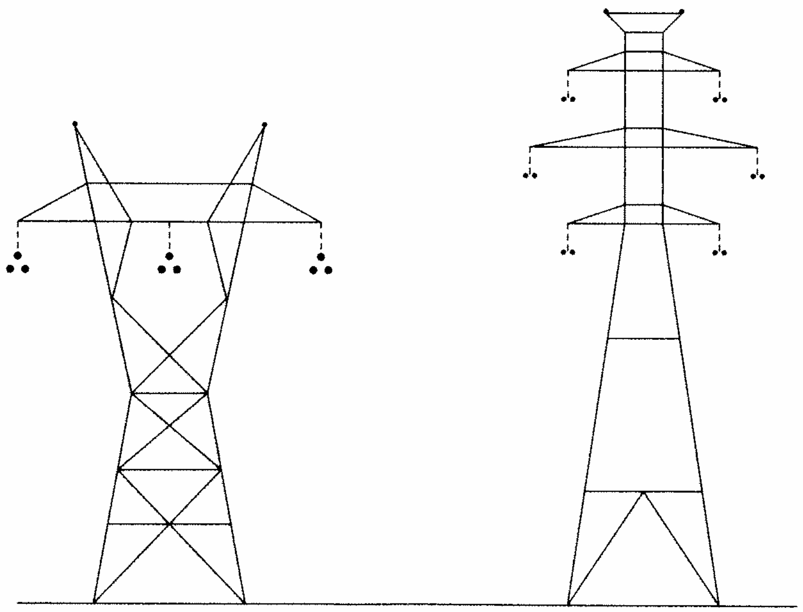


FIGURE 4.1
Typical lattice-type structure for 345-kV transmission line.

The selection of an economical voltage level for the transmission line is based on the amount of power and the distance of transmission. The voltage choice together with the selection of conductor size is mainly a process of weighing RI^2 losses, audible noise, and radio interference level against fixed charges on the investment. Standard transmission voltages are established in the United States by the American National Standards Institute (ANSI). Transmission voltage lines operating at more than 60 kV are standardized at 69 kV, 115 kV, 138 kV, 161 kV, 230 kV, 345 kV, 500 kV, 765 kV line-to-line. Transmission voltages above 230 kV are usually referred to as *extra-high voltage* (EHV) and those at 765 kV and above are referred to as *ultra-high voltage* (UHV). The most commonly used conductor materials for high voltage transmission lines are ACSR (aluminum conductor steel-reinforced), AAC (all-aluminum conductor), AAAC (all-aluminum alloy conductor), and ACAR (aluminum conductor alloy-reinforced). The reason for their popularity is their low relative cost and high strength-to-weight ratio as compared to copper conductors. Also, aluminum is in abundant supply, while copper is limited in quantity. A table of the most commonly used ACSR conductors is stored in file **acsr.m** Characteristics of other conductors can be found in conductor handbooks or manufacturer's literature. The conductors are stranded to have flexibility. The ACSR conductor consists of a center core of steel strands surrounded by layers of aluminum as shown in Figure 4.2. Each layer of strands is spiraled in the opposite direction of its adjacent layer. This spiraling holds the strands in place. The script file **acsr.m** has been updated to the GUI program named **acsrgui**.

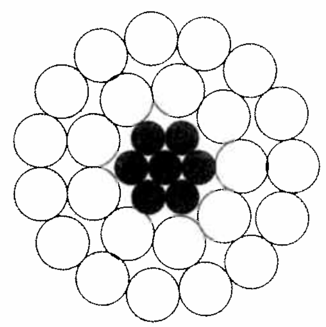


FIGURE 4.2
Cross-sectional view of a 24/7 ACSR conductor.

Conductor manufacturers provide the characteristics of the standard conductors with conductor sizes expressed in *circular mils* (cmil). One mil equals 0.001 inch, and for a solid round conductor the area in circular mils is defined as the square of diameter in mils. As an example, 1,000,000 cmil represents an area of a solid round conductor 1 inch in diameter. In addition, code words (bird names) have been assigned to each conductor for easy reference.

At voltages above 230 kV, it is preferable to use more than one conductor per phase, which is known as *bundling* of conductors. The bundle consists of two, three, or four conductors. Bundling increases the effective radius of the line's conductor and reduces the electric field strength near the conductors, which reduces corona power loss, audible noise, and radio interference. Another important advantage of bundling is reduced line reactance.

4.3 LINE RESISTANCE

The resistance of the conductor is very important in transmission efficiency evaluation and economic studies. The dc resistance of a solid round conductor at a specified temperature is given by

$$R_{dc} = \frac{\rho l}{A} \tag{4.1}$$

where ρ = conductor resistivity
l = conductor length
A = conductor cross-sectional area

The conductor resistance is affected by three factors: frequency, spiraling, and temperature.

When ac flows in a conductor, the current distribution is not uniform over the conductor cross-sectional area and the current density is greatest at the surface of the conductor. This causes the ac resistance to be somewhat higher than the dc resistance. This behavior is known as *skin effect*. At 60 Hz, the ac resistance is about 2 percent higher than the dc resistance.

Since a stranded conductor is spiraled, each strand is longer than the finished conductor. This results in a slightly higher resistance than the value calculated from 4.1.

The conductor resistance increases as temperature increases. This change can be considered linear over the range of temperature normally encountered and may be calculated from

$$R_2 = R_1 \frac{T + t_2}{T + t_1} \tag{4.2}$$

where R_2 and R_1 are conductor resistances at t_2 and t_1-C°, respectively. T is a temperature constant that depends on the conductor material. For aluminum $T \simeq$ 228.

Because of the above effects, the conductor resistance is best determined from manufacturers' data.

4.4 INDUCTANCE OF A SINGLE CONDUCTOR

A current-carrying conductor produces a magnetic field around the conductor. The magnetic flux lines are concentric closed circles with direction given by the right-hand rule. With the thumb pointing in the direction of the current, the fingers of the right hand encircled the wire point in the direction of the magnetic field. When the current changes, the flux changes and a voltage is induced in the circuit. By definition, for nonmagnetic material, the inductance L is the ratio of its total magnetic flux linkage to the current I, given by

$$L = \frac{\lambda}{I} \tag{4.3}$$

where λ = flux linkages, in Weber turns.

Consider a long round conductor with radius r, carrying a current I as shown in Figure 4.3.

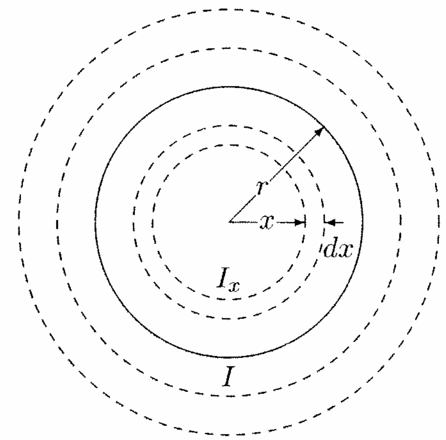


FIGURE 4.3
Flux linkage of a long round conductor.

The magnetic field intensity H_x, around a circle of radius x, is constant and tangent to the circle. The Ampere's law relating H_x to the current I_x is given by

$$\int_0^{2\pi x} H_x \cdot dl = I_x \tag{4.4}$$

or

$$H_x = \frac{I_x}{2\pi x} \tag{4.5}$$

where I_x is the current enclosed at radius x. As shown in Figure 4.3, Equation (4.5) is all that is required for evaluating the flux linkage λ of a conductor. The inductance of the conductor can be defined as the sum of contributions from flux linkages internal and external to the conductor.

4.4.1 INTERNAL INDUCTANCE

A simple expression can be obtained for the internal flux linkage by neglecting the skin effect and assuming uniform current density throughout the conductor cross section, i.e.,

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \tag{4.6}$$

Substituting for I_x in (4.5) yields

$$H_x = \frac{I}{2\pi r^2}x \tag{4.7}$$

For a nonmagnetic conductor with constant permeability μ_0, the magnetic flux density is given by $B_x = \mu_0 H_x$, or

$$B_x = \frac{\mu_0 I}{2\pi r^2}x \tag{4.8}$$

where μ_0 is the permeability of free space (or air) and is equal to $4\pi \times 10^{-7}$H/m. The differential flux $d\phi$ for a small region of thickness dx and one meter length of the conductor is

$$d\phi_x = B_x dx \cdot 1 = \frac{\mu_0 I}{2\pi r^2}x dx \tag{4.9}$$

The flux $d\phi_x$ links only the fraction of the conductor from the center to radius x. Thus, on the assumption of uniform current density, only the fraction $\pi x^2/\pi r^2$ of the total current is linked by the flux, i.e.,

$$d\lambda_x = (\frac{x^2}{r^2})d\phi_x = \frac{\mu_0 I}{2\pi r^4}x^3 dx \tag{4.10}$$

The total flux linkage is found by integrating $d\lambda_x$ from 0 to r.

$$\begin{aligned}\lambda_{int} &= \frac{\mu_0 I}{2\pi r^4}\int_0^r x^3 dx \\ &= \frac{\mu_0 I}{8\pi} \quad \text{Wb/m}\end{aligned} \tag{4.11}$$

From (4.3), the inductance due to the internal flux linkage is

$$L_{int} = \frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7} \quad \text{H/m} \tag{4.12}$$

Note that L_{int} is independent of the conductor radius r.

4.4.2 INDUCTANCE DUE TO EXTERNAL FLUX LINKAGE

Consider H_x external to the conductor at radius $x > r$ as shown in Figure 4.4. Since the circle at radius x encloses the entire current, $I_x = I$ and in (4.5) I_x is replaced by I and the flux density at radius x becomes

$$B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi x} \tag{4.13}$$

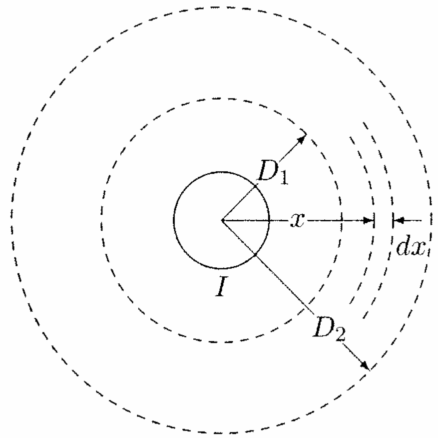


FIGURE 4.4
Flux linkage between D_1 and D_2.

Since the entire current I is linked by the flux outside the conductor, the flux linkage $d\lambda_x$ is numerically equal to the flux $d\phi_x$. The differential flux $d\phi_x$ for a small region of thickness dx and one meter length of the conductor is then given by

$$d\lambda_x = d\phi_x = B_x dx \cdot 1 = \frac{\mu_0 I}{2\pi x}dx \tag{4.14}$$

The external flux linkage between two points D_1 and D_2 is found by integrating $d\lambda_x$ from D_1 to D_2.

$$\begin{aligned}\lambda_{ext} &= \frac{\mu_0 I}{2\pi}\int_{D_1}^{D_2}\frac{1}{x}dx \\ &= 2 \times 10^{-7} I \ln\frac{D_2}{D_1} \quad \text{Wb/m}\end{aligned} \tag{4.15}$$

The inductance between two points external to a conductor is then

$$L_{ext} = 2 \times 10^{-7}\ln\frac{D_2}{D_1} \quad \text{H/m} \tag{4.16}$$

4.5 INDUCTANCE OF SINGLE-PHASE LINES

Consider one meter length of a single-phase line consisting of two solid round conductors of radius r_1 and r_2 as shown in Figure 4.5. The two conductors are separated by a distance D. Conductor 1 carries the phasor current I_1 referenced into the page and conductor 2 carries return current $I_2 = -I_1$. These currents set up magnetic field lines that links between the conductors as shown.

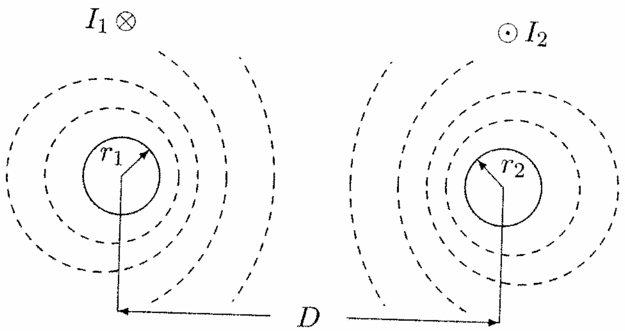


FIGURE 4.5
Single-phase two-wire line.

Inductance of conductor 1 due to internal flux is given by (4.12). The flux beyond D links a net current of zero and does not contribute to the net magnetic flux linkages in the circuit. Thus, to obtain the inductance of conductor 1 due to the net external flux linkage, it is necessary to evaluate (4.16) from $D_1 = r_1$ to $D_2 = D$.

$$L_{1(ext)} = 2 \times 10^{-7} \ln \frac{D}{r_1} \quad \text{H/m} \tag{4.17}$$

The total inductance of conductor 1 is then

$$L_1 = \frac{1}{2} \times 10^{-7} + 2 \times 10^{-7} \ln \frac{D}{r_1} \quad \text{H/m} \tag{4.18}$$

Equation (4.18) is often rearranged as follows:

$$\begin{aligned} L_1 &= 2 \times 10^{-7} \left(\frac{1}{4} + \ln \frac{D}{r_1} \right) \\ &= 2 \times 10^{-7} \left(\ln e^{\frac{1}{4}} + \ln \frac{1}{r_1} + \ln \frac{D}{1} \right) \\ &= 2 \times 10^{-7} \left(\ln \frac{1}{r_1 e^{-1/4}} + \ln \frac{D}{1} \right) \end{aligned} \tag{4.19}$$

Let $r_1' = r_1 e^{-\frac{1}{4}}$, the inductance of conductor 1 becomes

$$L_1 = 2 \times 10^{-7} \ln \frac{1}{r_1'} + 2 \times 10^{-7} \ln \frac{D}{1} \quad \text{H/m} \tag{4.20}$$

Similarly, the inductance of conductor 2 is

$$L_2 = 2 \times 10^{-7} \ln \frac{1}{r_2'} + 2 \times 10^{-7} \ln \frac{D}{1} \quad \text{H/m} \tag{4.21}$$

If the two conductors are identical, $r_1 = r_2 = r$ and $L_1 = L_2 = L$, and the inductance per conductor per meter length of the line is given by

$$L = 2 \times 10^{-7} \ln \frac{1}{r'} + 2 \times 10^{-7} \ln \frac{D}{1} \quad \text{H/m} \tag{4.22}$$

Examination of (4.22) reveals that the first term is only a function of the conductor radius. This term is considered as the inductance due to both the internal flux and that external to conductor 1 to a radius of 1 m. The second term of (4.22) is dependent only upon conductor spacing. This term is known as the *inductance spacing factor*. The above terms are usually expressed as inductive reactances at 60 Hz and are available in the manufacturers table in English units.

The term $r' = re^{-\frac{1}{4}}$ is known mathematically as the *self-geometric mean distance* of a circle with radius r and is abbreviated by GMR. r' can be considered as the radius of a fictitious conductor assumed to have no internal flux but with the same inductance as the actual conductor with radius r. GMR is commonly referred to as *geometric mean radius* and will be designated by D_s. Thus, the inductance per conductor in millihenries per kilometer becomes

$$L = 0.2 \ln \frac{D}{D_s} \quad \text{mH/km} \tag{4.23}$$

4.6 FLUX LINKAGE IN TERMS OF SELF- AND MUTUAL INDUCTANCES

The series inductance per phase for the above single-phase two-wire line can be expressed in terms of self-inductance of each conductor and their mutual inductance. Consider one meter length of the single-phase circuit represented by two coils characterized by the self-inductances L_{11} and L_{22} and the mutual inductance L_{12}. The magnetic polarity is indicated by dot symbols as shown in Figure 4.6.

The flux linkages λ_1 and λ_2 are given by

$$\begin{aligned} \lambda_1 &= L_{11} I_1 + L_{12} I_2 \\ \lambda_2 &= L_{21} I_1 + L_{22} I_2 \end{aligned} \tag{4.24}$$

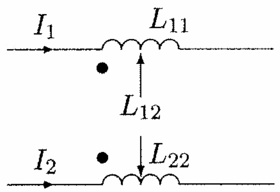


FIGURE 4.6
The single-phase line viewed as two magnetically coupled coils.

Since $I_2 = -I_1$, we have

$$\lambda_1 = (L_{11} - L_{12})I_1$$
$$\lambda_2 = (-L_{21} + L_{22})I_2 \tag{4.25}$$

Comparing (4.25) with (4.20) and (4.21), we conclude the following equivalent expressions for the self- and mutual inductances:

$$L_{11} = 2 \times 10^{-7} \ln \frac{1}{r_1'}$$
$$L_{22} = 2 \times 10^{-7} \ln \frac{1}{r_2'}$$
$$L_{12} = L_{21} = 2 \times 10^{-7} \ln \frac{1}{D} \tag{4.26}$$

The concept of self- and mutual inductance can be extended to a group of n conductors. Consider n conductors carrying phasor currents $I_1, I_2, \ldots, I_n$, such that

$$I_1 + I_2 + \cdots + I_i + \cdots + I_n = 0 \tag{4.27}$$

Generalizing (4.24), the flux linkages of conductor i are

$$\lambda_i = L_{ii}I_i + \sum_{j=1}^{n} L_{ij}I_j \qquad j \neq i \tag{4.28}$$

or

$$\lambda_i = 2 \times 10^{-7} \left(I_i \ln \frac{1}{r_i'} + \sum_{j=1}^{n} I_j \ln \frac{1}{D_{ij}} \right) \qquad j \neq i \tag{4.29}$$

4.7 INDUCTANCE OF THREE-PHASE TRANSMISSION LINES

4.7.1 SYMMETRICAL SPACING

Consider one meter length of a three-phase line with three conductors, each with radius r, symmetrically spaced in a triangular configuration as shown in Figure 4.7.

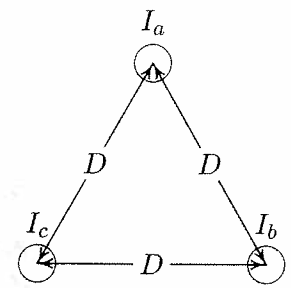


FIGURE 4.7
Three-phase line with symmetrical spacing.

Assuming balanced three-phase currents, we have

$$I_a + I_b + I_c = 0 \tag{4.30}$$

From (4.29) the total flux linkage of phase a conductor is

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D} \right) \tag{4.31}$$

Substituting for $I_b + I_c = -I_a$

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D} \right)$$
$$= 2 \times 10^{-7} I_a \ln \frac{D}{r'} \tag{4.32}$$

Because of symmetry, $\lambda_b = \lambda_c = \lambda_a$, and the three inductances are identical. Therefore, the inductance per phase per kilometer length is

$$L = 0.2 \ln \frac{D}{D_s} \text{ mH/km} \tag{4.33}$$

where r' is the geometric mean radius, GMR, and is shown by D_s. For a solid round conductor, $D_s = re^{-\frac{1}{4}}$ for stranded conductor D_s can be evaluated from (4.50). Comparison of (4.33) with (4.23) shows that inductance per phase for a three-phase circuit with equilateral spacing is the same as for one conductor of a single-phase circuit.

4.7.2 ASYMMETRICAL SPACING

Practical transmission lines cannot maintain symmetrical spacing of conductors because of construction considerations. With asymmetrical spacing, even with balanced currents, the voltage drop due to line inductance will be unbalanced. Consider one meter length of a three-phase line with three conductors, each with radius r. The Conductors are asymmetrically spaced with distances shown in Figure 4.8.

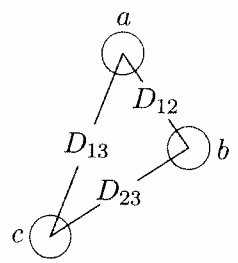


FIGURE 4.8
Three-phase line with asymmetrical spacing.

The application of (4.29) will result in the following flux linkages.

$$\lambda_a = 2\times 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{13}}\right)$$
$$\lambda_b = 2\times 10^{-7}\left(I_a \ln\frac{1}{D_{12}} + I_b \ln\frac{1}{r'} + I_c \ln\frac{1}{D_{23}}\right)$$
$$\lambda_c = 2\times 10^{-7}\left(I_a \ln\frac{1}{D_{13}} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{r'}\right) \qquad (4.34)$$

or in matrix form

$$\boldsymbol{\lambda} = \boldsymbol{LI} \qquad (4.35)$$

where the symmetrical inductance matrix L is given by

$$L = 2\times 10^{-7}\begin{bmatrix} \ln\frac{1}{r'} & \ln\frac{1}{D_{12}} & \ln\frac{1}{D_{13}} \\ \ln\frac{1}{D_{12}} & \ln\frac{1}{r'} & \ln\frac{1}{D_{23}} \\ \ln\frac{1}{D_{13}} & \ln\frac{1}{D_{23}} & \ln\frac{1}{r'} \end{bmatrix} \qquad (4.36)$$

For balanced three-phase currents with I_a as reference, we have

$$I_b = I_a\angle 240^\circ = a^2 I_a$$
$$I_c = I_a\angle 120^\circ = a I_a \qquad (4.37)$$

where the operator $a = 1\angle 120^\circ$ and $a^2 = 1\angle 240^\circ$. Substituting in (4.34) results in

$$L_a = \frac{\lambda_a}{I_a} = 2\times 10^{-7}\left(\ln\frac{1}{r'} + a^2 \ln\frac{1}{D_{12}} + a \ln\frac{1}{D_{13}}\right)$$
$$L_b = \frac{\lambda_b}{I_b} = 2\times 10^{-7}\left(a \ln\frac{1}{D_{12}} + \ln\frac{1}{r'} + a^2 \ln\frac{1}{D_{23}}\right)$$
$$L_c = \frac{\lambda_c}{I_c} = 2\times 10^{-7}\left(a^2 \ln\frac{1}{D_{13}} + a \ln\frac{1}{D_{23}} + \ln\frac{1}{r'}\right) \qquad (4.38)$$

Examination of (4.38) shows that the phase inductances are not equal and they contain an imaginary term due to the mutual inductance.

4.7.3 TRANSPOSE LINE

A per-phase model of the transmission line is required in most power system analysis. One way to regain symmetry in good measure and obtain a per-phase model is to consider transposition. This consists of interchanging the phase configuration every one-third the length so that each conductor is moved to occupy the next physical position in a regular sequence. Such a transposition arrangement is shown in Figure 4.9.

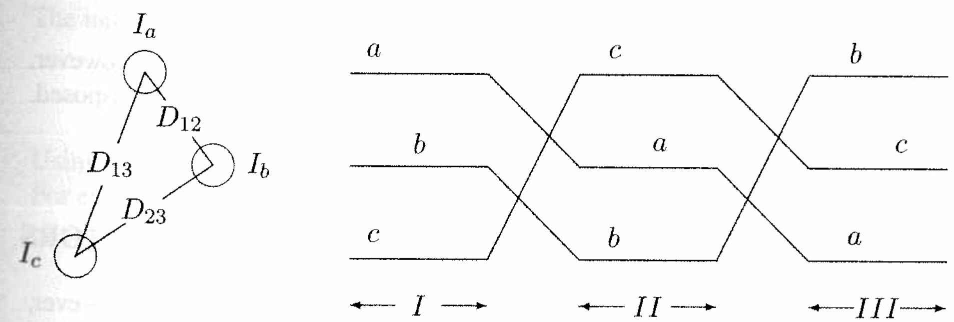


FIGURE 4.9
A transposed three-phase line.

Since in a transposed line each phase takes all three positions, the inductance per phase can be obtained by finding the average value of (4.38).

$$L = \frac{L_a + L_b + L_c}{3} \qquad (4.39)$$

Noting $a + a^2 = 1\angle 120^\circ + 1\angle 240^\circ = -1$, the average of (4.38) becomes

$$L \;=\; \frac{2\times 10^{-7}}{3}\left(3\ln\frac{1}{r'} - \ln\frac{1}{D_{12}} - \ln\frac{1}{D_{23}} - \ln\frac{1}{D_{13}}\right)$$

or

$$L = 2 \times 10^{-7}\left(\ln\frac{1}{r'} - \ln\frac{1}{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}\right)$$

$$= 2 \times 10^{-7}\ln\frac{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}{r'} \quad (4.40)$$

or the inductance per phase per kilometer length is

$$L = 0.2\ln\frac{GMD}{D_S} \text{ mH/km} \quad (4.41)$$

where

$$GMD = \sqrt[3]{D_{12}D_{23}D_{13}} \quad (4.42)$$

This again is of the same form as the expression for the inductance of one phase of a single-phase line. GMD (geometric mean distance) is the equivalent conductor spacing. For the above three-phase line this is the cube root of the product of the three-phase spacings. D_s is the geometric mean radius, GMR. For stranded conductor D_s is obtained from the manufacturer's data. For solid conductor, $D_s = r' = re^{-\frac{1}{4}}$.

In modern transmission lines, transposition is not generally used. However, for the purpose of modeling, it is most practical to treat the circuit as transposed. The error introduced as a result of this assumption is very small.

4.8 INDUCTANCE OF BUNDLED OR STRANDED CONDUCTORS

In the evaluation of inductance, solid round conductors were considered. However, in practical transmission lines, stranded conductors are used. Also, for reasons of economy, most EHV lines are constructed with bundled conductors. In this section an expression is found for the inductance of stranded or bundled conductors. The result can be used for evaluating the GMR of stranded or bundled conductors. It is also useful in finding the equivalent GMR and GMD of parallel circuits. Consider a single-phase line consisting of two bundled conductors x and y as shown in Figure 4.10. The current in x is I referenced into the page, and the return current in y is $-I$. Conductor x consists of n identical strands or subconductors, each with radius r_x. Conductor y consists of m identical strands or subconductors, each with radius r_y. The current is assumed to be equally divided among the subconductors. The current per strand is I/n in x and I/m in y. The application of (4.29) will result in the following expression for the total flux linkage of conductor a

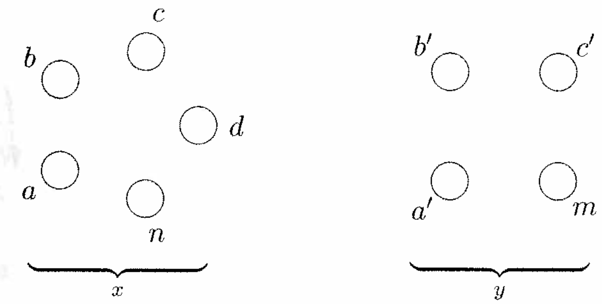


FIGURE 4.10
Single-phase line with two bundled conductors.

$$\lambda_a = 2 \times 10^{-7}\frac{I}{n}\left(\ln\frac{1}{r'_x} + \ln\frac{1}{D_{ab}} + \ln\frac{1}{D_{ac}} + \cdots + \ln\frac{1}{D_{an}}\right)$$
$$-2 \times 10^{-7}\frac{I}{m}\left(\ln\frac{1}{D_{aa'}} + \ln\frac{1}{D_{ab'}} + \ln\frac{1}{D_{ac'}} + \cdots + \ln\frac{1}{D_{am}}\right)$$

or

$$\lambda_a = 2 \times 10^{-7} I \ln\frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}\cdots D_{am}}}{\sqrt[n]{r'_x D_{ab}D_{ac}\cdots D_{an}}} \quad (4.43)$$

The inductance of subconductor a is

$$L_a = \frac{\lambda_a}{I/n} = 2n \times 10^{-7}\ln\frac{\sqrt[m]{D_{aa'}D_{ab'}D_{ac'}\cdots D_{am}}}{\sqrt[n]{r'_x D_{ab}D_{ac}\cdots D_{an}}} \quad (4.44)$$

Using (4.29), the inductance of other subconductors in x are similarly obtained. For example, the inductance of the subconductor n is

$$L_n = \frac{\lambda_n}{I/n} = 2n \times 10^{-7}\ln\frac{\sqrt[m]{D_{na'}D_{nb'}D_{nc'}\cdots D_{nm}}}{\sqrt[n]{r'_x D_{na}D_{nb}\cdots D_{nc}}} \quad (4.45)$$

The average inductance of any one subconductor in group x is

$$L_{av} = \frac{L_a + L_b + L_c + \cdots + L_n}{n} \quad (4.46)$$

Since all the subconductors of conductor x are electrically parallel, the inductance of x will be

$$L_x = \frac{L_{av}}{n} = \frac{L_a + L_b + L_c + \cdots + L_n}{n^2} \quad (4.47)$$

substituting the values of L_a, L_b, $L_c, \cdots, L_n$ in (4.47) results in

$$L_x = 2 \times 10^{-7}\ln\frac{GMD}{GMR_x} \text{ H/meter} \quad (4.48)$$

where

$$GMD = \sqrt[mn]{(D_{aa'}D_{ab'}\cdots D_{am})\cdots(D_{na'}D_{nb'}\cdots D_{nm})} \quad (4.49)$$

and

$$GMR_x = \sqrt[n^2]{(D_{aa}D_{ab}\cdots D_{an})\cdots(D_{na}D_{nb}\cdots D_{nn})} \quad (4.50)$$

where $D_{aa} = D_{bb}\cdots = D_{nn} = r'_x$

GMD is the mnth root of the product of the mnth distances between n strands of conductor x and m strands of conductor y. GMR_x is the n^2 root of the product of n^2 terms consisting of r' of every strand times the distance from each strand to all other strands within group x.

The inductance of conductor y can also be similarly obtained. The geometric mean radius GMR_y will be different. The geometric mean distance GMD, however, is the same.

Example 4.1 (chp4ex1)

A stranded conductor consists of seven identical strands each having a radius r as shown in Figure 4.11. Determine the GMR of the conductor in terms of r.

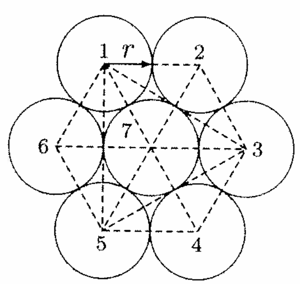


FIGURE 4.11
Cross section of a stranded conductor.

From Figure 4.11, the distance from strand 1 to all other strands is:

$$D_{12} = D_{16} = D_{17} = 2r$$
$$D_{14} = 4r$$
$$D_{13} = D_{15} = \sqrt{D_{14}^2 - D_{45}^2} = 2\sqrt{3}\,r$$

From (4.50) the GMR of the above conductor is

$$GMR = \sqrt[49]{(r' \cdot 2r \cdot 2\sqrt{3}\,r \cdot 4r \cdot 2\sqrt{3}\,r \cdot 2r \cdot 2r)^6 \cdot r'(2r)^6}$$

$$= r\sqrt[7]{(e)^{-\frac{1}{4}}\,(2)^6\,(3)^{\frac{6}{7}}\,(2)^{\frac{6}{7}}}$$
$$= 2.1767r$$

With a large number of strands the calculation of GMR can become very tedious. Usually these are available in the manufacturer's data.

4.8.1 *GMR* OF BUNDLED CONDUCTORS

Extra-high voltage transmission lines are usually constructed with bundled conductors. Bundling reduces the line reactance, which improves the line performance and increases the power capability of the line. Bundling also reduces the voltage surface gradient, which in turn reduces corona loss, radio interference, and surge impedance. Typically, bundled conductors consist of two, three, or four subconductors symmetrically arranged in configuration as shown in Figure 4.12. The subconductors within a bundle are separated at frequent intervals by spacer-dampers. Spacer-dampers prevent clashing, provide damping, and connect the subconductors in parallel.

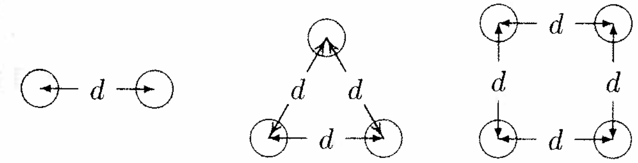


FIGURE 4.12
Examples of bundled arrangements.

The GMR of the equivalent single conductor is obtained by using (4.50). If D_s is the GMR of each subconductor and d is the bundle spacing, we have

for the two-subconductor bundle

$$D_s^b = \sqrt[4]{(D_s \times d)^2} = \sqrt{D_s \times d} \quad (4.51)$$

for the three-subconductor bundle

$$D_s^b = \sqrt[9]{(D_s \times d \times d)^3} = \sqrt[3]{D_s \times d^2} \quad (4.52)$$

for the four-subconductor bundle

$$D_s^b = \sqrt[16]{(D_s \times d \times d \times d \times 2^{\frac{1}{2}})^4} = 1.09\sqrt[4]{D_s \times d^3} \quad (4.53)$$

4.9 INDUCTANCE OF THREE-PHASE DOUBLE-CIRCUIT LINES

A three-phase double-circuit line consists of two identical three-phase circuits. The circuits are operated with a_1–a_2, b_1–b_2, and c_1–c_2 in parallel. Because of geometrical differences between conductors, voltage drop due to line inductance will be unbalanced. To achieve balance, each phase conductor must be transposed within its group and with respect to the parallel three-phase line. Consider a three-phase double-circuit line with relative phase positions $a_1b_1c_1$–$c_2b_2a_2$, as shown in Figure 4.13.

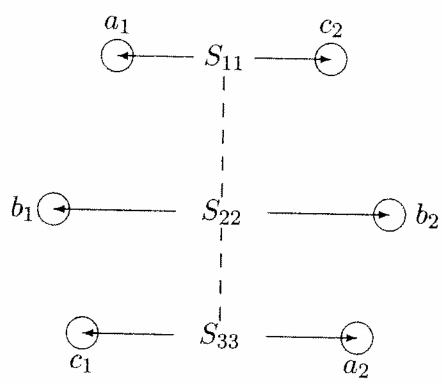


FIGURE 4.13
Transposed double-circuit line.

The method of GMD can be used to find the inductance per phase. To do this, we group identical phases together and use (4.49) to find the GMD between each phase group

$$D_{AB} = \sqrt[4]{D_{a_1b_1}D_{a_1b_2}D_{a_2b_1}D_{a_2b_2}}$$
$$D_{BC} = \sqrt[4]{D_{b_1c_1}D_{b_1c_2}D_{b_2c_1}D_{b_2c_2}}$$
$$D_{AC} = \sqrt[4]{D_{a_1c_1}D_{a_1c_2}D_{a_2c_1}D_{a_2c_2}} \tag{4.54}$$

The equivalent GMD per phase is then

$$GMD = \sqrt[3]{D_{AB}D_{BC}D_{AC}} \tag{4.55}$$

Similarly, from (4.50), the GMR of each phase group is

$$D_{SA} = \sqrt[4]{(D_s^bD_{a_1a_2})^2} = \sqrt{D_s^bD_{a_1a_2}}$$
$$D_{SB} = \sqrt[4]{(D_s^bD_{b_1b_2})^2} = \sqrt{D_s^bD_{b_1b_2}}$$
$$D_{SC} = \sqrt[4]{(D_s^bD_{c_1c_2})^2} = \sqrt{D_s^bD_{c_1c_2}} \tag{4.56}$$

where D_s^b is the geometric mean radius of the bundled conductors given by (4.51)–(4.53). The equivalent geometric mean radius for calculating the per-phase inductance to neutral is

$$GMR_L = \sqrt[3]{D_{SA}D_{SB}D_{SC}} \tag{4.57}$$

The inductance per phase in millihenries per kilometer is

$$L = 0.2 \ln \frac{GMD}{GMR_L} \quad \text{mH/km} \tag{4.58}$$

4.10 LINE CAPACITANCE

Transmission line conductors exhibit capacitance with respect to each other due to the potential difference between them. The amount of capacitance between conductors is a function of conductor size, spacing, and height above ground. By definition, the capacitance C is the ratio of charge q to the voltage V, given by

$$C = \frac{q}{V} \tag{4.59}$$

Consider a long round conductor with radius r, carrying a charge of q coulombs per meter length as shown in Figure 4.14.

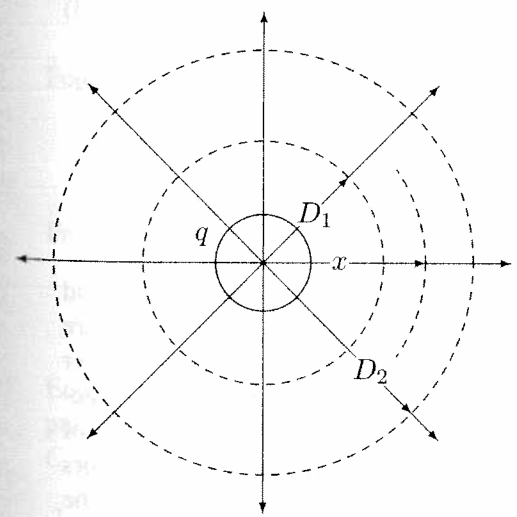


FIGURE 4.14
Electric field around a long round conductor.

The charge on the conductor gives rise to an electric field with radial flux lines. The total electric flux is numerically equal to the value of charge on the conductor. The intensity of the field at any point is defined as the force per unit charge and is termed *electric field intensity* designated as E. Concentric cylinders surrounding the conductor are equipotential surfaces and have the same electric flux density. From Gauss's law, for one meter length of the conductor, the electric flux density at a cylinder of radius x is given by

$$D = \frac{q}{A} = \frac{q}{2\pi x(1)} \tag{4.60}$$

The electric field intensity E may be found from the relation

$$E = \frac{D}{\varepsilon_0} \tag{4.61}$$

where ε_0 is the permittivity of free space and is equal to 8.85×10^{-12} F/m. Substituting (4.60) in (4.61) results in

$$E = \frac{q}{2\pi\varepsilon_0 x} \tag{4.62}$$

The potential difference between cylinders from position D_1 to D_2 is defined as the work done in moving a unit charge of one coulomb from D_2 to D_1 through the electric field produced by the charge on the conductor. This is given by

$$V_{12} = \int_{D_1}^{D_2} E dx = \int_{D_1}^{D_2} \frac{q}{2\pi\varepsilon_0 x} dx = \frac{q}{2\pi\varepsilon_0} \ln \frac{D_2}{D_1} \tag{4.63}$$

The notation V_{12} implies the voltage drop from 1 relative to 2, that is, 1 is understood to be positive relative to 2. The charge q carries its own sign.

4.11 CAPACITANCE OF SINGLE-PHASE LINES

Consider one meter length of a single-phase line consisting of two long solid round conductors each having a radius r as shown in Figure 4.15. The two conductors are separated by a distance D. Conductor 1 carries a charge of q_1 coulombs/meter and conductor 2 carries a charge of q_2 coulombs/meter. The presence of the second conductor and ground disturbs the field of the first conductor. The distance of separation of the wires D is great with respect to r and the height of conductors is much larger compared with D. Therefore, the distortion effect is small and the charge is assumed to be uniformly distributed on the surface of the conductors.

Assuming conductor 1 alone to have a charge of q_1, the voltage between conductor 1 and 2 is

$$V_{12(q_1)} = \frac{q_1}{2\pi\varepsilon_0} \ln \frac{D}{r} \tag{4.64}$$

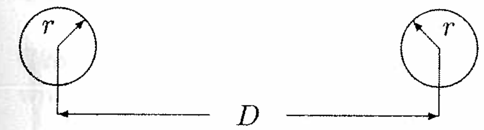


FIGURE 4.15
Single-phase two-wire line.

Now assuming only conductor 2, having a charge of q_2, the voltage between conductors 2 and 1 is

$$V_{21(q_2)} = \frac{q_2}{2\pi\varepsilon_0} \ln \frac{D}{r}$$

Since $V_{12(q_2)} = -V_{21(q_2)}$, we have

$$V_{12(q_2)} = \frac{q_2}{2\pi\varepsilon_0} \ln \frac{r}{D} \tag{4.65}$$

From the principle of superposition, the potential difference due to presence of both charges is

$$V_{12} = V_{12(q_1)} + V_{12(q_2)} = \frac{q_1}{2\pi\varepsilon_0} \ln \frac{D}{r} + \frac{q_2}{2\pi\varepsilon_0} \ln \frac{r}{D} \tag{4.66}$$

For a single-phase line $q_2 = -q_1 = -q$, and (4.66) reduces to

$$V_{12} = \frac{q}{\pi\varepsilon_0} \ln \frac{D}{r} \tag{4.67}$$

From (4.59), the capacitance between conductors is

$$C_{12} = \frac{\pi\varepsilon_0}{\ln \frac{D}{r}} \text{ F/m} \tag{4.68}$$

Equation (4.68) gives the line-to-line capacitance between the conductors. For the purpose of transmission line modeling, we find it convenient to define a capacitance C between each conductor and a neutral as illustrated in Figure 4.16. Since the

FIGURE 4.16
Illustration of capacitance to neutral.

voltage to neutral is half of V_{12}, the capacitance to neutral $C = 2C_{12}$, or

$$C = \frac{2\pi\varepsilon_0}{\ln \frac{D}{r}} \text{ F/m} \tag{4.69}$$

Recalling $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m and converting to μF per kilometer, we have

$$C = \frac{0.0556}{\ln \frac{D}{r}} \ \mu\text{F/km} \tag{4.70}$$

The capacitance per phase contains terms analogous to those derived for inductance per phase. However, unlike inductance where the conductor geometric mean radius (GMR) is used, in capacitance formula the actual conductor radius r is used.

4.12 POTENTIAL DIFFERENCE IN A MULTICONDUCTOR CONFIGURATION

Consider n parallel long conductors with charges $q_1, q_2, \ldots, q_n$ coulombs/meter as shown in Figure 4.17.

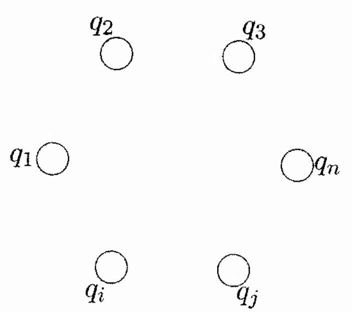


FIGURE 4.17
Multiconductor configuration.

Assume that the distortion effect is negligible and the charge is uniformly distributed around the conductor, with the following constraint

$$q_1 + q_2 + \cdots + q_n = 0 \tag{4.71}$$

Using superposition and (4.63), potential difference between conductors i and j due to the presence of all charges is

$$V_{ij} = \frac{1}{2\pi\varepsilon_0} \sum_{k=1}^{n} q_k \ln \frac{D_{kj}}{D_{ki}} \tag{4.72}$$

When $k = i$, D_{ii} is the distance between the surface of the conductor and its center, namely its radius r.

4.13 CAPACITANCE OF THREE-PHASE LINES

Consider one meter length of a three-phase line with three long conductors, each with radius r, with conductor spacing as shown Figure 4.18.

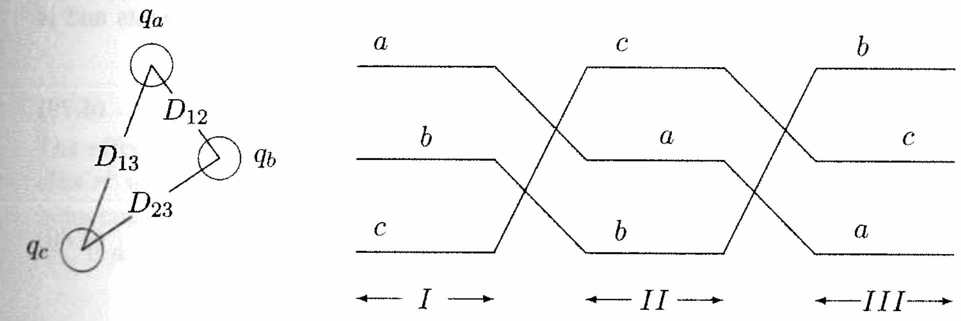


FIGURE 4.18
Three-phase transmission line.

Since we have a balanced three-phase system

$$q_a + q_b + q_c = 0 \tag{4.73}$$

We shall neglect the effect of ground and the shield wires. Assume that the line is transposed. We proceed with the calculation of the potential difference between a and b for each section of transposition. Applying (4.72) to the first section of the transposition, V_{ab} is

$$V_{ab(I)} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{D_{12}}{r} + q_b \ln \frac{r}{D_{12}} + q_c \ln \frac{D_{23}}{D_{13}}\right) \tag{4.74}$$

Similarly, for the second section of the transposition, we have

$$V_{ab(II)} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{D_{23}}{r} + q_b \ln \frac{r}{D_{23}} + q_c \ln \frac{D_{13}}{D_{12}}\right) \tag{4.75}$$

and for the last section

$$V_{ab(III)} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{D_{13}}{r} + q_b \ln \frac{r}{D_{13}} + q_c \ln \frac{D_{12}}{D_{23}}\right) \tag{4.76}$$

The average value of V_{ab} is

$$V_{ab} = \frac{1}{(3)2\pi\varepsilon_0}\left(q_a \ln \frac{D_{12}D_{23}D_{13}}{r^3} + q_b \ln \frac{r^3}{D_{12}D_{23}D_{13}} + q_c \ln \frac{D_{12}D_{23}D_{13}}{D_{12}D_{23}D_{13}}\right) \tag{4.77}$$

or

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}{r} + q_b \ln \frac{r}{(D_{12}D_{23}D_{13})^{\frac{1}{3}}}\right) \tag{4.78}$$

Note that the GMD of the conductor appears in the logarithm arguments and is given by

$$GMD = \sqrt[3]{D_{12}D_{23}D_{13}} \tag{4.79}$$

Therefore, V_{ab} is

$$V_{ab} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{GMD}{r} + q_b \ln \frac{r}{GMD}\right) \tag{4.80}$$

Similarly, we find the average voltage V_{ac} as

$$V_{ac} = \frac{1}{2\pi\varepsilon_0}\left(q_a \ln \frac{GMD}{r} + q_c \ln \frac{r}{GMD}\right) \tag{4.81}$$

Adding (4.80) and (4.81) and substituting for $q_b + q_c = -q_a$, we have

$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon}\left(2q_a \ln \frac{GMD}{r} - q_a \ln \frac{r}{GMD}\right) = \frac{3q_a}{2\pi\varepsilon_0} \ln \frac{GMD}{r} \tag{4.82}$$

For balanced three-phase voltages,

$$\begin{aligned} V_{ab} &= V_{an}\angle 0^\circ - V_{an}\angle -120^\circ \\ V_{ac} &= V_{an}\angle 0^\circ - V_{an}\angle -240^\circ \end{aligned} \tag{4.83}$$

Therefore,

$$V_{ab} + V_{ac} = 3V_{an} \tag{4.84}$$

Substituting in (4.82) the capacitance per phase to neutral is

$$C = \frac{q_a}{V_{an}} = \frac{2\pi\varepsilon_0}{\ln \frac{GMD}{r}} \quad \text{F/m} \tag{4.85}$$

or capacitance to neutral in μF per kilometer is

$$C = \frac{0.0556}{\ln \frac{GMD}{r}} \quad \mu\text{F/km} \tag{4.86}$$

This is of the same form as the expression for the capacitance of one phase of a single-phase line. GMD (geometric mean distance) is the equivalent conductor spacing. For the above three-phase line this is the cube root of the product of the three-phase spacings.

4.14 EFFECT OF BUNDLING

The procedure for finding the capacitance per phase for a three-phase transposed line with bundle conductors follows the same steps as the procedure in Section 4.13. The capacitance per phase is found to be

$$C = \frac{2\pi\varepsilon_0}{\ln \frac{GMD}{r^b}} \quad \text{F/m} \tag{4.87}$$

The effect of bundling is to introduce an equivalent radius r^b. The equivalent radius r^b is similar to the GMR (geometric mean radius) calculated earlier for the inductance with the exception that radius r of each subconductor is used instead of D_s. If d is the bundle spacing, we obtain for the two-subconductor bundle

$$r^b = \sqrt{r \times d} \tag{4.88}$$

for the three-subconductor bundle

$$r^b = \sqrt[3]{r \times d^2} \tag{4.89}$$

for the four-subconductor bundle

$$r^b = 1.09\sqrt[4]{r \times d^3} \tag{4.90}$$

4.15 CAPACITANCE OF THREE-PHASE DOUBLE-CIRCUIT LINES

Consider a three-phase double-circuit line with relative phase positions $a_1b_1c_1$ - $c_2b_2a_2$, as shown in Figure 4.13. Each phase conductor is transposed within its group and with respect to the parallel three-phase line. The effect of shield wires and the ground are considered to be negligible for this balanced condition. Following the procedure of section 4.13, the average voltages V_{ab}, V_{ac} and V_{an} are calculated and the per-phase equivalent capacitance to neutral is obtained to be

$$C = \frac{2\pi\varepsilon_0}{\ln \frac{GMD}{GMR_c}} \quad \text{F/m} \tag{4.91}$$

or capacitance to neutral in μF per kilometer is

$$C = \frac{0.0556}{\ln \frac{GMD}{GMR_c}} \quad \mu\text{F/km} \tag{4.92}$$

The expression for GMD is the same as was found for inductance calculation and is given by (4.55). The GMR_c of each phase group is similar to the GMR_L, with the exception that in (4.56) r^b is used instead of D_s^b. This will result in the following equations

$$r_A = \sqrt{r^b\, D_{a_1a_2}}$$
$$r_B = \sqrt{r^b\, D_{b_1b_2}}$$
$$r_C = \sqrt{r^b\, D_{c_1c_2}} \tag{4.93}$$

where r^b is the geometric mean radius of the bundled conductors given by (4.88) – (4.90). The equivalent geometric mean radius for calculating the per-phase capacitance to neutral is

$$GMR_C = \sqrt[3]{r_A\, r_B\, r_C} \tag{4.94}$$

4.16 EFFECT OF EARTH ON THE CAPACITANCE

For an isolated charged conductor the electric flux lines are radial and are orthogonal to the cylindrical equipotential surfaces. The presence of earth will alter the distribution of electric flux lines and equipotential surfaces, which will change the effective capacitance of the line.

The earth level is an equipotential surface, therefore the flux lines are forced to cut the surface of the earth orthogonally. The effect of the presence of earth can be accounted for by the method of *image charges* introduced by Kelvin. To illustrate this method, consider a conductor with a charge q coulombs/meter at a height H above ground. Also, imagine a charge $-q$ placed at a depth H below the surface of earth. This configuration without the presence of the earth surface will produce the same field distribution as a single charge and the earth surface. Thus, the earth can be replaced for the calculation of electric field potential by a fictitious charged conductor with charge equal and opposite to the charge on the actual conductor and at a depth below the surface of the earth the same as the height of the actual conductor above earth. This imaginary conductor is called the image of the actual conductor. The procedure of Section 4.13 can now be used for the computation of the capacitance.

The effect of the earth is to increase the capacitance. But normally the height of the conductor is large as compared to the distance between the conductors, and the earth effect is negligible. Therefore, for all line models used for balanced steady-state analysis, the effect of earth on the capacitance can be neglected. However, for unbalanced analysis such as unbalanced faults, the earth's effect as well as the shield wires should be considered.

Example 4.2 (chp4ex2)

A 500-kV three-phase transposed line is composed of one $ACSR$ 1,272,000–cmil, 45/7 Bittern conductor per phase with horizontal conductor configuration as shown in Figure 4.19. The conductors have a diameter of 1.345 in and a GMR of 0.5328 in. Find the inductance and capacitance per phase per kilometer of the line.

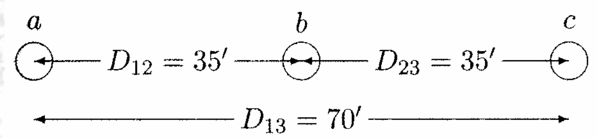


FIGURE 4.19
Conductor layout for Example 4.2.

Conductor radius is $r = \frac{1.345}{2\times 12} = 0.056$ ft, and $GMR_L = 0.5328/12 = 0.0444$ ft. GMD is obtained using (4.42)

$$GMD = \sqrt[3]{35 \times 35 \times 70} = 44.097 \quad \text{ft}$$

From (4.58) the inductance per phase is

$$L = 0.2 \ln \frac{44.097}{0.0444} = 1.38 \quad \text{mH/km}$$

and from (4.92) the capacitance per phase is

$$C = \frac{0.0556}{\ln \frac{44.097}{0.056}} = 0.0083 \quad \mu\text{F/km}$$

Example 4.3 (chp4ex3)

The line in Example 4.2 is replaced by two $ACSR$ 636,000-cmil, 24/7 Rook conductors which have the same total cross-sectional area of aluminum as one Bittern conductor. The line spacing as measured from the center of the bundle is the same as before and is shown in Figure 4.20.

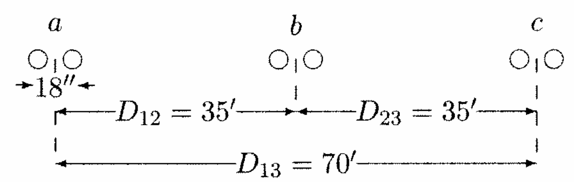


FIGURE 4.20
Conductor layout for Example 4.3.

The conductors have a diameter of 0.977 in and a GMR of 0.3924 in. Bundle spacing is 18 in. Find the inductance and capacitance per phase per kilometer of the line and compare it with that of Example 4.2.

Conductor radius is $r = \frac{0.977}{2} = 0.4885$ in, and from Example 4.2 $GMD = 44.097$ ft. The equivalent geometric mean radius with two conductors per bundle, for calculating inductance and capacitance, are given by (4.51) and (4.88)

$$GMR_L = \frac{\sqrt{d \times D_s}}{12} = \frac{\sqrt{18 \times 0.3924}}{12} = 0.22147 \text{ ft}$$

and

$$GMR_c = \frac{\sqrt{d \times r}}{12} = \frac{\sqrt{18 \times 0.4885}}{12} = 0.2471 \text{ ft}$$

From (4.58) the inductance per phase is

$$L = 0.2 \ln \frac{44.097}{0.22147} = 1.0588 \text{ mH/km}$$

and from (4.92) the capacitance per phase is

$$C = \frac{0.0556}{\ln \frac{44.097}{0.2471}} = 0.0107 \quad \mu\text{F/km}$$

Comparing with the results of Example 4.2, there is a 23.3 percent reduction in the inductance and a 28.9 percent increase in the capacitance.

The function **[GMD, GMRL, GMRC] = gmd** is developed for the computation of GMD, GMR_L, and GMR_C for single-circuit, double-circuit vertical, and horizontal transposed lines with up to four bundled conductors. A menu is displayed for the selection of any of the above three circuits. The user is prompted to input the phase spacing, number of bundled conductors and their spacing, conductor diameter, and the GMR of the individual conductor. The specifications for some common $ACSR$ conductors are contained in a file named **acsr.m**. The command **acsr** will display the characteristics of $ACSR$ conductors. Also, the function **[L, C] = gmd2lc** in addition to the geometric mean values returns the inductance in mH per km and the capacitance in μF per km.

A new GUI program named **lcgui** is developed for the computation of transmission line parameters. This is a user-friendly program, which makes the data entry for various configurations very easy.

Example 4.4 (Run lcgui)

A 735-kV three-phase transposed line is composed of four $ACSR$, 954, 000-cmil, 45/7 Rail conductors per phase with horizontal conductor configuration as shown in Figure 4.21. Bundle spacing is 46 cm. Use **acsrgui** in *MATLAB* to obtain the conductor size and the electrical characteristics for the Rail conductor. Find the inductance and capacitance per phase per kilometer of the line.

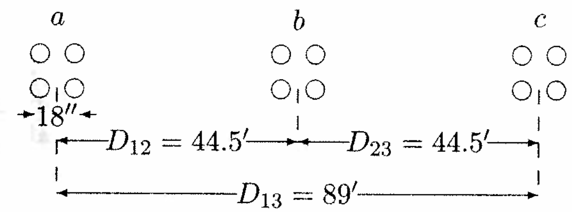


FIGURE 4.21
Conductor layout for Example 4.4.

The command **acsr** displays the conductor code name and the area in cmils for the ACSR conductors. The user is then prompted to enter the conductor code name within single quotes.

```
Enter ACSR code name within single quotes -> 'rail'
Al Area Strand  Diameter GMR  Resistance Ohm/km  Ampacity
 cmil   Al/St     cm      cm    60Hz 25C  60Hz 50C  Ampere
954000   45/7    2.959   1.173  0.0624   0.0683     1000
```

The following commands

```
[GMD, GMRL, GMRC] = gmd;
L=0.2*log(GMD/GMRL)          % mH/km        Eq. (4.58)
C = 0.0556/log(GMD/GMRC)     % micro F/km   Eq. (4.92)
```

result in

```
  Number of three-phase circuits                  Enter
  Single-circuit                                    1
  Double-circuit vertical configuration             2
  Double-circuit horizontal configuration           3
  To quit                                           0

Select number of menu → 1
Enter spacing unit within quotes 'm' or 'ft' → 'ft'
Enter row vector [D12, D23, D13] = [44.5  44.5  89]
Cond. size, bundle spacing unit: 'cm' or 'in' → 'cm'
Conductor diameter in cm = 2.959
Geometric Mean Radius in cm = 1.173
No. of bundled cond. (enter 1 for single cond.) = 4
Bundle spacing in cm = 46
GMD = 56.06649 ft
GMRL = 0.65767 ft     GMRC = 0.69696 ft
L = 0.8891
C = 0.0127
```

Example 4.5 (Run lcgui)

A 345-kV double-circuit three-phase transposed line is composed of two $ACSR$, 1,431,000-cmil, 45/7 Bobolink conductors per phase with vertical conductor configuration as shown in Figure 4.22. The conductors have a diameter of 1.427 in and a GMR of 0.564 in. The bundle spacing in 18 in. Find the inductance and capacitance per phase per kilometer of the line. The following commands

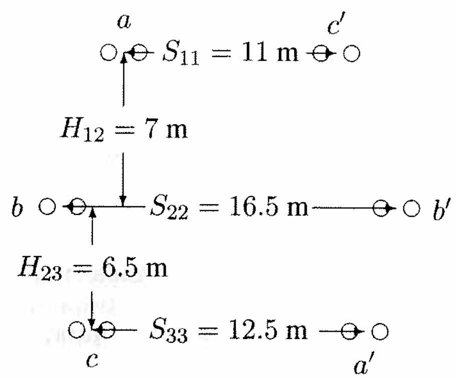


FIGURE 4.22
Conductor layout for Example 4.5.

```
[GMD, GMRL, GMRC] = gmd;
L=0.2*log(GMD/GMRL)          % mH/km        Eq. (4.58)
C = 0.0556/log(GMD/GMRC)     % micro F/km   Eq. (4.92)
```

result in

```
Number of three-phase circuits               Enter
Single-circuit                                 1
Double-circuit vertical configuration          2
Double-circuit horizontal configuration        3
To quit                                        0

Select number of menu → 2

Circuit Arrangements
(1) abc-c'b'a'
(2) abc-a'b'c'

Enter (1 or 2) → 1
Enter spacing unit within quotes 'm' or 'ft' → 'm'
Enter row vector [S11, S22, S33] = [11  16.5  12.5]
Enter row vector [H12, H23] = [7  6.5]
Cond. size, bundle spacing unit: 'cm' or 'in' → 'in'
Conductor diameter in inch = 1.427
Geometric Mean Radius in inch = 0.564
No. of bundled cond. (enter 1 for single cond.) = 2
Bundle spacing in inch = 18
GMD = 11.21352 m
GMRL = 1.18731 m     GMRC = 1.25920 m
L = 0.4491
C = 0.0254
```

Example 4.6 (Run lcgui)

A 345-kV double-circuit three-phase transposed line is composed of one $ACSR$, 556,500-cmil, 26/7 Dove conductor per phase with horizontal conductor configuration as shown in Figure 4.23. The conductors have a diameter of 0.927 in and a GMR of 0.3768 in. Bundle spacing is 18 in. Find the inductance and capacitance per phase per kilometer of the line. The following commands

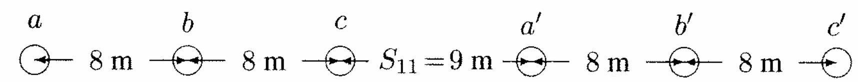


FIGURE 4.23
Conductor layout for Example 4.6.

```
[GMD, GMRL, GMRC] = gmd;
L=0.2*log(GMD/GMRL)          % mH/km        Eq. (4.58)
C = 0.0556/log(GMD/GMRC)     % micro F/km   Eq. (4.92)
```

result in

```
Number of three-phase circuits               Enter
Single-circuit                                 1
Double-circuit vertical configuration          2
Double-circuit horizontal configuration        3
To quit                                        0

Select number of menu → 3

Circuit Arrangements
(1) abc-a'b'c'
(2) abc-c'b'a'

Enter (1 or 2) → 1
Enter spacing unit within quotes 'm' or 'ft' → 'm'
Enter row vector [D12, D23, S13] = [8  8  16]
Enter distance between two circuits, S11 = 9
```

```
Cond. size, bundle spacing unit: 'cm' or 'in' → 'in'
Conductor diameter in inch = 0.927
Geometric Mean Radius in inch = 0.3768
No. of bundled cond. (enter 1 for single cond.) = 1
GMD = 14.92093 m
GMRL = 0.48915 m      GMRC = 0.54251 m
L = 0.6836
C = 0.0168
```

4.17 MAGNETIC FIELD INDUCTION

Transmission line magnetic fields affect objects in the proximity of the line. The magnetic fields, related to the currents in the line, induces voltage in objects that have a considerable length parallel to the line, such as fences, pipelines, and telephone wires.

The magnetic field is affected by the presence of earth return currents. Carson [14] presents an equation for computation of mutual resistance and inductance which are functions of the earth's resistivity. For balanced three-phase systems the total earth return current is zero. Under normal operating conditions, the magnetic field in proximity to balanced three-phase lines may be calculated considering the currents in the conductors and neglecting earth currents.

Magnetic fields have been reported to affect blood composition, growth, behavior, immune systems, and neural functions. There are general concerns regarding the biological effects of electromagnetic and electrostatic fields on people. Long-term effects are the subject of several worldwide research efforts.

Example 4.7 (chp4ex7)

A three-phase untransposed transmission line and a telephone line are supported on the same towers as shown in Figure 4.24. The power line carries a 60-Hz balanced current of 200 A per phase. The telephone line is located directly below phase b. Assuming balanced three-phase currents in the power line, find the voltage per kilometer induced in the telephone line.

From (4.15) the flux linkage between conductors 1 and 2 due to current I_a is

$$\lambda_{12(I_a)} = 0.2 I_a \ln \frac{D_{a2}}{D_{a1}} \quad \text{mWb/km}$$

Since $D_{b1} = D_{b2}$, λ_{12} due to I_b is zero. The flux linkage between conductors 1 and 2 due to current I_c is

$$\lambda_{12(I_c)} = 0.2 I_c \ln \frac{D_{c2}}{D_{c1}} \quad \text{mWb/km}$$

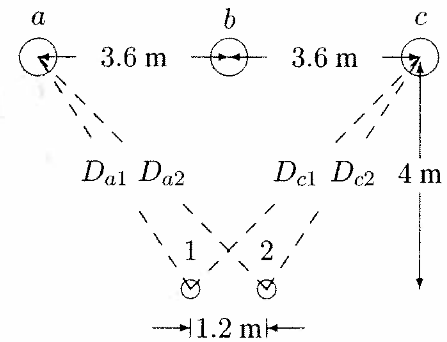


FIGURE 4.24
Conductor layout for Example 4.6.

Total flux linkage between conductors 1 and 2 due to all currents is

$$\lambda_{12} = 0.2 I_a \ln \frac{D_{a2}}{D_{a1}} + 0.2 I_c \ln \frac{D_{c2}}{D_{c1}} \quad \text{mWb/km}$$

For positive phase sequence, with I_a as reference, $I_c = I_a \angle -240^\circ$ and we have

$$\lambda_{12} = 0.2 I_a \left(\ln \frac{D_{a2}}{D_{a1}} + 1\angle -240^\circ \ln \frac{D_{c2}}{D_{c1}} \right) \quad \text{mH/km}$$

With I_a as reference, the instantaneous flux linkage is

$$\lambda_{12}(t) = \sqrt{2}\, |\lambda_{12}| \cos(\omega t + \alpha)$$

Thus, the induced voltage in the telephone line per kilometer length is

$$v = \frac{d\lambda_{12}(t)}{dt} = \sqrt{2}\,\omega |\lambda_{12}| \cos(\omega t + \alpha + 90^\circ)$$

The rms voltage induced in the telephone line per kilometer is

$$V = \omega |\lambda_{12}| \angle \alpha + 90^\circ = j\omega \lambda_{12}$$

From the circuits geometry

$$D_{a1} = D_{c2} = (3^2 + 4^2)^{\frac{1}{2}} = 5 \ \text{m}$$
$$D_{a2} = D_{c1} = (4.2^2 + 4^2)^{\frac{1}{2}} = 5.8 \ \text{m}$$

The total flux linkage is

$$\lambda_{12} = 0.2 \times 200\angle 0^\circ \ln \frac{5.8}{5} + 0.2 \times 200\angle -240^\circ \ln \frac{5}{5.8}$$
$$= 10.283\angle -30^\circ \ \text{mWb/km}$$

The voltage induced in the telephone line per kilometer is

$$V = j\omega\lambda_{12} = j2\pi 60 (10.283\angle -30^\circ)(10^{-3}) = 3.88\angle 60^\circ \quad \text{V/km}$$

4.18 ELECTROSTATIC INDUCTION

Transmission line electric fields affect objects in the proximity of the line. The electric field produced by high voltage lines induces current in objects which are in the area of the electric fields. The effects of electric fields becomes of increasing concern at higher voltages. Electric fields, related to the voltage of the line, are the primary cause of induction to vehicles, buildings, and objects of comparable size. The human body is affected with exposure to electric discharges from charged objects in the field of the line. These may be steady current or spark discharges. The current densities in humans induced by electric fields of transmission lines are known to be much higher than those induced by magnetic fields.

The resultant electric field in proximity to a transmission line can be obtained by representing the earth effect by image charges located below the conductors at a depth equal to the conductor height.

4.19 CORONA

When the surface potential gradient of a conductor exceeds the dielectric strength of the surrounding air, ionization occurs in the area close to the conductor surface. This partial ionization is known as *corona*. The dielectric strength of air during fair weather and at NTP (25°C and 76 cm of Hg) is about 30 kV/cm.

Corona produces power loss, audible hissing sound in the vicinity of the line, ozone and radio and television interference. The audible noise is an environmental concern and occurs in foul weather. Radio interference occurs in the AM band. Rain and snow may produce moderate TVI in a low signal area. Corona is a function of conductor diameter, line configuration, type of conductor, and condition of its surface. Atmospheric conditions such as air density, humidity, and wind influence the generation of corona. Corona losses in rain or snow are many times the losses during fair weather. On a conductor surface, an irregularity such as a contaminating particle causes a voltage gradient that may become the point source of a discharge. Also, insulators are contaminated by dust or chemical deposits which will lower the disruptive voltage and increase the corona loss. The insulators are cleaned periodically to reduce the extent of the problem. Corona can be reduced by increasing the conductor size and the use of conductor bundling.

The power loss associated with corona can be represented by shunt conductance. However, under normal operating conditions g, which represents the resistive leakage between a phase and ground, has negligible effect on performance and is customarily neglected. (i.e., $g = 0$).

PROBLEMS

4.1. A solid cylindrical aluminum conductor 25 km long has an area of 336,400 circular mils. Obtain the conductor resistance at (a) 20°C and (b) 50°C. The resistivity of aluminum at 20°C is 2.8×10^{-8} Ω-m.

4.2. A transmission-line cable consists of 12 identical strands of aluminum, each 3 mm in diameter. The resistivity of aluminum strand at 20°C is 2.8×10^{-8} Ω-m. Find the 50°C ac resistance per km of the cable. Assume a skin-effect correction factor of 1.02 at 60 Hz.

4.3. A three-phase transmission line is designed to deliver 190.5 MVA at 220 kV over a distance of 63 km. The total transmission line loss is not to exceed 2.5 percent of the rated line MVA. If the resistivity of the conductor material is 2.84×10^{-8} Ω-m, determine the required conductor diameter and the conductor size in circular mils.

4.4. A single-phase transmission line 35 km long consists of two solid round conductors, each having a diameter of 0.9 cm. The conductor spacing is 2.5 m. Calculate the equivalent diameter of a fictitious hollow, thin-walled conductor having the same equivalent inductance as the original line. What is the value of the inductance per conductor?

4.5. Find the geometric mean radius of a conductor in terms of the radius r of an individual strand for

(a) Three equal strands as shown in Figure 4.25(a)
(b) Four equal strands as shown in Figure 4.25(b)

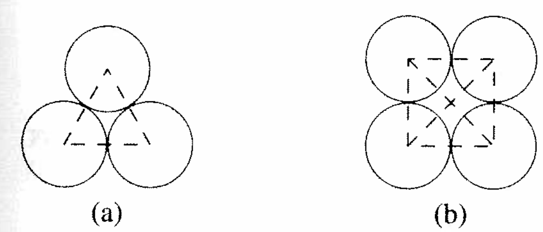


FIGURE 4.25
Cross section of the stranded conductor for Problem 4.5.

4.6. One circuit of a single-phase transmission line is composed of three solid 0.5-cm radius wires. The return circuit is composed of two solid 2.5-cm radius wires. The arrangement of conductors is as shown in Figure 4.26. Applying the concept of the GMD and GMR, find the inductance of the complete line in millihenry per kilometer.

4.18 ELECTROSTATIC INDUCTION

Transmission line electric fields affect objects in the proximity of the line. The electric field produced by high voltage lines induces current in objects which are in the area of the electric fields. The effects of electric fields becomes of increasing concern at higher voltages. Electric fields, related to the voltage of the line, are the primary cause of induction to vehicles, buildings, and objects of comparable size. The human body is affected with exposure to electric discharges from charged objects in the field of the line. These may be steady current or spark discharges. The current densities in humans induced by electric fields of transmission lines are known to be much higher than those induced by magnetic fields.

The resultant electric field in proximity to a transmission line can be obtained by representing the earth effect by image charges located below the conductors at a depth equal to the conductor height.

4.19 CORONA

When the surface potential gradient of a conductor exceeds the dielectric strength of the surrounding air, ionization occurs in the area close to the conductor surface. This partial ionization is known as *corona*. The dielectric strength of air during fair weather and at NTP (25°C and 76 cm of Hg) is about 30 kV/cm.

Corona produces power loss, audible hissing sound in the vicinity of the line, ozone and radio and television interference. The audible noise is an environmental concern and occurs in foul weather. Radio interference occurs in the AM band. Rain and snow may produce moderate TVI in a low signal area. Corona is a function of conductor diameter, line configuration, type of conductor, and condition of its surface. Atmospheric conditions such as air density, humidity, and wind influence the generation of corona. Corona losses in rain or snow are many times the losses during fair weather. On a conductor surface, an irregularity such as a contaminating particle causes a voltage gradient that may become the point source of a discharge. Also, insulators are contaminated by dust or chemical deposits which will lower the disruptive voltage and increase the corona loss. The insulators are cleaned periodically to reduce the extent of the problem. Corona can be reduced by increasing the conductor size and the use of conductor bundling.

The power loss associated with corona can be represented by shunt conductance. However, under normal operating conditions g, which represents the resistive leakage between a phase and ground, has negligible effect on performance and is customarily neglected. (i.e., $g = 0$).

PROBLEMS

4.1. A solid cylindrical aluminum conductor 25 km long has an area of 336,400 circular mils. Obtain the conductor resistance at (a) 20°C and (b) 50°C. The resistivity of aluminum at 20°C is 2.8×10^{-8} Ω-m.

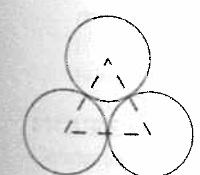
4.2. A transmission-line cable consists of 12 identical strands of aluminum, each 3 mm in diameter. The resistivity of aluminum strand at 20°C is 2.8×10^{-8} Ω-m. Find the 50°C ac resistance per km of the cable. Assume a skin-effect correction factor of 1.02 at 60 Hz.

4.3. A three-phase transmission line is designed to deliver 190.5 MVA at 220 kV over a distance of 63 km. The total transmission line loss is not to exceed 2.5 percent of the rated line MVA. If the resistivity of the conductor material is 2.84×10^{-8} Ω-m, determine the required conductor diameter and the conductor size in circular mils.

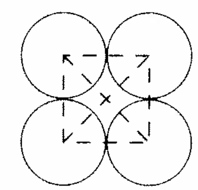
4.4. A single-phase transmission line 35 km long consists of two solid round conductors, each having a diameter of 0.9 cm. The conductor spacing is 2.5 m. Calculate the equivalent diameter of a fictitious hollow, thin-walled conductor having the same equivalent inductance as the original line. What is the value of the inductance per conductor?

4.5. Find the geometric mean radius of a conductor in terms of the radius r of an individual strand for

(a) Three equal strands as shown in Figure 4.25(a)
(b) Four equal strands as shown in Figure 4.25(b)



(a)



(b)

FIGURE 4.25
Cross section of the stranded conductor for Problem 4.5.

4.6. One circuit of a single-phase transmission line is composed of three solid 0.5-cm radius wires. The return circuit is composed of two solid 2.5-cm radius wires. The arrangement of conductors is as shown in Figure 4.26. Applying the concept of the GMD and GMR, find the inductance of the complete line in millihenry per kilometer.

FIGURE 4.26
Conductor layout for Problem 4.6.

4.7. A three-phase, 60-Hz transposed transmission line has a flat horizontal configuration as shown in Figure 4.27. The line reactance is 0.486 Ω per kilometer. The conductor geometric mean radius is 2.0 cm. Determine the phase spacing D in meters.

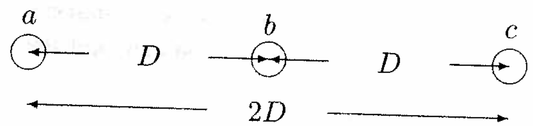


FIGURE 4.27
Conductor layout for Problem 4.7.

4.8. A three-phase transposed line is composed of one ACSR 159,000-cmil, 54/19 Lapwing conductor per phase with flat horizontal spacing of 8 m as shown in Figure 4.28. The GMR of each conductor is 1.515 cm.
(a) Determine the inductance per phase per kilometer of the line.
(b) This line is to be replaced by a two-conductor bundle with 8 m spacing measured from the center of the bundles as shown in Figure 4.29. The spacing between the conductors in the bundle is 40 cm. If the line inductance per phase is to be 77 percent of the inductance in part (a), what would be the GMR of each new conductor in the bundle?

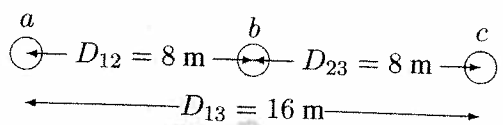


FIGURE 4.28
Conductor layout for Problem 4.8 (a).

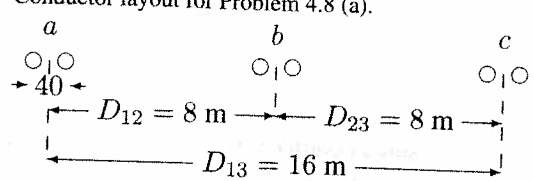


FIGURE 4.29
Conductor layout for Problem 4.8 (b).

4.9. A three-phase transposed line is composed of one ACSR, 1,431,000-cmil, 47/7 Bobolink conductor per phase with flat horizontal spacing of 11 m as shown in Figure 4.30. The conductors have a diameter of 3.625 cm and a GMR of 1.439 cm. The line is to be replaced by a three-conductor bundle of ACSR, 477,000-cmil, 26/7 Hawk conductors having the same cross-sectional area of aluminum as the single-conductor line. The conductors have a diameter of 2.1793 cm and a GMR of 0.8839 cm. The new line will also have a flat horizontal configuration, but it is to be operated at a higher voltage and therefore the phase spacing is increased to 14 m as measured from the center of the bundles as shown in Figure 4.31. The spacing between the conductors in the bundle is 45 cm. Determine

(a) The percentage change in the inductance.
(b) The percentage change in the capacitance.

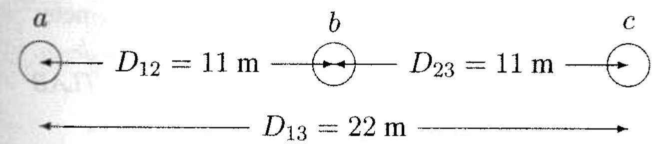


FIGURE 4.30
Conductor layout for Problem 4.9 (a).

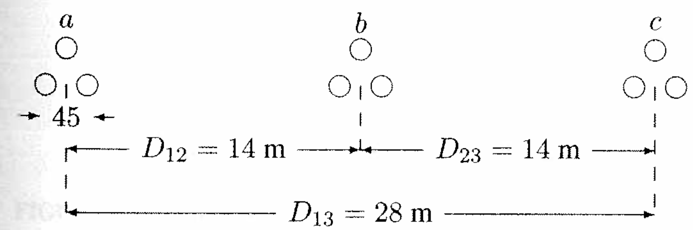


FIGURE 4.31
Conductor layout for Problem 4.9 (b).

4.10. A single-circuit three-phase transposed transmission line is composed of four ACSR, 1,272,000-cmil conductor per phase with horizontal configuration as shown in Figure 4.32. The bundle spacing is 45 cm. The conductor code name is *pheasant*. In *MATLAB*, use command **acsrgui** to find the conductor diameter and its GMR. Determine the inductance and capacitance per phase per kilometer of the line. Use function **[GMD, GMRL, GMRC] =gmd,** (4.58) and (4.92) in *MATLAB* to verify your results.

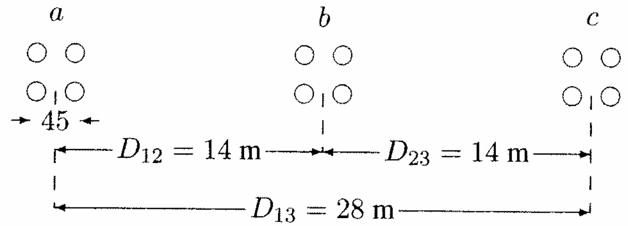


FIGURE 4.32
Conductor layout for Problem 4.10.

4.11. A double circuit three-phase transposed line is composed of two ACSR, 2, 167, 000-cmil, 72/7 Kiwi conductor per phase with vertical configuration as shown in Figure 4.33. The conductors have a diameter of 4.4069 cm and a *GMR* of 1.7374 cm. The bundle spacing is 45 cm. The circuit arrangement is $a_1b_1c_1$, $c_2b_2a_2$. Find the inductance and capacitance per phase per kilometer of the line. Find these values when the circuit arrangement is $a_1b_1c_1$, $a_2b_2c_2$. Use function **[GMD, GMRL, GMRC] =gmd**, (4.58) and (4.92) in *MATLAB* to verify your results.

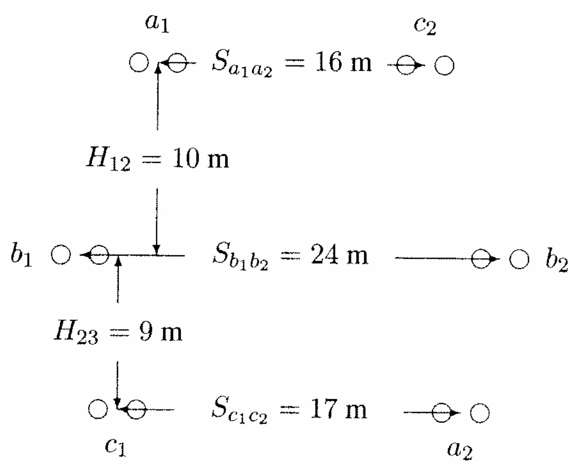


FIGURE 4.33
Conductor layout for Problem 4.11.

4.12. The conductors of a double-circuit three-phase transmission line are placed on the corner of a hexagon as shown in Figure 4.34. The two circuits are in parallel and are sharing the balanced load equally. The conductors of the circuits are identical, each having a radius r. Assume that the line is symmetrically transposed. Using the method of GMD, determine an expression for the capacitance per phase per meter of the line.

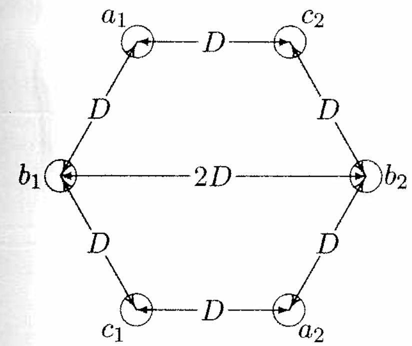


FIGURE 4.34
Conductor layout for Problem 4.12.

4.13. A 60-Hz, single-phase power line and a telephone line are parallel to each other as shown in Figure 4.35. The telephone line is symmetrically positioned directly below phase b. The power line carries an rms current of 226 A. Assume zero current flows in the ungrounded telephone wires. Find the magnitude of the voltage per km induced in the telephone line.

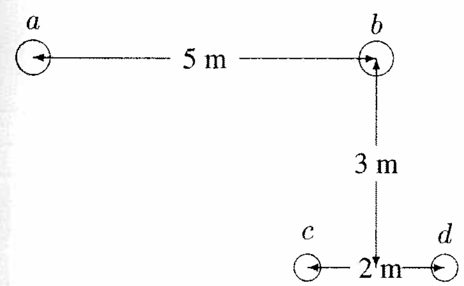


FIGURE 4.35
Conductor layout for Problem 4.13.

4.14. A three-phase, 60-Hz untransposed transmission line runs in parallel with a telephone line for 20 km. The power line carries a balanced three-phase rms current of $I_a = 320\angle 0°$ A, $I_b = 320\angle -120°$ A, and $I_c = 320\angle -240°$ A. The line configuration is as shown in Figure 4.36. Assume zero current flows in the ungrounded telephone wires. Find the magnitude of the voltage induced in the telephone line.

4.15. Since earth is an equipotential plane, the electric flux lines are forced to cut the surface of the earth orthogonally. The earth effect can be represented by placing an oppositely charged conductor a depth H below the surface of the earth as shown in Figure 4.37(a). This configuration without the presence

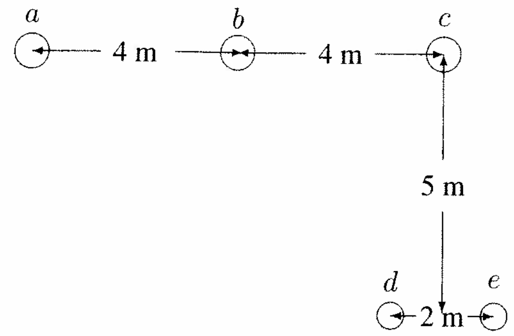
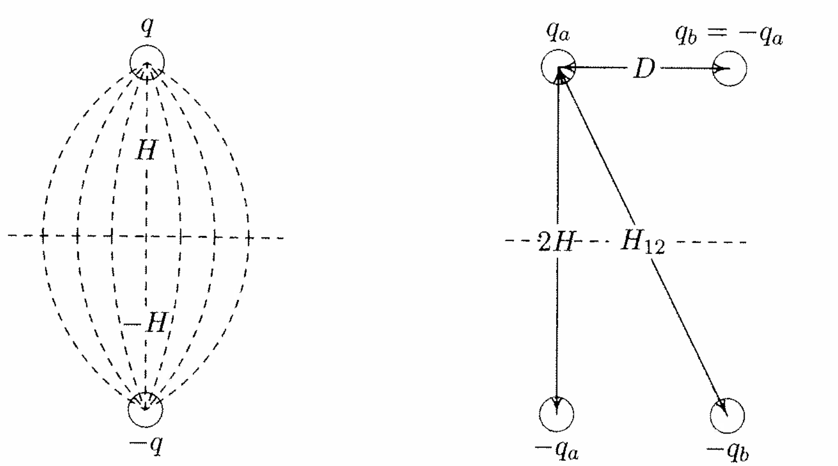


FIGURE 4.36
Conductor layout for Problem 4.14.

of the earth will produce the same field as a single charge and the earth surface. This imaginary conductor is called the image conductor. Figure 4.37(b) shows a single-phase line with its image conductors. Find the potential difference V_{ab} and show that the equivalent capacitance to neutral is given by

$$C_{an} = C_{bn} = \frac{2\pi\varepsilon}{\ln(\frac{D}{r}\frac{2H}{H_{12}})}$$



(a) Earth plane replaced by image conductor

(b) Single-phase line and its image

FIGURE 4.37
Conductor layout for Problem 4.15.

CHAPTER 5

LINE MODEL AND PERFORMANCE

5.1 INTRODUCTION

In Chapter 4 the per-phase parameters of transmission lines were obtained. This chapter deals with the representation and performance of transmission lines under normal operating conditions. Transmission lines are represented by an equivalent model with appropriate circuit parameters on a "per-phase" basis. The terminal voltages are expressed from one line to neutral, the current for one phase and, thus, the three-phase system is reduced to an equivalent single-phase system.

The model used to calculate voltages, currents, and power flows depends on the length of the line. In this chapter the circuit parameters and voltage and current relations are first developed for "short" and "medium" lines. Problems relating to the regulation and losses of lines and their operation under conditions of fixed terminal voltages are then considered.

Next, long line theory is presented and expressions for voltag[illegible] along the distributed line model are obtained. Propagation const[illegible] istic impedance are defined, and it is demonstrated that the electri[illegible] transmitted over the lines at approximately the speed of light. Si[illegible] conditions at the two ends of the line are of primary importance[illegible]