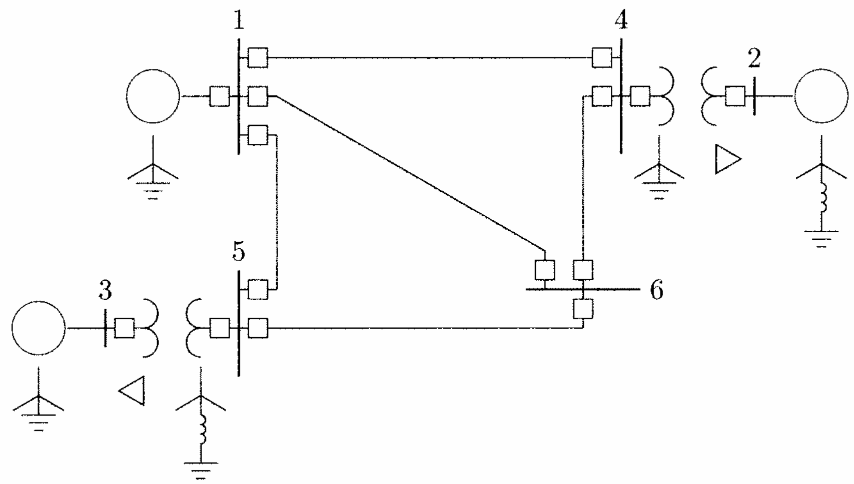

FIGURE 10.32
One-line diagram for Problem 10.21.

The transformer connections are shown in Figure 10.32. The Δ-Y transformer between buses 3 and 5 is grounded through a reactor of reactance 0.10 per unit. The generator's positive- and zero-sequence reactances including the reactance of grounding neutrals on a 100-MVA base is tabulated below.

GENERATOR TRANSIENT IMPEDANCE, PU			
Gen. No.	X^1	X^0	X_n
1	0.20	0.06	0.00
2	0.15	0.04	0.05
3	0.25	0.08	0.00

Resistances, shunt reactances, and loads are neglected, and all negative-sequence reactances are assumed equal to the positive-sequence reactances. Use **zbuild** function to obtain the positive- and zero-sequence bus impedance matrices. Assume all the prefault bus voltages are equal to $1\angle 0°$, use **lgfault**, **llfault**, and **dlgfault** to compute the fault current, bus voltages, and line currents for the following unbalanced faults.

(a) A bolted single line-to-ground fault at bus 6.
(b) A bolted line-to-line fault at bus 6.
(c) A bolted double line-to-ground fault at bus 6.

CHAPTER 11

STABILITY

11.1 INTRODUCTION

The tendency of a power system to develop restoring forces equal to or greater than the disturbing forces to maintain the state of equilibrium is known as *stability*. If the forces tending to hold machines in synchronism with one another are sufficient to overcome the disturbing forces, the system is said to remain stable (to stay in synchronism).

The stability problem is concerned with the behavior of the synchronous machines after a disturbance. For convenience of analysis, stability problems are generally divided into two major categories — *steady-state stability* and *transient stability*. Steady-state stability refers to the ability of the power system to regain synchronism after small and slow disturbances, such as gradual power changes. An extension of the steady-state stability is known as the *dynamic stability*. The dynamic stability is concerned with small disturbances lasting for a long time with the inclusion of automatic control devices. Transient stability studies deal with the effects of large, sudden disturbances such as the occurrence of a fault, the sudden outage of a line or the sudden application or removal of loads. Transient stability studies are needed to ensure that the system can withstand the transient condition following a major disturbance. Frequently, such studies are conducted when new generating and transmitting facilities are planned. The studies are helpful in determining such things as the nature of the relaying system needed, critical clearing time of circuit breakers, voltage level of, and transfer capability between systems.

11.2 SWING EQUATION

Under normal operating conditions, the relative position of the rotor axis and the resultant magnetic field axis is fixed. The angle between the two is known as the *power angle* or *torque angle*. During any disturbance, rotor will decelerate or accelerate with respect to the synchronously rotating air gap mmf, and a relative motion begins. The equation describing this relative motion is known as the *swing equation*. If, after this oscillatory period, the rotor locks back into synchronous speed, the generator will maintain its stability. If the disturbance does not involve any net change in power, the rotor returns to its original position. If the disturbance is created by a change in generation, load, or in network conditions, the rotor comes to a new operating power angle relative to the synchronously revolving field.

In order to understand the significance of the power angle we refer to the combined phasor/vector diagram of a two-pole cylindrical rotor generator illustrated in Figure 3.2. From this figure we see that the power angle δ_r is the angle between the rotor mmf F_r and the resultant air gap mmf F_{sr}, both rotating at synchronous speed. It is also the angle between the no-load generated emf E and the resultant stator voltage E_{sr}. If the generator armature resistance and leakage flux are neglected, the angle between E and the terminal voltage V, denoted by δ, is considered as the power angle.

Consider a synchronous generator developing an electromagnetic torque T_e and running at the synchronous speed ω_{sm}. If T_m is the driving mechanical torque, then under steady-state operation with losses neglected we have

$$T_m = T_e \tag{11.1}$$

A departure from steady state due to a disturbance results in an accelerating ($T_m > T_e$) or decelerating ($T_m < T_e$) torque T_a on the rotor.

$$T_a = T_m - T_e \tag{11.2}$$

If J is the combined moment of inertia of the prime mover and generator, neglecting frictional and damping torques, from law's of rotation we have

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \tag{11.3}$$

where θ_m is the angular displacement of the rotor with respect to the stationary reference axis on the stator. Since we are interested in the rotor speed relative to synchronous speed, the angular reference is chosen relative to a synchronously rotating reference frame moving with constant angular velocity ω_{sm}, that is

$$\theta_m = \omega_{sm}t + \delta_m \tag{11.4}$$

where δ_m is the rotor position before disturbance at time $t = 0$, measured from the synchronously rotating reference frame. Derivative of (11.4) gives the rotor angular velocity

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{ms} + \frac{d\delta_m}{dt} \tag{11.5}$$

and the rotor acceleration is

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \tag{11.6}$$

Substituting (11.6) into (11.3), we have

$$J\frac{d^2\delta_m}{dt^2} = T_m - T_e \tag{11.7}$$

Multiplying (11.7) by ω_m, results in

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e \tag{11.8}$$

Since angular velocity times torque is equal to the power, we write the above equation in terms of power

$$J\omega_m\frac{d^2\delta_m}{dt^2} = P_m - P_e \tag{11.9}$$

The quantity $J\omega_m$ is called the inertia constant and is denoted by M. It is related to kinetic energy of the rotating masses, W_k.

$$W_k = \frac{1}{2}J\omega_m^2 = \frac{1}{2}M\omega_m \tag{11.10}$$

or

$$M = \frac{2W_k}{\omega_m} \tag{11.11}$$

Although M is called inertia constant, it is not really constant when the rotor speed deviates from the synchronous speed. However, since ω_m does not change by a large amount before stability is lost, M is evaluated at the synchronous speed and is considered to remain constant, i.e.,

$$M = \frac{2W_k}{\omega_{sm}} \tag{11.12}$$

The swing equation in terms of the inertia constant becomes

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_e \tag{11.13}$$

It is more convenient to write the swing equation in terms of the electrical power angle δ. If p is the number of poles of a synchronous generator, the electrical power angle δ is related to the mechanical power angle δ_m by

$$\delta = \frac{p}{2}\delta_m \tag{11.14}$$

also,

$$\omega = \frac{p}{2}\omega_m \tag{11.15}$$

Swing equation in terms of electrical power angle is

$$\frac{2}{p}M\frac{d^2\delta}{dt^2} = P_m - P_e \tag{11.16}$$

Since power system analysis is done in per unit system, the swing equation is usually expressed in per unit. Dividing (11.16) by the base power S_B, and substituting for M from (11.12) results in

$$\frac{2}{p}\frac{2W_K}{\omega_{sm}S_B}\frac{d^2\delta}{dt^2} = \frac{P_m}{S_B} - \frac{P_e}{S_B} \tag{11.17}$$

We now define the important quantity known as the H *constant* or *per unit inertia constant*.

$$H = \frac{\text{kinetic energy in MJ at rated speed}}{\text{machine rating in MVA}} = \frac{W_K}{S_B} \tag{11.18}$$

The unit of H is seconds. The value of H ranges from 1 to 10 seconds, depending on the size and type of machine. Substituting in (11.17), we get

$$\frac{2}{p}\frac{2H}{\omega_{sm}}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \tag{11.19}$$

where $P_{m(pu)}$ and $P_{e(pu)}$ are the per unit mechanical power and electrical power, respectively. The electrical angular velocity is related to the mechanical angular velocity by $\omega_{sm} = (2/p)\omega_s$. (11.19) in terms of electrical angular velocity is

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_{m\,(pu)} - P_{e\,(pu)} \tag{11.20}$$

The above equation is often expressed in terms of frequency f_0, and to simplify the notation, the subscript *pu* is omitted and the powers are understood to be in per unit.

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{11.21}$$

where δ is in electrical radian. If δ is expressed in electrical degrees, the swing equation becomes

$$\frac{H}{180 f_0}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{11.22}$$

11.3 SYNCHRONOUS MACHINE MODELS FOR STABILITY STUDIES

The representation of a synchronous machine during transient conditions was discussed in Chapter 8. In Section 8.6 the cylindrical rotor machine was modeled with a constant voltage source behind proper reactances, which may be X''_d, X'_d, or X_d. The simplest model for stability analysis is the classical model, where saliency is ignored, and the machine is represented by a constant voltage E' behind the direct axis transient reactance X'_d.

Consider a generator connected to a major substation of a very large system through a transmission line as shown in Figure 11.1.

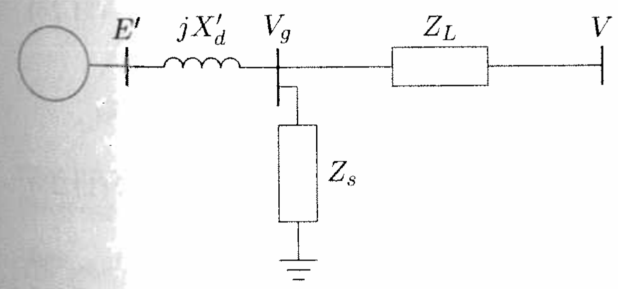

FIGURE 11.1
One machine connected to an infinite bus.

The substation bus voltage and frequency is assumed to remain constant. This is commonly referred to as an *infinite bus*, since its characteristics do not change regardless of the power supplied or consumed by any device connected to it. The generator is represented by a constant voltage behind the direct axis transient reactance X'_d. The node representing the generator terminal voltage V_g can be eliminated by converting the Y-connected impedances to an equivalent Δ with admittances given by

$$y_{10} = \frac{Z_L}{jX'_d Z_s + jX'_d Z_L + Z_L Z_s}$$
$$y_{20} = \frac{jX'_d}{jX'_d Z_s + jX'_d Z_L + Z_L Z_s} \quad (11.23)$$
$$y_{12} = \frac{Z_s}{jX'_d Z_s + jX'_d Z_L + Z_L Z_s}$$

The equivalent circuit with internal voltage represented by node 1 and the infinite bus by node 2 is shown in Figure 11.2. Writing the nodal equations, we have

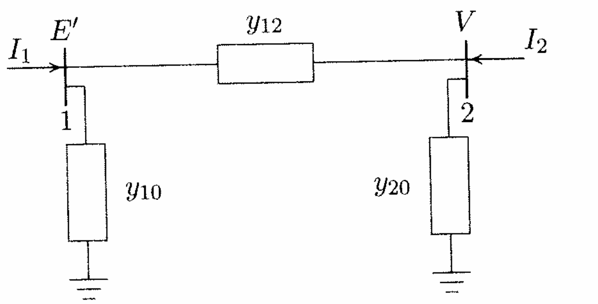

FIGURE 11.2
Equivalent circuit of one machine connected to an infinite bus.

$$I_1 = (y_{10} + y_{12})E' - y_{12}V \quad (11.24)$$
$$I_2 = -y_{12}E' + (y_{20} + y_{12})V$$

The above equations can be written in terms of the bus admittance matrix as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} E' \\ V \end{bmatrix} \quad (11.25)$$

The diagonal elements of the bus admittance matrix are $Y_{11} = y_{10} + y_{12}$, and $Y_{22} = y_{20} + y_{12}$. The off-diagonal elements are $Y_{12} = Y_{21} = -y_{12}$. Expressing the voltages and admittances in polar form, the real power at node 1 is given by

$$P_e = \Re[E'I_1^*]$$
$$= \Re[|E'|\angle\delta(|Y_{11}|\angle-\theta_{11}|E'|\angle-\delta + |Y_{12}|\angle-\theta_{12}|V|\angle 0)]$$

or

$$P_e = |E'|^2|Y_{11}|\cos\theta_{11} + |E'||V||Y_{12}|\cos(\delta - \theta_{12}) \quad (11.26)$$

The power flow equation given by (6.25) when applied to the above two-bus power system results in the same expression as (11.26). In most systems, Z_L and Z_s are predominantly inductive. If all resistances are neglected, $\theta_{11} = \theta_{12} = 90°$, $Y_{12} = B_{12} = 1/X_{12}$, and we obtain a simplified expression for power

$$P_e = |E'||V||B_{12}|\cos(\delta - 90°)$$

or

$$P_e = \frac{|E'||V|}{X_{12}} \sin\delta \quad (11.27)$$

This is the simplest form of the power flow equation and is basic to an understanding of all stability problems. The relation shows that the power transmitted depends upon the transfer reactance and the angle between the two voltages. The curve P_e versus δ is known as the *power angle curve* and is shown in Figure 11.3.

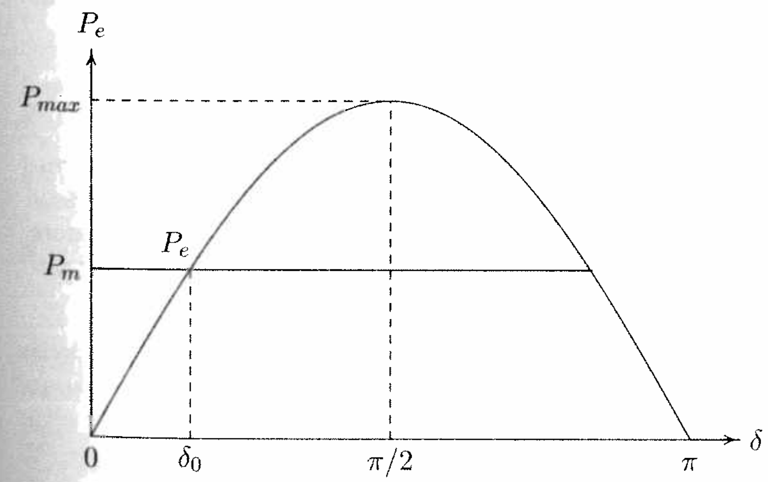

FIGURE 11.3
Power-angle curve.

The gradual increase of the generator power output is possible until the maximum electrical power is transferred. This maximum power is referred to as the *steady-state stability limit*, and occurs at an angular displacement of 90°.

$$P_{max} = \frac{|E'||V|}{X_{12}} \quad (11.28)$$

If an attempt were made to advance δ further by further increasing the shaft input, the electrical power output will decrease from the P_{max} point. The machine will accelerate, causing loss of synchronism with the infinite bus bar. The electric power equation in terms of P_{max} is

$$P_e = P_{max} \sin \delta \tag{11.29}$$

When a generator is suddenly short-circuited, the current during the transient period is limited by its transient reactance X'_d. Thus, for transient stability problems, with the saliency neglected, the machine is represented by the voltage E' behind the reactance X'_d. If V_g is the generator terminal voltage and I_a is the pre-fault steady state generator current, E' is computed from

$$E' = V_g + jX'_d I_a \tag{11.30}$$

Since the field winding has a small resistance, the field flux linkages will tend to remain constant during the initial disturbance, and thus the voltage E' is assumed constant. The transient power-angle curve has the same general form as the steady-state curve; however, it attains larger peak compared to the steady-state peak value.

11.3.1 SYNCHRONOUS MACHINE MODEL INCLUDING SALIENCY

In Section 3.4 we developed the two-axis model of a synchronous machine under steady state conditions taking into account the effect of saliency. The phasor diagram of the salient-pole machine under steady state conditions, with armature resistance neglected, was presented in Figure 3.8. This phasor diagram is represented in Figure 11.4.

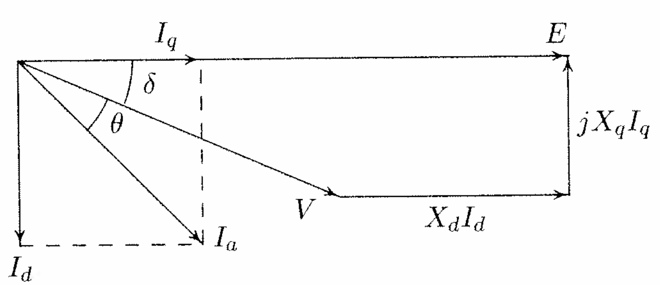

FIGURE 11.4
Phasor diagram during transient period.

The power-angle equation was given by (3.32). This equation presented in per unit is

$$P_e = \frac{|E||V|}{X_d} \sin \delta + |V|^2 \frac{X_d - X_q}{2X_d X_q} \sin 2\delta \tag{11.31}$$

and

$$|E| = |V| \cos \delta + X_d I_d$$

or

$$|E| = |V| \cos \delta + X_d |I_a| \sin(\delta + \theta) \tag{11.32}$$

where E is the no-load generated emf in per unit and V is the generator terminal voltage in per unit. X_d and X_q are direct and quadrature axis reactances of the synchronous machine. For a derivation of the above formula, refer to Section 3.4. For a given power delivered at a given terminal voltage, we must compute E. In order to do that, we must first compute δ as follows:

$$\begin{aligned}|V| \sin \delta &= X_q I_q \\ &= X_q |I_a| \cos(\delta + \theta) \\ &= X_q |I_a| (\cos \delta \cos \theta - \sin \delta \sin \theta)\end{aligned}$$

From the above relation, δ is found to be

$$\delta = \tan^{-1} \frac{X_q |I_a| \cos \theta}{|V| + X_q |I_a| \sin \theta} \tag{11.33}$$

Substituting for δ from (11.33) into (11.32) will result in the voltage E.

A logical extension of the model would be to include the effect of transient saliency. Since the machine circuits are largely inductive, the flux linkages tend to remain constant during the early part of the transient period. During the transient period, the direct axis transient reactance is X'_d. Since the field is on the direct axis X'_q, the quadrature axis transient reactance remains the same as X_q. The phasor diagram under transient condition is shown in Figure 11.5. Following the procedure

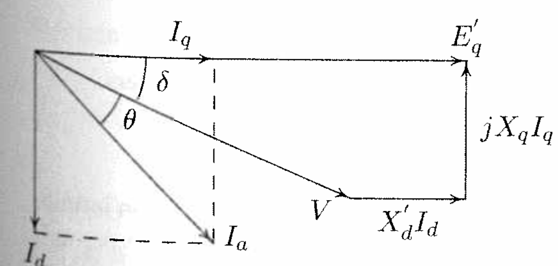

FIGURE 11.5
Phasor diagram during transient period.

in Section 3.4, the transient power-angle equation expressed in per unit becomes

$$P_e = \frac{|E_q'||V|}{X_d'}\sin\delta + |V|^2\frac{X_d' - X_q}{2X_d'X_q}\sin 2\delta \tag{11.34}$$

This equation represents the approximate behavior of the synchronous machine during the early part of transient period. We now have to determine E_q'. From the phasor diagram in Figure 11.5, we have

$$|E_q'| = |V|\cos\delta + X_d'I_d$$

or

$$|E_q'| = |V|\cos\delta + X_d'|I_a|\sin(\delta+\theta)$$

From (11.32), we find

$$|I_a|\sin(\delta+\theta) = \frac{|E| - |V|\cos\delta}{X_d}$$

and substitute it in the above equation to get

$$|E_q'| = \frac{X_d'|E| + (X_d - X_d')|V|\cos\delta}{X_d} \tag{11.35}$$

The prefault excitation voltage and power angles are computed from (11.32) and (11.33).

In this section we presented two simple models for cylindrical rotor and salient rotor synchronous machines. The choice of model for a given situation must, in general, depend upon the type of study being conducted as well as the data available. Although these models are useful for many stability studies, it is not adequate for many situations. More accurate models must include the effects of the various rotor circuits.

Example 11.1 (chp11ex1)

Consider a synchronous machine characterized by the following parameters:

$$X_d = 1.0 \quad X_q = 0.6 \quad X_d' = 0.3 \quad \text{per unit}$$

and negligible armature resistance. The machine is connected directly to an infinite bus of voltage 1.0 per unit. The generator is delivering a real power of 0.5 per unit at 0.8 power factor lagging. Determine the voltage behind transient reactance and the transient power-angle equation for the following cases.

(a) Neglecting the saliency effect
(b) Including the effect of saliency

$$\theta = \cos^{-1} 0.8 = 36.87^\circ$$
$$S = \frac{0.5}{0.8}\angle 36.87^\circ = 0.625\angle 36.87^\circ \quad \text{pu}$$

The prefault steady state current is

$$I_a = \frac{S^*}{V^*} = 0.625\angle -36.87^\circ \quad \text{pu}$$

(a) With saliency neglected, the voltage behind transient reactance is

$$E' = V + jX_d'I_a = 1.0 + (j0.3)(0.625\angle -36.87^\circ) = 1.1226\angle 7.679^\circ \quad \text{pu}$$

The transient power-angle curve is given by

$$P_e = \frac{|E'||V|}{X_d'}\sin\delta = \frac{(1.1226)(1)}{0.3}\sin\delta$$

or

$$P_e = 3.7419\sin\delta$$

(b) When the saliency effect is considered, the initial steady state power angle given by (11.33) is

$$\delta = \tan^{-1}\frac{X_q|I_a|\cos\theta}{|V| + X_q|I_a|\sin\theta} = \tan^{-1}\frac{(0.6)(0.625)(0.8)}{1.0 + (0.6)(.625)(.6)} = 13.7608^\circ$$

The steady state excitation voltage E, given by (11.32), is

$$|E| = |V|\cos\delta + X_d|I_a|\sin(\delta+\theta)$$
$$= (1.0)\cos(13.7608^\circ) + (1.0)(0.625)\sin(13.7608^\circ + 36.87^\circ) = 1.4545 \quad \text{pu}$$

The transient voltage E_q' given by (11.35) is

$$|E_q'| = \frac{X_d'|E| + (X_d - X_d')|V|\cos\delta}{X_d}$$
$$= \frac{(0.3)(1.4545) + (1.0 - 0.3)(1.0)(\cos 13.7608^\circ)}{1.0} = 1.1162 \quad \text{pu}$$

and from (11.34) the transient power-angle equation is

$$P_e = \frac{(1.1162)(1)}{0.3}\sin\delta + \frac{(1.0)^2(0.3 - 0.6)}{2(0.3)(0.6)}\sin 2\delta$$

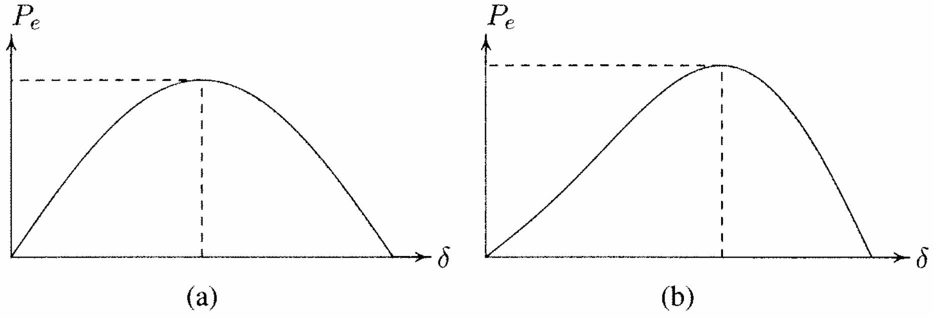

FIGURE 11.6
Transient power-angle curve for Example 11.1.

or

$$P_e = 3.7208 \sin\delta - 0.8333 \sin 2\delta$$

Using *MATLAB*, the power-angle equations obtained in (a) and (b) are plotted as shown in Figure 11.6. We use the function **[Pmax, k] = max(P)** to find the maximum power in case (b). The maximum power is found to be 4.032, occurring at angle $\delta(k) = 110.01°$.

The coefficient of $\sin 2\delta$ is relatively small, and since $X'_d < X_q$, it is negative. Thus, the $\sin 2\delta$ term has the property of subtracting from the $\sin\delta$ term in the region $0° < \delta < 90°$, but adding to it in the region $90° < \delta < 180°$. During sudden impact, when δ swings from its initial value to the maximum value for marginal stability, the overall effect of the $\sin 2\delta$ term has the tendency to average out to zero. For this reason, the $\sin 2\delta$ term is often ignored in the approximate power-angle equation.

11.4 STEADY-STATE STABILITY — SMALL DISTURBANCES

The steady-state stability refers to the ability of the power system to remain in synchronism when subjected to small disturbances. It is convenient to assume that the disturbances causing the changes disappear. The motion of the system is free, and stability is assured if the system returns to its original state. Such a behavior can be determined in a linear system by examining the characteristic equation of the system. It is assumed that the automatic controls, such as voltage regulator and governor, are not active. The actions of governor and excitation system and control devices are discussed in Chapter 12 when dealing with dynamic stability.

To illustrate the steady-state stability problem, we consider the dynamic behavior of a one-machine system connected an infinite bus bar as shown in Figure 11.1. Substituting for the electrical power from (11.29) into the swing equation given in (11.21) results in

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta \tag{11.36}$$

The swing equation is a nonlinear function of the power angle. However, for small disturbances, the swing equation may be linearized with little loss of accuracy as follows. Consider a small deviation $\Delta\delta$ in power angle from the initial operating point δ_0, i.e.,

$$\delta = \delta_0 + \Delta\delta \tag{11.37}$$

Substituting in (11.36), we get

$$\frac{H}{\pi f_0}\frac{d^2(\delta_0 + \Delta\delta)}{dt^2} = P_m - P_{max}\sin(\delta_0 + \Delta\delta)$$

or

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} + \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} = P_m - P_{max}(\sin\delta_0\cos\Delta\delta + \cos\delta_0\sin\Delta\delta)$$

Since $\Delta\delta$ is small, $\cos\Delta\delta \cong 1$ and $\sin\Delta\delta \cong \Delta\delta$, and we have

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} + \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} = P_m - P_{max}\sin\delta_0 - P_{max}\cos\delta_0\,\Delta\delta$$

Since at the initial operating state

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} = P_m - P_{max}\sin\delta_0$$

The above equation reduces to linearized equation in terms of incremental changes in power angle, i.e.,

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_{max}\cos\delta_0\,\Delta\delta = 0 \tag{11.38}$$

The quantity $P_{max}\cos\delta_0$ in (11.38) is the slope of the power-angle curve at δ_0. It is known as the *synchronizing power coefficient*, denoted by P_s. This coefficient plays an important part in determining the system stability, and is given by

$$P_s = \left.\frac{dP}{d\delta}\right|_{\delta_0} = P_{max}\cos\delta_0 \tag{11.39}$$

Substituting in (11.38), we have

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_s\Delta\delta = 0 \tag{11.40}$$

The solution of the above second-order differential equation depends on the roots of the characteristic equation given by

$$s^2 = -\frac{\pi f_0}{H}P_s \tag{11.41}$$

When P_s is negative, we have one root in the right-half s-plane, and the response is exponentially increasing and stability is lost. When P_s is positive, we have two roots on the j-ω axis, and the motion is oscillatory and undamped. The system is marginally stable with a natural frequency of oscillation given by

$$\omega_n = \sqrt{\frac{\pi f_0}{H}P_s} \tag{11.42}$$

It can be seen from Figure 11.3 that the range where P_s (i.e., the slope $dP/d\delta$) is positive lies between 0 and 90° with a maximum value at no-load ($\delta_0 = 0$).

As long as there is a difference in angular velocity between the rotor and the resultant rotating air gap field, induction motor action will take place between them, and a torque will be set up on the rotor tending to minimize the difference between the two angular velocities. This is called the *damping torque*. The damping power is approximately proportional to the speed deviation.

$$P_d = D\frac{d\delta}{dt} \tag{11.43}$$

The damping coefficient D may be determined either from design data or by test. Additional damping torques are caused by the speed/torque characteristic of the prime mover and the load dynamic, which are not considered here. When the synchronizing power coefficient P_s is positive, because of the damping power, oscillations will damp out eventually, and the operation at the equilibrium angle will be restored. No loss of synchronism occurs and the system is stable.

If damping is accounted for, the linearized swing equation becomes

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_s\,\Delta\delta = 0 \tag{11.44}$$

or

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi f_0}{H}D\frac{d\Delta\delta}{dt} + \frac{\pi f_0}{H}P_s\,\Delta\delta = 0 \tag{11.45}$$

or in terms of the standard second-order differential equation, we have

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\,\Delta\delta = 0 \tag{11.46}$$

where ω_n, the natural frequency of oscillation is given by (11.42), and ζ is defined as the dimensionless damping ratio, given by

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{HP_s}} \tag{11.47}$$

The characteristic equation is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{11.48}$$

For normal operating conditions, $\zeta = D/2\sqrt{\frac{\pi f_0}{HP_s}} < 1$, and roots of the characteristic equation are complex

$$\begin{aligned} s_1, s_2 &= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \\ &= -\zeta\omega_n + j\omega_d \end{aligned} \tag{11.49}$$

where ω_d is the damped frequency of oscillation given by

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{11.50}$$

It is clear that for positive damping, roots of the characteristic equation have negative real part if synchronizing power coefficient P_s is positive. The response is bounded and the system is stable.

We now write (11.46) in state variable form. This makes it possible to extend the analysis to multimachine systems. Let

$$x_1 = \Delta\delta \quad \text{and} \quad x_2 = \Delta\omega = \dot{\Delta\delta} \quad \text{then}$$
$$\dot{x}_1 = x_2 \quad \text{and} \quad \dot{x}_2 = -\omega_n^2 x_1 - 2\zeta\omega_n x_2$$

Writing the above equations in matrix, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11.51}$$

or

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \tag{11.52}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \tag{11.53}$$

This is the unforced state variable equation or the *homogeneous* state equation. If the state variables x_1 and x_2 are the desired response, we define the output vector $\mathbf{y}(t)$ as

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11.54}$$

or

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{11.55}$$

Taking the Laplace transform, we have

$$s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s)$$

or

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(0) \tag{11.56}$$

where

$$(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -1 \\ \omega_n^2 & s + 2\zeta\omega_n \end{bmatrix} \tag{11.57}$$

Substituting for $(s\mathbf{I} - \mathbf{A})^{-1}$, we have

$$\mathbf{X}(s) = \frac{\begin{bmatrix} s + 2\zeta\omega_n & 1 \\ -\omega_n^2 & s \end{bmatrix} \mathbf{x}(0)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

When the rotor is suddenly perturbed by a small angle $\Delta\delta_0$, $x_1(0) = \Delta\delta_0$ and $x_2(0) = \Delta\omega_0 = 0$, and we obtain

$$\Delta\delta(s) = \frac{(s + 2\zeta\omega_n)\Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

and

$$\Delta\omega(s) = -\frac{\omega_n^2\,\Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Taking inverse Laplace transforms results in the *zero-input* response

$$\Delta\delta = \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta) \tag{11.58}$$

and

$$\Delta\omega = -\frac{\omega_n\,\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\omega_d t \tag{11.59}$$

where ω_d is the damped frequency of oscillation, and θ is given by

$$\theta = \cos^{-1}\zeta \tag{11.60}$$

The motion of rotor relative to the synchronously revolving field is

$$\delta = \delta_0 + \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta) \tag{11.61}$$

and the rotor angular frequency is

$$\omega = \omega_0 - \frac{\omega_n\,\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\omega_d t \tag{11.62}$$

The response time constant is

$$\tau = \frac{1}{\zeta\omega_n} = \frac{2H}{\pi f_0 D} \tag{11.63}$$

and the response settles in approximately four time constants, and the settling time is

$$t_s \cong 4\tau \tag{11.64}$$

From (11.42) and (11.47), we note that as inertia constant H increases, the natural frequency and the damping ratio decreases, resulting in a longer settling time. An increase in the synchronizing power coefficient P_s results in an increase in the natural frequency and a decrease in the damping ratio.

Example 11.2 (chp11ex2)

A 60-Hz synchronous generator having inertia constant H = 9.94 MJ/MVA and a transient reactance $X'_d = 0.3$ per unit is connected to an infinite bus through a purely reactive circuit as shown in Figure 11.7. Reactances are marked on the diagram on a common system base. The generator is delivering real power of 0.6 per unit, 0.8 power factor lagging to the infinite bus at a voltage of $V = 1$ per unit.

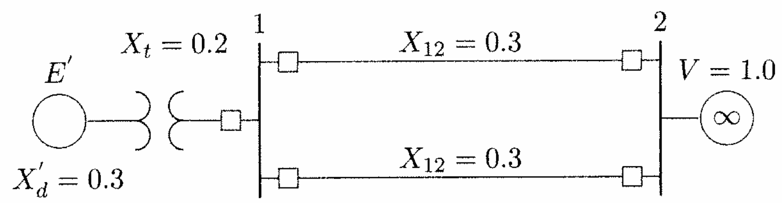

FIGURE 11.7
One-line diagram for Example 11.2.

Assume the per unit damping power coefficient is $D = 0.138$. Consider a small disturbance of $\Delta\delta = 10° = 0.1745$ radian. For example, the breakers open and then quickly close. Obtain equations describing the motion of the rotor angle and the generator frequency.

The transfer reactance between the generated voltage and the infinite bus is

$$X = 0.3 + 0.2 + \frac{0.3}{2} = 0.65$$

The per unit apparent power is

$$S = \frac{0.6}{0.8}\angle\cos^{-1} 0.8 = 0.75\angle 36.87°$$

The current is

$$I = \frac{S^*}{V^*} = \frac{0.75\angle-36.87°}{1.0\angle 0°} = 0.75\angle-36.87°$$

The excitation voltage is

$$E' = V + jXI = 1.0\angle 0° + (j0.65)(0.75\angle-36.87°) = 1.35\angle 16.79°$$

Thus, the initial operating power angle is $16.79° = 0.2931$ radian. The synchronizing power coefficient given by (11.39) is

$$P_s = P_{max}\cos\delta_0 \quad = \quad \frac{(1.35)(1)}{0.65}\cos 16.79° = 1.9884$$

The undamped angular frequency of oscillation and damping ratio are

$$\omega_n = \sqrt{\frac{\pi f_0}{H}P_s} = \sqrt{\frac{(\pi)(60)}{9.94}1.9884} = 6.1405 \text{ rad/sec}$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{HP_s}} = \frac{0.138}{2}\sqrt{\frac{(\pi)(60)}{(9.94)(1.9884)}} = 0.2131$$

The linearized force-free equation which determines the mode of oscillation given by (11.46) with δ in radian is

$$\frac{d^2\Delta\delta}{dt^2} + 2.62\frac{d\Delta\delta}{dt} + 37.7\Delta\delta = 0$$

From (11.50), the damped angular frequency of oscillation is

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 6.1405\sqrt{1-(0.2131)^2} = 6.0 \text{ rad/sec}$$

corresponding to a damped oscillation frequency of

$$f_d = \frac{6.0}{2\pi} = 0.9549 \text{ Hz}$$

From (11.61) and (11.62), the motion of rotor relative to the synchronously revolving field in electrical degrees and the frequency excursion in Hz are given by equations

$$\delta = 16.79° + 10.234e^{-1.3t}\sin(6.0t + 77.6966°)$$
$$f = 60 - 0.1746e^{-1.3t}\sin 6.0t$$

The above equations are written in *MATLAB* commands as follows

```
E=1.35; V=1.0; H=9.94; X=0.65; Pm=0.6; D= 0.138; f0 = 60;
Pmax = E*V/X, d0 = asin(Pm/Pmax)              % Max. power
Ps = Pmax*cos(d0)       % Synchronizing power coefficient
wn = sqrt(pi*60/H*Ps)% Undamped frequency of  oscillation
z = D/2*sqrt(pi*60/(H*Ps))                 % Damping ratio
wd=wn*sqrt(1-z^2), fd=wd/(2*pi) %Damped frequency oscill.
tau = 1/(z*wn)                              % Time constant
th = acos(z)                              % Phase angle theta
Dd0 = 10*pi/180;                  % Initial angle in radian
t = 0:.01:3;
Dd = Dd0/sqrt(1-z^2)*exp(-z*wn*t).*sin(wd*t + th);
d = (d0+Dd)*180/pi;                  % Power angle in degree
Dw = -wn*Dd0/sqrt(1-z^2)*exp(-z*wn*t).*sin(wd*t);
f = f0 + Dw/(2*pi);                        % Frequency in Hz
subplot(2,1,1), plot(t, d), grid
xlabel('t  sec'), ylabel('Delta    degree')
subplot(2,1,2), plot(t,f), grid
xlabel('t  sec'), ylabel('Frequency    Hz')
subplot(111)
```

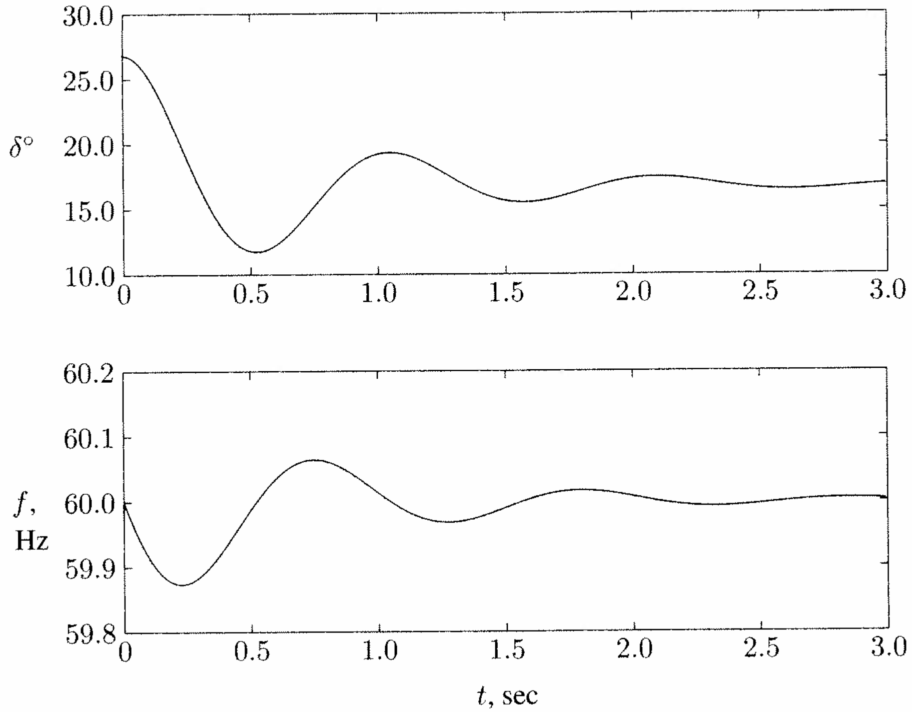

FIGURE 11.8
Natural responses of the rotor angle and frequency for machine of Example 11.2.

The result is shown in Figure 11.8.

The response shows that a small disturbance will be followed by a relatively slowly damped oscillation, or swing, of the rotor, before steady state operation at synchronous speed is resumed. In the case of a steam turbine generator, oscillations subside in a matter of two to three seconds. In the above example, the response settles in about $t_s \simeq 4\tau = 4(1/1.3) \simeq 3.1$ seconds. We also observe that the oscillations are fairly low in frequency, in the order of 0.955 Hz.

The formulation of the one-machine system with all control devices inactive resulted in a second-order differential equation or a two-dimensional state equation. Later on, when the analysis is extended to a multimachine system, an n-dimensional state variables equation is obtained. *MATLAB* Control Toolbox provides a function named **initial** for simulating continuous-time linear systems due to an initial condition on the states. Given the system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y} &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned} \tag{11.65}$$

$[\mathbf{y}, \mathbf{x}] = \mathbf{initial}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{x_0}, \mathbf{t})$ returns the output and state responses of the system to the initial condition $\mathbf{x_0}$. The matrices $\mathbf{y}$ and $\mathbf{x}$ contain the output and state response of the system at the regularly spaced time vector t.

From (11.52)–(11.54), the zero-input state equation for Example 11.2 is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -37.705 & -2.617 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The initial state variables are $\Delta\delta_0 = 10° = 0.1745$ radian, and $\Delta\dot{\delta}_0 = 0$. The following *MATLAB* commands are used to obtain the zero-input response for Example 11.2.

```
A = [0  1; -37.705  -2.617];
B = [0; 0];                               %  Column B zero-input
C=[1 0; 0 1];%Unity matrix defining output y as x1 and x2
D = [0; 0];
Dx0 = [0.1745; 0];                         % Initial conditions
[y, x] = initial(A, B, C, D, Dx0, t);
Dd = x(:, 1); Dw = x(:, 2);    % State variables x1 and x2
d = (d0 + Dd)*180/pi;                    % Power angle in degree
f = f0 + Dw/(2*pi);                           % Frequency in Hz
subplot(2,1,1), plot(t, d), grid
xlabel('t  sec'), ylabel('Delta    Degree')
subplot(2,1,2), plot(t, f), grid
xlabel('t  sec'), ylabel('Frequency    Hz'),subplot(111)
```

The simulation results are exactly the same as the graphs shown in Figure 11.8.

Although it is convenient to assume that the disturbances causing the changes disappear, we will now investigate the system response to small power impacts. Assume the power input is increased by a small amount ΔP. Then the linearized swing equation becomes

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_s\,\Delta\delta = \Delta P \tag{11.66}$$

or

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi f_0}{H}D\frac{d\Delta\delta}{dt} + \frac{\pi f_0}{H}P_s\,\Delta\delta = \frac{\pi f_0}{H}\Delta P \tag{11.67}$$

or in terms of the standard second-order differential equation, we have

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\omega_n \frac{d\Delta\delta}{dt} + \omega_n^2\,\Delta\delta = \Delta u \tag{11.68}$$

where

$$\Delta u = \frac{\pi f_0}{H}\Delta P \tag{11.69}$$

and ω_n and ζ are given by (11.42) and (11.47), respectively. Transforming to the state variable form, we have

$$x_1 = \Delta\delta \quad \text{and} \quad x_2 = \Delta\omega = \dot{\Delta\delta} \quad \text{then}$$
$$\dot{x}_1 = x_2 \quad \text{and} \quad \dot{x}_2 = -\omega_n^2 x_1 - 2\zeta\omega_n x_2$$

Writing the above equations in matrix, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta u \tag{11.70}$$

or

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\Delta u(t) \tag{11.71}$$

This is the forced state variable equation or the *zero-state* equation, and with x_1 and x_2 the desired response, the output vector $\mathbf{y}(t)$ is given by (11.55). Taking the Laplace transform of the state equation (11.71) with zero initial states results in

$$s\mathbf{X}(s) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\,\Delta U(s)$$

or

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\,\Delta U(s) \tag{11.72}$$

where

$$\Delta U(s) = \frac{\Delta u}{s}$$

Substituting for $(s\mathbf{I} - \mathbf{A})^{-1}$, we have

$$\mathbf{X}(s) = \frac{\begin{bmatrix} s + 2\zeta\omega_n & 1 \\ -\omega_n^2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{\Delta u}{s}}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

or

$$\Delta\delta(s) = \frac{\Delta u}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

and

$$\Delta\omega(s) = \frac{\Delta u}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

Taking inverse Laplace transforms results in the step response

$$\Delta\delta = \frac{\Delta u}{\omega_n^2}[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta)] \tag{11.73}$$

where $\theta = \cos^{-1}\zeta$ and

$$\Delta\omega = \frac{\Delta u}{\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\omega_d t \tag{11.74}$$

Substituting for Δu from (11.69), the motion of rotor relative to the synchronously revolving field in electrical radian becomes

$$\delta = \delta_0 + \frac{\pi f_0\,\Delta P}{H\omega_n^2}[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta)] \tag{11.75}$$

and the rotor angular frequency in radian per second is

$$\omega = \omega_0 + \frac{\pi f_0\,\Delta P}{H\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\omega_d t \tag{11.76}$$

Example 11.3 (chp11ex3), (sim11ex3.mdl)

The generator of Example 11.2 is operating in the steady state at $\delta_0 = 16.79°$ when the input power is increased by a small amount $\Delta P = 0.2$ per unit. The generator excitation and the infinite bus bar voltage are the same as before, i.e., $E' = 1.35$ per unit and $V = 1.0$ per unit.
(a) Using (11.75) and (11.76), obtain the step response for the rotor angle and the generator frequency.
(b) Obtain the response using the *MATLAB* **step** function.
(c) Obtain a *SIMULINK* block diagram representation of the state-space model and simulate to obtain the response.

(a) Substituting for H, δ_0, ζ, and ω_n evaluated in Example 11.2 in (11.75), and expressing the power angle in degree, we get

$$\delta = 16.79° + \frac{(180)(60)(0.2)}{(9.94)(6.1405)^2}[1 - \frac{1}{\sqrt{1-(0.2131)^2}} e^{-1.3t} \sin(6t + 77.6966°)]$$

or

$$\delta = 16.79° + 5.7631[1 - 1.0235e^{-1.3t}\sin(6t + 77.6966°)]$$

Also, substituting the values in (11.76) and expressing the frequency in Hz, we get

$$f = 60 + \frac{(60)(0.2)}{2(9.94)(6.1405)\sqrt{1-(0.2131)^2}}e^{-1.3t}\sin 6t$$

or

$$f = 60 + 0.10e^{-1.3t}\sin 6t$$

The above functions are plotted over a range of 0 to 3 seconds and the result is shown in Figure 11.9.

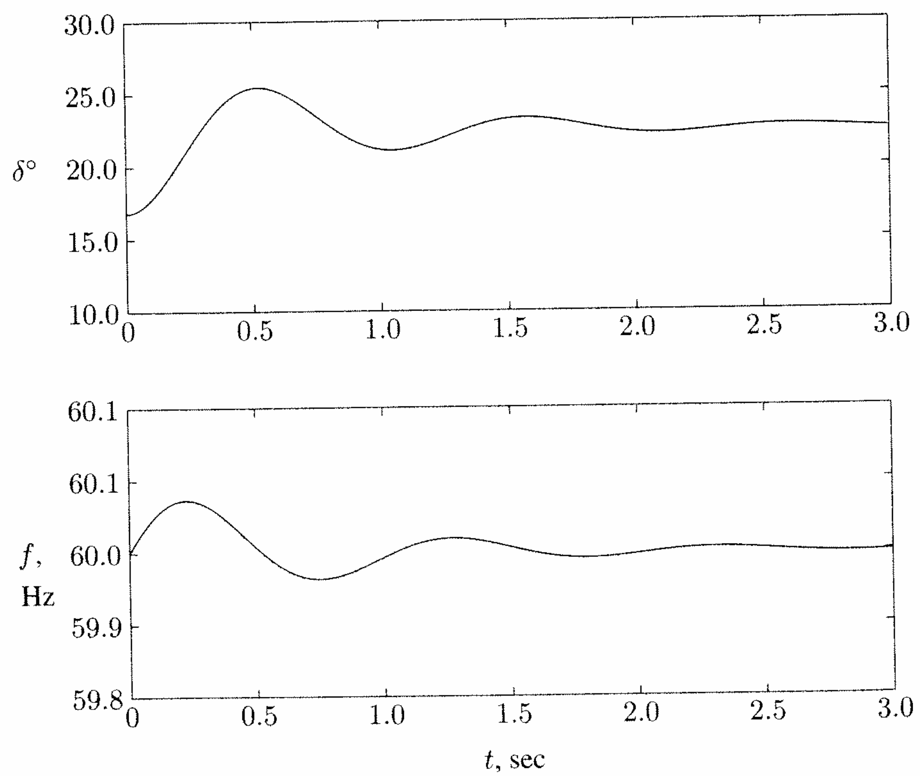

FIGURE 11.9
Step responses of the rotor angle and frequency for machine of Example 11.3.

(b) The step response of the state equation can be obtained conveniently, using the *MATLAB* Control Toolbox **[y, x] = lsim(A, B, C, D, u, t)** function or **[y,x] = step(A, B, C, D, iu, t)** function. These functions are particularly useful when dealing with multimachine systems. $[\mathbf{y}, \mathbf{x}] = \mathbf{step}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{iu}, \mathbf{t})$ returns the output and step responses of the system. The index **iu** specifies which input to be used for the step response. With only one input **iu** = 1. The matrices **y** and **x** contain the output and state response of the system at the regularly spaced time vector t.

From (11.70), the state equation for Example 11.3 is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -37.705 & -2.617 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta u$$

and

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

From (11.69), $\Delta u = (60\pi/9.94)(0.2) = 3.79$. The following *MATLAB* commands are used to obtain the step response for Example 11.3.

```
A = [0  1; -37.705  -2.617];
Dp = 0.2; Du = 3.79;    % Small step change in power input
B = [0; 1]*Du;
C=[1 0; 0 1];%Unity matrix defining output y as x1 and x2
D = [0; 0];
[y, x] = step(A, B, C, D, 1, t);
Dd = x(:, 1); Dw = x(:, 2);    % State variables x1 and x2
d = (d0 + Dd)*180/pi;                 % Power angle in degree
f = f0 + Dw/(2*pi);                      % Frequency in Hz
subplot(2,1,1), plot(t, d), grid
xlabel('t  sec'), ylabel('Delta    degree')
subplot(2,1,2), plot(t, f), grid
xlabel('t  sec'), ylabel('Frequency    Hz'),subplot(111)
```

The simulation results are exactly the same as the analytical solution and the plots are shown in Figure 11.9.

(c) A *SIMULINK* model named **sim11ex3.mdl** is constructed as shown in Figure 11.10. The file is opened and is run in the *SIMULINK WINDOW*. The simulation results in the same response as shown in Figure 11.9.

The response shows that the oscillation subsides in approximately 3.1 seconds and a new steady state operating point is attained at $\delta = 22.5°$. For the linearized swing equation, the stability is entirely independent of the input, and for a positive damping coefficient the system is always stable as long as the synchronizing power coefficient is positive. Theoretically, power can be increased gradually

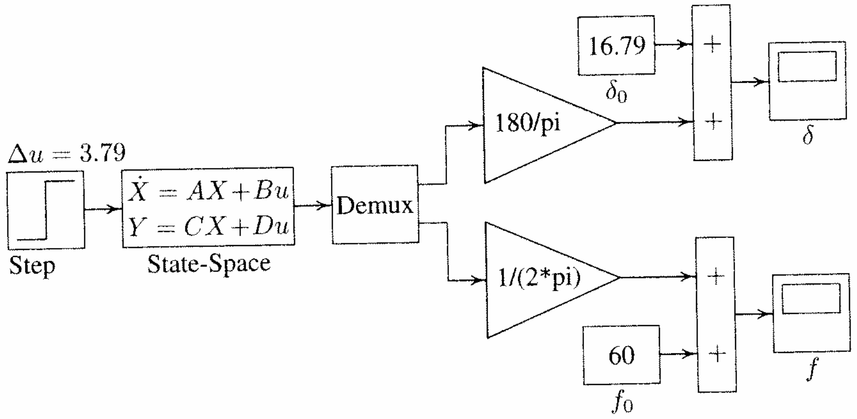

FIGURE 11.10
Simulation block diagram for Example 11.3.

up to the steady-state limit. It is important to note that the linearized equation is only valid for very small power impact and deviation from the operating state. Indeed, for a large sudden impact the nonlinear equation may result in unstable solution and stability is lost even if the impact is less than the steady-state power limit.

An important characteristic of the linear system is that the response is asymptotically stable if all roots of the characteristic equation have negative real part. The polynomial characteristic equation is obtained from the determinant of $(s\mathbf{I} - \mathbf{A})$ or eigenvalues of A. The eigenvalues provide very important results regarding the nature of response. The reciprocal of the real component of the eigenvalues gives the time constants, and the imaginary component gives the damped frequency of oscillations. Thus, the linear system expressed in the state variable form is asymptotically stable if and only if all of the eigenvalues of $\mathbf{A}$ lie in the left half of the complex plane. Therefore, to investigate the system stability of a multimachine system when subjected to small disturbances, all we need to do is to examine the eigenvalues of the $\mathbf{A}$ matrix. If the homogeneous state equation is written as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{11.77}$$

we note that matrix $\mathbf{A}$ is the *Jacobian matrix* whose elements are partial derivatives of rows of $\mathbf{f}(\mathbf{x})$ with respect to state variables $x_1, x_2, \cdots, x_n$, evaluated at the equilibrium point. In *MATLAB* we can use the function $\mathbf{r} = \text{eig}(\mathbf{A})$, which returns the eigenvalues of the $\mathbf{A}$ matrix. In Example 11.2, the $\mathbf{A}$ matrix was found to be

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -37.705 & -2.617 \end{bmatrix}$$

and the commands


```
A = [0  1; -37.705  -2.617];
r = eig(A)
```

result in

```
r =
     -1.3  +  6.00i
     -1.3  +  6.00i
```

The linearized model for small disturbances is very useful when the system is extended to include the governor action and the effect of automatic voltage regulators in a multimachine system. The linearized model allows the application of the linear control system analysis and compensation, which will be dealt with in Chapter 12.

11.5 TRANSIENT STABILITY — EQUAL-AREA CRITERION

The transient stability studies involve the determination of whether or not synchronism is maintained after the machine has been subjected to severe disturbance. This may be sudden application of load, loss of generation, loss of large load, or a fault on the system. In most disturbances, oscillations are of such magnitude that linearization is not permissible and the nonlinear swing equation must be solved.

A method known as the *equal-area criterion* can be used for a quick prediction of stability. This method is based on the graphical interpretation of the energy stored in the rotating mass as an aid to determine if the machine maintains its stability after a disturbance. The method is only applicable to a one-machine system connected to an infinite bus or a two-machine system. Because it provides physical insight to the dynamic behavior of the machine, application of the method to analysis of a single machine connected to a large system is considered here.

Consider a synchronous machine connected to an infinite bus. The swing equation with damping neglected as given by (11.21) is

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a$$

where P_a is the accelerating power. From the above equation, we have

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(P_m - P_e)$$

Multiplying both sides of the above equation by $2d\delta/dt$, we get

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{2\pi f_0}{H}(P_m - P_e)\frac{d\delta}{dt}$$

This may be written as

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = \frac{2\pi f_0}{H}(P_m - P_e)\frac{d\delta}{dt}$$

or

$$d\left[\left(\frac{d\delta}{dt}\right)^2\right] = \frac{2\pi f_0}{H}(P_m - P_e)d\delta$$

Integrating both sides,

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)d\delta$$

or

$$\frac{d\delta}{dt} = \sqrt{\frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)d\delta} \tag{11.78}$$

Equation (11.78) gives the relative speed of the machine with respect to the synchronously revolving reference frame. For stability, this speed must become zero at some time after the disturbance. Therefore, from (11.78), we have for the stability criterion,

$$\int_{\delta_0}^{\delta}(P_m - P_e)d\delta = 0 \tag{11.79}$$

Consider the machine operating at the equilibrium point δ_0, corresponding to the mechanical power input $P_{m0} = P_{e0}$ as shown in Figure 11.11. Consider a sudden step increase in input power represented by the horizontal line P_{m1}. Since $P_{m1} > P_{e0}$, the accelerating power on the rotor is positive and the power angle δ increases. The excess energy stored in the rotor during the initial acceleration is

$$\int_{\delta_0}^{\delta_1}(P_{m1} - P_e)d\delta = \text{area } abc = \text{area } A_1 \tag{11.80}$$

With increase in δ, the electrical power increases, and when $\delta = \delta_1$, the electrical power matches the new input power P_{m1}. Even though the accelerating power is zero at this point, the rotor is running above synchronous speed; hence, δ and

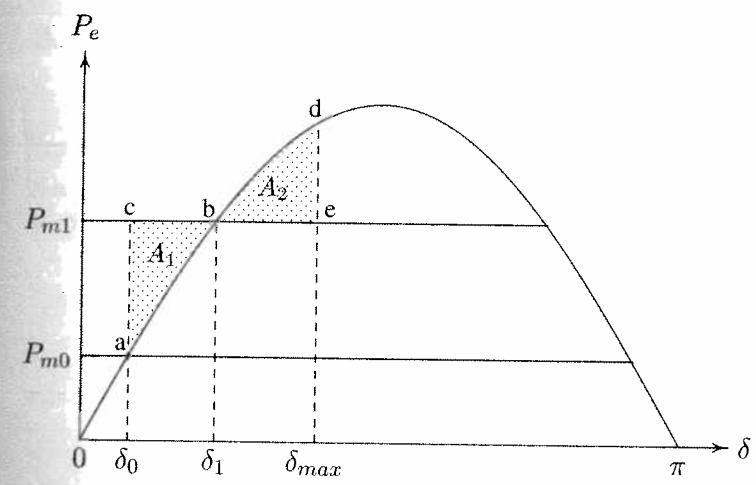

FIGURE 11.11
Equal-area criterion—sudden change of load.

electrical power P_e will continue to increase. Now $P_m < P_e$, causing the rotor to decelerate toward synchronous speed until $\delta = \delta_{max}$. According to (11.79), the rotor must swing past point b until an equal amount of energy is given up by the rotating masses. The energy given up by the rotor as it decelerates back to synchronous speed is

$$\int_{\delta_1}^{\delta_{max}}(P_{m1} - P_e)d\delta = \text{area } bde = \text{area } A_2 \tag{11.81}$$

The result is that the rotor swings to point b and the angle δ_{max}, at which point

$$|\text{area } A_1| = |\text{area } A_2| \tag{11.82}$$

This is known as the *equal-area criterion*. The rotor angle would then oscillate back and forth between δ_0 and δ_{max} at its natural frequency. The damping present in the machine will cause these oscillations to subside and the new steady state operation would be established at point b.

11.5.1 APPLICATION TO SUDDEN INCREASE IN POWER INPUT

The equal-area criterion is used to determine the maximum additional power P_m which can be applied for stability to be maintained. With a sudden change in the power input, the stability is maintained only if area A_2 at least equal to A_1 can be located above P_m. If area A_2 is less than area A_1, the accelerating momentum can never be overcome. The limit of stability occurs when δ_{max} is at the intersection of line P_m and the power-angle curve for $90° < \delta < 180°$, as shown in Figure 11.12.

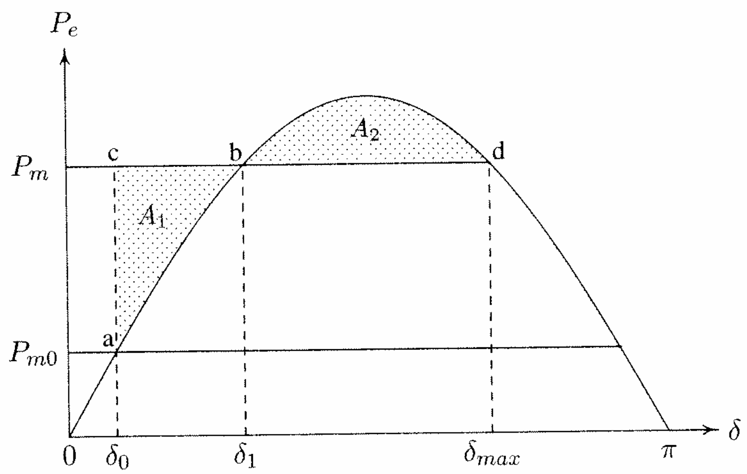

FIGURE 11.12
Equal-area criterion—maximum power limit.

Applying the equal-area criterion to Figure 11.12, we have

$$P_m(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{max} \sin\delta \, d\delta = \int_{\delta_1}^{\delta_{max}} P_{max} \sin\delta \, d\delta \; - \; P_m(\delta_{max} - \delta_1)$$

Integrating the above expression yields

$$(\delta_{max} - \delta_0)P_m = P_{max}(\cos\delta_0 - \cos\delta_{max})$$

Substituting for P_m, from

$$P_m = P_{max} \sin\delta_{max}$$

into the above equation results in

$$(\delta_{max} - \delta_0)\sin\delta_{max} + \cos\delta_{max} = \cos\delta_0 \tag{11.83}$$

The above nonlinear algebraic equation can be solved by an iterative technique for δ_{max}. Once δ_{max} is obtained, the maximum permissible power or the transient stability limit is found from

$$P_m = P_{max} \sin\delta_1 \tag{11.84}$$

where

$$\delta_1 = \pi - \delta_{max} \tag{11.85}$$

Equation (11.83) is a nonlinear function of angle δ_{max}, written as

$$f(\delta_{max}) = c \tag{11.86}$$

An iterative solution is obtained, using the Newton-Raphson method, described in Section 6.3. Starting with an initial estimate of $\pi/2 < \delta_{max}^{(k)} < \pi$, the Newton-Raphson algorithm gives

$$\Delta\delta_{max}^{(k)} = \frac{c - f(\delta_{max}^{(k)})}{\frac{df}{d\delta_{max}}\Big|_{\delta_{max}^{(k)}}} \tag{11.87}$$

where $df/d\delta_{max}$ is the derivative of (11.83) and is given by

$$\frac{df}{d\delta_{max}}\Big|_{\delta_{max}^{(k)}} = (\delta_{max}^{(k)} - \delta_0)\cos\delta_{max}^{(k)} \tag{11.88}$$

and

$$\delta_{max}^{(k+1)} = \delta_{max}^{(k)} + \Delta\delta_{max}^{(k)} \tag{11.89}$$

A solution is obtained when the difference between the absolute value of the successive iteration is less than a specified accuracy, i.e.,

$$|\delta_{max}^{(k+1)} - \delta_{max}^{(k)}| \leq \epsilon \tag{11.90}$$

A function named **eacpower**(P_0, E, V, X) is developed for a one-machine system connected to an infinite bus. The function uses the above algorithm to find the sudden maximum permissible power that can be applied for critical stability. The function plots the power-angle curve and displays the shaded equal-areas. P_0, E, V, and X are the initial power, the transient internal voltage, the infinite bus bar voltage, and the transfer reactance, respectively, all in per unit. If **eacpower** is used without arguments, the user is prompted to enter the above quantities.

Example 11.4 (chp11ex4)

The machine of Example 11.2 is delivering a real power of 0.6 per unit, at 0.8 power factor lagging to the infinite bus bar. The infinite bus bar voltage is 1.0 per unit. Determine
(a) The maximum power input that can be applied without loss of synchronism.
(b) Repeat (a) with zero initial power input. Assume the generator internal voltage remains constant at the value computed in (a).

In Example 11.2 the transfer reactance and the generator internal voltage were found to be $X = 0.65$ pu, and $E' = 1.35$ pu.

(a) We use the following command:

```
P0 = 0.6; E = 1.35; V = 1.0; X = 0.65;
eacpower(P0, E, V, X)
```

which displays the graph shown in Figure 11.13 and results in

```
Initial power                      =   0.600 pu
Initial power angle                =  16.791 degree
Sudden initial power               =   1.084 pu
Total power for critical stability =   1.684 pu
Maximum angle swing                = 125.840 pu
New operating angle                =  54.160 degree
```

(b) The initial power input is set to zero, i.e., $P_o = 0$, and using **eacpower(Po, E, V, X)** displays the graph shown in Figure 11.14 with the following results:

```
Initial power                      =   0.00 pu
Initial power angle                =   0.00 degree
Sudden initial power               =   1.505 pu
Total power for critical stability =   1.505 pu
Maximum angle swing                = 133.563 pu
New operating angle                =  46.437 degree
```

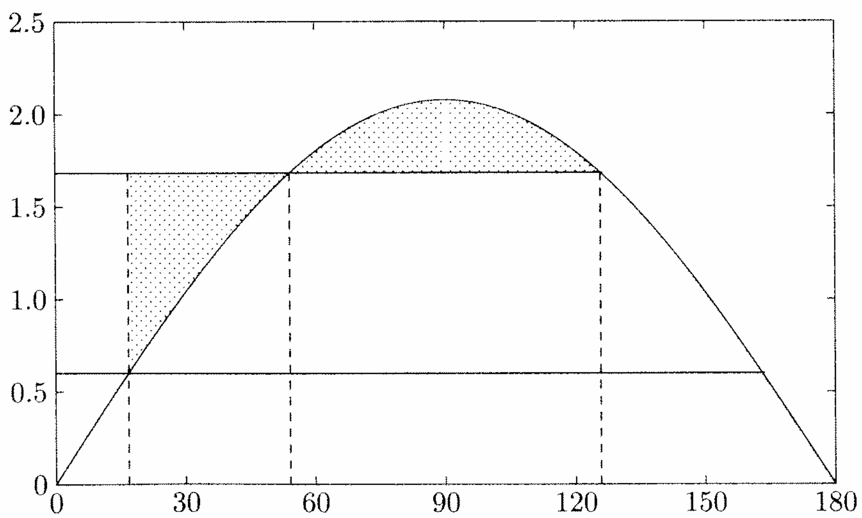

FIGURE 11.13
Maximum power limit by equal-area criterion for Example 11.4 (a).

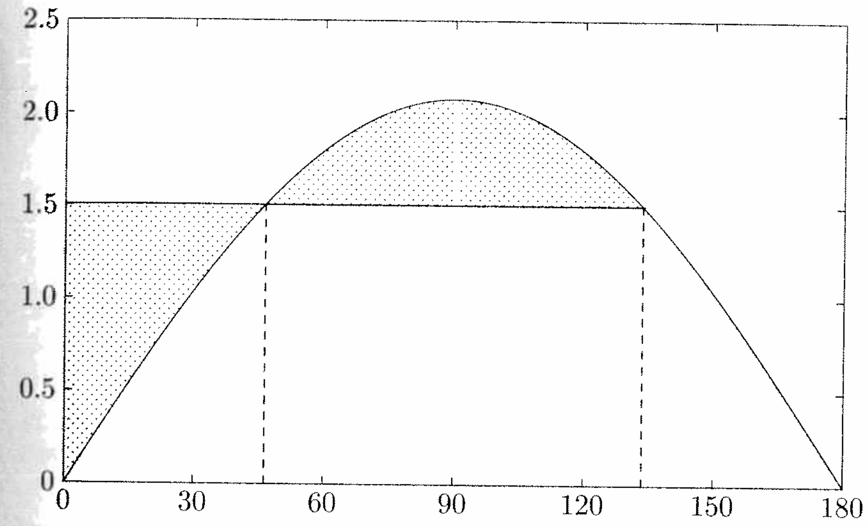

FIGURE 11.14
Maximum power limit by equal-area criterion for Example 11.4 (b).

11.6 APPLICATION TO THREE-PHASE FAULT

Consider Figure 11.15 where a generator is connected to an infinite bus bar through two parallel lines. Assume that the input power P_m is constant and the machine is operating steadily, delivering power to the system with a power angle δ_0 as shown in Figure 11.16. A temporary three-phase bolted fault occurs at the sending end of one of the line at bus 1.

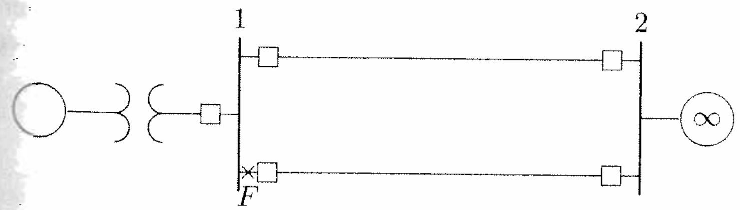

FIGURE 11.15
One-machine system connected to infinite bus, three-phase fault at F.

When the fault is at the sending end of the line, point F, no power is transmitted to the infinite bus. Since the resistances are neglected, the electrical power P_e is zero, and the power-angle curve corresponds to the horizontal axis. The machine accelerates with the total input power as the accelerating power, thereby increasing

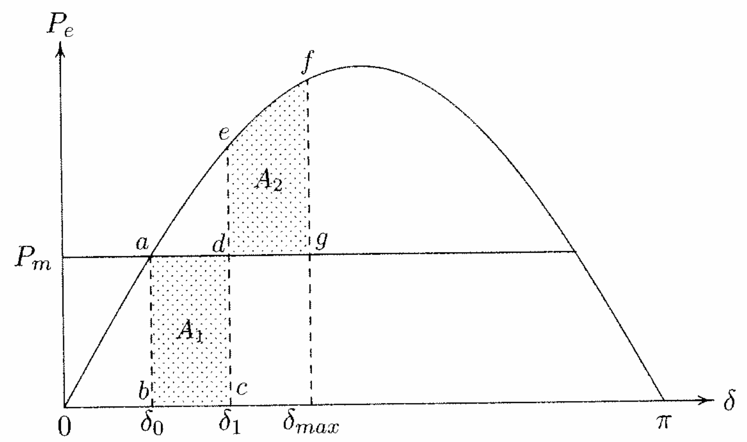

FIGURE 11.16
Equal-area criterion for a three-phase fault at the sending end.

its speed, storing added kinetic energy, and increasing the angle δ. When the fault is cleared, both lines are assumed to be intact. The fault is cleared at δ_1, which shifts the operation to the original power-angle curve at point e. The net power is now decelerating, and the previously stored kinetic energy will be reduced to zero at point f when the shaded area $(defg)$, shown by A_2, equals the shaded area $(abcd)$, shown by A_1. Since P_e is still greater than P_m, the rotor continues to decelerate and the path is retraced along the power-angle curve passing through points e and a. The rotor angle would then oscillate back and forth around δ_0 at its natural frequency. Because of the inherent damping, oscillation subsides and the operating point returns to the original power angle δ_0.

The critical clearing angle is reached when any further increase in δ_1 causes the area A_2, representing decelerating energy to become less than the area representing the accelerating energy. This occurs when δ_{max}, or point f, is at the intersection of line P_m and curve P_e, as shown in Figure 11.17. Applying equal-area criterion to Figure 11.17, we have

$$\int_{\delta_0}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{max}} (P_{max} \sin\delta - P_m) d\delta$$

Integrating both sides, we have

$$P_m(\delta_c - \delta_0) = P_{max}(\cos\delta_c - \cos\delta_{max}) - P_m(\delta_{max} - \delta_c)$$

Solving for δ_c, we get

$$\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max} \qquad (11.91)$$

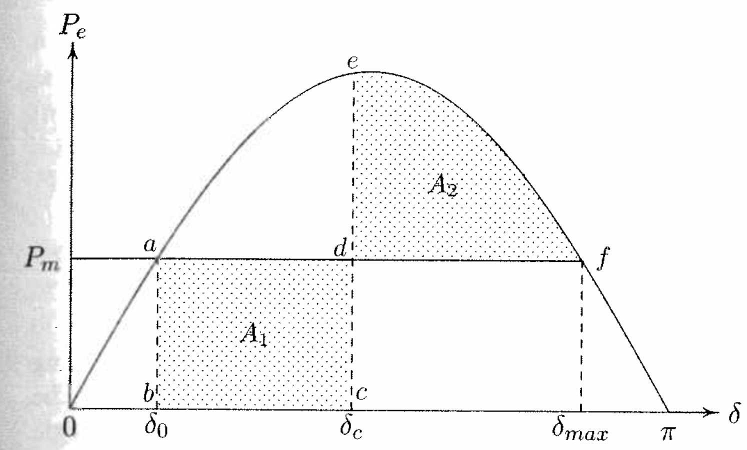

FIGURE 11.17
Equal-area criterion for critical clearing angle.

The application of equal-area criterion made it possible to find the critical clearing angle for the machine to remain stable. To find the critical clearing time, we still need to solve the nonlinear swing equation. For this particular case where the electrical power P_e during fault is zero, an analytical solution for critical clearing time can be obtained. The swing equation as given by (11.21), during fault with $P_e = 0$ becomes

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m$$

or

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H} P_m$$

Integrating both sides

$$\frac{d\delta}{dt} = \frac{\pi f_0}{H} P_m \int_0^t dt = \frac{\pi f_0}{H} P_m t$$

Integrating again, we get

$$\delta = \frac{\pi f_0}{2H} P_m t^2 + \delta_0$$

Thus, if δ_c is the critical clearing angle, the corresponding critical clearing time is

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}} \qquad (11.92)$$

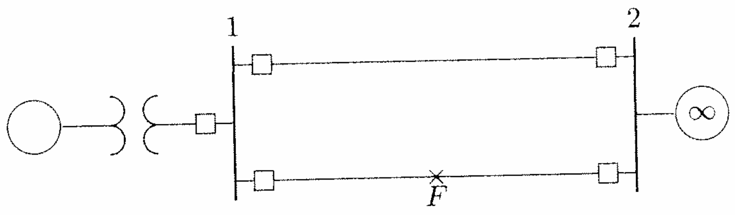

FIGURE 11.18
One-machine system connected to infinite bus, three-phase fault at F.

Now consider the fault location F at some distance away from the sending end as shown in Figure 11.18. Assume that the input power P_m is constant and the machine is operating steadily, delivering power to the system with a power angle δ_0 as shown in Figure 11.19. The power-angle curve corresponding to the prefault condition is given by curve A.

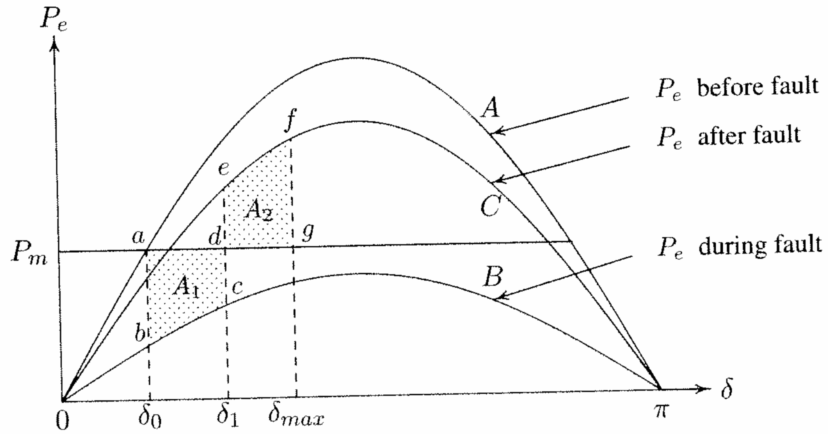

FIGURE 11.19
Equal-area criterion for a three-phase fault at the away from the sending end.

With fault location at F, away from the sending end, the equivalent transfer reactance between bus bars is increased, lowering the power transfer capability and the power-angle curve is represented by curve B. Finally, curve C represents the postfault power-angle curve, assuming the faulted line is removed. When the three-phase fault occurs, the operating point shifts immediately to point b on curve B. An excess of the mechanical input over its electrical output accelerates the rotor, thereby storing excess kinetic energy, and the angle δ increases. Assume the fault is cleared at δ_1 by isolating the faulted line. This suddenly shifts the operating point to e on curve C. The net power is now decelerating, and the previously stored kinetic energy will be reduced to zero at point f when the shaded area ($defg$) equals the shaded area ($abcd$). Since P_e is still greater than P_m, the rotor continues

to decelerate, and the path is retraced along the power-angle curve passing through point e. The rotor angle will then oscillate back and forth around e at its natural frequency. The damping present in the machine will cause these oscillations to subside and a new steady state operation will be established at the intersection of P_m and curve C.

The critical clearing angle is reached when any further increase in δ_1 causes the area A_2, representing decelerating energy, to become less than the area representing the accelerating energy. This occurs when δ_{max}, or point f, is at the intersection of line P_m and curve C as shown in Figure 11.20.

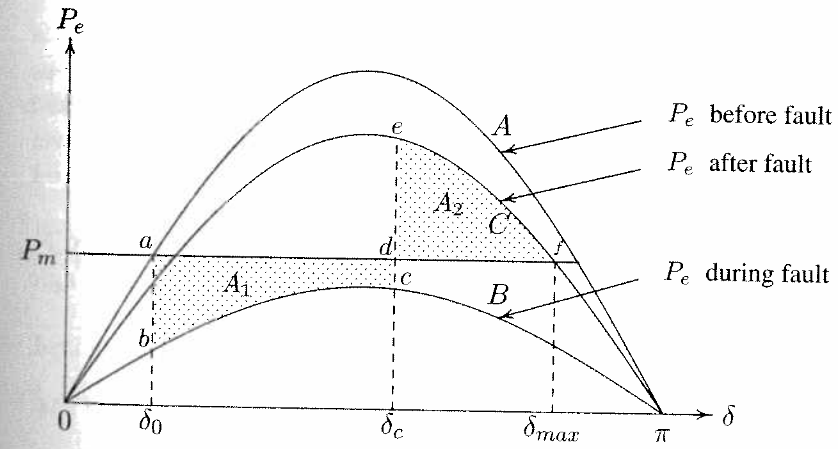

FIGURE 11.20
Equal-area criterion for critical clearing angle.

Applying equal-area criterion to Figure 11.20, we get

$$P_m(\delta_c - \delta_0) - \int_{\delta_0}^{\delta_c} P_{2max} \sin\delta \, d\delta = \int_{\delta_c}^{\delta_{max}} P_{3max} \sin\delta \, d\delta - P_m(\delta_{max} - \delta_c)$$

Integrating both sides, and solving for δ_c, we obtain

$$\cos\delta_c = \frac{P_m(\delta_{max} - \delta_0) + P_{3max}\cos\delta_{max} - P_{2max}\cos\delta_0}{P_{3max} - P_{2max}} \tag{11.93}$$

The application of equal-area criterion gives the critical clearing angle to maintain stability. However, because of the nonlinearity of the swing equation, an analytical solution for critical clearing time is not possible. In the next section we will discuss the numerical solution, which can readily be extended to large systems.

A function named **eacfault**(P_0, E, V, X_1, X_2, X_3) is developed for a one-machine system connected to an infinite bus. This function obtains the power-angle curve before fault, during fault, and after the fault clearance. The function uses equal-area criterion to find the critical clearing angle. For the case when power transfer during fault is zero, (11.92) is used to find the critical clearing time. The function plots the power-angle curve and displays the shaded equal-areas. P_0, E, and V are the initial power, the generator transient internal voltage, and the infinite bus bar voltage, all in per unit. X_1 is the transfer reactance before fault. X_2 is the transfer reactance during fault. If power transfer during fault is zero, **inf** must be used for X_2. Finally, X_3 is the postfault transfer reactance. If **eacfault** is used without arguments, the user is prompted to enter the above quantities.

Example 11.5 (chp11ex5)

A 60-Hz synchronous generator having inertia constant H = 5 MJ/MVA and a direct axis transient reactance $X'_d = 0.3$ per unit is connected to an infinite bus through a purely reactive circuit as shown in Figure 11.21. Reactances are marked on the diagram on a common system base. The generator is delivering real power $P_e = 0.8$ per unit and $Q = 0.074$ per unit to the infinite bus at a voltage of $V = 1$ per unit.
(a) A temporary three-phase fault occurs at the sending end of the line at point F. When the fault is cleared, both lines are intact. Determine the critical clearing angle and the critical fault clearing time.
(b) A three-phase fault occurs at the middle of one of the lines, the fault is cleared, and the faulted line is isolated. Determine the critical clearing angle.

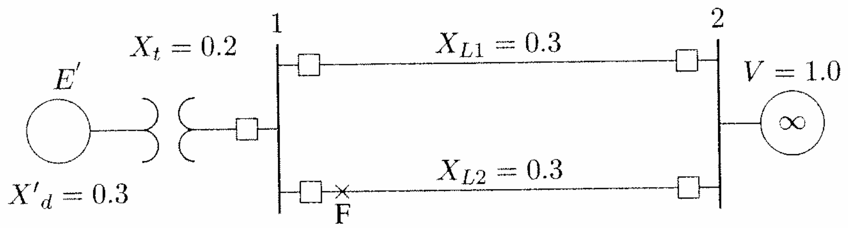

FIGURE 11.21
One-line diagram for Example 11.5.

The current flowing into the infinite bus is

$$I = \frac{S^*}{V^*} = \frac{0.8 - j0.074}{1.0\angle 0^\circ} = 0.8 - j0.074 \text{ pu}$$

The transfer reactance between internal voltage and the infinite bus before fault is

$$X_1 = 0.3 + 0.2 + \frac{0.3}{2} = 0.65$$

The transient internal voltage is

$$E' = V + jX_1 I = 1.0 + (j0.65)(0.8 - j0.074) = 1.17\angle 26.387^\circ \text{ pu}$$

(a) Since both lines are intact when the fault is cleared, the power-angle equation before and after the fault is

$$P_{max}\sin\delta = \frac{(1.17)(1.0)}{0.65}\sin\delta = 1.8\sin\delta$$

The initial operating angle is given by

$$1.8\sin\delta_0 = 0.8$$

or

$$\delta_0 = 26.388^\circ = 0.46055 \text{ rad}$$

and referring to Figure 11.17

$$\delta_{max} = 180^\circ - \delta_0 = 153.612^\circ = 2.681 \text{ rad}$$

Since the fault is at the beginning of the transmission line, the power transfer during fault is zero, and the critical clearing angle as given by (11.91) is

$$\cos\delta_c = \frac{0.8}{1.8}(2.681 - 0.46055) + \cos 153.61^\circ = 0.09106$$

Thus, the critical clearing angle is

$$\delta_c = \cos^{-1}(0.09106) = 84.775^\circ = 1.48 \text{ rad}$$

From (11.92), the critical clearing time is

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}} = \sqrt{\frac{(2)(5)(1.48 - 0.46055)}{(\pi)(60)(.8)}} = 0.26 \text{ second}$$

The use of function **eacfault**(P_m, E, V, X_1, X_2, X_3) to solve the above problem and to display power-angle plot with the shaded equal-areas is demonstrated below. We use the following commands

```
Pm = 0.8; E = 1.17; V = 1.0;
X1 = 0.65; X2 = inf; X3 = 0.65;
eacfault(Pm, E, V, X1, X2, X3)
```

The graph is displayed as shown in Figure 11.22 and the result is

```
Initial power angle      =  26.388
Maximum angle swing      = 153.612
Critical clearing angle  =  84.775
Critical clearing time   =   0.260 sec
```

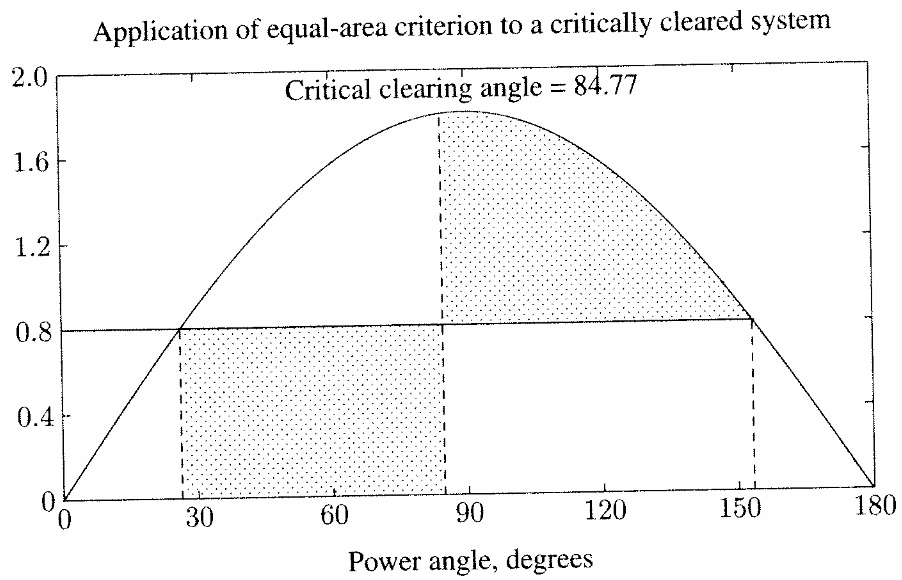

FIGURE 11.22
Equal-area criterion for Example 11.5 (a).

(b) The power-angle curve before the occurrence of the fault is the same as before, given by

$$P_{1\,max} = 1.8 \sin \delta$$

and the generator is operating at the initial power angle $\delta_0 = 26.4^\circ = 0.4605$ rad. The fault occurs at point F at the middle of one line, resulting in the circuit shown in Figure 11.23. The transfer reactance during fault may be found most readily by converting the Y-circuit ABF to an equivalent delta, eliminating junction C. The resulting circuit is shown in Figure 11.24.

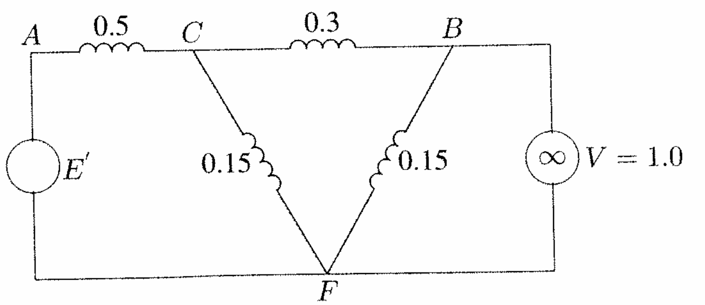

FIGURE 11.23
Equivalent circuit with three-phase fault at the middle of one line.

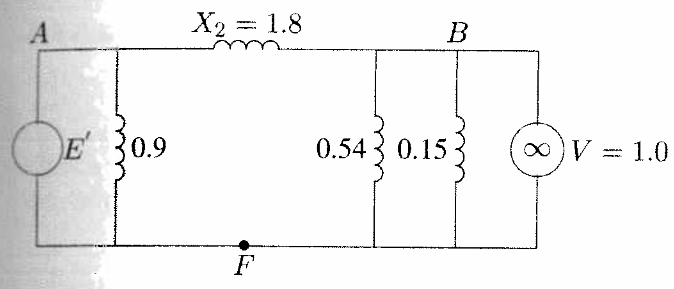

FIGURE 11.24
Equivalent circuit after Y-Δ transformation.

The equivalent reactance between generator and the infinite bus is

$$X_2 = \frac{(0.5)(0.3) + (0.5)(0.15) + (0.3)(0.15)}{0.15} = 1.8 \text{ pu}$$

Thus, the power-angle curve during fault is

$$P_{2\,max} \sin \delta = \frac{(1.17)(1.0)}{1.8} \sin \delta = 0.65 \sin \delta$$

When fault is cleared the faulted line is isolated. Therefore, the postfault transfer reactance is

$$X_3 = 0.3 + 0.2 + 0.3 = 0.8 \text{ pu}$$

and the power-angle curve is

$$P_{3\,max} \sin \delta = \frac{(1.17)(1.0)}{0.8} \sin \delta = 1.4625 \sin \delta$$

Referring to Figure 11.20

$$\delta_{max} = 180^\circ - \sin^{-1}\left(\frac{0.8}{1.4625}\right) = 146.838^\circ = 2.5628 \text{ rad}$$

Applying (11.93), the critical clearing angle is given by

$$\cos \delta_c = \frac{0.8(2.5628 - 0.46055) + 1.4625 \cos 146.838^\circ - 0.65 \cos 26.388^\circ}{1.4625 - 0.65} = -0.15356$$

Thus, the critical clearing angle is

$$\delta_c = \cos^{-1}(-0.15356) = 98.834^\circ$$

Function **eacfault**(P_m, E, V, X_1, X_2, X_3) is used to solve part (b) and to display power-angle plot. We use the following commands

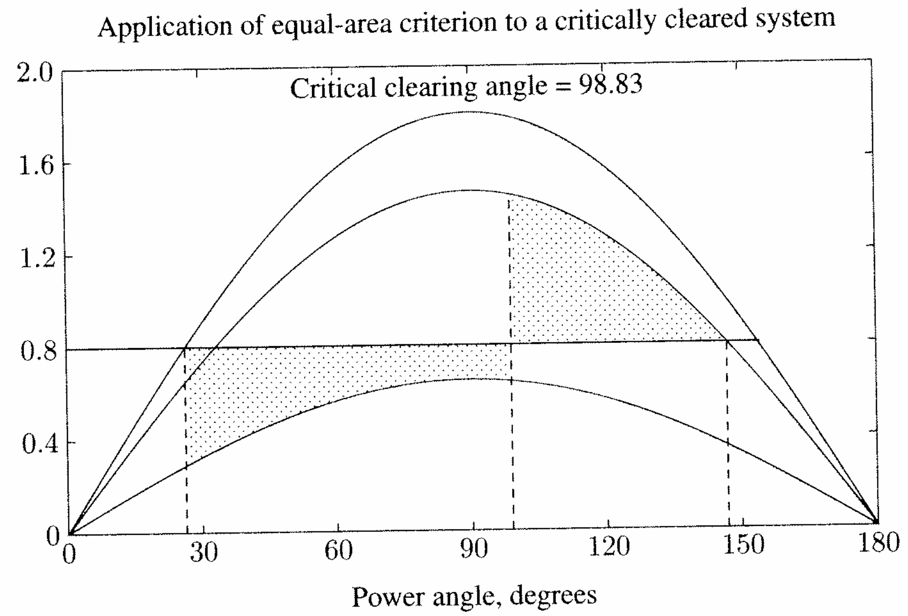

FIGURE 11.25
Equal-area criterion for Example 11.5 (b).

```
Pm = 0.8; E = 1.17; V = 1.0;
X1 = 0.65; X2 = 1.8; X3 = 0.8;
eacfault(Pm, E, V, X1, X2, X3)
```

The graph is displayed as shown in Figure 11.25 and the result is

```
Initial power angle      =  26.388
Maximum angle swing      = 146.838
Critical clearing angle  =  98.834
```

11.7 NUMERICAL SOLUTION OF NONLINEAR EQUATION

Numerical integration techniques can be applied to obtain approximate solutions of nonlinear differential equations. Many algorithms are available for numerical integration. Euler's method is the simplest and the least accurate of all numerical methods. It is presented here because of its simplicity. By studying this method, we will be able to grasp the basic ideas involved in numerical solutions of ODE and can more easily understand the more powerful methods such as the Runge-Kutta procedure.

Consider the first-order differential equation

$$\frac{dx}{dt} = f(x) \tag{11.94}$$

Euler's method is illustrated in Figure 11.26, where the curve shown represents the solution $x(t)$. If at t_0 the value of $x(t_0)$ denoted by x_0 is given, the curve can

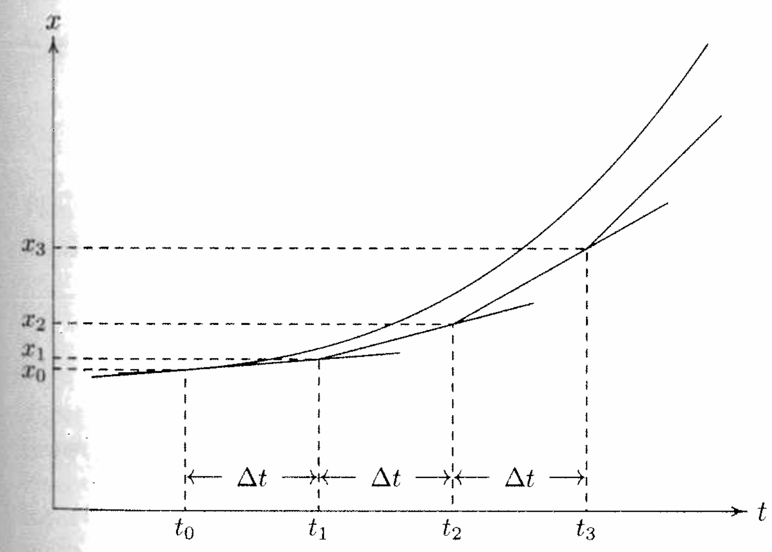

FIGURE 11.26
Graphical interpretation of Euler's method.

be approximated by the tangent evaluated at this point. For a small increment in t denoted by Δt, the increment in x is given by

$$\Delta x \approx \left.\frac{dx}{dt}\right|_{x_0} \Delta t$$

where $\left.\frac{dx}{dt}\right|_{x_0}$ is the slope of the curve at (t_0, x_0), which can be determined from (11.94). Thus, the value of x at $t_0 + \Delta t$ is

$$x_1 = x_0 + \Delta x = x_0 + \left.\frac{dx}{dt}\right|_{x_0} \Delta t$$

The above approximation is the Taylor series expansion of x around point (t_0, x_0), where higher-order terms have been discarded.

The subsequent values of x can be similarly determined. Hence, the computational algorithm is

$$x_{i+1} = x_i + \left.\frac{dx}{dt}\right|_{x_i} \Delta t \tag{11.95}$$

By applying the above algorithm successively, we can find approximate values of $x(t)$ at enough points from an initial state (t_0, x_0) to a final state (t_f, x_f). A graphical illustration is shown in Figure 11.26. Euler's method assumes that the slope is constant over the entire interval Δt causing the points to fall below the curve. An improvement can be obtained by calculating the slope at both the beginning and end of interval, and then averaging these slopes. This procedure is known as the modified Euler's method and is described as follows.

By using the derivative at the beginning of the step, the value at the end of the step ($t_1 = t_0 + \Delta t$) is predicted from

$$x_1^p = x_0 + \left.\frac{dx}{dt}\right|_{x_0} \Delta t$$

Using the predicted value of x_1^p, the derivative at the end of interval is determined by

$$\left.\frac{dx}{dt}\right|_{x_1^p} = f(t_1, x_1^p)$$

Then, the average value of the two derivatives is used to find the corrected value

$$x_1^c = x_0 + \left(\frac{\left.\frac{dx}{dt}\right|_{x_0} + \left.\frac{dx}{dt}\right|_{x_1^p}}{2} \right) \Delta t$$

Hence, the computational algorithm for the successive values is

$$x_{i+1}^c = x_i + \left(\frac{\left.\frac{dx}{dt}\right|_{x_i} + \left.\frac{dx}{dt}\right|_{x_{i+1}^p}}{2} \right) \Delta t \qquad (11.96)$$

The problem with Euler's method is that there is numerical error introduced when discarding the higher-order terms in the series Taylor expansion. But by using a reasonably small value of Δt, we can decrease the error between successive points. If the step size is decreased too much, the number of steps increases and the computer round-off error increases directly with the number of operations. Thus, the step size must be selected small enough to obtain a reasonably accurate solution, but at the same time, large enough to avoid the numerical limitations of the computer.

The above technique can be applied to the solution of higher-order differential equations. An nth order differential equation can be expressed in terms of n first-order differential equations by the introduction of auxiliary variables. These

variables are referred to as *state variables*, which may be physical quantities in a system. For example, given the second-order differential equation

$$a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = c$$

and the initial conditions x_0 and $\left.\frac{dx}{dt}\right|_{x_0}$ at t_0, we introduce the following state variables.

$$x_1 = x$$
$$x_2 = \frac{dx}{dt}$$

Thus, the above second-order differential equation can be written as the two following simultaneous first-order differential equations.

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{c}{a_2} - \frac{a_0}{a_2} x_1 - \frac{a_1}{a_2} x_2$$

There are many other more powerful techniques for the numerical solution of nonlinear equations. A popular technique is the Runge-Kutta method, which is based on formulas derived by using an approximation to replace the truncated Taylor series expansion. The interested reader should refer to textbooks on numerical techniques. *MATLAB* provides two powerful functions for the numerical solution of differential equations employing the Runge-Kutta-Fehlberg methods. These are **ode23** and **ode45**, based on the Fehlberg second- and third-order pair of formulas for medium accuracy and forth- and fifth-order pair for higher accuracy. The nth-order differential equation must be transformed into n first order differential equations and must be placed in an M-file that returns the state derivative of the equations. The formats for these functions are

```
[t, x] = ode23('xprime', tspan, x0)
[t, x] = ode45('xprime', tspan, x0)
```

where **tspan =[t0, tfinal]** is the time interval for the integration and **x0** is a column vector of initial conditions at time **t0**. **xprime** is the state derivative of the equations, defined in a file named **xprime.m**

11.8 NUMERICAL SOLUTION OF THE SWING EQUATION

To demonstrate the solution of the swing equation, consider Figure 11.18 where a generator is connected to an infinite bus bar through two parallel lines. Assume that the input power P_m is constant. Under steady state operation $P_e = P_m$, and the initial power angle is given by

$$\delta_0 = \sin^{-1}\frac{P_m}{P_{1\,max}}$$

where

$$P_{1\,max} = \frac{|E'||V|}{X_1}$$

and X_1 is the transfer reactance before the fault. The rotor is running at synchronous speed, and the change in the angular velocity is zero, i.e.,

$$\Delta\omega_0 = 0$$

Now consider a three-phase fault at the middle of one line as shown in Figure 11.18. The equivalent transfer reactance between bus bars is increased, lowering the power transfer capability, and the amplitude of the power-angle equation becomes

$$P_{2\,max} = \frac{|E'||V|}{X_2}$$

where X_2 is the transfer reactance during fault. The swing equation given by (11.21) is

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}(P_m - P_{2\,max}\sin\delta) = \frac{\pi f_0}{H}P_a$$

The above swing equation is transformed into the state variable form as

$$\frac{d\delta}{dt} = \Delta\omega \qquad (11.97)$$
$$\frac{d\Delta\omega}{dt} = \frac{\pi f_0}{H}P_a$$

We now apply the modified Euler's method to the above equations. By using the derivatives at the beginning of the step, the value at the end of the step ($t_1 = t_0 + \Delta t$) is predicted from

$$\delta^p_{i+1} = \delta_i + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_i}\Delta t$$
$$\Delta\omega^p_{i+1} = \Delta\omega_i + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_i}\Delta t$$

Using the predicted value of δ^p_{i+1}, and $\Delta\omega^p_{i+1}$ the derivatives at the end of interval are determined by

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega^p_{i+1}} = \Delta\omega^p_{i+1}$$
$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta^p_{i+1}} = \left.\frac{\pi f_0}{H}P_a\right|_{\delta^p_{i+1}}$$

Then, the average value of the two derivatives is used to find the corrected value

$$\delta^c_{i+1} = \delta_i + \left(\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_i} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega^p_{i+1}}}{2}\right)\Delta t$$
$$\Delta\omega^c_{i+1} = \Delta\omega_i + \left(\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_i} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta^p_{i+1}}}{2}\right)\Delta t \qquad (11.98)$$

Based on the above algorithm, a function named **swingmeu**(P_m, E, V, X_1, X_2, X_3, H, f, t_c, t_f, Dt) is written for the transient stability analysis of a one-machine system. The function arguments are

- $\mathbf{P_m}$ Per unit mechanical power input, assumed to remain constant
- **E** Constant voltage back of transient reactance in per unit
- **V** Infinite bus bar voltage in per unit
- $\mathbf{X_1}$ Per unit reactance between buses E and V before fault
- $\mathbf{X_2}$ Per unit reactance between buses E and V during fault
- $\mathbf{X_3}$ Per unit reactance between buses E and V after fault clearance
- **H** Generator inertia constant in second, (MJ/MVA)
- **f** System nominal frequency
- $\mathbf{t_c}$ Fault clearing time
- $\mathbf{t_f}$ Final time for integration
- **Dt** Integration time interval, required for modified Euler

If **swingmeu** is used without the arguments, the user is prompted to enter the required data. In addition, based on the *MATLAB* automatic step size Runge-Kutta **ode23** and **ode45** functions, two more functions are developed for the transient stability analysis of a one-machine system. These are **swingrk2**(P_m, E, V, X_1, X_2, X_3, H, f, t_c, t_f), based on **ode23**, and **swingrk4**(P_m, E, V, X_1, X_2, X_3, H, f, t_c, t_f), based on **ode45**. The function arguments are as defined above, except since these techniques use automatic step size, the argument **Dt** is not required. Again, if **swingrk2** and **swingrk4** are used without arguments, the user is prompted to enter the required data. All the functions above use a function named **cctime**(P_m, E, V, X_1, X_2, X_3, H, f), which obtains the critical clearing time of fault for critical stability.

Example 11.6 (chp11ex6), (sim11ex6.mdl)

In the system of Example 11.5 a three-phase fault at the middle of one line is cleared by isolating the faulted circuit simultaneously at both ends.

(a) The fault is cleared in 0.3 second. Obtain the numerical solution of the swing equation for 1.0 second using the modified Euler method (function **swingmeu**) with a step size of $\Delta t = 0.01$ second. From the swing curve, determine the system stability.
(b) The **swingmeu** function automatically calls upon the **cctime** function and determines the critical clearing time. Repeat the simulation and obtain the swing plots for the critical clearing time, and when fault is cleared in 0.5 second.
(c) Obtain a *SIMULINK* block diagram model for the swing equation, and simulate for a fault clearing time of 0.3 and 0.5 second. Repeat the simulation until a critical clearing time is obtained.

(a) For the purpose of understanding the procedure, the computations are performed for one step. From Example 11.5, the power-angle curve before the occurrence of the fault is given by

$$P_{1\,max} = 1.8 \sin \delta$$

and the generator is operating at the initial power angle

$$\delta_0 = 26.388^\circ = 0.46055 \text{ rad}$$
$$\Delta\omega_0 = 0$$

The fault occurs at point F at the middle of one line, resulting in the circuit shown in Figure 11.23 (page 499). From the results obtained in Example 11.5, the accelerating power equation is

$$P_a = 0.8 - 0.65 \sin \delta$$

Applying the modified Euler's method, the derivatives at the beginning of the step are

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_0} = 0$$
$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_0} = \frac{\pi(60)}{5}(0.8 - 0.65 \sin 26.388^\circ) = 19.2684 \;\; rad/sec^2$$

At the end of the first step ($t_1 = 0.01$), the predicted values are

$$\delta_1^p = 0.46055 + (0)(0.01) = 0.46055 \text{ rad} = 26.388^\circ$$
$$\Delta\omega_1^p = 0 + (19.2684)(0.01) = 0.1927 \text{ rad/sec}$$

Using the predicted value of δ_1^p, and $\Delta\omega_1^p$, the derivatives at the end of interval are determined by

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_1^p} = \Delta\omega_1^p = 0.1927 \text{ rad/sec}$$
$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_1^p} = \frac{\pi(60)}{5}(0.8 - \sin 26.388^\circ) = 19.2684 \text{ rad/sec}^2$$

Then, the average value of the two derivatives is used to find the corrected value

$$\delta_1^c = 0.46055 + \frac{0 + 0.1927}{2}(0.01) = 0.4615 \text{ rad}$$
$$\Delta\omega_1^c = 0.0 + \frac{19.2684 + 19.2684}{2}(0.01) = 0.1927 \text{ rad/sec}$$

The process is continued for the successive steps, until at $t = 0.3$ second when the fault is cleared. From Example 11.5, the postfault accelerating power equation is

$$P_a = 0.8 - 1.4625 \sin \delta$$

The process is continued with the new accelerating equation until the specified final time $t_f = 1.0$ second. The complete computations are obtained using the **swingmeu** function as follows

```
Pm = 0.80;  E = 1.17;  V = 1.0;
X1 = 0.65; X2 = 1.80; X3 = 0.8;
H = 5; f = 60; tc = 0.3; tf = 1.0; Dt = 0.01;
swingmeu(Pm, E, V, X1, X2, X3, H, f, tc, tf, Dt)
```

The time interval and the corresponding power angle δ in degrees and the speed deviation $\Delta\omega$ in rad/sec are displayed in a tabular form. The swing plot is displayed as shown in Figure 11.27.

The swing curve shows that the power angle returns after a maximum swing indicating that with inclusion of system damping, the oscillations will subside and a new operating angle is attained. Hence, the system is found to be stable for this fault clearing time. The critical clearing time is determined by the program to be

```
Critical clearing time  = 0.4 second
Critical clearing angle = 98.83 degrees
```

(b) The above program is run for a clearing time of $t_c = 0.4$ second and $t_c = 0.5$ second with the results shown in Figure 11.28. The swing curve for $t_c = 0.4$ second corresponds to the critical clearing time. The swing curve for $t_c = 0.5$

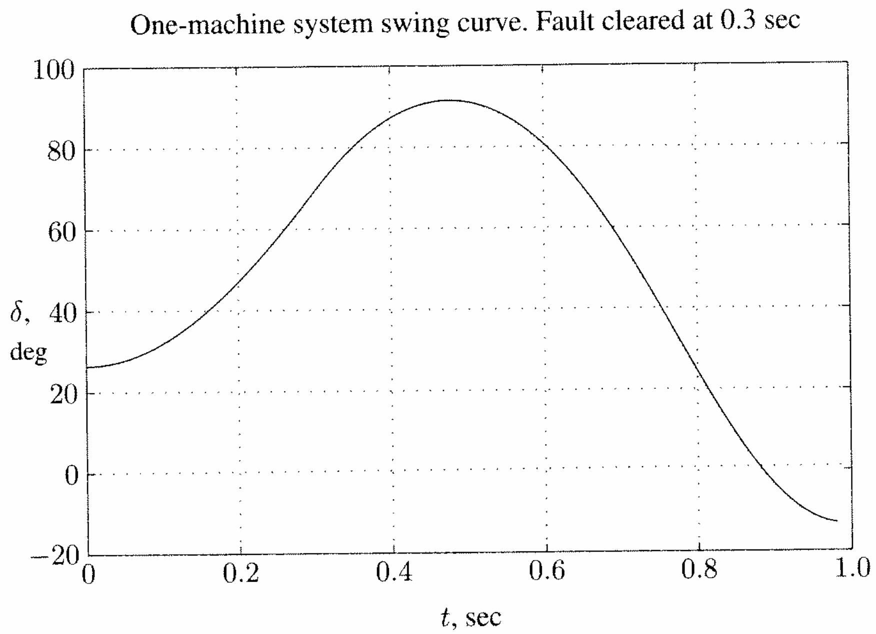

FIGURE 11.27
Swing curve for machine of Example 11.6. Fault cleared at 0.3 sec.

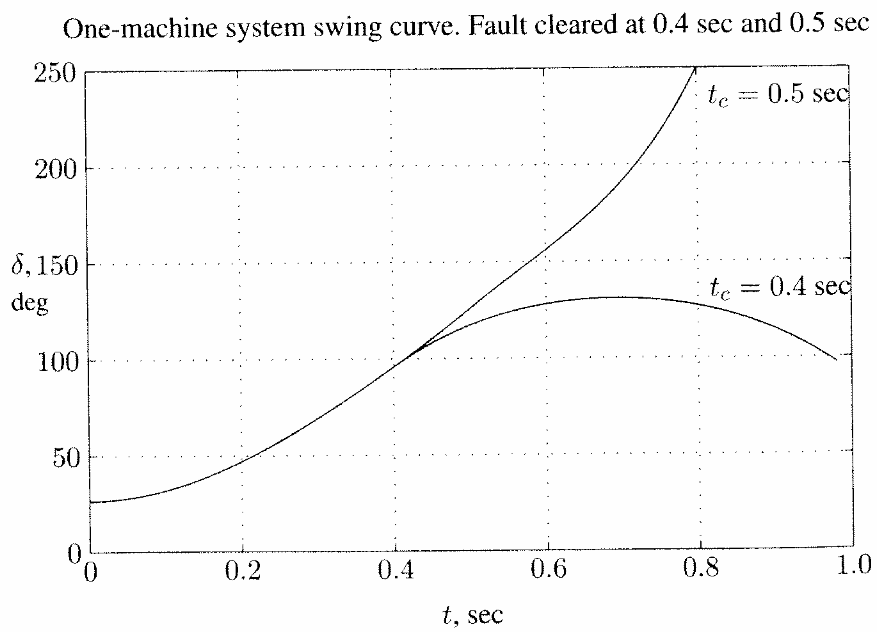

FIGURE 11.28
Swing curves for machine of Example 11.6, for fault clearance at 0.4 sec and 0.5 sec.

second shows that the power angle δ is increasing without limit. Hence, the system is unstable for this clearing time.

The swing curves for the machine in Example 11.6 are obtained for fault clearing times of $t_c = 0.3$, $t_c = 0.4$, and $t_c = 0.5$, with **swingrk4** function, which uses the *MATLAB* **ode45** function. We use the following statements.

```
Pm = 0.80; E = 1.17; V = 1.0; H = 5.0; f = 60;
X1 = 0.65; X2 = 1.80; X3 = 0.8;
tc = 0.3; tf = 1;
swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)
tc = .5;
swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)
tc = .4;
swingrk4(Pm, E, V, X1, X2, X3, H, f, tc, tf)
```

The same numerical solutions are obtained and the swing curves are the same as the ones shown in Figures 11.27 and 11.28.

(c) Using the state-space representation of the swing equation, given in (11.97), a *SIMULINK* model named **sim11ex6.mdl** is constructed as shown in Figure 11.29.

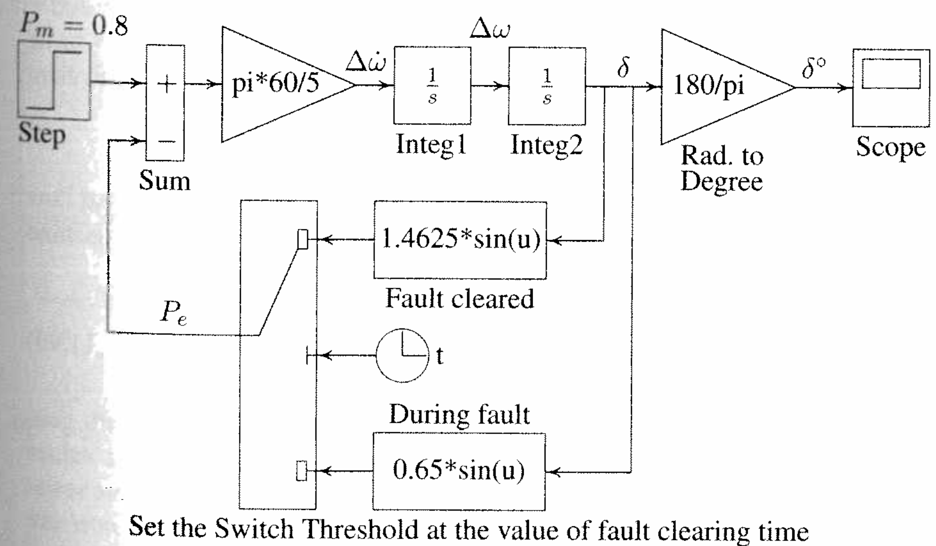

FIGURE 11.29
Simulation block diagram for Example 11.6.

The file is opened and is run in the *SIMULINK WINDOW*. Open the Switch Dialog Box and change the Switch Threshold setting for different values of fault clearing time. The simulation results in the same response as shown in Figure 11.27.

11.9 MULTIMACHINE SYSTEMS

Multimachine equations can be written similar to the one-machine system connected to the infinite bus. In order to reduce the complexity of the transient stability analysis, similar simplifying assumptions are made as follows.

1. Each synchronous machine is represented by a constant voltage source behind the direct axis transient reactance. This representation neglects the effect of saliency and assumes constant flux linkages.
2. The governor's actions are neglected and the input powers are assumed to remain constant during the entire period of simulation.
3. Using the prefault bus voltages, all loads are converted to equivalent admittances to ground and are assumed to remain constant.
4. Damping or asynchronous powers are ignored.
5. The mechanical rotor angle of each machine coincides with the angle of the voltage behind the machine reactance.
6. Machines belonging to the same station swing together and are said to be *coherent*. A group of coherent machines is represented by one equivalent machine.

The first step in the transient stability analysis is to solve the initial load flow and to determine the initial bus voltage magnitudes and phase angles. The machine currents prior to disturbance are calculated from

$$I_i = \frac{S_i^*}{V_i^*} = \frac{P_i - jQ_i}{V_i^*} \qquad i = 1, 2, \ldots, m \tag{11.99}$$

where m is the number of generators. V_i is the terminal voltage of the ith generator, P_i and Q_i are the generator real and reactive powers. All unknown values are determined from the initial power flow solution. The generator armature resistances are usually neglected and the voltages behind the transient reactances are then obtained.

$$E_i' = V_i + jX_d' I_i \tag{11.100}$$

Next, all loads are converted to equivalent admittances by using the relation

$$y_{i0} = \frac{S_i^*}{|V_i|^2} = \frac{P_i - jQ_i}{|V_i|^2} \tag{11.101}$$

To include voltages behind transient reactances, m buses are added to the n-bus power system network. The equivalent network with all loads converted to admittances is shown in Figure 11.30.

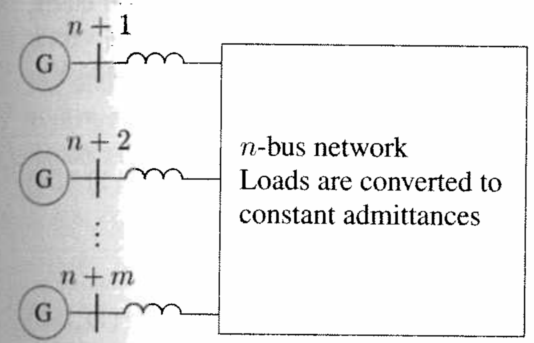

FIGURE 11.30
Power system representation for transient stability analysis.

Nodes $n+1, n+2, \ldots, n+m$ are the internal machine buses, i.e., the buses behind the transient reactances. The node voltage equation with node 0 as reference for this network, as given by (6.2), is

$$\left[\begin{array}{c} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_{n+1} \\ \vdots \\ I_{n+m} \end{array}\right] = \left[\begin{array}{ccc|ccc} Y_{11} & \cdots & Y_{1n} & Y_{1(n+1)} & \cdots & Y_{1(n+m)} \\ Y_{21} & \cdots & Y_{2n} & Y_{2(n+1)} & \cdots & Y_{2(n+m)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Y_{n1} & \cdots & Y_{nn} & Y_{n(n+1)} & \cdots & Y_{n(n+m)} \\ \hline Y_{(n+1)1} & \cdots & Y_{(n+1)n} & Y_{(n+1)(n+1)} & \cdots & Y_{(n+1)(n+m)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Y_{(n+m)1} & \cdots & Y_{(n+m)n} & Y_{(n+m)(n+1)} & \cdots & Y_{(n+m)(n+m)} \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ \vdots \\ V_n \\ \hline E'_{n+1} \\ \vdots \\ E'_{n+m} \end{array}\right] \tag{11.102}$$

or

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\,\mathbf{V}_{bus} \tag{11.103}$$

where $\mathbf{I}_{bus}$ is the vector of the injected bus currents and $\mathbf{V}_{bus}$ is the vector of bus voltages measured from the reference node. The diagonal elements of the bus admittance matrix are the sum of admittances connected to it, and the off-diagonal elements are equal to the negative of the admittance between the nodes. This is similar to the **lfybus** used in the power flow analysis. The difference is that additional nodes are added to include the machine voltages behind transient reactances. Also, diagonal elements are modified to include the load admittances.

To simplify the analysis, all nodes other than the generator internal nodes are eliminated using the Kron reduction formula. To eliminate the load buses, the bus admittance matrix in (11.102) is partitioned such that the n buses to be removed are represented in the upper n rows. Since no current enters or leaves the load buses, currents in the n rows are zero. The generator currents are denoted by the vector $\mathbf{I}_m$ and the generator and load voltages are represented by the vectors $\mathbf{E'}_m$ and $\mathbf{V}_n$, respectively. Then, Equation (11.102), in terms of submatrices, becomes

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{I}_m \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{nn} & \mathbf{Y}_{nm} \\ \mathbf{Y}^t_{nm} & \mathbf{Y}_{mm} \end{bmatrix} \begin{bmatrix} \mathbf{V}_n \\ \mathbf{E'}_m \end{bmatrix} \tag{11.104}$$

The voltage vector $\mathbf{V}_n$ may be eliminated by substitution as follows.

$$\mathbf{0} = \mathbf{Y}_{nn}\mathbf{V}_n + \mathbf{Y}_{nm}\mathbf{E'}_m \tag{11.105}$$

$$\mathbf{I}_m = \mathbf{Y}^t_{nm}\mathbf{V}_n + \mathbf{Y}_{mm}\mathbf{E'}_m \tag{11.106}$$

from (11.105),

$$\mathbf{V}_n = -\mathbf{Y}_{nn}{}^{-1}\mathbf{Y}_{nm}\mathbf{E'}_m \tag{11.107}$$

Now substituting into (11.106), we have

$$\begin{aligned}\mathbf{I}_m &= [\mathbf{Y}_{mm} - \mathbf{Y}^t_{nm}\mathbf{Y}_{nn}{}^{-1}\mathbf{Y}_{nm}]\mathbf{E'}_m \\ &= \mathbf{Y}^{red}_{bus}\mathbf{E'}_m\end{aligned} \tag{11.108}$$

The reduced admittance matrix is

$$\mathbf{Y}^{red}_{bus} = \mathbf{Y}_{mm} - \mathbf{Y}^t_{nm}\mathbf{Y}_{nn}{}^{-1}\mathbf{Y}_{nm} \tag{11.109}$$

The reduced bus admittance matrix has the dimensions $(m \times m)$, where m is the number of generators.

The electrical power output of each machine can now be expressed in terms of the machine's internal voltages.

$$S^*_{ei} = E'^*_i I_i$$

or

$$P_{ei} = \Re[E'^*_i I_i] \tag{11.110}$$

where

$$I_i = \sum_{j=1}^{m} E'_j Y_{ij} \tag{11.111}$$

Expressing voltages and admittances in polar form, i.e., $E'_i = |E'_i|\angle\delta_i$ and $Y_{ij} = |Y_{ij}|\angle\theta_{ij}$, and substituting for I_i in (11.110), results in

$$P_{ei} = \sum_{j=1}^{m} |E'_i||E'_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{11.112}$$

The above equation is the same as the power flow equation given by (6.52). Prior to disturbance, there is equilibrium between the mechanical power input and the electrical power output, and we have

$$P_{mi} = \sum_{j=1}^{m} |E'_i||E'_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{11.113}$$

11.10 MULTIMACHINE TRANSIENT STABILITY

The classical transient stability study is based on the application of a three-phase fault. A solid three-phase fault at bus k in the network results in $V_k = 0$. This is simulated by removing the kth row and column from the prefault bus admittance matrix. The new bus admittance matrix is reduced by eliminating all nodes except the internal generator nodes. The generator excitation voltages during the fault and postfault modes are assumed to remain constant. The electrical power of the ith generator in terms of the new reduced bus admittance matrices are obtained from (11.112). The swing equation with damping neglected, as given by (11.21), for machine i becomes

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m} |E'_i||E'_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \tag{11.114}$$

where Y_{ij} are the elements of the faulted reduced bus admittance matrix, and H_i is the inertia constant of machine i expressed on the common MVA base S_B. If H_{Gi} is the inertia constant of machine i expressed on the machine rated MVA S_{Gi}, then H_i is given by

$$H_i = \frac{S_{Gi}}{S_B}H_{Gi} \tag{11.115}$$

Showing the electrical power of the ith generator by P^f_e and transforming (11.114) into state variable model yields

$$\begin{aligned}\frac{d\delta_i}{dt} &= \Delta\omega_i \qquad i = 1, \ldots, m \\ \frac{d\Delta\omega_i}{dt} &= \frac{\pi f_0}{H_i}(P_m - P^f_e)\end{aligned} \tag{11.116}$$

We have two state equations for each generator, with initial power angles δ_{0_i} and $\Delta\omega_{0_i} = 0$. The *MATLAB* function **ode23** is employed to solve the above $2m$ first-order differential equations. When the fault is cleared, which may involve the removal of the faulty line, the bus admittance matrix is recomputed to reflect the change in the network. Next, the postfault reduced bus admittance matrix is evaluated and the postfault electrical power of the ith generator shown by P_i^{pf} is readily determined from (11.112). Using the postfault power P_i^{pf}, the simulation is continued to determine the system stability, until the plots reveal a definite trend as to stability or instability. Usually the slack generator is selected as the reference, and the phase angle difference of all other generators with respect to the reference machine are plotted. Usually, the solution is carried out for two swings to show that the second swing is not greater than the first one. If the angle differences do not increase, the system is stable. If any of the angle differences increase indefinitely, the system is unstable.

Based on the above procedure, a program named **trstab** is developed for transient stability analysis of a multimachine network subjected to a balanced three-phase fault. The program **trstab** must be preceded by the power flow program. Any of the power flow programs **lfgauss**, **lfnewton**, or **decouple** can be used. In addition to the power flow data, generator data must be specified in a matrix named **gendata**. The first column contains the generator bus number terminal. Columns 2 and 3 contain resistance and transient reactance in per unit on the specified common MVA base, and the last column contain the machine inertia constant in seconds, expressed on the common MVA base. The program **trstab** automatically adds additional buses to include the generator impedances in the power flow line data. Also, the bus admittance matrix is modified to include the load admittances **yload**, returned by the power flow program. The program prompts the user to enter the faulted bus number, fault clearing time, and the line numbers of the removed faulty line. The program displays the prefault, faulted, and postfault reduced bus admittance matrices. The machine phase angles are tabulated and a plot of the swing curves is obtained. The program inquires for other fault clearing times and fault locations. The use of **trstab** program is demonstrated in the following example.

Example 11.7 (chp11ex7)

The power system network of an electric utility company is shown in Figure 11.31. The load data and voltage magnitude, generation schedule, and the reactive power limits for the regulated buses are tabulated on the next page. Bus 1, whose voltage is specified as $V_1 = 1.06\angle 0°$, is taken as the slack bus. The line data containing the series resistance and reactance in per unit, and one-half of the total capacitance in per unit susceptance on a 100-MVA base is also tabulated as shown.

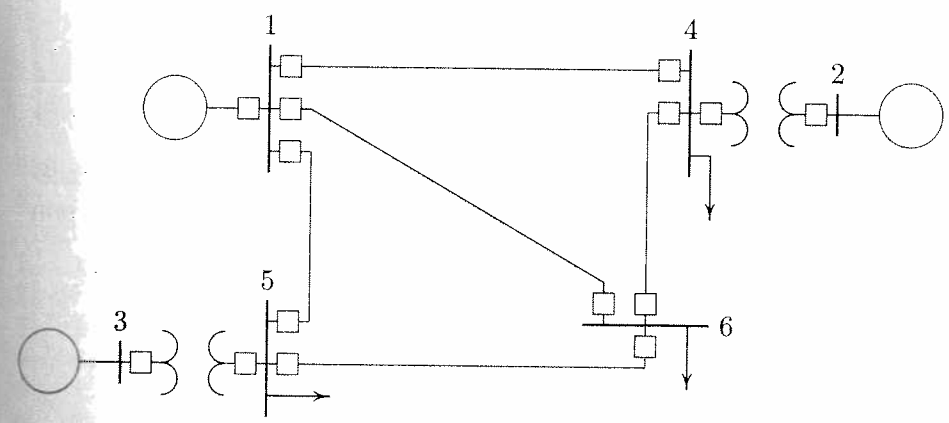

FIGURE 11.31
One-line diagram for Example 11.7.

LOAD DATA

Bus No.	Load MW	Load Mvar
1	0	0
2	0	0
3	0	0
4	100	70
5	90	30
6	160	110

GENERATION SCHEDULE

Bus No.	Voltage Mag.	Generation, MW	Mvar Limits Min.	Mvar Limits Max.
1	1.06			
2	1.04	150	0	140
3	1.03	100	0	90

LINE DATA

Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}B$, PU
1	4	0.035	0.225	0.0065
1	5	0.025	0.105	0.0045
1	6	0.040	0.215	0.0055
2	4	0.000	0.035	0.0000
3	5	0.000	0.042	0.0000
4	6	0.028	0.125	0.0035
5	6	0.026	0.175	0.0300

The generator's armature resistances and transient reactances in per unit, and the inertia constants in seconds expressed on a 100-MVA base are given below:

MACHINE DATA			
Gen.	R_a	X'_d	H
1	0	0.20	20
2	0	0.15	4
3	0	0.25	5

A three-phase fault occurs on line 5–6 near bus 6, and is cleared by the simultaneous opening of breakers at both ends of the line. Using the **trstab** program, perform a transient stability analysis. Determine the system stability for
(a) When the fault is cleared in 0.4 second
(b) When the fault is cleared in 0.5 second
(c) Repeat the simulation to determine the critical clearing time.

The required data and commands are as follows:

```
basemva = 100;  accuracy = 0.0001;  maxiter = 10;
%      Bus Bus  Voltage Angle --Load-- --Generator-- Injected
%       No code Mag. degree MW   Mvar   MW   Mvar Qmin Qmax Mvar
busdata=[1  1   1.06   0      0    0    0    0    0    0   0
         2  2   1.04   0      0    0  150    0    0  140   0
         3  2   1.03   0      0    0  100    0    0   90   0
         4  0   1.0    0    100   70    0    0    0    0   0
         5  0   1.0    0     90   30    0    0    0    0   0
         6  0   1.0    0    160  110    0    0    0    0   0];
% Line data
%         Bus  bus   R       X       1/B     1 for line code or
%         nl   nr    pu      pu      pu      tap setting value
linedata=[1    4    0.035   0.225   0.0065     1.0
          1    5    0.025   0.105   0.0045     1.0
          1    6    0.040   0.215   0.0055     1.0
          2    4    0.000   0.035   0.0000     1.0
          3    5    0.000   0.042   0.0000     1.0
          4    6    0.028   0.125   0.0035     1.0
          5    6    0.026   0.175   0.0300     1.0];
lfybus          % form the bus admittance matrix for power flow
lfnewton        % Power flow solution by Newton-Raphson method
busout          % Prints the power flow solution on the screen
% Generator data
%         Gen.  Ra    Xd'    H
gendata=[ 1     0    0.20   20
          2     0    0.15    4
          3     0    0.25    5];
trstab                      % Performs the stability analysis.
      % User is prompted to enter the clearing time of fault.
```


The power flow result is

```
                Power Flow Solution by Newton-Raphson Method
                   Maximum Power Mismatch = 1.80187e-007
                           No. of Iterations = 4

Bus  Voltage  Angle   -----Load-----  ---Generation--- Injected
No.  Mag.     degree   MW       Mvar    MW       Mvar     Mvar

1    1.060    0.000    0.000    0.00   105.287 107.335    0.00
2    1.040    1.470    0.000    0.00   150.000  99.771    0.00
3    1.030    0.800    0.000    0.00   100.000  35.670    0.00
4    1.008   -1.401  100.000   70.00     0.000   0.000    0.00
5    1.016   -1.499   90.000   30.00     0.000   0.000    0.00
6    0.941   -5.607  160.000  110.00     0.000   0.000    0.00

Total                350.000  210.00   355.287 242.776    0.00
```

The **trstab** result is

```
Reduced prefault bus admittance matrix

  Ybf =
       0.3517 - 2.8875i   0.2542 + 1.1491i   0.1925 + 0.9856i
       0.2542 + 1.1491i   0.5435 - 2.8639i   0.1847 + 0.6904i
       0.1925 + 0.9856i   0.1847 + 0.6904i   0.2617 - 2.2835i

      G(i)    E'(i)      d0(i)      Pm(i)
       1     1.2781     8.9421     1.0529
       2     1.2035    11.8260     1.5000
       3     1.1427    13.0644     1.0000

Enter faulted bus No. -> 6

Reduced faulted bus admittance matrix

  Ypf =
       0.1913 - 3.5849i   0.0605 + 0.3644i   0.0523 + 0.4821i
       0.0605 + 0.3644i   0.3105 - 3.7467i   0.0173 + 0.1243i
       0.0523 + 0.4821i   0.0173 + 0.1243i   0.1427 - 2.6463i

Fault is cleared by opening a line.  The bus to bus numbers of
line to be removed must be entered within brackets, e.g. [5,7]
Enter the bus to bus Nos. of line to be removed -> [5, 6]
```

```
Reduced postfault bus admittance matrix

 Yaf =
     0.3392 - 2.8879i   0.2622 + 1.1127i   0.1637 + 1.0251i
     0.2622 + 1.1127i   0.6020 - 2.7813i   0.1267 + 0.5401i
     0.1637 + 1.0251i   0.1267 + 0.5401i   0.2859 - 2.0544i

Enter clearing time of fault in sec. tc = 0.4
Enter final simulation time in sec.  tf = 1.5
```

The phase angle differences of each machine with respect to the slack bus are printed in a tabular form on the screen, which is not presented here. The program also obtains a plot of the swing curves which is presented in Figure 11.32.

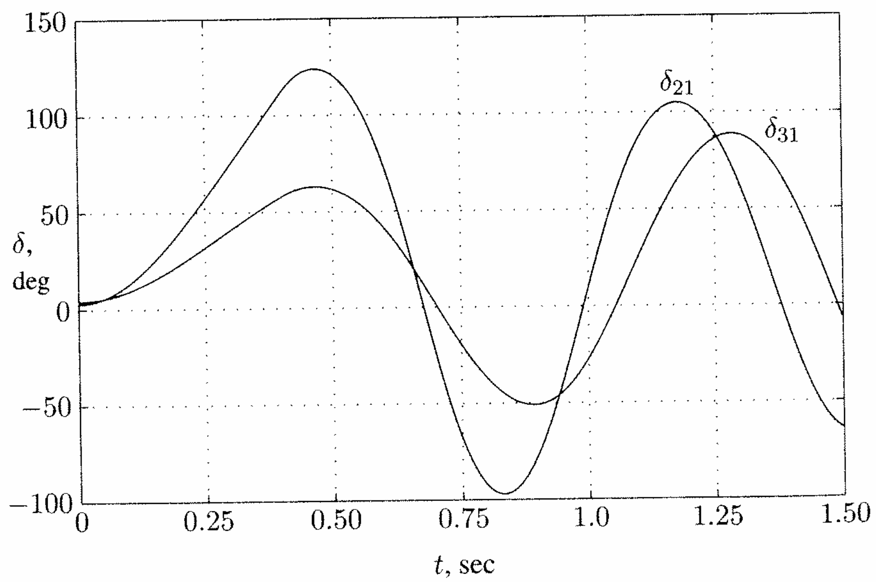

FIGURE 11.32
Plots of angle differences for machines 2 and 3 of Example 11.7.

Again the tabulated result is printed on the screen, and plots of the swing curves are obtained as shown in Figure 11.33. Figure 11.32 shows that the phase angle differences, after reaching a maximum of $\delta_{21} = 123.9°$ and $\delta_{31} = 62.95°$ will decrease, and the machines swing together. Hence, the system is found to be stable when fault is cleared in 0.4 second.

The program inquires for another fault clearing time, and the results continue as follows:


```
Another clearing time of fault?
Enter 'y' or 'n' within quotes -> 'y'
Enter clearing time of fault in sec. tc = 0.5
Enter final simulation time in sec.  tf = 1.5
```

The swing curves shown in Figure 11.33 show that machine 2 phase angle increases without limit. Thus, the system is unstable when fault is cleared in 0.5 second. The simulation is repeated for a clearing time of 0.45 second, which is found to be critically stable.

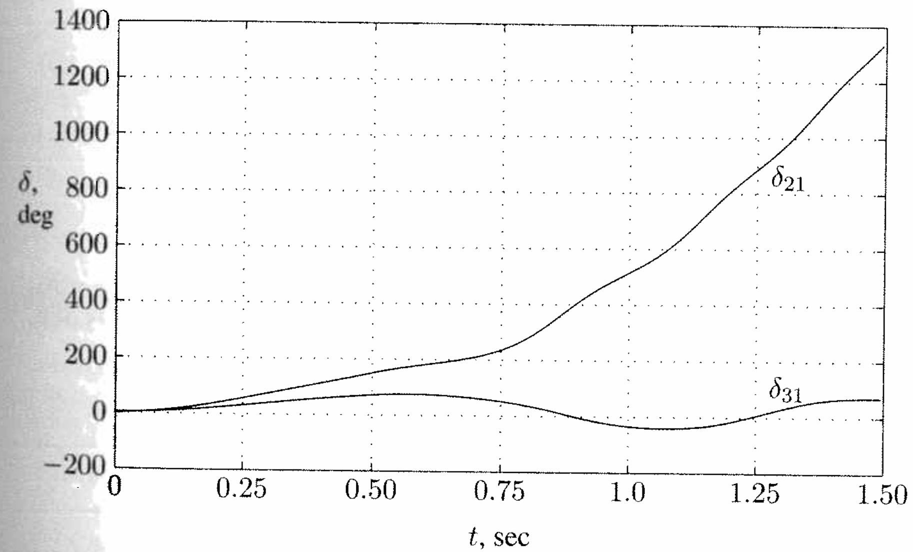

FIGURE 11.33
Plots of angle differences for machines 2 and 3 of Example 11.7.

PROBLEMS

11.1. A four-pole, 60-Hz synchronous generator has a rating of 200 MVA, 0.8 power factor lagging. The moment of inertia of the rotor is 45,100 kg·m^2. Determine M and H.

11.2. A two-pole, 60-Hz synchronous generator has a rating of 250 MVA, 0.8 power factor lagging. The kinetic energy of the machine at synchronous speed is 1080 MJ. The machine is running steadily at synchronous speed and delivering 60 MW to a load at a power angle of 8 electrical degrees. The load is suddenly removed. Determine the acceleration of the rotor. If the acceleration computed for the generator is constant for a period of 12 cycles, determine the value of the power angle and the rpm at the end of this time.

11.3. Determine the kinetic energy stored by a 250-MVA, 60-Hz, two-pole synchronous generator with an inertia constant H of 5.4 MJ/MVA. Assume the machine is running steadily at synchronous speed with a shaft input of 331,100 hp. The electrical power developed suddenly changes from its normal value to a value of 200 MW. Determine the acceleration or deceleration of the rotor. If the acceleration computed for the generator is constant for a period of 9 cycles, determine the change in the power angle in that period and the rpm at the end of 9 cycles.

11.4. The swing equations of two interconnected synchronous machines are written as

$$\frac{H_1}{\pi f_0}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$

$$\frac{H_2}{\pi f_0}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

Denote the relative power angle between the two machines by $\delta = \delta_1 - \delta_2$. Obtain a swing equation equivalent to that of a single machine in terms of δ, and show that

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where

$$H = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_m = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2}$$

$$P_e = \frac{H_2 P_{e1} - H_1 P_{e2}}{H_1 + H_2}$$

11.5. Two synchronous generators represented by a constant voltage behind transient reactance are connected by a pure reactance $X = 0.3$ per unit, as shown in Figure 11.34. The generator inertia constants are $H_1 = 4.0$ MJ/MVA and $H_2 = 6$ MJ/MVA, and the transient reactances are $X_1' = 0.16$ and $X_2' = 0.20$ per unit. The system is operating in the steady state with $E_1' = 1.2$, $P_{m1} = 1.5$ and $E_2' = 1.1$, $P_{m2} = 1.0$ per unit. Denote the relative power

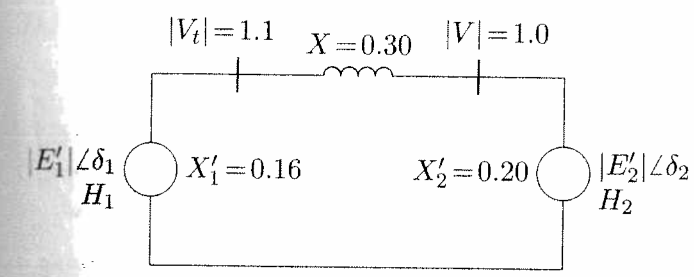

FIGURE 11.34
System of Problem 11.5.

angle between the two machines by $\delta = \delta_1 - \delta_2$. Referring to Problem 11.4, reduce the two-machine system to an equivalent one-machine against an infinite bus. Find the inertia constant of the equivalent machine, the mechanical input power, and the amplitude of its power angle curve, and obtain the equivalent swing equation in terms of δ.

11.6. A 60-Hz synchronous generator has a transient reactance of 0.2 per unit and an inertia constant of 5.66 MJ/MVA. The generator is connected to an infinite bus through a transformer and a double circuit transmission line, as shown in Figure 11.35. Resistances are neglected and reactances are expressed on a common MVA base and are marked on the diagram. The generator is delivering a real power of 0.77 per unit to bus bar 1. Voltage magnitude at bus 1 is 1.1. The infinite bus voltage $V = 1.0\angle 0°$ per unit. Determine the generator excitation voltage and obtain the swing equation as given by (11.36).

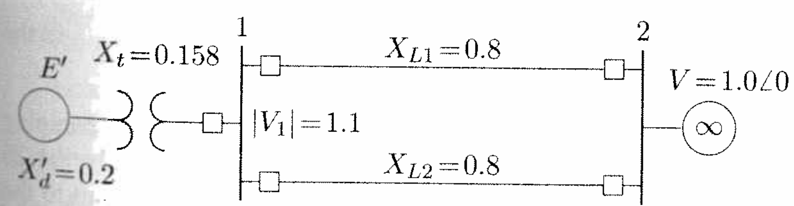

FIGURE 11.35
System of Problem 11.6.

11.7. A three-phase fault occurs on the system of Problem 11.6 at the sending end of the transmission lines. The fault occurs through an impedance of 0.082 per unit. Assume the generator excitation voltage remains constant at $E' = 1.25$ per unit. Obtain the swing equation during the fault.

11.8. The power-angle equation for a salient-pole generator is given by

$$P_e = P_{max} \sin\delta + P_K \sin 2\delta$$

Consider a small deviation in power angle from the initial operating point δ_0, i.e., $\delta = \delta + \delta_0$. Obtain an expression for the synchronizing power coefficient, similar to (11.39). Also, find the linearized swing equation in terms of $\Delta\delta$.

11.9. Consider the displacement x for a unit mass supported by a nonlinear spring as shown in Figure 11.36. The equation of motion is described by

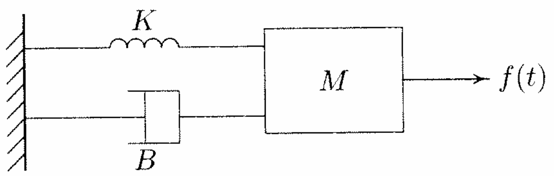

FIGURE 11.36
System of Problem 11.9.

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + K \sin x(t) = f(t)$$

where M is the mass, B is the frictional coefficient and K is the spring constant. The system is at the steady state $x(0) = 0$ for $f(0) = 0$. A small perturbation $f(t) = f(0) + \Delta f(t)$ results in the displacement $x(t) = x(0) + \Delta x(t)$.

(a) Obtain a linearized expression for the motion of the system in terms of the system parameters, $\Delta x(t)$ and $\Delta f(t)$.

(b) For $M = 1.6$, $B = 9.6$, and $K = 40$, find the damping ratio ζ and the damped frequency of oscillation ω_d.

11.10. The machine in the power system of Problem 11.6 has a per unit damping coefficient of $D = 0.15$. The generator excitation voltage is $E' = 1.25$ per unit and the generator is delivering a real power of 0.77 per unit to the infinite bus at a voltage of $V = 1.0$ per unit. Write the linearized swing equation for this power system. Use (11.61) and (11.62) to find the equations describing the motion of the rotor angle and the generator frequency for a small disturbance of $\Delta\delta = 15°$. Use *MATLAB* to obtain the plots of rotor angle and frequency.

11.11. Write the linearized swing equation of Problem 11.10 in state variable form. Use $[y, x]$ = **initial**(A, B, C, D, x_0, t) and **plot** commands to obtain the zero-input response for the initial conditions $\Delta\delta = 15°$, and $\Delta\omega_n = 0$.

11.12. The generator of Problem 11.10 is operating in the steady state at $\delta_0 = 27.835°$ when the input power is increased by a small amount $\Delta P = 0.15$ per unit. The generator excitation and the infinite bus voltage are the same as before. Use (11.75) and (11.76) to find the equations describing the motion of the rotor angle and the generator frequency for a small disturbance of $\Delta P = 0.15$ per unit. Use *MATLAB* to obtain the plots of rotor angle and frequency.

11.13. Write the linearized swing equation of Problem 11.10 in state variable form. Use $[y, x]$ = **step**$(A, B, C, D, 1, t)$ and **plot** commands to obtain the zero-state response when the input power is increased by a small amount $\Delta P = 0.15$ per unit.

11.14. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit. Use **eacpower**(P_m, E, V, X) to find

(a) The maximum power input that can be added without loss of synchronism.

(b) Repeat (a) with zero initial power input. Assume the generator internal voltage remains constant at the value computed in (a).

11.15. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit.

(a) A temporary three-phase fault occurs at the sending end of one of the transmission lines. When the fault is cleared, both lines are intact. Using equal area criterion, determine the critical clearing angle and the critical fault clearing time. Use **eacfault**$(P_m, E, V, X_1, X_2, X_3)$ to check the result and to display the power-angle plot.

(b) A three-phase fault occurs at the middle of one of the lines, the fault is cleared, and the faulted line is isolated. Determine the critical clearing angle. Use **eacfault**$(P_m, E, V, X_1, X_2, X_3)$ to check the results and to display the power-angle plot.

11.16. The machine of Problem 11.6 is delivering a real power input of 0.77 per unit to the infinite bus at a voltage of 1.0 per unit. The generator excitation voltage is $E' = 1.25$ per unit. A three-phase fault at the middle of one line is cleared by isolating the faulted circuit simultaneously at both ends.

(a) The fault is cleared in 0.2 second. Obtain the numerical solution of the swing equation for 1.5 seconds. Select one of the functions **swingmeu**, **swingrk2**, or **swingrk4**.

(b) Repeat the simulation and obtain the swing plots when fault is cleared in 0.4 second, and for the critical clearing time.

11.17. Consider the power system network of Example 11.7 with the described operating condition. A three-phase fault occurs on line 1–5 near bus 5 and is cleared by the simultaneous opening of breakers at both ends of the line. Using the **trstab** program, perform a transient stability analysis. Determine the system stability for

(a) When the fault is cleared in 0.2 second
(b) When the fault is cleared in 0.4 second
(c) Repeat the simulation to determine the critical clearing time.

11.18. The power system network of an electric company is shown in Figure 11.37. The load data is as follows.

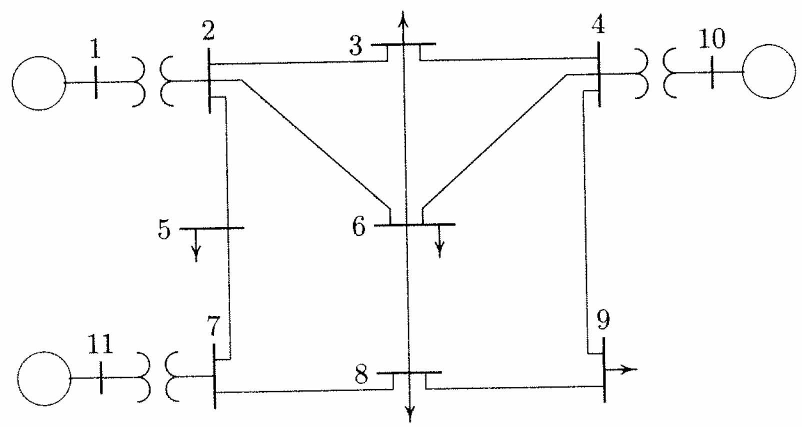

FIGURE 11.37
System of Problem 11.18.

LOAD DATA

Bus No.	Load MW	Load Mvar	Bus No.	Load MW	Load Mvar
1	0.0	0.0	7	0.0	0.0
2	0.0	0.0	8	110.0	90.0
3	150.0	120.0	9	80.0	50.0
4	0.0	0.0	10	0.0	0.0
5	120.0	60.0	11	0.0	0.0
6	140.0	90.0			

Voltage magnitude, generation schedule, and the reactive power limits for the regulated buses are tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.04\angle 0°$, is taken as the slack bus.

GENERATION SCHEDULE

Bus No.	Voltage Mag.	Generation, MW	Mvar Limits Min.	Mvar Limits Max.
1	1.040			
10	1.035	200.0	0.0	180.0
11	1.030	160.0	0.0	120.0

The line data containing the per unit series impedance, and one-half of the shunt capacitive susceptance on a 100-MVA base is tabulated below.

LINE AND TRANSFORMER DATA

Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}B$, PU
1	2	0.000	0.006	0.000
2	3	0.008	0.030	0.004
2	5	0.004	0.015	0.002
2	6	0.012	0.045	0.005
3	4	0.010	0.040	0.005
3	6	0.004	0.040	0.005
4	6	0.015	0.060	0.008
4	9	0.018	0.070	0.009
4	10	0.000	0.008	0.000
5	7	0.005	0.043	0.003
6	8	0.006	0.048	0.000
7	8	0.006	0.035	0.004
7	11	0.000	0.010	0.000
8	9	0.005	0.048	0.000

The generator's armature resistance and transient reactances in per unit, and the inertia constants expressed on a 100-MVA base are given below.

MACHINE DATA

Gen.	R_a	X'_d	H
1	0	0.20	12
10	0	0.15	10
11	0	0.25	9

A three-phase fault occurs on line 4–9, near bus 4, and is cleared by the simultaneous opening of breakers at both ends of the line. Using the **trstab** program, perform a transient stability analysis. Determine the stability for

(a) When the fault is cleared in 0.4 second
(b) When the fault is cleared in 0.8 second
(c) Repeat the simulation to determine the critical clearing time.

CHAPTER 12

POWER SYSTEM CONTROL

12.1 INTRODUCTION

So far, this text has concentrated on the problems of establishing a normal operating state and optimum scheduling of generation for a power system. This chapter deals with the control of active and reactive power in order to keep the system in the steady-state. In addition, simple models of the essential components used in the control systems are presented. The objective of the control strategy is to generate and deliver power in an interconnected system as economically and reliably as possible while maintaining the voltage and frequency within permissible limits.

Changes in real power affect mainly the system frequency, while reactive power is less sensitive to changes in frequency and is mainly dependent on changes in voltage magnitude. Thus, real and reactive powers are controlled separately. The *load frequency control* (LFC) loop controls the real power and frequency and the *automatic voltage regulator* (AVR) loop regulates the reactive power and voltage magnitude. Load frequency control (LFC) has gained in importance with the growth of interconnected systems and has made the operation of interconnected systems possible. Today, it is still the basis of many advanced concepts for the control of large systems.

The methods developed for control of individual generators, and eventually control of large interconnections, play a vital role in modern energy control centers. Modern energy control centers (ECC) are equipped with on-line computers

performing all signal processing through the remote acquisition systems known as *supervisory control and data acquisition* (SCADA) systems. Only an introduction to power system control is presented here. This chapter utilizes some of the concepts of feedback control systems. Some students may not be fully versed in feedback theory. Therefore, a brief review of the fundamentals of linear control systems analysis and design is included in Appendix B. The use of *MATLAB CONTROL TOOLBOX* functions and some useful custom-made functions are also described in this appendix.

The role of *automatic generation control* (AGC) in power system operation, with reference to tie-line power control under normal operating conditions, is first analyzed. Typical responses to real power demand are illustrated using the latest simulation technique available by the *MATLAB SIMULINK* package. Finally, the requirement of reactive power and voltage regulation and the influence on stability of both speed and excitation controls, with use of suitable feedback signals, are examined.

12.2 BASIC GENERATOR CONTROL LOOPS

In an interconnected power system, load frequency control (LFC) and automatic voltage regulator (AVR) equipment are installed for each generator. Figure 12.1 represents the schematic diagram of the load frequency control (LFC) loop and the automatic voltage regulator (AVR) loop. The controllers are set for a particular operating condition and take care of small changes in load demand to maintain the frequency and voltage magnitude within the specified limits. Small changes in real power are mainly dependent on changes in rotor angle δ and, thus, the frequency. The reactive power is mainly dependent on the voltage magnitude (i.e., on the generator excitation). The excitation system time constant is much smaller than the prime mover time constant and its transient decay much faster and does not affect the LFC dynamic. Thus, the cross-coupling between the LFC loop and the AVR loop is negligible, and the load frequency and excitation voltage control are analyzed independently.

12.3 LOAD FREQUENCY CONTROL

The operation objectives of the LFC are to maintain reasonably uniform frequency, to divide the load between generators, and to control the tie-line interchange schedules. The change in frequency and tie-line real power are sensed, which is a measure of the change in rotor angle δ, i.e., the error $\Delta\delta$ to be corrected. The error signal, i.e., Δf and ΔP_{tie}, are amplified, mixed, and transformed into a real power command signal ΔP_V, which is sent to the prime mover to call for an increment in the torque.

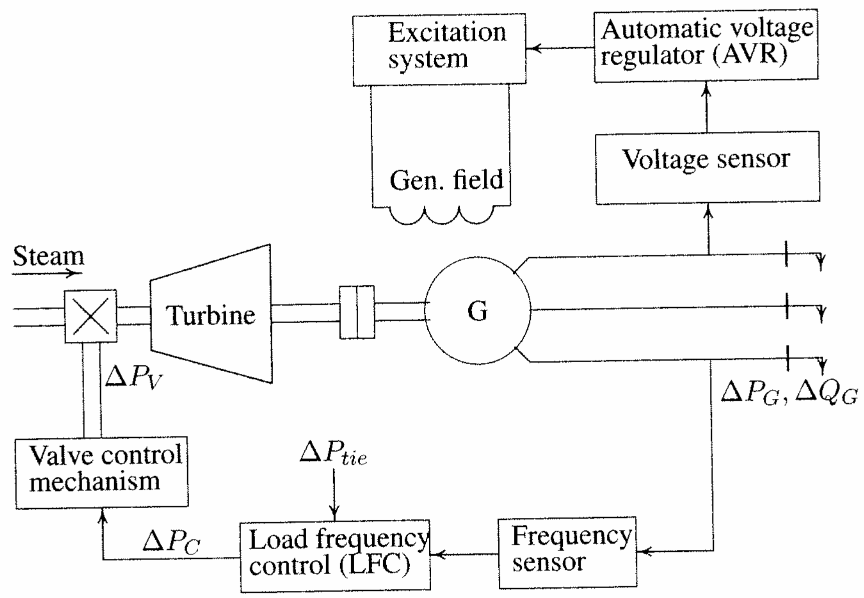

FIGURE 12.1
Schematic diagram of LFC and AVR of a synchronous generator.

The prime mover, therefore, brings change in the generator output by an amount ΔP_g which will change the values of Δf and ΔP_{tie} within the specified tolerance.

The first step in the analysis and design of a control system is mathematical modeling of the system. The two most common methods are the transfer function method and the state variable approach. The state variable approach can be applied to portray linear as well as nonlinear systems. In order to use the transfer function and linear state equations, the system must first be linearized. Proper assumptions and approximations are made to linearize the mathematical equations describing the system, and a transfer function model is obtained for the following components.

12.3.1 GENERATOR MODEL

Applying the swing equation of a synchronous machine given by (11.21) to small perturbation, we have

$$\frac{2H}{\omega_s}\frac{d^2\Delta\delta}{dt^2} = \Delta P_m - \Delta P_e \qquad (12.1)$$

or in terms of small deviation in speed

$$\frac{d\Delta\frac{\omega}{\omega_s}}{dt} = \frac{1}{2H}(\Delta P_m - \Delta P_e) \qquad (12.2)$$

With speed expressed in per unit, without explicit per unit notation, we have

$$\frac{d\Delta\omega}{dt} = \frac{1}{2H}(\Delta P_m - \Delta P_e) \qquad (12.3)$$

Taking Laplace transform of (12.3), we obtain

$$\Delta\Omega(s) = \frac{1}{2Hs}[\Delta P_m(s) - \Delta P_e(s)] \qquad (12.4)$$

The above relation is shown in block diagram form in Figure 12.2.

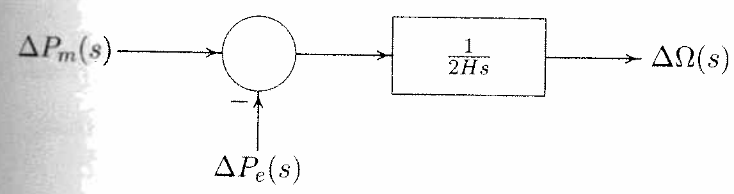

FIGURE 12.2
Generator block diagram.

12.3.2 LOAD MODEL

The load on a power system consists of a variety of electrical devices. For resistive loads, such as lighting and heating loads, the electrical power is independent of frequency. Motor loads are sensitive to changes in frequency. How sensitive it is to frequency depends on the composite of the speed-load characteristics of all the driven devices. The speed-load characteristic of a composite load is approximated by

$$\Delta P_e = \Delta P_L + D\Delta\omega \qquad (12.5)$$

where ΔP_L is the nonfrequency-sensitive load change, and $D\Delta\omega$ is the frequency-sensitive load change. D is expressed as percent change in load divided by percent change in frequency. For example, if load is changed by 1.6 percent for a 1 percent change in frequency, then $D = 1.6$. Including the load model in the generator block diagram, results in the block diagram of Figure 12.3. Eliminating the simple feedback loop in Figure 12.3, results in the block diagram shown in Figure 12.4.

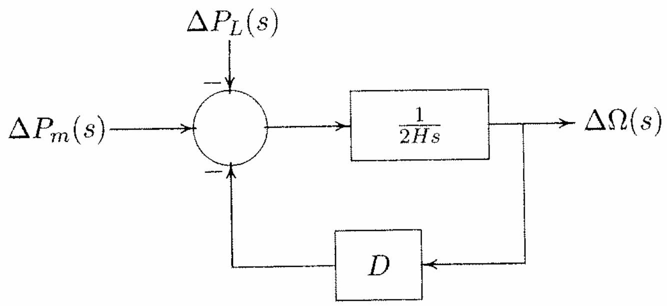

FIGURE 12.3
Generator and load block diagram.

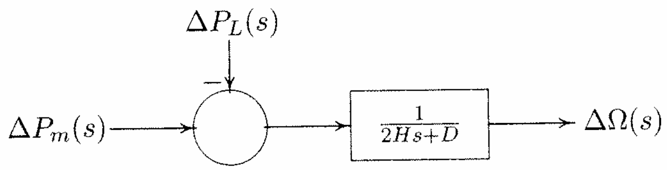

FIGURE 12.4
Generator and load block diagram.

12.3.3 PRIME MOVER MODEL

The source of mechanical power, commonly known as the *prime mover*, may be hydraulic turbines at waterfalls, steam turbines whose energy comes from the burning of coal, gas, nuclear fuel, and gas turbines. The model for the turbine relates changes in mechanical power output ΔP_m to changes in steam valve position ΔP_V. Different types of turbines vary widely in characteristics. The simplest prime mover model for the nonreheat steam turbine can be approximated with a single time constant τ_T, resulting in the following transfer function

$$G_T(s) = \frac{\Delta P_m(s)}{\Delta P_V(s)} = \frac{1}{1 + \tau_T s} \quad (12.6)$$

The block diagram for a simple turbine is shown in Figure 12.5.

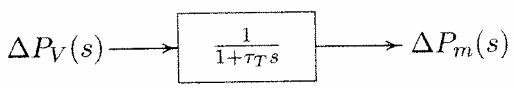

FIGURE 12.5
Block diagram for a simple nonreheat steam turbine.

The time constant τ_T is in the range of 0.2 to 2.0 seconds.

12.3.4 GOVERNOR MODEL

When the generator electrical load is suddenly increased, the electrical power exceeds the mechanical power input. This power deficiency is supplied by the kinetic energy stored in the rotating system. The reduction in kinetic energy causes the turbine speed and, consequently, the generator frequency to fall. The change in speed is sensed by the turbine governor which acts to adjust the turbine input valve to change the mechanical power output to bring the speed to a new steady-state. The earliest governors were the Watt governors which sense the speed by means of rotating *flyballs* and provides mechanical motion in response to speed changes. However, most modern governors use electronic means to sense speed changes. Figure 12.6 shows schematically the essential elements of a conventional Watt governor, which consists of the following major parts.

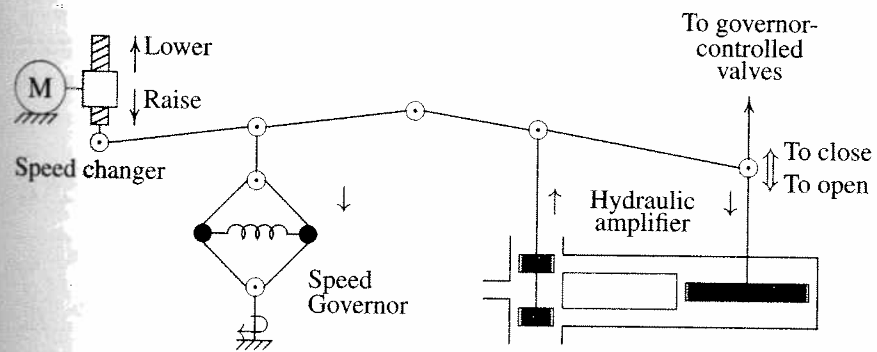

FIGURE 12.6
Speed governing system.

1. Speed Governor: The essential part are centrifugal flyballs driven directly or through gearing by the turbine shaft. The mechanism provides upward and downward vertical movements proportional to the change in speed.
2. Linkage Mechanism: These are links for transforming the flyballs movement to the turbine valve through a hydraulic amplifier and providing a feedback from the turbine valve movement.
3. Hydraulic Amplifier: Very large mechanical forces are needed to operate the steam valve. Therefore, the governor movements are transformed into high power forces via several stages of hydraulic amplifiers.
4. Speed Changer: The speed changer consists of a servomotor which can be operated manually or automatically for scheduling load at nominal frequency.

By adjusting this set point, a desired load dispatch can be scheduled at nominal frequency.

For stable operation, the governors are designed to permit the speed to drop as the load is increased. The steady-state characteristics of such a governor is shown in Figure 12.7.

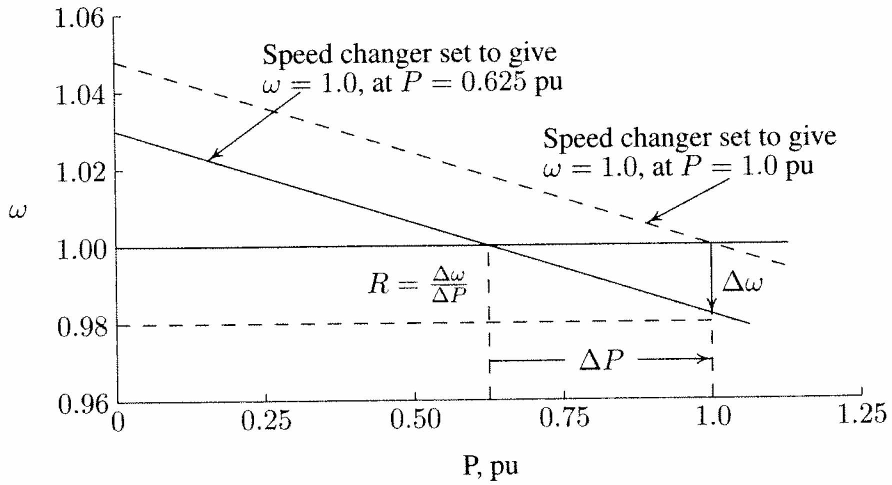

FIGURE 12.7
Governor steady-state speed characteristics.

The slope of the curve represents the speed regulation R. Governors typically have a speed regulation of 5–6 percent from zero to full load. The speed governor mechanism acts as a comparator whose output ΔP_g is the difference between the reference set power ΔP_{ref} and the power $\frac{1}{R}\Delta\omega$ as given from the governor speed characteristics, i.e.,

$$\Delta P_g = \Delta P_{ref} - \frac{1}{R}\Delta\omega \tag{12.7}$$

or in s-domain

$$\Delta P_g(s) = \Delta P_{ref}(s) - \frac{1}{R}\Delta\Omega(s) \tag{12.8}$$

The command ΔP_g is transformed through the hydraulic amplifier to the steam valve position command ΔP_V. Assuming a linear relationship and considering a simple time constant τ_g, we have the following s-domain relation

$$\Delta P_V(s) = \frac{1}{1+\tau_g s}\Delta P_g(s) \tag{12.9}$$

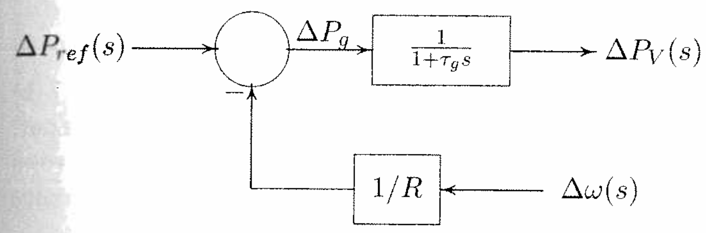

FIGURE 12.8
Block diagram representation of speed governing system for steam turbine.

Equations (12.8) and (12.9) are represented by the block diagram shown in Figure 12.8. Combining the block diagrams of Figures 12.4, 12.5, and 12.8 results in the complete block diagram of the load frequency control of an isolated power station shown in Figure 12.9. Redrawing the block diagram of Figure 12.9 with the load

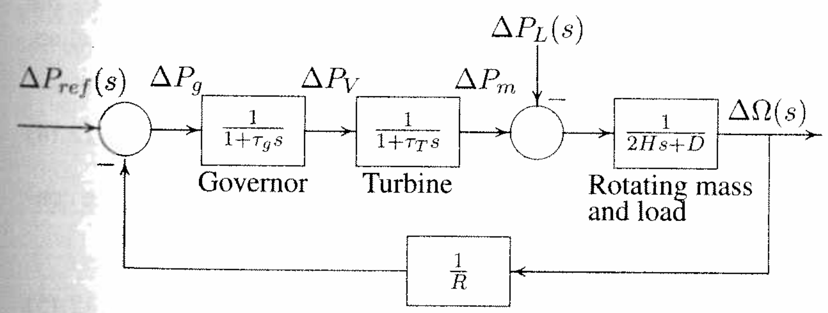

FIGURE 12.9
Load frequency control block diagram of an isolated power system.

change $-\Delta P_L(s)$ as the input and the frequency deviation $\Delta\Omega(s)$ as the output results in the block diagram shown in Figure 12.10. The open-loop transfer function of the block diagram in Figure 12.10 is

$$KG(s)H(s) = \frac{1}{R}\frac{1}{(2Hs+D)(1+\tau_g s)(1+\tau_T s)} \tag{12.10}$$

and the closed-loop transfer function relating the load change ΔP_L to the frequency deviation $\Delta\Omega$ is

$$\frac{\Delta\Omega(s)}{-\Delta P_L(s)} = \frac{(1+\tau_g s)(1+\tau_T s)}{(2Hs+D)(1+\tau_g s)(1+\tau_T s)+1/R} \tag{12.11}$$

or

$$\Delta\Omega(s) = -\Delta P_L(s)T(s) \tag{12.12}$$

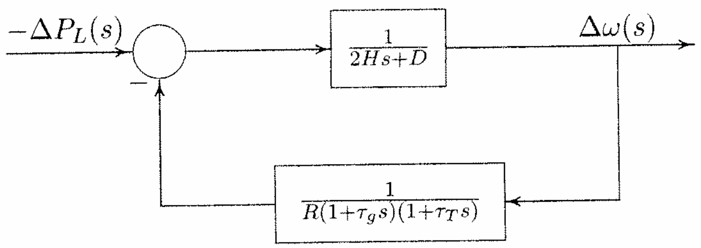

FIGURE 12.10
LFC block diagram with input $\Delta P_L(s)$ and output $\Delta\Omega(s)$.

The load change is a step input, i.e., $\Delta P_L(s) = \Delta P_L/s$. Utilizing the final value theorem, the steady-state value of $\Delta\omega$ is

$$\Delta\omega_{ss} = \lim_{s\to 0} s\Delta\Omega(s) = (-\Delta P_L)\frac{1}{D+1/R} \tag{12.13}$$

It is clear that for the case with no frequency-sensitive load (i.e., with $D = 0$), the steady-state deviation in frequency is determined by the governor speed regulation, and is

$$\Delta\omega_{ss} = (-\Delta P_L)R \tag{12.14}$$

When several generators with governor speed regulations $R_1, R_2, \ldots, R_n$ are connected to the system, the steady-state deviation in frequency is given by

$$\Delta\omega_{ss} = (-\Delta P_L)\frac{1}{D+1/R_1+1/R2+\cdots 1/R_n} \tag{12.15}$$

Example 12.1 (chp12ex1)

An isolated power station has the following parameters

Turbine time constant $\tau_T = 0.5$ sec
Governor time constant $\tau_g = 0.2$ sec
Generator inertia constant $H = 5$ sec
Governor speed regulation = R per unit

The load varies by 0.8 percent for a 1 percent change in frequency, i.e., $D = 0.8$
(a) Use the Routh-Hurwitz array (Appendix B.2.1) to find the range of R for control system stability.
(b) Use *MATLAB* **rlocus** function to obtain the root locus plot.
(c) The governor speed regulation of Example 12.1 is set to $R = 0.05$ per unit. The turbine rated output is 250 MW at nominal frequency of 60 Hz. A sudden load change of 50 MW ($\Delta P_L = 0.2$ per unit) occurs.

(i) Find the steady-state frequency deviation in Hz.

(ii) Use *MATLAB* to obtain the time-domain performance specifications and the frequency deviation step response.

(d) Construct the *SIMULINK* block diagram (see Appendix A.17) and obtain the frequency deviation response for the condition in part (c).
Substituting the system parameters in the LFC block diagram of Figure 12.10 results in the block diagram shown in Figure 12.11. The open-loop transfer function

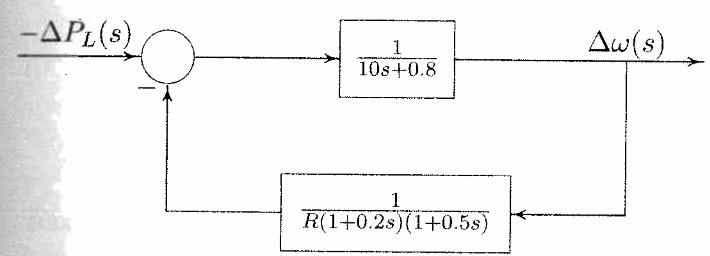

FIGURE 12.11
LFC block diagram for Example 12.1.

is

$$KG(s)H(s) = \frac{K}{(10s+0.8)(1+0.2s)(1+0.5s)} = \frac{K}{s^3+7.08s^2+10.56s+0.8}$$

where $K = \frac{1}{R}$

(a) The characteristic equation is given by

$$1 + KG(s)H(s) = 1 + \frac{K}{s^3+7.08s^2+10.56s+0.8} = 0$$

which results in the characteristic polynomial equation

$$s^3 + 7.08s^2 + 10.56s + 0.8 + K = 0$$

The Routh-Hurwitz array for this polynomial is then (see Appendix B.2.1)

$$\begin{array}{c|cc} s^3 & 1 & 10.56 \\ s^2 & 7.08 & 0.8+K \\ s^1 & \frac{73.965-K}{7.08} & 0 \\ s^0 & 0.8+K & 0 \end{array}$$

From the s^1 row, we see that for control system stability, K must be less than 73.965. Also from the s^0 row, K must be greater than -0.8. Thus, with positive values of K, for control system stability

$$K < 73.965$$

Since $R = \frac{1}{K}$, for control system stability, the governor speed regulation must be

$$R > \frac{1}{73.965} \qquad \text{or} \qquad R > 0.0135$$

For $K = 73.965$, the auxiliary equation from the s^2 row is

$$7.08s^2 + 74.765 = 0$$

or $s = \pm j3.25$. That is, for $R = 0.0135$, we have a pair of conjugate poles on the $j\omega$ axis, and the control system is marginally stable.

(b) To obtain the root-locus, we use the following commands.

```
num=1;
den = [1  7.08  10.56 .8];
figure(1), rlocus(num, den)
```

The result is shown in Figure 12.12.

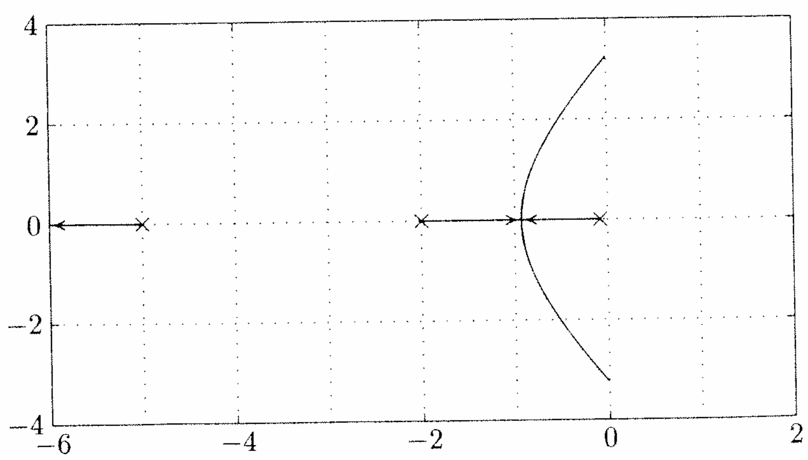

FIGURE 12.12
Root-locus plot for Example 12.1.

The loci intersect the $j\omega$ axis at $s = \pm j3.25$ for $K = 73.965$. Thus, the system is marginally stable for $R = \frac{1}{73.965} = 0.0135$.

(c) The closed-loop transfer function of the system shown in Figure 12.11 is

$$\frac{\Delta\Omega(s)}{-\Delta P_L(s)} = T(s) = \frac{(1+0.2s)(1+0.5s)}{(10s+0.8)(1+0.2s)(1+0.5s)+1/.05}$$
$$= \frac{0.1s^2+0.7s+1}{s^3+7.08s^2+10.56s+20.8}$$

(i) The steady-state frequency deviation due to a step input is

$$\Delta\omega_{ss} = \lim_{s\to 0} s\Delta\Omega(s) = \frac{1}{20.8}(-0.2) = -0.0096 \quad \text{pu}$$

Thus, the steady-state frequency deviation in hertz due to the sudden application of a 50-MW load is $\Delta f = (-0.0096)(60) = 0.576$ Hz.

(ii) To obtain the step response and the time-domain performance specifications, we use the following commands

```
PL = 0.2; numc = [0.1  0.7  1];
denc = [1   7.08  10.56  20.8];
t = 0:.02:10; c = -PL*step(num, den, t);
figure(2), plot(t, c), xlabel('t,  sec'), ylabel('pu')
title('Frequency deviation step response'), grid
  timespec(num, den)
```

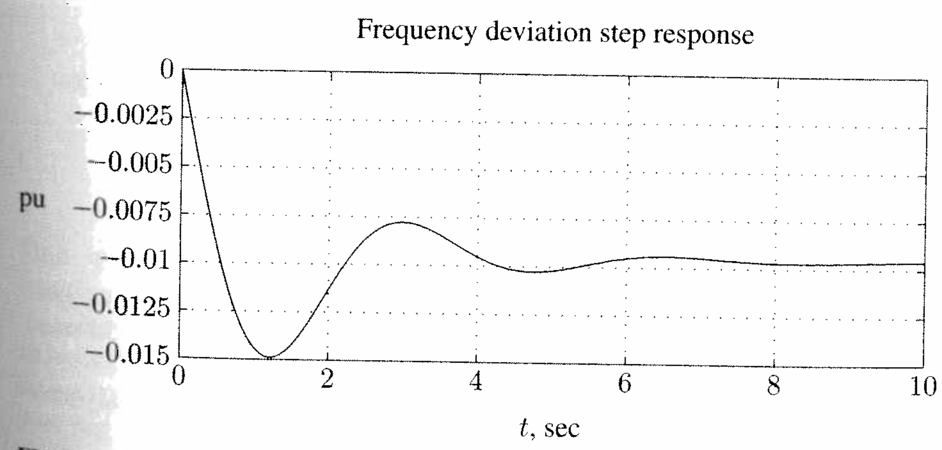

FIGURE 12.13
Frequency deviation step response for Example 12.1.

The frequency deviation step response is shown in Figure 12.13, and the time-domain performance specifications are

```
Peak time = 1.223     Percent overshoot = 54.80
Rise time = 0.418
Settling time = 6.8
```

(d) A *SIMULINK* model named **sim12ex1.mdl** is constructed as shown in Figure 12.14. The file is opened and is run in the *SIMULINK* window. The simulation results in the same response as shown in Figure 12.13.

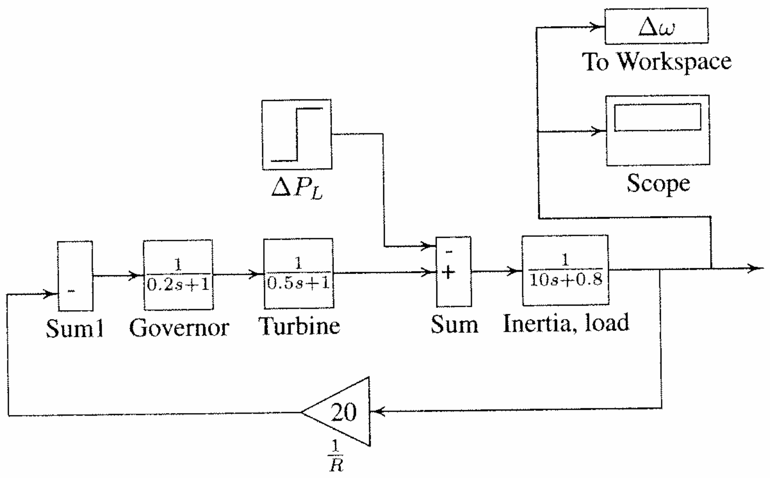

FIGURE 12.14
Simulation block diagram for Example 12.1.

Example 12.2 (chp12ex2)

A single area consists of two generating units with the following characteristics.

Unit	Rating	Speed regulation R (pu on unit MVA base)
1	600 MVA	6%
2	500 MVA	4%

The units are operating in parallel, sharing 900 MW at the nominal frequency. Unit 1 supplies 500 MW and unit 2 supplies 400 MW at 60 Hz. The load is increased by 90 MW.

(a) Assume there is no frequency-dependent load, i.e., $D = 0$. Find the steady-state frequency deviation and the new generation on each unit.

(b) The load varies 1.5 percent for every 1 percent change in frequency, i.e., $D = 1.5$. Find the steady-state frequency deviation and the new generation on each unit.

First we express the governor speed regulation of each unit to a common MVA base. Select 1000 MVA for the apparent power base, then

$$R_1 = \frac{1000}{600}(0.06) = 0.1 \text{ pu}$$
$$R_2 = \frac{1000}{500}(0.05) = 0.08 \text{ pu}$$

The per unit load change is

$$\Delta P_L = \frac{90}{1000} = 0.09 \text{ pu}$$

(a) From (12.15) with $D = 0$, the per unit steady-state frequency deviation is

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{-0.09}{10 + 12.5} = -0.004 \text{ pu}$$

Thus, the steady-state frequency deviation in Hz is

$$\Delta f = (-0.004)(60) = -0.24 \text{ Hz}$$

and the new frequency is

$$f = f_0 + \Delta f = 60 - 0.24 = 59.76 \text{ Hz}$$

The change in generation for each unit is

$$\Delta P_1 = -\frac{\Delta\omega}{R_1} = -\frac{-0.004}{0.1} = 0.04 \text{ pu} = 40 \text{ MW}$$

$$\Delta P_2 = -\frac{\Delta\omega}{R_2} = -\frac{-0.004}{0.08} = 0.05 \text{ pu} = 50 \text{ MW}$$

Thus, unit 1 supplies 540 MW and unit 2 supplies 450 MW at the new operating frequency of 59.76 Hz.

MATLAB is used to plot the per unit speed characteristics of each governor as shown in Figure 12.15. As we can see from this figure, the initial generations are 0.5 and 0.40 per unit at the nominal frequency of 1.0 per unit. With the addition of 0.09 per unit power speed drops to 0.996 per unit. The new generations are 0.54 and 0.45 per unit.

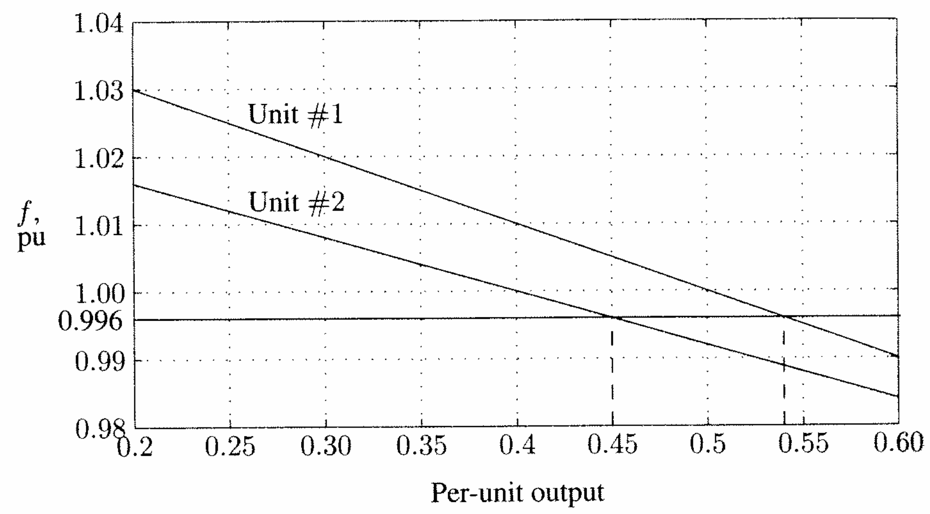

FIGURE 12.15
Load division between the two units of Example 12.2

(b) D expressed to the base of 1000 MVA is: $D = \frac{990}{1000}(1.5) = 1.485$
The per unit steady-state frequency deviation is

$$\Delta\omega_{ss} = \frac{-\Delta P_L}{\frac{1}{R_1} + \frac{1}{R_2} + D} = \frac{-0.09}{10 + 12.5 + 1.485} = -0.0037523 \text{ pu}$$

Thus, the steady-state frequency deviation in Hz is

$$\Delta f = (-0.0037523)(60) = -0.22514 \text{ Hz}$$

and the new frequency is

$$f = f_0 + \Delta f = 60 - 0.22514 = 59.7748 \text{ Hz}$$

The change in generation for each unit is

$$\Delta P_1 = -\frac{\Delta\omega}{R_1} = -\frac{-0.0037523}{0.1} = 0.037523 \text{ pu}$$
$$= 37.523 \text{ MW}$$

$$\Delta P_2 = -\frac{\Delta\omega}{R_2} = -\frac{-0.0037523}{0.08} = 0.046904 \text{ pu}$$
$$= 46.904 \text{ MW}$$

Thus, unit 1 supplies 537.523 MW and unit 2 supplies 446.904 MW at the new operating frequency of 59.7748 Hz. The total change in generation is 84.427, which is 5.573 MW less than the 90 MW load change. This is because of the change in load due to frequency drop which is given by

$$\Delta\omega D = (-0.0037523)(1.485) = -0.005572 \text{ pu}$$
$$= -5.572 \text{ MW}$$

12.4 AUTOMATIC GENERATION CONTROL

If the load on the system is increased, the turbine speed drops before the governor can adjust the input of the steam to the new load. As the change in the value of speed diminishes, the error signal becomes smaller and the position of the governor flyballs gets closer to the point required to maintain a constant speed. However, the constant speed will not be the set point, and there will be an offset. One way to restore the speed or frequency to its nominal value is to add an integrator. The integral unit monitors the average error over a period of time and will overcome the offset. Because of its ability to return a system to its set point, integral action is also known as the *rest action*. Thus, as the system load changes continuously, the generation is adjusted automatically to restore the frequency to the nominal value. This scheme is known as the *automatic generation control* (AGC). In an interconnected system consisting of several pools, the role of the AGC is to divide the loads among system, stations, and generators so as to achieve maximum economy and correctly control the scheduled interchanges of tie-line power while maintaining a reasonably uniform frequency. Of course, we are implicitly assuming that the system is stable, so the steady-state is achievable. During large transient disturbances and emergencies, AGC is bypassed and other emergency controls are applied. In the following section, we consider the AGC in a single area system and in an interconnected power system.

12.4.1 AGC IN A SINGLE AREA SYSTEM

With the primary LFC loop, a change in the system load will result in a steady-state frequency deviation, depending on the governor speed regulation. In order to reduce the frequency deviation to zero, we must provide a reset action. The rest action can be achieved by introducing an integral controller to act on the load reference setting to change the speed set point. The integral controller increases the system type by 1 which forces the final frequency deviation to zero. The LFC system, with the addition of the secondary loop, is shown in Figure 12.16. The integral controller gain K_I must be adjusted for a satisfactory transient response. Combining the parallel branches results in the equivalent block diagram shown in

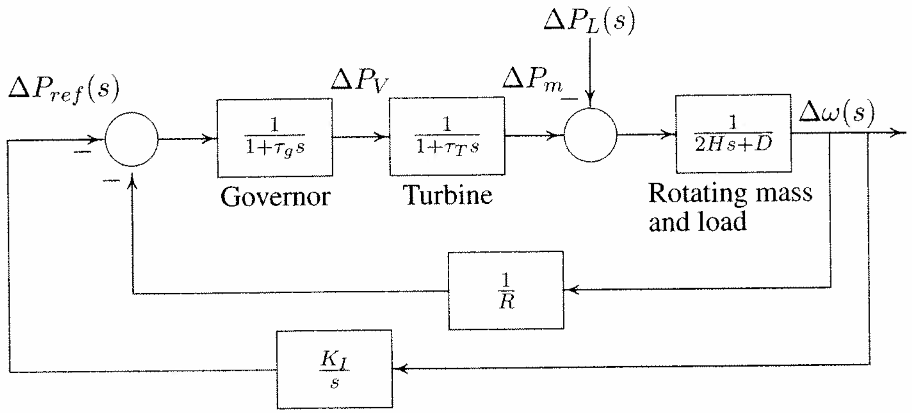

FIGURE 12.16
AGC for an isolated power system.

Figure 12.17.

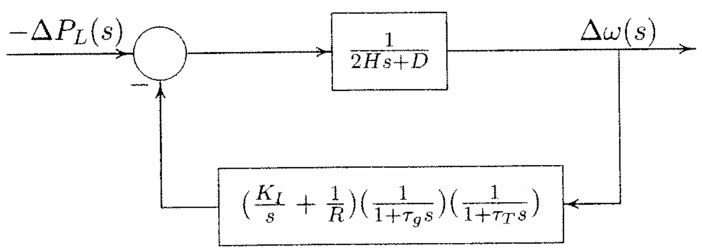

FIGURE 12.17
The equivalent block diagram of AGC for an isolated power system.

The closed-loop transfer function of the control system shown in Figure 12.17 with only $-\Delta P_L$ as input becomes

$$\frac{\Delta\Omega(s)}{-\Delta P_L(s)} = \frac{s(1+\tau_g s)(1+\tau_T s)}{s(2Hs+D)(1+\tau_g s)(1+\tau_T s) + K_I + s/R} \tag{12.16}$$

Example 12.3 (chp12ex3), (sim12ex3.mdl)

The LFC system in Example 12.1 is equipped with the secondary integral control loop for automatic generation control.
(a) Use the *MATLAB* **step** function to obtain the frequency deviation step response for a sudden load change of $\Delta P_L = 0.2$ per unit. Set the integral controller gain to $K_I = 7$.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (a).

(a) Substituting for the system parameters in (12.16), with speed regulation adjusted to $R = 0.05$ per unit, results in the following closed-loop transfer function

$$T(s) = \frac{0.1s^3 + 0.7s^2 + s}{s^4 + 7.08s^3 + 10.56s^2 + 20.8s + 7}$$

To find the step response, we use the following commands

```
PL = 0.2;
KI = 7;
num = [0.1  0.7  1 0];
den = [1   7.08  10.56  20.8 KI];
t = 0:.02:12;
c = -PL*step(num, den, t);
plot(t, c), grid
xlabel('t, sec'), ylabel('pu')
title('Frequency deviation step response')
```

The step response is shown in Figure 12.18.

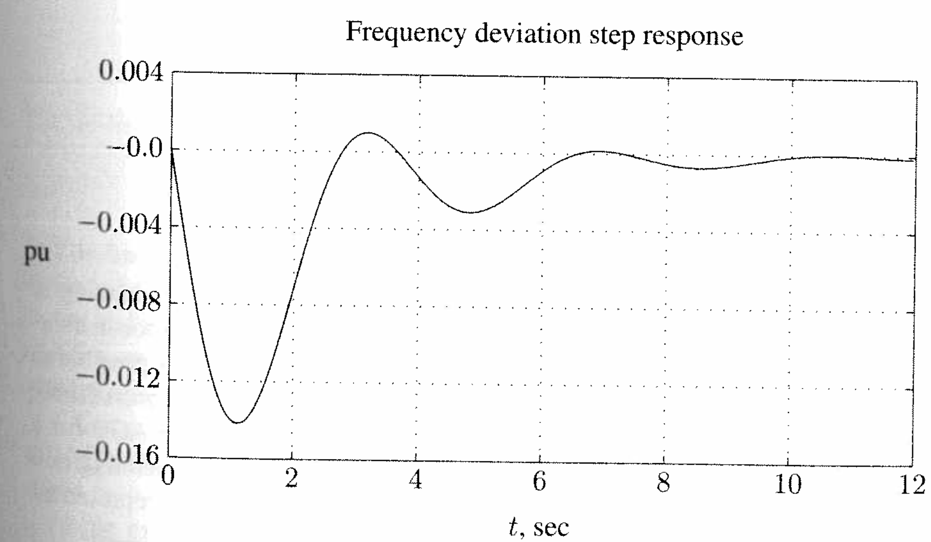

FIGURE 12.18
Frequency deviation step response for Example 12.3.

From the step response, we observe that the steady-state frequency deviation $\Delta\omega_{ss}$ is zero, and the frequency returns to its nominal value in approximately 10 seconds.

(b) A *SIMULINK* model named **sim12ex3.mdl** is constructed as shown in Figure 12.19. The file is opened and is run in the *SIMULINK* window. The simulation results in the same response as shown in Figure 12.18.

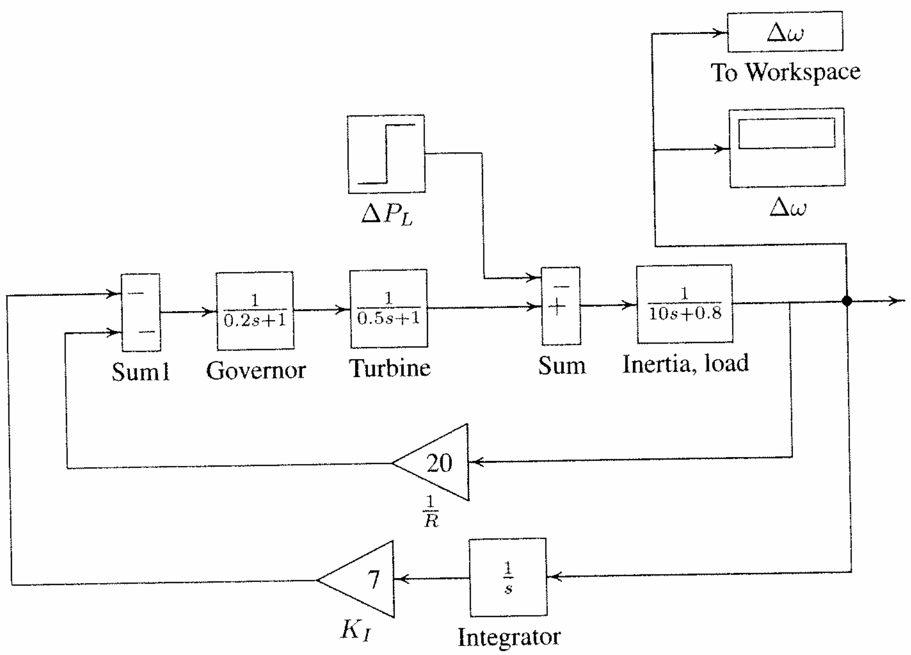

FIGURE 12.19
Simulation block diagram for Example 12.3.

12.4.2 AGC IN THE MULTIAREA SYSTEM

In many cases, a group of generators are closely coupled internally and swing in unison. Furthermore, the generator turbines tend to have the same response characteristics. Such a group of generators are said be *coherent*. Then it is possible to let the LFC loop represent the whole system, which is referred to as a *control area*. The AGC of a multiarea system can be realized by studying first the AGC for a two-area system. Consider two areas represented by an equivalent generating unit interconnected by a lossless tie line with reactance X_{tie}. Each area is represented by a voltage source behind an equivalent reactance as shown in Figure 12.20.

During normal operation, the real power transferred over the tie line is given by

$$P_{12} = \frac{|E_1||E_2|}{X_{12}} sin\delta_{12} \qquad (12.17)$$

where $X_{12} = X_1 + X_{tie} + X_2$, and $\delta_{12} = \delta_1 - \delta_2$. Equation (12.17) can be linearized

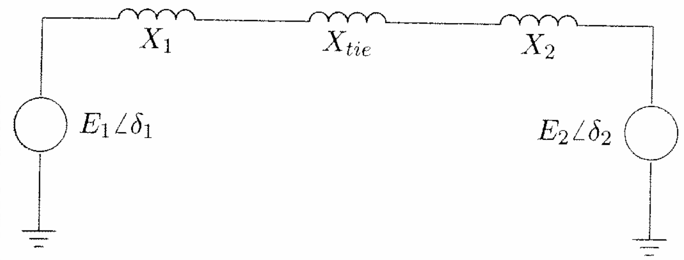

FIGURE 12.20
Equivalent network for a two-area power system.

for a small deviation in the tie-line flow ΔP_{12} from the nominal value, i.e.,

$$\Delta P_{12} = \left.\frac{dP_{12}}{d\delta_{12}}\right|_{\delta_{12_0}} \Delta\delta_{12} = P_s\Delta\delta_{12} \qquad (12.18)$$

The quantity P_s is the slope of the power angle curve at the initial operating angle $\delta_{12_0} = \delta_{1_0} - \delta_{2_0}$. This was defined as the synchronizing power coefficient by (11.39) in Section 11.4. Thus we have

$$P_s = \left.\frac{dP_{12}}{d\delta_{12}}\right|_{\delta 12_0} = \frac{|E_1||E_2|}{X_{12}} \cos\Delta\delta_{12_0} \qquad (12.19)$$

The tie-line power deviation then takes on the form

$$\Delta P_{12} = P_s(\Delta\delta_1 - \Delta\delta_2) \qquad (12.20)$$

The tie-line power flow appears as a load increase in one area and a load decrease in the other area, depending on the direction of the flow. The direction of flow is dictated by the phase angle difference; if $\Delta\delta_1 > \Delta\delta_2$, the power flows from area 1 to area 2. A block diagram representation for the two-area system with LFC containing only the primary loop is shown in Figure 12.21.

Let us consider a load change ΔP_{L1} in area 1. In the steady-state, both areas will have the same steady-state frequency deviation, i.e.,

$$\Delta\omega = \Delta\omega_1 = \Delta\omega_2 \qquad (12.21)$$

and

$$\begin{aligned} \Delta P_{m1} - \Delta P_{12} - \Delta P_{L1} &= \Delta\omega D_1. \\ \Delta P_{m2} + \Delta P_{12} &= \Delta\omega D_2 \end{aligned} \qquad (12.22)$$

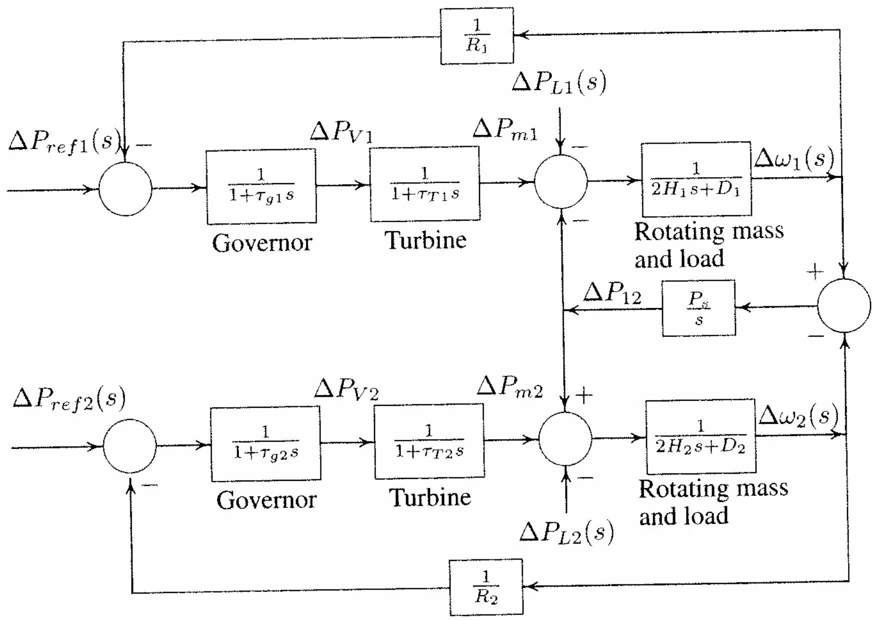

FIGURE 12.21
Two-area system with only primary LFC loop.

The change in mechanical power is determined by the governor speed characteristics, given by

$$\Delta P_{m1} = \frac{-\Delta\omega}{R_1}$$
$$\Delta P_{m2} = \frac{-\Delta\omega}{R_2} \tag{12.23}$$

Substituting from (12.23) into (12.22), and solving for $\Delta\omega$, we have

$$\Delta\omega = \frac{-\Delta P_{L1}}{(\frac{1}{R_1} + D_1) + (\frac{1}{R_2} + D_2)} = \frac{-\Delta P_{L1}}{B_1 + B2} \tag{12.24}$$

where

$$B_1 = \frac{1}{R_1} + D_1$$
$$B_2 = \frac{1}{R_2} + D_2 \tag{12.25}$$

B_1 and B_2 are known as the *frequency bias factors*. The change in the tie-line power is

$$\Delta P_{12} = -\frac{(\frac{1}{R_2} + D_2)\Delta P_{L1}}{(\frac{1}{R_1} + D_1) + (\frac{1}{R_2} + D_2)} = \frac{B_2}{B_1 + B_2}(-\Delta P_{L1}) \tag{12.26}$$

Example 12.4 (chp12ex4), (sim12ex4.mdl)

A two-area system connected by a tie line has the following parameters on a 1000-MVA common base

Area	1	2
Speed regulation	$R_1 = 0.05$	$R_2 = 0.0625$
Frequency-sens. load coeff.	$D_1 = 0.6$	$D_2 = 0.9$
Inertia constant	$H_1 = 5$	$H_2 = 4$
Base power	1000 MVA	1000 MVA
Governor time constant	$\tau_{g1} = 0.2$ sec	$\tau_{g2} = 0.3$ sec
Turbine time constant	$\tau_{T1} = 0.5$ sec	$\tau_{T2} = 0.6$ sec

The units are operating in parallel at the nominal frequency of 60 Hz. The synchronizing power coefficient is computed from the initial operating condition and is given to be $P_s = 2.0$ per unit. A load change of 187.5 MW occurs in area 1.
(a) Determine the new steady-state frequency and the change in the tie-line flow.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (a).

(a) The per unit load change in area 1 is

$$\Delta P_{L1} = \frac{187.5}{1000} = 0.1875 \text{ pu}$$

The per unit steady-state frequency deviation is

$$\Delta\omega_{ss} = \frac{-\Delta P_{L1}}{(\frac{1}{R_1} + D_1) + (\frac{1}{R_2} + D_2)} = \frac{-0.1875}{(20 + 0.6) + (16 + 0.9)} = -0.005 \text{ pu}$$

Thus, the steady-state frequency deviation in Hz is

$$\Delta f = (-0.005)(60) = -0.3 \text{ Hz}$$

and the new frequency is

$$f = f_0 + \Delta f = 60 - 0.3 = 59.7 \text{ Hz}$$

The change in mechanical power in each area is

$$\Delta P_{m1} = -\frac{\Delta\omega}{R_1} = -\frac{-0.005}{0.05} = 0.10 \text{ pu}$$
$$= 100 \text{ MW}$$

$$\Delta P_{m2} = -\frac{\Delta\omega}{R_2} = -\frac{-0.005}{0.0625} = 0.080 \text{ pu}$$
$$= 80 \text{ MW}$$

Thus, area 1 increases the generation by 100 MW and area 2 by 80 MW at the new operating frequency of 59.7 Hz. The total change in generation is 180 MW, which is 7.5 MW less than the 187.5 MW load change because of the change in the area loads due to frequency drop.

The change in the area 1 load is $\Delta\omega D_1 = (-0.005)(0.6) = -0.003$ per unit (−3.0 MW), and the change in the area 2 load is $\Delta\omega D_1 = (-0.005)(0.9) = -0.0045$ per unit (−4.5 MW). Thus, the change in the total area load is −7.5 MW. The tie-line power flow is

$$\Delta P_{12} = \Delta\omega\left(\frac{1}{R_2} + D2\right) = -0.005(16.9) = 0.0845 \text{ pu}$$
$$= -84.5 \text{ MW}$$

That is, 84.5 MW flows from area 2 to area 1. 80 MW comes from the increased generation in area 2, and 4.5 MW comes from the reduction in area 2 load due to frequency drop.

(b) A *SIMULINK* model named **sim12ex4.mdl** is constructed as shown in Figure 12.22. The file is opened and is run in the *SIMULINK* window. The simulation result is shown in Figure 12.23. The simulation diagram returns the vector DP, containing t, P_{m1}, P_{m2}, and P_{12}. A plot of the per unit power response is obtained in *MATLAB* as shown in Figure 12.24.

12.4.3 TIE-LINE BIAS CONTROL

In Example 12.4, where LFCs were equipped with only the primary control loop, a change of power in area 1 was met by the increase in generation in both areas associated with a change in the tie-line power, and a reduction in frequency. In the normal operating state, the power system is operated so that the demands of areas are satisfied at the nominal frequency. A simple control strategy for the normal mode is

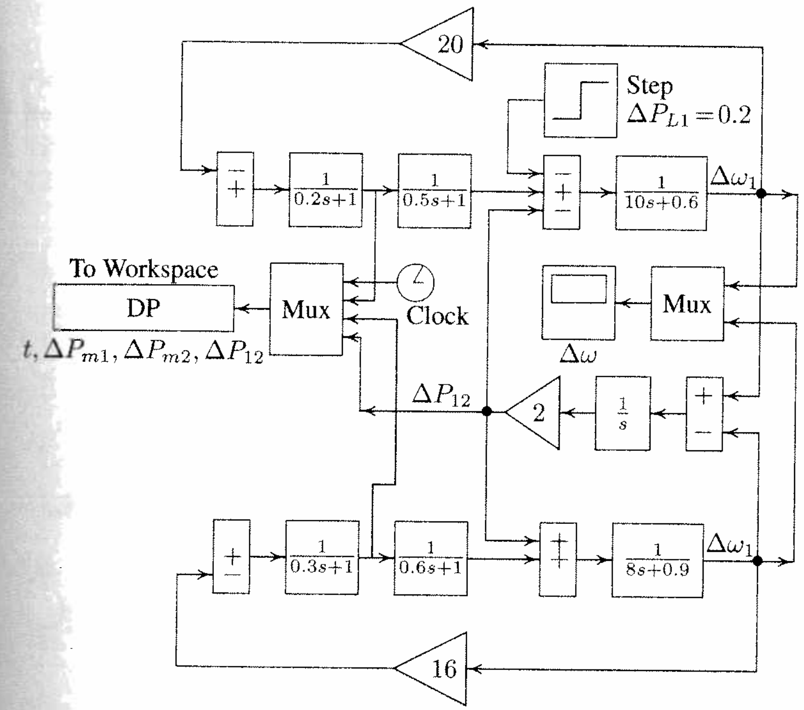

FIGURE 12.22
Simulation block diagram for Example 12.4.

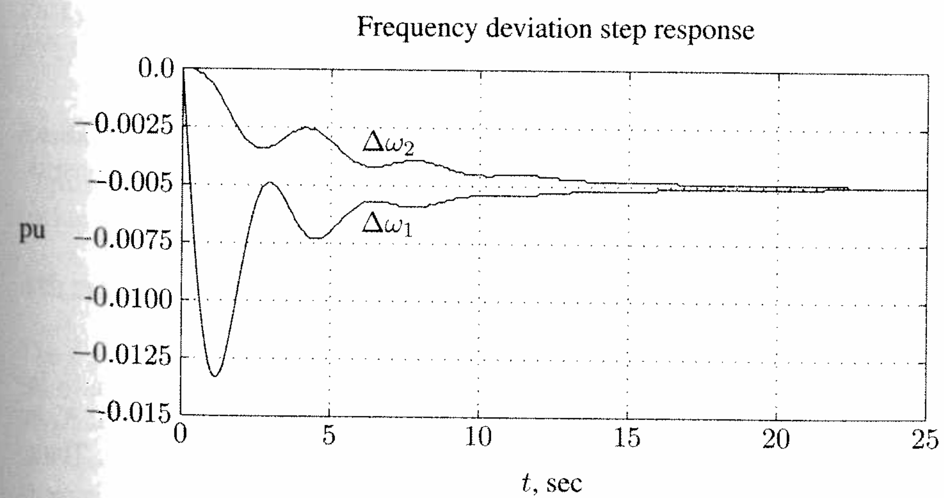

FIGURE 12.23
Frequency deviation step response for Example 12.4.

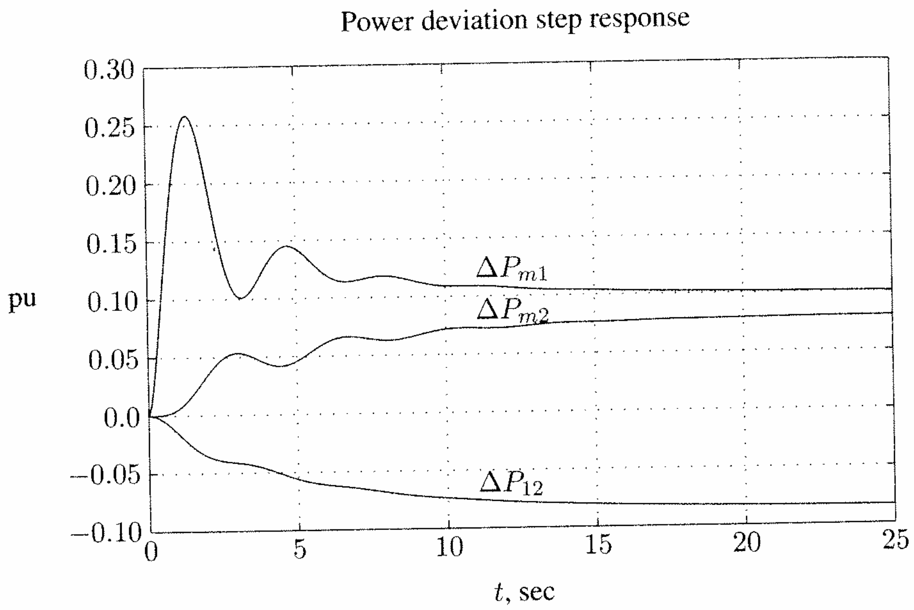

FIGURE 12.24
Power deviation step response for Example 12.4.

- Keep frequency approximately at the nominal value (60 Hz).
- Maintain the tie-line flow at about schedule.
- Each area should absorb its own load changes.

Conventional LFC is based upon tie-line bias control, where each area tends to reduce the area control error (ACE) to zero. The control error for each area consists of a linear combination of frequency and tie-line error.

$$\text{ACE}_i = \sum_{j=1}^{n} \Delta P_{ij} + K_i \, \Delta\omega \tag{12.27}$$

The area bias K_i determines the amount of interaction during a disturbance in the neighboring areas. An overall satisfactory performance is achieved when K_i is selected equal to the frequency bias factor of that area, i.e., $B_i = \frac{1}{R_i} + D_i$. Thus, the ACEs for a two-area system are

$$\begin{aligned} \text{ACE}_1 &= \Delta P_{12} + B_1 \, \Delta\omega_1 \\ \text{ACE}_2 &= \Delta P_{21} + B_2 \, \Delta\omega_2 \end{aligned} \tag{12.28}$$

where ΔP_{12} and ΔP_{21} are departures from scheduled interchanges. ACEs are used as actuating signals to activate changes in the reference power set points, and when steady-state is reached, ΔP_{12} and $\Delta\omega$ will be zero. The integrator gain constant must be chosen small enough so as not to cause the area to go into a chase mode. The block diagram of a simple AGC for a two-area system is shown in Figure 12.25. We can easily extend the tie-line bias control to an n-area system.

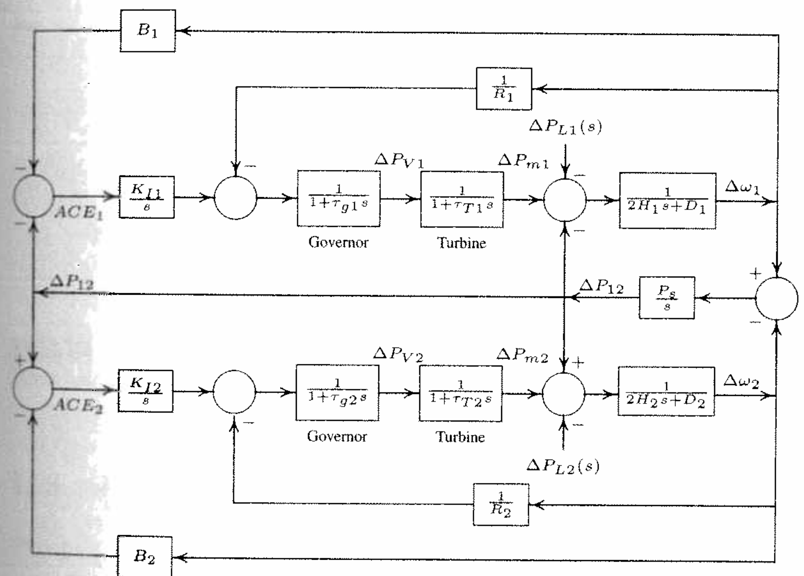

FIGURE 12.25
AGC block diagram for a two-area system.

Example 12.5 (sim12ex5.mdl)

Construct the *SIMULINK* model for the two-area system of Example 12.4 with the inclusion of the ACEs, and obtain the frequency and power response for each area.

A *SIMULINK* model named **sim12ex5.mdl** is constructed as shown in Figure 12.26.

The file is opened and is run in the *SIMULINK* window. The integrator gain constants are adjusted for a satisfactory response. The simulation result for $K_{I1} = K_{I2} = 0.3$ is shown in Figure 12.27. The simulation diagram returns the vector ΔP, containing t, ΔP_{m1}, ΔP_{m2}, and ΔP_{12}. A plot of the per unit power response is obtained in *MATLAB* as shown in Figure 12.28. As we can see from Figure 12.27, the frequency deviation returns to zero with a settling time of approximately 20 seconds. Also, the tie-line power change reduces to zero, and the increase in area 1 load is met by the increase in generation ΔP_{m1}.

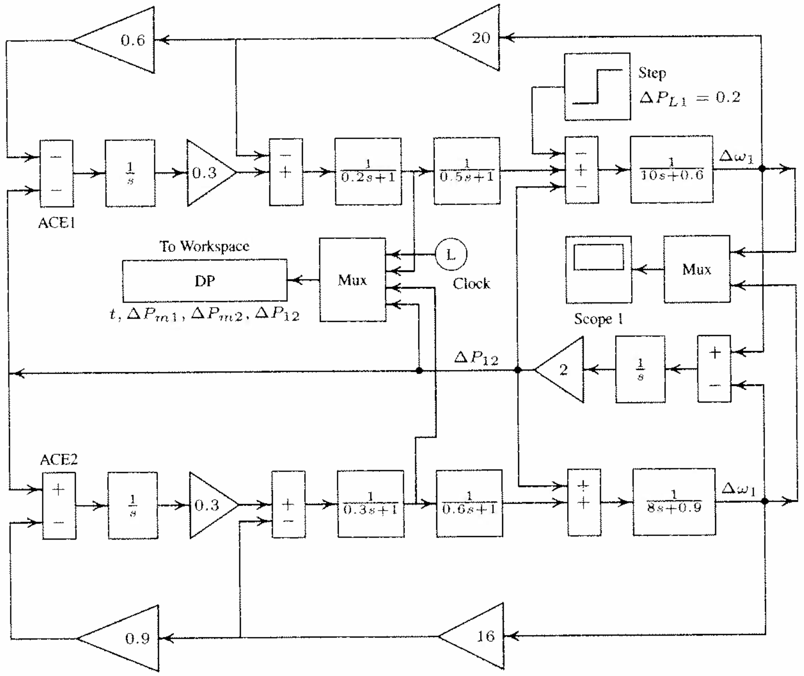

FIGURE 12.26
Simulation block diagram for Example 12.5.

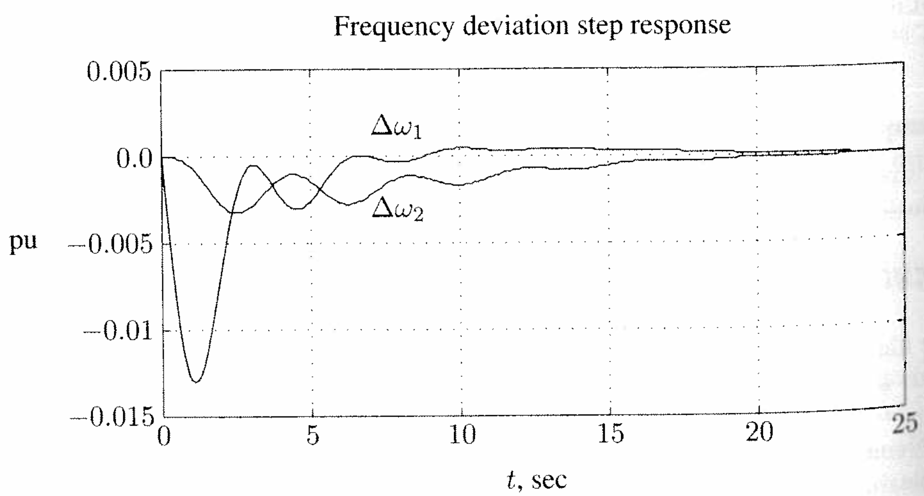

FIGURE 12.27
Frequency deviation step response for Example 12.5.

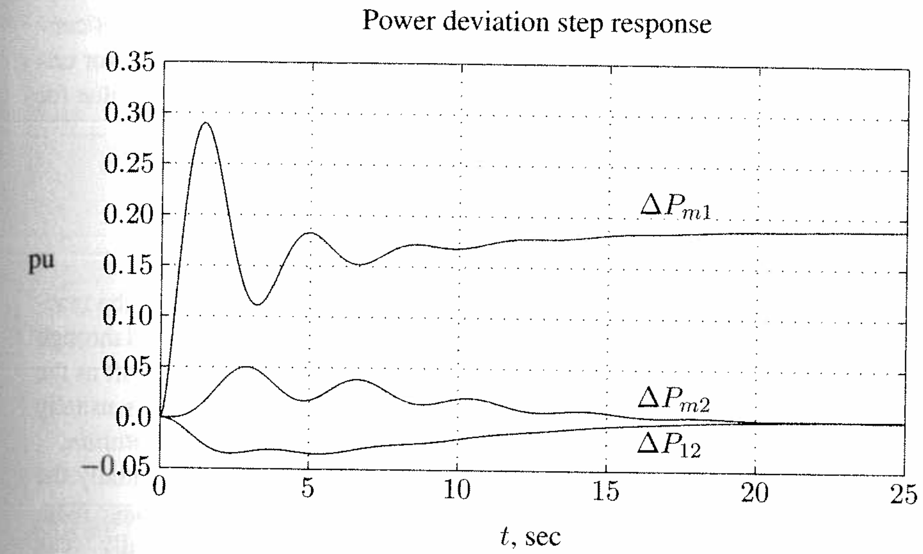

FIGURE 12.28
Power deviation step response for Example 12.5.

12.5 AGC WITH OPTIMAL DISPATCH OF GENERATION

The factors influencing power generation at minimum cost are operating efficiencies, fuel cost, and transmission losses. The optimal dispatch of generation was discussed in Chapter 7, and a program named **dispatch** was developed to find the optimal dispatch of generation for an interconnected power system.

The optimal dispatch of generation may be treated within the framework of LFC. In direct digital control systems, the digital computer is included in the control loop which scans the unit generation and tie-line flows. These settings are compared with the optimal settings derived from the solution of the optimal dispatch program, such as **dispatch** program developed in Chapter 7. If the actual settings are off from the optimal values, the computer generates the raise/lower pulses which are sent to the individual units. The allocation program will also take into account the tie-line power contracts between the areas.

With the development of modern control theory, several concepts are included in the AGC which go beyond the simple tie-line bias control. The fundamental approach is the use of more extended mathematical models. In retrospect, the AGC can be used to include the representation of the dynamics of the area, or even of the complete system.

Other concepts of the modern control theory are being employed, such as state estimation and optimal control with linear regulator utilizing constant feedback gains. In addition to the structures which aim at the control of deterministic signals and disturbances, there are schemes which employ stochastic control concepts, e.g., minimization of some expected value of an integral quadratic error criterion. Usually, this results in the design of the Kalman filter, which is of value for the control of small random disturbances.

12.6 REACTIVE POWER AND VOLTAGE CONTROL

The generator excitation system maintains generator voltage and controls the reactive power flow. The generator excitation of older systems may be provided through slip rings and brushes by means of dc generators mounted on the same shaft as the rotor of the synchronous machine. However, modern excitation systems usually use ac generators with rotating rectifiers, and are known as *brushless excitation*.

As we have seen, a change in the real power demand affects essentially the frequency, whereas a change in the reactive power affects mainly the voltage magnitude. The interaction between voltage and frequency controls is generally weak enough to justify their analysis separately.

The sources of reactive power are generators, capacitors, and reactors. The generator reactive powers are controlled by field excitation. Other supplementary methods of improving the voltage profile on electric transmission systems are transformer load-tap changers, switched capacitors, step-voltage regulators, and static var control equipment. The primary means of generator reactive power control is the generator excitation control using *automatic voltage regulator* (AVR), which is discussed in this chapter. The role of an (AVR) is to hold the terminal voltage magnitude of a synchronous generator at a specified level. The schematic diagram of a simplified AVR is shown in Figure 12.29.

An increase in the reactive power load of the generator is accompanied by a drop in the terminal voltage magnitude. The voltage magnitude is sensed through a potential transformer on one phase. This voltage is rectified and compared to a dc set point signal. The amplified error signal controls the exciter field and increases the exciter terminal voltage. Thus, the generator field current is increased, which results in an increase in the generated emf. The reactive power generation is increased to a new equilibrium, raising the terminal voltage to the desired value. We will look briefly at the simplified models of the component involved in the AVR system.

12.6.1 AMPLIFIER MODEL

The excitation system amplifier may be a magnetic amplifier, rotating amplifier, or modern electronic amplifier. The amplifier is represented by a gain K_A and a time

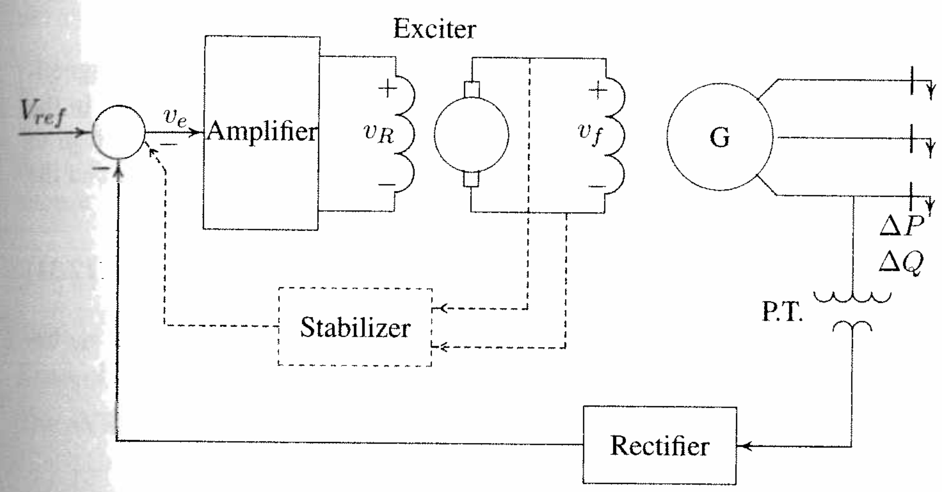

FIGURE 12.29
A typical arrangement of a simple AVR.

constant τ_A, and the transfer function is

$$\frac{V_R(s)}{V_e(s)} = \frac{K_A}{1 + \tau_A s} \tag{12.29}$$

Typical values of K_A are in the range of 10 to 400. The amplifier time constant is very small, in the range of 0.02 to 0.1 second, and often is neglected.

12.6.2 EXCITER MODEL

There is a variety of different excitation types. However, modern excitation systems uses ac power source through solid-state rectifiers such as SCR. The output voltage of the exciter is a nonlinear function of the field voltage because of the saturation effects in the magnetic circuit. Thus, there is no simple relationship between the terminal voltage and the field voltage of the exciter. Many models with various degrees of sophistication have been developed and are available in the IEEE recommendation publications. A reasonable model of a modern exciter is a linearized model, which takes into account the major time constant and ignores the saturation or other nonlinearities. In the simplest form, the transfer function of a modern exciter may be represented by a single time constant τ_E and a gain K_E, i.e.,

$$\frac{V_F(s)}{V_R(s)} = \frac{K_E}{1 + \tau_E s} \tag{12.30}$$

The time constant of modern exciters are very small.

12.6.3 GENERATOR MODEL

The synchronous machine generated emf is a function of the machine magnetization curve, and its terminal voltage is dependent on the generator load. In the linearized model, the transfer function relating the generator terminal voltage to its field voltage can be represented by a gain K_G and a time constant τ_G, and the transfer function is

$$\frac{V_t(s)}{V_F(s)} = \frac{K_G}{1 + \tau_G s} \tag{12.31}$$

These constants are load dependent, K_G may vary between 0.7 to 1, and τ_G between 1.0 and 2.0 seconds from full-load to no-load.

12.6.4 SENSOR MODEL

The voltage is sensed through a potential transformer and, in one form, it is rectified through a bridge rectifier. The sensor is modeled by a simple first order transfer function, given by

$$\frac{V_S(s)}{V_t(s)} = \frac{K_R}{1 + \tau_R s} \tag{12.32}$$

τ_R is very small, and we may assume a range of 0.01 to 0.06 second. Utilizing the above models results in the AVR block diagram shown in Figure 12.30.

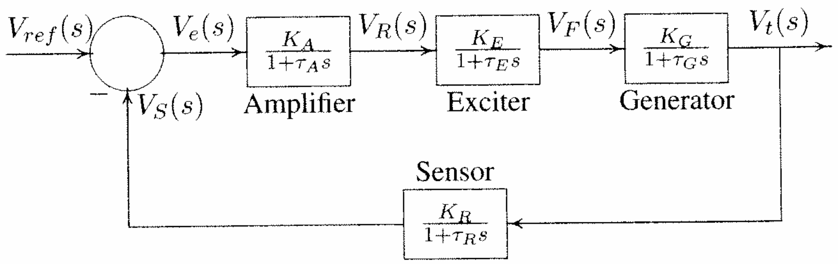

FIGURE 12.30
A simplified automatic voltage regulator block diagram.

The open-loop transfer function of the block diagram in Figure 12.30 is

$$KG(s)H(s) = \frac{K_A K_E K_G K_R}{(1 + \tau_A s)(1 + \tau_E s)(1 + \tau_G s)(1 + \tau_R s)} \tag{12.33}$$

and the closed-loop transfer function relating the generator terminal voltage $V_t(s)$ to the reference voltage $V_{ref}(s)$ is

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{K_A K_E K_G K_R (1 + \tau_R s)}{(1 + \tau_A s)(1 + \tau_E s)(1 + \tau_G s)(1 + \tau_R s) + K_A K_E K_G K_R} \tag{12.34}$$

or

$$V_t(s) = T(s)V_{ref}(s) \tag{12.35}$$

For a step input $V_{ref}(s) = \frac{1}{s}$, using the final value theorem, the steady-state response is

$$V_{t_{ss}} = \lim_{s \to 0} sV_t(s) = \frac{K_A}{1 + K_A} \tag{12.36}$$

Example 12.6 (chp12ex6), (sim12ex6.mdl)

The AVR system of a generator has the following parameters

	Gain	Time constant
Amplifier	K_A	$\tau_A = 0.1$
Exciter	$K_E = 1$	$\tau_E = 0.4$
Generator	$K_G = 1$	$\tau_G = 1.0$
Sensor	$K_R = 1$	$\tau_R = 0.05$

(a) Use the Routh-Hurwitz array (Appendix B.2.1) to find the range of K_A for control system stability.
(b) Use *MATLAB* **rlocus** function to obtain the root locus plot.
(c) The amplifier gain is set to $K_A = 10$
  (i) Find the steady-state step response.
  (ii) Use *MATLAB* to obtain the step response and the time-domain performance specifications.
(d) Construct the *SIMULINK* block diagram and obtain the step response.

Substituting the system parameters in the AVR block diagram of Figure 12.30 results in the block diagram shown in Figure 12.31.

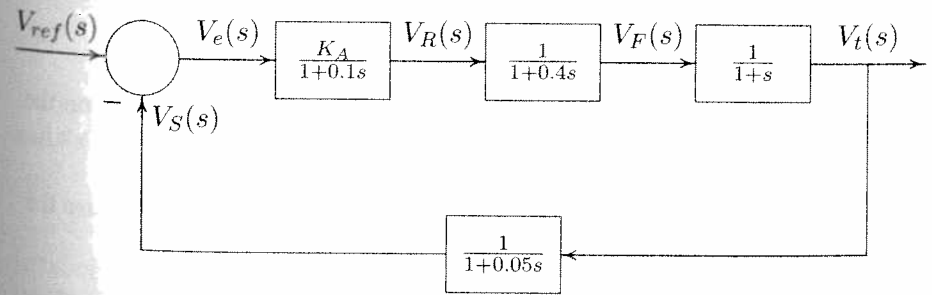

FIGURE 12.31
AVR block diagram for Example 12.6.

The open-loop transfer function of the AVR system shown in Figure 12.31 is

$$KG(s)H(s) = \frac{K_A}{(1+0.1s)(1+0.4s)(1+s)(1+0.05s)} = \frac{500K_A}{(s+10)(s+2.5)(s+1)(s+20)} = \frac{500K_A}{s^4+33.5s^3+307.5s^2+775s+500}$$

(a) The characteristic equation is given by

$$1 + KG(s)H(s) = 1 + \frac{500K_A}{s^4+33.5s^3+307.5s^2+775ss+500} = 0$$

which results in the characteristic polynomial equation

$$s^4 + 33.5s^3 + 307.5s^2 + 775s + 500 + 500K_A = 0$$

The Routh-Hurwitz array for this polynomial is then (see Appendix B.2.1)

s^4	1	307.5	$500+500K_A$
s^3	33.5	775	0
s^2	284.365	$500+500K_A$	0
s^1	$58.9K_A - 716.1$	0	0
s^0	$500+500K_A$		

From the s^1 row we see that, for control system stability, K_A must be less than 12.16, also from the s^0 row, K_A must be greater than -1. Thus, with positive values of K_A, for control system stability, the amplifier gain must be

$$K_A < 12.16$$

For $K = 12.16$, the auxiliary equation from the s^2 row is

$$284.365s^2 + 6580 = 0$$

or $s = \pm j4.81$. That is, for $K = 12.16$, we have a pair of conjugate poles on the $j\omega$ axis, and the control system is marginally stable.

(b) To obtain the root-locus plot for the range of K from 0 to 12.16, we use the following commands.

```
num=500;
den=[1  33.5  307.5  775  500];
figure(1), rlocus(num, den);
```


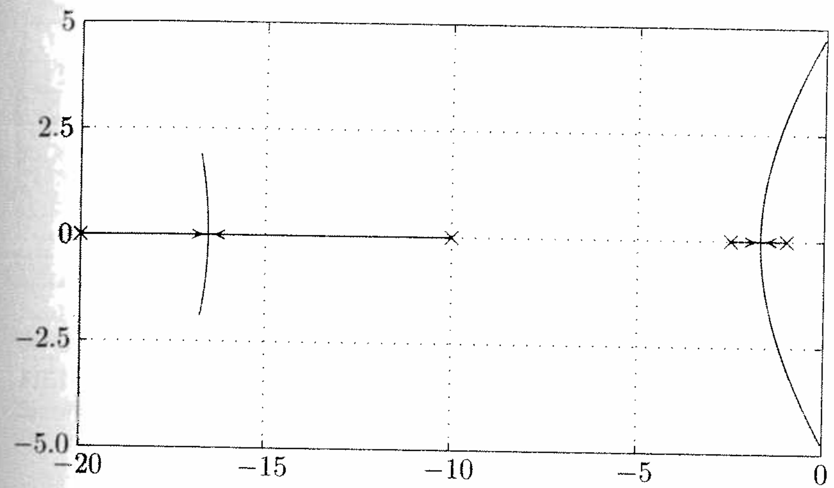

FIGURE 12.32
Root-locus plot for Example 12.6.

The result is shown in Figure 12.32. The loci intersect the $j\omega$ axis at $s = \pm j4.81$ for $K_A = 12.16$. Thus, the system is marginally stable for $K_A = 12.16$.
(c) The closed-loop transfer function of the system shown in Figure 12.31 is

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{25K_A(s+20)}{s^4+33.5s^3+307.5s^2+775s+500+500K_A}$$

(i) The steady-state response is

$$V_{t_{ss}} = \lim_{s\to 0} sV_t(s) = \frac{K_A}{1+K_A}$$

For the amplifier gain of $K_A = 10$, the steady-state response is

$$V_{t_{ss}} = \frac{10}{1+10} = 0.909$$

and the steady-state error is

$$V_{e_{ss}} = 1.0 - 0.909 = 0.091$$

In order to reduce the steady-state error, the amplifier gain must be increased, which results in an unstable control system.
(ii) To obtain the step response and the time-domain performance specifications, we use the following commands

```
KA= 10;
numc=KA*[25  500];
denc=[1  33.5  307.5  775  500+ 500*KA];
t=0:.05:20;
c=step(numc, denc, t);
figure(2), plot(t, c), grid
timespec(numc, denc)
```

The time-domain performance specifications are

```
Peak time = 0.791        Percent overshoot = 82.46
Rise time = 0.247
Settling time = 19.04
```

The terminal voltage step response is shown in Figure 12.33.

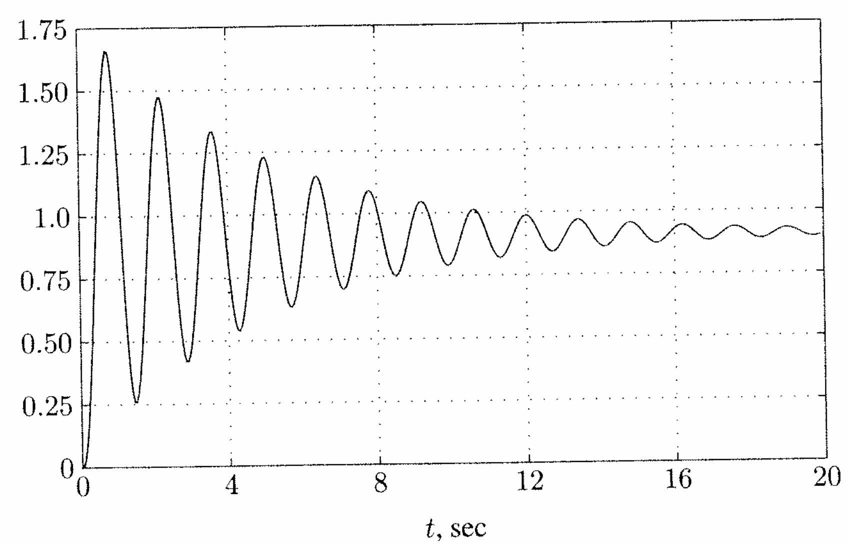

FIGURE 12.33
Terminal voltage step response for Example 12.6.

(d) A *SIMULINK* model named **sim12ex6.mdl** is constructed as shown in Figure 12.34. The file is opened and is run in the *SIMULINK* window. The simulation results in the same response as shown in Figure 12.33. From the results, we see that for an amplifier gain $K_A = 10$, the response is highly oscillatory, with a very large overshoot and a long settling time. Furthermore, the steady-state error is over 9 percent. We cannot have a small steady-state error and a satisfactory transient response at the same time.

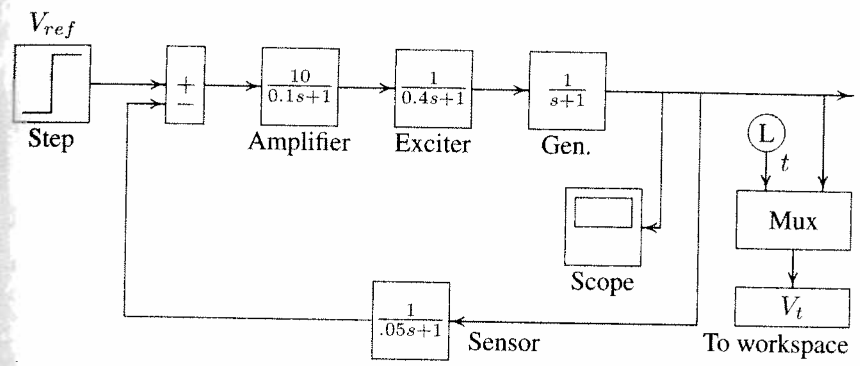

FIGURE 12.34
Simulation block diagram for Example 12.6.

12.6.5 EXCITATION SYSTEM STABILIZER — RATE FEEDBACK

As we have seen in Example 12.6, even for an small amplifier gain of $K_A = 10$, AVR step response is not satisfactory, and a value exceeding 12.16 results in an unbounded response. Thus, we must increase the relative stability by introducing a controller, which would add a zero to the AVR open-loop transfer function. One way to do this is to add a rate feedback to the control system as shown in Figure 12.35. By proper adjustment of K_F and τ_F, a satisfactory response can be obtained.

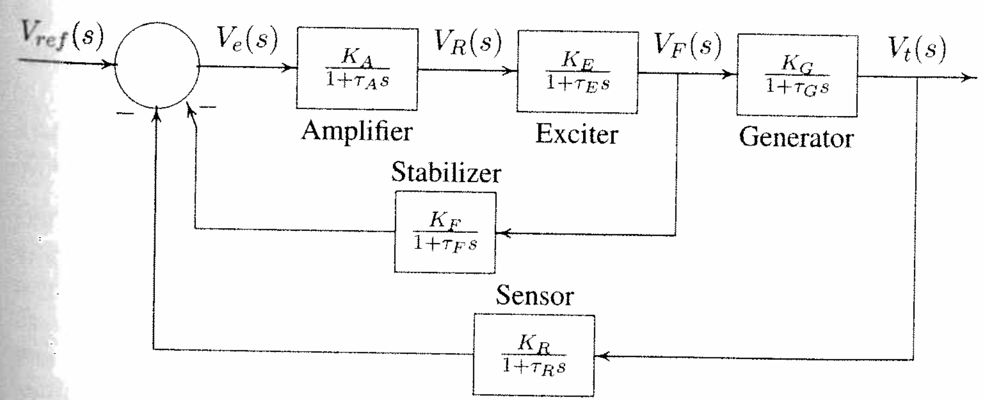

FIGURE 12.35
Block diagram of the compensated AVR system.

Example 12.7 (chp12ex7), (sim12ex7.mdl)

A rate feedback stabilizer is added to the AVR system of Example 12.6. The stabilizer time constant is $\tau_F = 0.04$ second, and the derivative gain is adjusted to $K_F = 2$.
(a) Obtain the step response and the time-domain performance specifications.
(b) Construct the *SIMULINK* model and obtain the step response.

(a) Substituting for the parameters in the block diagram of Figure 12.35 and applying the Mason's gain formula, we obtain the closed-loop transfer function

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{250(s^2 + 45s + 500)}{s^5 + 58.5s^4 + 13,645s^3 + 270,962.5s^2 + 274,875s + 137,500}$$

(i) The steady-state response is

$$V_{t_{ss}} = \lim_{s \to 0} sV_t(s) = \frac{(250)(500)}{137,500} = 0.909$$

To find the step response, we use the following commands

```
numc=250*[1 45 500];
denc=[1  58.5  13645  270962.5  274875 137500];
t=0:.05:10;
c=step(numc, denc, t); plot(t, c), grid
timespec(numc, denc)
```

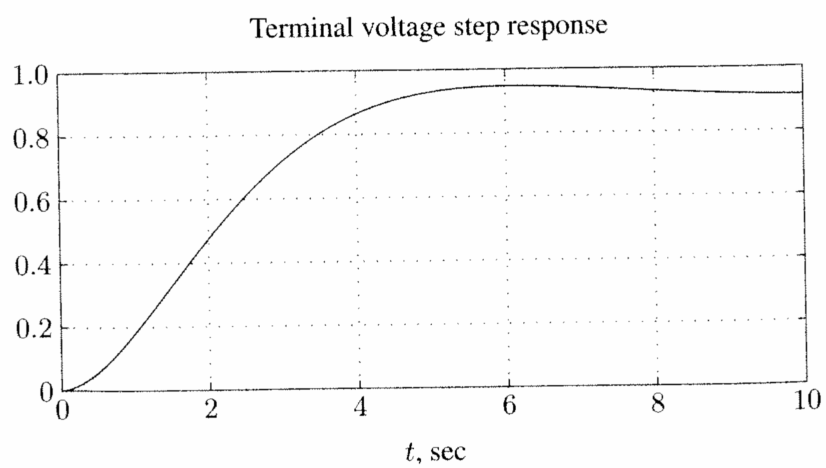

FIGURE 12.36
Terminal voltage step response for Example 12.7.

The step response is shown in Figure 12.36. The time-domain performance specifications are

```
Peak time = 6.08        Percent overshoot = 4.13
Rise time = 2.95
Settling time = 8.08
```

(b) A *SIMULINK* model named **sim12ex7.mdl** is constructed as shown in Figure 12.37.

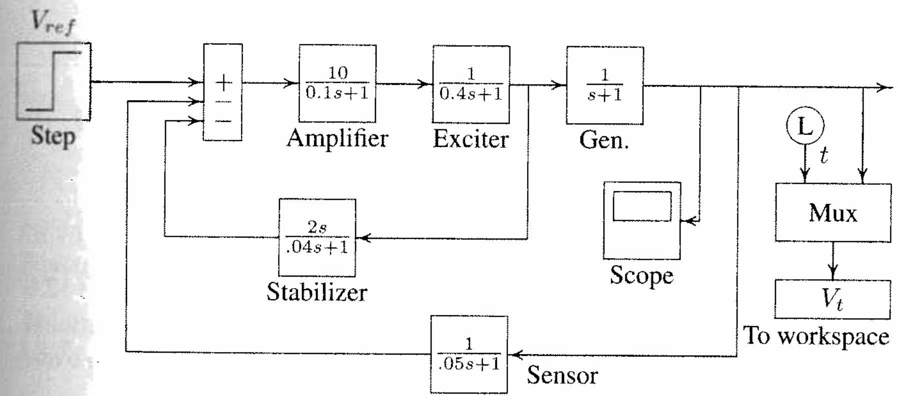

FIGURE 12.37
Simulation block diagram for Example 12.7.

The file is opened and is run in the *SIMULINK* window. The simulation results in the same response as shown in Figure 12.36. The results show a very satisfactory transient response with an overshoot of 4.13 percent and a settling time of approximately 8 seconds.

12.6.6 EXCITATION SYSTEM STABILIZER — PID CONTROLLER

One of the most common controllers available commercially is the *proportional integral derivative* (PID) controller. The PID controller is used to improve the dynamic response as well as to reduce or eliminate the steady-state error. The derivative controller adds a finite zero to the open-loop plant transfer function and improves the transient response. The integral controller adds a pole at origin and increases the system type by one and reduces the steady-state error due to a step function to zero. The PID controller transfer function is

$$G_C(s) = K_P + \frac{K_I}{s} + K_D s \tag{12.37}$$

The block diagram of an AVR compensated with a PID controller is shown in Figure 12.38.

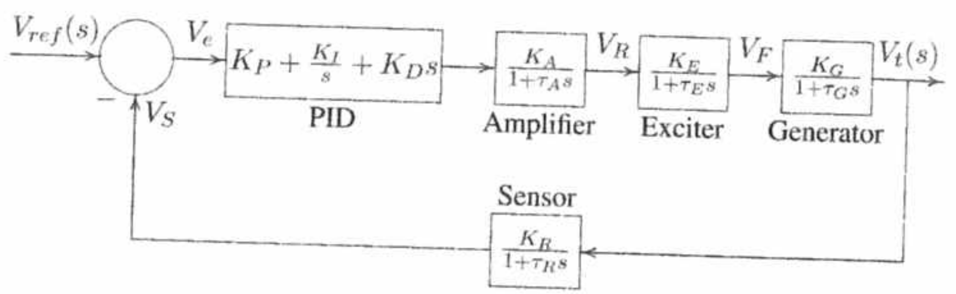

FIGURE 12.38
AVR system with PID controller.

Example 12.8 (sim12ex8.mdl)

A PID controller is added in the forward path of the AVR system of Example 12.6 as shown in Figure 12.38. Construct the *SIMULINK* model. Set the proportional gain K_P to 1.0 and adjust K_I and K_D until a step response with a minimum overshoot and a very small settling time is obtained.

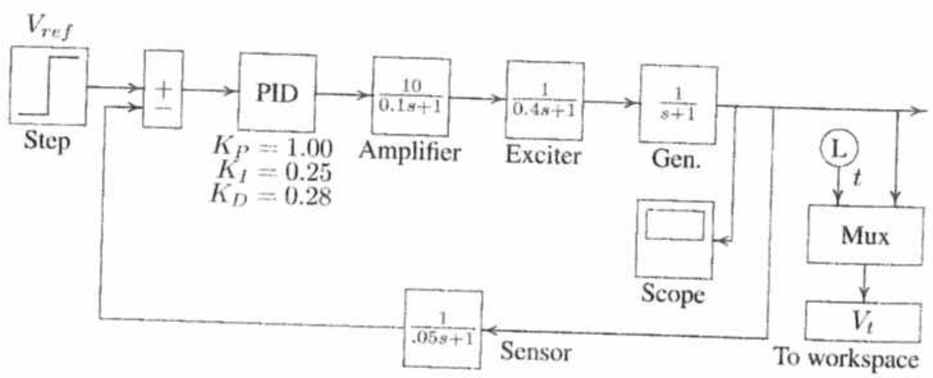

FIGURE 12.39
Simulation block diagram for Example 12.8.

A *SIMULINK* model named **sim12ex8.mdl** is constructed as shown in Figure 12.39. The file is opened and is run in the *SIMULINK* window. An integral gain of $K_I = 0.25$ and a derivative gain of $K_D = 0.28$ is found to be satisfactory. The response settles in about 1.4 seconds with a negligibly small overshoot. Note that the PID controller reduces the steady-state error to zero. The simulation result for the above settings is shown in Figure 12.40.

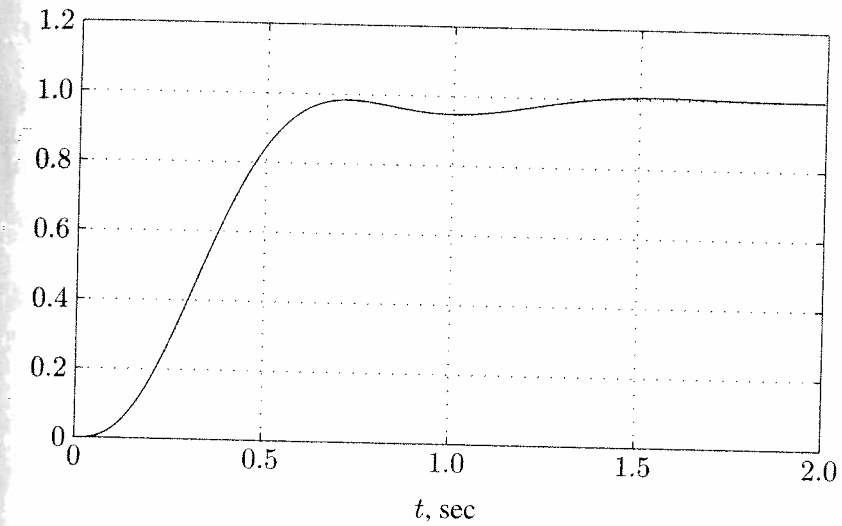

FIGURE 12.40
Terminal voltage step response for Example 12.8.

12.7 AGC INCLUDING EXCITATION SYSTEM

Since there is a weak coupling between the LFC and AVR systems, the frequency and voltage were controlled separately. We can study the coupling effect by extending the linearized AGC system to include the excitation system. In (12.17), we found that a small change in the real power is the product of the synchronizing power coefficient P_S and the change in the power angle $\Delta\delta$. If we include the small effect of voltage upon real power, we obtain the following linearized equation

$$\Delta P_e = P_s\,\Delta\delta + K_2 E' \tag{12.38}$$

where K_2 is the change in electrical power for a small change in the stator emf. Also, including the small effect of rotor angle upon the generator terminal voltage, we may write

$$\Delta V_t = K_5\,\Delta\delta + K_6 E' \tag{12.39}$$

where K_5 is the change in the terminal voltage for a small change in rotor angle at constant stator emf, and K_6 is the change in terminal voltage for a small change in the stator emf at constant rotor angle. Finally, modifying the generator field transfer function to include the effect of rotor angle, we may express the stator emf as

$$E' = \frac{K_G}{1+\tau_G}(V_f - K_4\,\Delta\delta) \tag{12.40}$$

The above constants depend upon the network parameters and the operating conditions. For the detailed derivation, see references 2 and 52. For a stable system, P_S is positive. Also, K_2, K_4, and K_6 are positive, but K_5 may be negative. Including (12.38)–(12.40) in the AGC system of Figure 12.16 and the AVR system of Figure 12.38, a linearized model for the combined LFC and AVR systems is obtained. A combined simulation block diagram is constructed in Example 12.9.

Example 12.9 (sim12ex9.mdl)

An isolated power station has the following parameters

	Gain	Time constant
Turbine	$K_T = 1$	$\tau_T = 0.5$
Governor	$K_g = 1$	$\tau_g = 0.2$
Amplifier	$K_A = 10$	$\tau_A = 0.1$
Exciter	$K_E = 1$	$\tau_E = 0.4$
Generator	$K_G = 0.8$	$\tau_G = 1.4$
Sensor	$K_R = 1$	$\tau_R = 0.05$
Inertia	$H = 5$	
Regulation	$R = 0.05$	

The load varies by 0.8 percent for a 1 percent change in frequency, i.e., $D = 0.8$. Assume the synchronizing coefficient P_S is 1.5, and the voltage coefficient K_6 is 0.5. Also, the coupling constants are $K_2 = 0.2$, $K_4 = 1.4$, and $K_5 = -0.1$. Construct the combined *SIMULINK* block diagram and obtain the frequency deviation and terminal voltage responses for a load change of $\Delta P_{L1} = 0.2$ per unit.

A *SIMULINK* model named **sim12ex9.mdl** is constructed as shown in Figure 12.41. The file is opened and is run in the *SIMULINK* window. The integrator gain in the secondary LFC loop is set to a value of 6.0. The excitation PID controller is tuned for $K_P = 1$, $K_I = 0.25$, and $K_D = 0.3$. The speed deviation step response and the terminal voltage step response are shown in Figures 12.42 and 12.43. It is observed that when the coupling coefficients are set to zero, there is little change in the transient response. Thus, separate treatment of frequency and voltage control loops is justified.

12.8 INTRODUCTORY MODERN CONTROL APPLICATION

The classical design techniques used so far are based on the root-locus that utilize only the plant output for feedback with a dynamic controller. In this section, we employ modern control designs that require the use of all state variables to form a linear static controller.

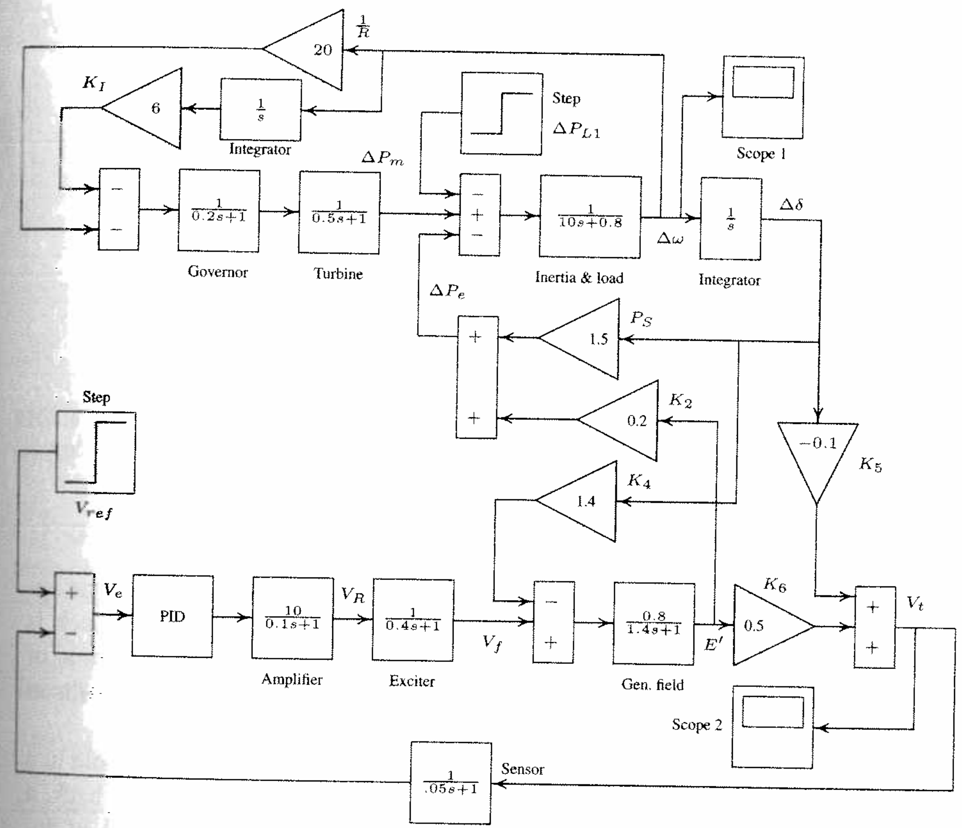

FIGURE 12.41
Simulation block diagram for Example 12.9.

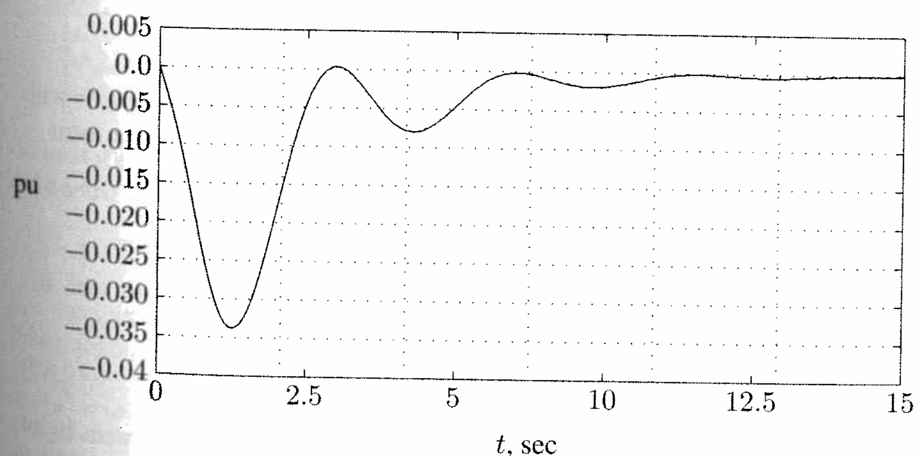

FIGURE 12.42
Frequency deviation step response for Example 12.8.

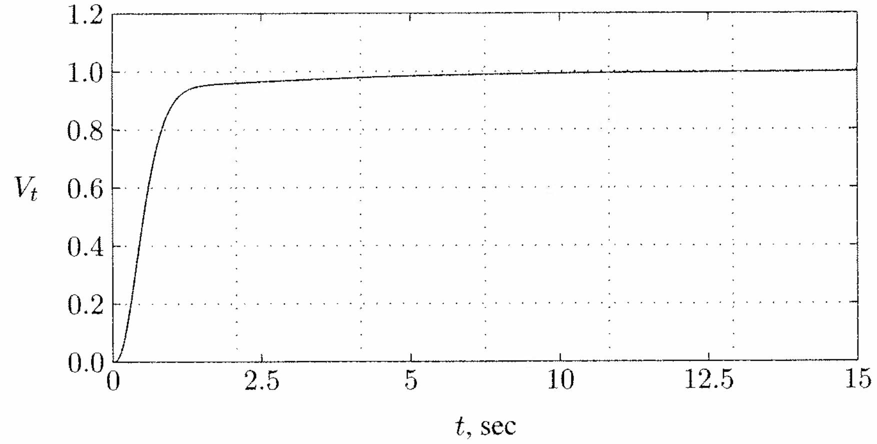

FIGURE 12.43
Terminal voltage step response for Example 12.8.

Modern control design is especially useful in multivariable systems. One approach in modern control systems accomplished by the use of state feedback is known as *pole-placement design*. The pole-placement design allows all roots of the system characteristic equation to be placed in desired locations. This results in a regulator with constant gain vector **K**.

The state-variable feedback concept requires that all states be accessible in a physical system. For systems in which all states are not available for feedback, a state estimator (observer) may be designed to implement the pole-placement design. The other approach to the design of regulator systems is the optimal control problem, where a specified mathematical performance criterion is minimized.

12.8.1 POLE-PLACEMENT DESIGN

The control is achieved by feeding back the state variables through a regulator with constant gains. Consider the control system presented in the state-variable form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \qquad (12.41)$$
$$y(t) = \mathbf{C}\mathbf{x}(\mathbf{t})$$

Now consider the block diagram of the system shown in Figure 12.44 with the following state feedback control

$$u(t) = -\mathbf{K}\mathbf{x}(t) \qquad (12.42)$$

where **K** is a $1 \times n$ vector of constant feedback gains. The control system input $r(t)$ is assumed to be zero. The purpose of this system is to return all state variables to values of zero when the states have been perturbed.

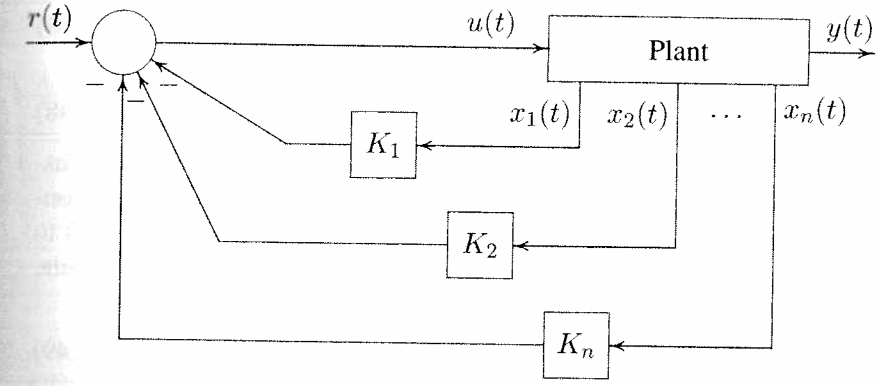

FIGURE 12.44
Control system design via pole placement.

Substituting (12.42) into (12.41), the compensated system state-variable representation becomes

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}(t) = \mathbf{A_f}\mathbf{x}(t) \qquad (12.43)$$

The compensated system characteristic equation is

$$|s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}| = 0 \qquad (12.44)$$

Assume the system is represented in the phase variable canonical form as follows.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(t) \qquad (12.45)$$

Substituting for **A** and **B** into (12.44), the compensated characteristic equation for the control system is found.

$$|s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}| = s^n + (a_{n-1} + k_n)s^{n-1} + \cdots + (a_1 + k_2)s + (a_0 + k_1) = 0 \qquad (12.46)$$

For the specified closed-loop pole locations $-\lambda_1, \ldots, -\lambda_n$, the desired characteristic equation is

$$\alpha_c(s) = (s + \lambda_1)\cdots(s + \lambda_n) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0 = 0 \qquad (12.47)$$

The design objective is to find the gain matrix **K** such that the characteristic equation for the controlled system is identical to the desired characteristic equation.

Thus, the gain vector $\mathbf{K}$ is obtained by equating coefficients of equations (12.46) and (12.47), and for the ith coefficient we get

$$k_i = \alpha_{i-1} - a_{i-1} \tag{12.48}$$

If the state model is not in the phase-variable canonical form, we can use the transformation technique to transform the given state model to the phase-variable canonical form. The gain factor is obtained for this model and then transformed back to conform with the original model. This procedure results in the following formula, known as *Ackermann's formula*.

$$\mathbf{K} = [\ 0 \quad 0 \quad \cdots \quad 0 \quad 1\ ]\,\mathbf{S}^{-1}\alpha_c(\mathbf{A}) \tag{12.49}$$

where the matrix $\mathbf{S}$ is given by

$$\mathbf{S} = [\ B \quad AB \quad A^2B \quad \ldots \quad A^{n-1}B\ ] \tag{12.50}$$

and the notation $\alpha_c(\mathbf{A})$ is given by

$$\alpha_c(\mathbf{A}) = \mathbf{A}^n + \alpha_{n-1}\mathbf{A}^{n-1} + \cdots + \alpha_1\mathbf{A} + \alpha_0\mathbf{I} \tag{12.51}$$

The function $[\mathbf{K}, \mathbf{A_f}] = \mathbf{placepol}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{p})$ is developed for the pole-placement design. $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are system matrices and $\mathbf{p}$ is a row vector containing the desired closed-loop poles. This function returns the gain vector $\mathbf{K}$ and the closed-loop system matrix $\mathbf{A_f}$. Also, the *MATLAB Control System Toolbox* contains two functions for pole-placement design. Function $\mathbf{K} = \mathbf{acker}(\mathbf{A}, \mathbf{B}, \mathbf{p})$ is for single input systems, and function $\mathbf{K} = \mathbf{place}(\mathbf{A}, \mathbf{B}, \mathbf{p})$, which uses a more reliable algorithm, is for multiinput systems. The condition that must exist to place the closed-loop poles at the desired location is to be able to transform the given state model into phase-variable canonical form.

We demonstrate the use of pole-placement design by applying it to the LFC of an isolated power system considered before, which is represented again in Figure 12.45. The s-domain equations describing the block diagram shown in Figure 12.45 are

$$\begin{aligned}(1 + \tau_g s)\Delta P_V(s) &= \Delta P_{ref} - \frac{1}{R}\,\Delta\Omega(s)\\ (1 + \tau_T s)\Delta P_m(s) &= \Delta P_V\\ (2Hs + D)\Delta\Omega(s) &= \Delta P_m - \Delta P_L\end{aligned} \tag{12.52}$$

Solving for the first derivative term, we have

$$\begin{aligned}s\Delta P_V(s) &= -\frac{1}{\tau_g}\,\Delta P_V - \frac{1}{R\tau_g}\,\Delta\Omega(s) + \frac{1}{\tau_g}\,\Delta P_{ref}(s)\\ s\Delta P_m(s) &= \frac{1}{\tau_T}\,\Delta P_V - \frac{1}{\tau_T}\,\Delta P_m\\ s\Delta\Omega(s) &= \frac{1}{2H}\,\Delta P_m - \frac{D}{2H}\,\Delta\Omega(s) - \frac{1}{2H}\,\Delta P_L\end{aligned} \tag{12.53}$$

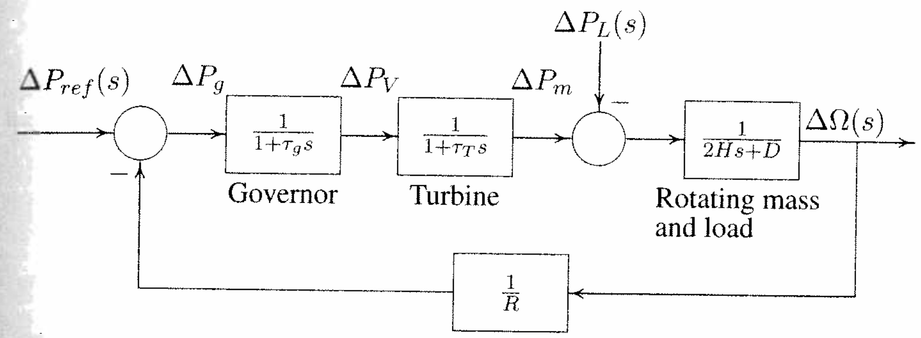

FIGURE 12.45
Load frequency control block diagram of an isolated power system.

Transforming into time-domain and expressing in matrix form the state equation becomes

$$\begin{bmatrix}\Delta\dot{P}_V\\ \Delta\dot{P}_m\\ \Delta\dot{\omega}\end{bmatrix} = \begin{bmatrix}\frac{-1}{\tau_g} & 0 & \frac{-1}{R\tau_g}\\ \frac{1}{\tau_T} & \frac{-1}{\tau_T} & 0\\ 0 & \frac{1}{2H} & \frac{-D}{2H}\end{bmatrix}\begin{bmatrix}\Delta P_V\\ \Delta P_m\\ \Delta\omega\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{-1}{2H}\end{bmatrix}\Delta P_L + \begin{bmatrix}\frac{1}{\tau_g}\\ 0\\ 0\end{bmatrix}\Delta P_{ref} \tag{12.54}$$

Example 12.10 (chp12xx.m), (sim12xxb.mdl), (sim12xxd.mdl)

Obtain the state variable representation of the LFC system of Example 12.1 with one input ΔP_L and perform the following analysis.
(a) Use the *MATLAB* **step** function to obtain the frequency deviation step response for a sudden load change of $\Delta P_L = 0.2$ per unit.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (a).
(c) Use **placepole(A, B, C, p)** function to place the compensated closed-loop pole at $-2 \pm j6$ and -3. Obtain the frequency deviation step response of the compensated system.
(d) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (c).

Substituting the parameters of the system in Example 12.1 in the state equation (12.51) with $\Delta P_{ref} = 0$, we have

$$\dot{\mathbf{x}} = \begin{bmatrix}-5 & 0 & -100\\ 2 & -2 & 0\\ 0 & 0.1 & -0.08\end{bmatrix}\mathbf{x} + \begin{bmatrix}0\\ 0\\ -0.1\end{bmatrix}u$$

and the output equation is

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{x}$$

where $y = \Delta\omega$ and

$$\mathbf{x} = \begin{bmatrix} \Delta P_V \\ \Delta P_m \\ \Delta\omega \end{bmatrix}$$

(a) We use the following commands

```
PL = 0.2;
A = [-5  0  -100; 2  -2  0; 0  0.1  -0.08];
B = [0;  0;  -0.1]; BPL = B*PL;
C = [0  0  1];  D = 0;
t=0:0.02:10;
[y, x] = step(A, BPL, C, D, 1, t);
figure(1), plot(t, y), grid
xlabel('t, sec'), ylabel('pu')
r =eig(A)
```

The frequency deviation step response result is shown in Figure 12.46, which is the same as the response obtained in Figure 12.13 using the transfer function method.

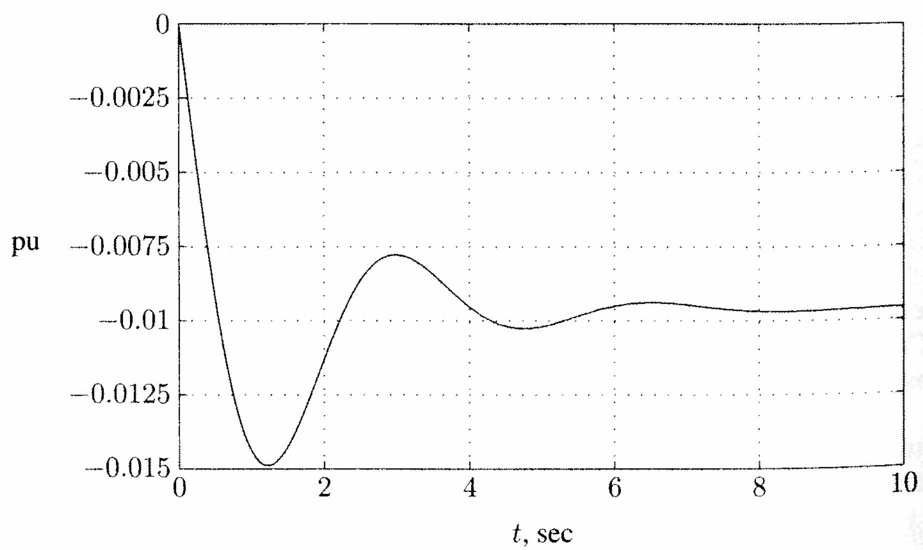

FIGURE 12.46
Uncompensated frequency deviation step response for Example 12.10.

The command **r = eig(A)** returns the roots of the characteristic equation, which are

```
r =
  -5.8863
  -0.5968 + 1.7825i
  -0.5968 - 1.7825i
```

(b) The *SIMULINK* state-space model can be used to obtain the response. A *SIMULINK* model named **sim12xxb.mdl** is constructed as shown in Figure 12.47. The state-space description dialog box is opened, and the **A**, **B**, **C**, and **D** constants are entered in the appropriate box in *MATLAB* matrix notation. The simulation

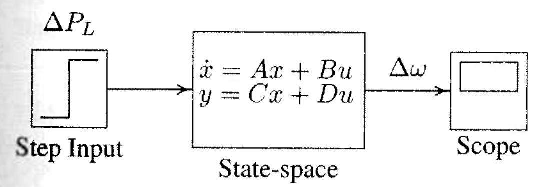

FIGURE 12.47
Simulation block diagram for Example 12.10 (b).

parameters are set to the appropriate values. The file is opened and is run in the *SIMULINK* window. The simulation results in the same response as shown in Figure 12.46.

(c) We are seeking the feedback gain vector **K** to place the roots of the system characteristic equation at $-2 \pm j6$ and -3. The following commands are added to the previous file.

```
P=[-2.0+j*6   -2.0-j*6   -3];
[K, Af] = placepol(A, B, C, P);
t=0:0.02:4;
[y, x] = step(Af, BPL, C, D, 1, t);
figure(2), plot(t, y), grid
xlabel('t,  sec'), ylabel('pu')
```

The result is

```
Feedback gain vector K
  4.2   0.8   0.8
Uncompensated Plant transfer function:
Numerator     0   -0.10   -0.70   -1.0
Denominator   1    7.08   10.56   20.8
```

```
Compensated system closed-loop transfer function:
Numerator       0    -0.1     -0.7     -1
Denominator     1     7.0     52.0    120

Compensated system matrix A - B*K
   -5.00     0.00   -100.00
    2.00    -2.00      0.00
    0.42     0.18      0.00
```

and the frequency deviation step response is shown in Figure 12.48.

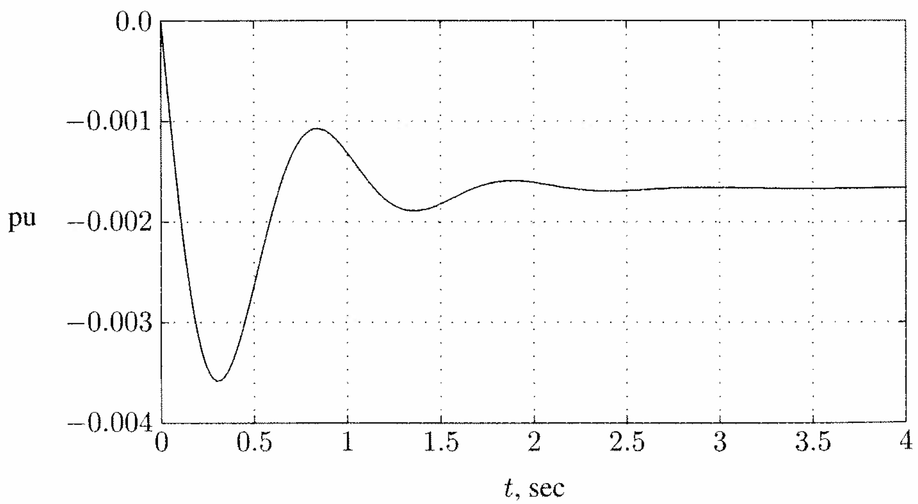

FIGURE 12.48
Compensated frequency deviation step response for Example 12.10.

Thus, the state feedback constants $K_1 = 4.2$, $K_2 = 0.8$, and $K_3 = 0.8$ result in the desired characteristic equation roots. The transient response is improved, and the response settles to a steady-state value of $\Delta_{ss} = -0.0017$ per unit in about 2.5 seconds.

(d) A *SIMULINK* model named **sim12xxd.mdl** is constructed as shown in Figure 12.49. In the state-space description dialog box, **C** is specified as an identity matrix of rank 3 to provide the three state variables as output. The file is opened and is run in the *SIMULINK* window. The simulation results in the same response as shown in Figure 12.48.

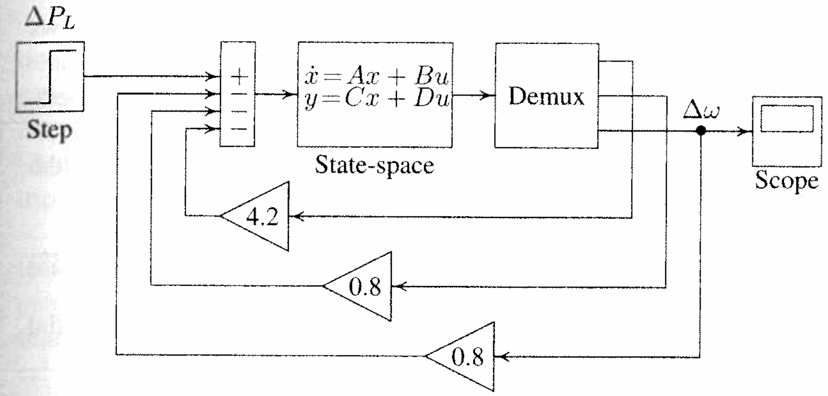

FIGURE 12.49
Simulation block diagram for Example 12.10 (d).

12.8.2 OPTIMAL CONTROL DESIGN

Optimal control is a branch of modern control theory that deals with designing controls for dynamic systems by minimizing a performance index that depends on the system variables. In this section, we will discuss the design of optimal controllers for linear systems with quadratic performance index, the so-called *linear quadratic regulator* (LQR) problem. The object of the optimal regulator design is to determine the optimal control law $\mathbf{u}^*(\mathbf{x}, t)$ which can transfer the system from its initial state to the final state such that a given performance index is minimized. The performance index is selected to give the best trade-off between performance and cost of control. The performance index that is widely used in optimal control design is known as the *quadratic performance index* and is based on minimum-error and minimum-energy criteria.

Consider the plant described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \tag{12.55}$$

The problem is to find the vector $\mathbf{K}(t)$ of the control law

$$\mathbf{u}(t) = -\mathbf{K}(t)\mathbf{x}(t) \tag{12.56}$$

which minimizes the value of a quadratic performance index $\mathbf{J}$ of the form

$$\mathbf{J} = \int_{t_0}^{t_f} (\mathbf{x}'\mathbf{Q}\mathbf{x} + \mathbf{u}'\mathbf{R}\mathbf{u})dt \tag{12.57}$$

subject to the dynamic plant equation in (12.55). In (12.57), $\mathbf{Q}$ is a positive semidefinite matrix, and $\mathbf{R}$ is a real symmetric matrix. $\mathbf{Q}$ is positive semidefinite, if all its principal minors are nonnegative. The choice of the elements of **Q** and **R** allows the relative weighting of individual state variables and individual control inputs.

To obtain a formal solution, we can use the method of Lagrange multipliers. The constraint problem is solved by augmenting (12.55) into (12.57) using an n-vector of *Lagrange multipliers*, λ. The problem reduces to the minimization of the following unconstrained function.

$$\mathcal{L}(\mathbf{x}, \lambda, u, t) = [\mathbf{x}'\mathbf{Q}\mathbf{x} + \mathbf{u}'\mathbf{R}\mathbf{u}] + \lambda'[\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} - \dot{\mathbf{x}}] \qquad (12.58)$$

The optimal values (denoted by the subscript $*$) are found by equating the partial derivatives to zero.

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \mathbf{A}\mathbf{X}^* + \mathbf{B}\mathbf{u}^* - \dot{\mathbf{x}}^* = \mathbf{0} \quad \Rightarrow \quad \dot{\mathbf{x}}^* = \mathbf{A}\mathbf{X}^* + \mathbf{B}\mathbf{u}^* \qquad (12.59)$$

$$\frac{\partial \mathcal{L}}{\partial u} = 2\mathbf{R}\mathbf{u}^* + \lambda'\mathbf{B} = \mathbf{0} \quad \Rightarrow \quad \mathbf{u}^* = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{R}^{-1}\lambda'\mathbf{B} \qquad (12.60)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 2\mathbf{x}'^*\mathbf{Q} + \dot{\lambda}' + \lambda'\mathbf{A} = \mathbf{0} \quad \Rightarrow \quad \dot{\lambda} = -\mathbf{2}\mathbf{Q}\mathbf{x}^* - \mathbf{A}'\lambda \qquad (12.61)$$

Assume that there exists a symmetric, time-varying positive definite matrix $\mathbf{p}(t)$ satisfying

$$\lambda = 2\mathbf{p}(t)\mathbf{x}^* \qquad (12.62)$$

Substituting (12.42) into (12.60) gives the optimal closed-loop control law

$$\mathbf{u}^*(t) = -\mathbf{R}^{-1}\mathbf{B}'\mathbf{p}(t)\mathbf{x}^* \qquad (12.63)$$

Obtaining the derivative of (12.62), we have

$$\dot{\lambda} = 2(\dot{\mathbf{p}}\mathbf{x}^* + \mathbf{p}\dot{\mathbf{x}}^*) \qquad (12.64)$$

Finally, equating (12.61) with (12.64), we obtain

$$\dot{\mathbf{p}}(t) = -\mathbf{p}(t)\mathbf{A} - \mathbf{A}'\mathbf{p}(t) - \mathbf{Q} + \mathbf{p}(t)\mathbf{B}\mathbf{R}^{-1}\mathbf{B}'\mathbf{p}(t) \qquad (12.65)$$

The above equation is referred to as the matrix *Riccati equation*. The boundary condition for (12.65) is $\mathbf{p}(t_f) = \mathbf{0}$. Therefore, (12.65) must be integrated backward in time. Since a numerical solution is performed forward in time, a dummy time variable $\tau = t_f - t$ is replaced for time t. Once the solution to (12.65) is obtained, the solution of the state equation (12.59) in conjunction with the optimum control equation (12.63) is obtained.

The function $[\tau, \mathbf{p}, \mathbf{K}, \mathbf{t}, \mathbf{x}]$= **riccati** is developed for the time-domain solution of the Riccati equation. The function returns the solution of the matrix Riccati equation, $\mathbf{p}(\tau)$, the optimal feedback gain vector $\mathbf{k}(\tau)$, and the initial state response $\mathbf{x}(t)$. In order to use this function, the user must declare the function $[\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R}, \mathbf{t_0}, \mathbf{t_f}, \mathbf{x_0}] = \mathbf{system}(\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R}, \mathbf{t_0}, \mathbf{t_f}, \mathbf{x_0})$ containing system matrices and the performance index matrices in an M-file named **system.m**.

The optimal controller gain is a time-varying state-variable feedback. Such feedback are inconvenient to implement, because they require the storage in computer memory of time-varying gains. An alternative control scheme is to replace the time-varying optimal gain $\mathbf{K}(t)$ by its constant steady-state value. In most practical applications, the use of the steady-state feedback gain is adequate. For linear time-invariant systems, since $\dot{\mathbf{p}} = \mathbf{0}$, when the process is of infinite duration, that is $t_f = \infty$, (12.65) reduces to the algebraic Riccati equation

$$\mathbf{p}\mathbf{A} + \mathbf{A}'\mathbf{p} + \mathbf{Q} - \mathbf{p}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}'\mathbf{p} = \mathbf{0} \qquad (12.66)$$

The *MATLAB Control System Toolbox* function **[k, p]=lqr2(A, B, Q, R)** can be used for the solution of the algebraic Riccati equation.

The LQR design procedure is in stark contrast to classical control design, where the gain matrix **K** is selected directly. To design the optimal LQR, the design engineer first selects the design parameter weight matrices **Q** and **R**. Then, the feedback gain **K** is automatically given by matrix design equations and the closed-loop time responses are found by simulation. If these responses are unsuitable, new values of **Q** and **R** are selected and the design is repeated. This has the significant advantages of allowing all the control loops in a multiloop system to be closed simultaneously, while guaranteeing closed-loop stability.

Example 12.11 (chp12xx1.m), (sim12xx1.mdl)

Design an LQR state feedback for the system described in Example 12.10.
(a) Find the optimal feedback gain vector to minimize the performance index

$$\mathbf{J} = \int_0^{\infty} \left(20{x_1}^2 + 10{x_2}^2 + 5{x_3}^2 + 0.15u^2\right) dt$$

The admissible states and control values are unconstrained. Obtain the frequency deviation step response for a sudden load change of $\Delta P_L = 0.2$ per unit.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (a).

For this system we have

$$\mathbf{A} = \begin{bmatrix} -5 & 0 & -100 \\ 2 & -2 & 0 \\ 0 & 0.1 & -0.08 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ -0.1 \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

and $R = 0.15$.

(a) We use the following commands

```
PL=0.2;
A = [-5  0  -100; 2  -2  0; 0  0.1  -0.08];
B = [0;  0;  -0.1]; BPL=PL*B;
C = [0  0  1];  D = 0;
Q = [20 0  0; 0  10 0; 0  0  5];  R = .15;
[K, P] = lqr2(A, B, Q, R)
Af = A - B*K
t=0:0.02:1;
[y, x] = step(Af, BPL, C, D, 1, t);
plot(t, y), grid, xlabel('t,  sec'), ylabel('pu')
```

The result is

```
K =
     6.4128     1.1004 -112.6003
P =
     1.5388     0.3891    -9.6192
     0.3891     2.3721    -1.6506
    -9.6192    -1.6506   168.9004
Af =
    -5.0000          0 -100.0000
     2.0000    -2.0000          0
     0.6413     0.2100   -11.3400
```

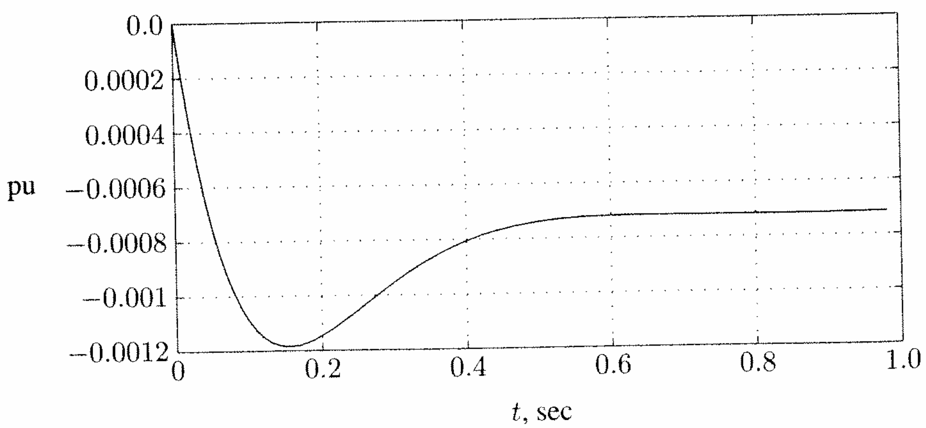

FIGURE 12.50
Frequency deviation step response for Example 12.11.

The frequency deviation step response is shown in Figure 12.50. The transient response settles to a steady-state value of $\Delta_{ss} = -0.0007$ per unit in about 0.6 second.

(b) A *SIMULINK* model named **sim12xx1.mdl** is constructed as shown in Figure 12.51. The state-space description dialog box is opened, and the **A**, **B**, **C**, and **D** constants are entered in the appropriate box in *MATLAB* matrix notation. Also, the LQR description dialog box is opened, and weighting matrix **Q** and weighting coefficient **R** are set to the given values. The simulation parameters are set to the appropriate values. The file is opened and is run in the *SIMULINK* window. The simulation results in the same response as shown in Figure 12.50.

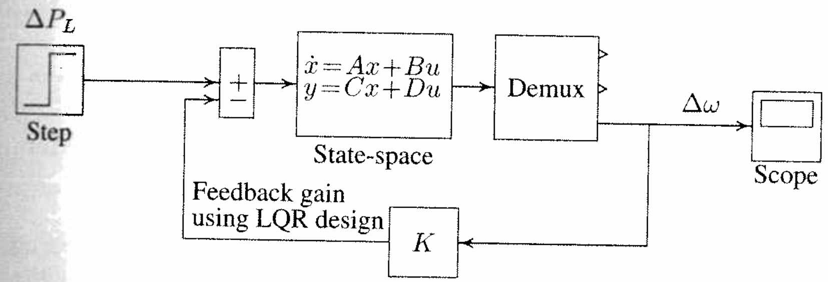

FIGURE 12.51
Simulation block diagram for Example 12.11.

PROBLEMS

12.1. A 250-MW, 60-Hz turbine generator set has a speed regulation of 5 percent based on its own rating. The generator frequency decreases from 60 Hz to a steady state value of 59.7Hz. Determine the increase in the turbine power output.

12.2. Two generating units rated for 250 MW and 400 MW have governor speed regulation of 6.0 and 6.4 percent, respectively, from no-load to full-load, respectively. They are operating in parallel and share a load of 500 MW. Assuming free governor action, determine the load shared by each unit.

12.3. A single area consists of two generating units, rated at 400 and 800 MVA, with speed regulation of 4 percent and 5 percent on their respective ratings. The units are operating in parallel, sharing 700 MW. Unit 1 supplies 200 MW and unit 2 supplies 500 MW at 1.0 per unit (60 Hz) frequency. The load is increased by 130 MW.

(a) Assume there is no frequency-dependent load, i.e., $D = 0$. Find the steady-state frequency deviation and the new generation on each unit.

(b) The load varies 0.804 percent for every 1 percent change in frequency, i.e., $D = 0.804$. Find the steady-state frequency deviation and the new generation on each unit.

12.4. An isolated power station has the LFC system as shown in Figure 12.9 with the following parameters

Turbine time constant $\tau_T = 0.5$ sec
Governor time constant $\tau_g = 0.25$ sec
Generator inertia constant $H = 8$ sec
Governor speed regulation = R per unit

The load varies by 1.6 percent for a 1 percent change in frequency, i.e., $D = 1.6$.
(a) Use the Routh-Hurwitz array (Appendix B.2.1) to find the range of R for control system stability.
(b) Use *MATLAB* **rlocus** function to obtain the root-locus plot.

12.5. The governor speed regulation of Problem 12.4 is set to $R = 0.04$ per unit. The turbine rated output is 200 MW at nominal frequency of 60 Hz. A sudden load change of 50 MW ($\Delta P_L = 0.25$ per unit) occurs.
(a) Find the steady-state frequency deviation in Hz.
(b) Obtain the closed-loop transfer function and use *MATLAB* to obtain the frequency deviation step response.
(c) Construct the *SIMULINK* block diagram and obtain the frequency deviation response.

12.6. The LFC system in Problem 12.5 is equipped with the secondary integral control loop for automatic generation control as shown in Figure 12.16.
(a) Use the *MATLAB* **step** function to obtain the frequency deviation step response for a sudden load change of $\Delta P_L = 0.25$ per unit. Set the integral controller gain to $K_I = 9$.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (a).

12.7. The load changes of 200 MW and 150 MW occur simultaneously in areas 1 and 2 of the two-area system of Example 12.4. Modify the *SIMULINK* block diagram (sim12ex4.mdl), and obtain the frequency deviation and the power responses.

12.8. Modify the *SIMULINK* model for the two-area system of Example 12.5 with the tie-line bias control (sim12ex5.mdl) to include the load changes specified in Problem 12.7. Obtain the frequency and power response deviation for each area.

12.9. A generating unit has a simplified linearized AVR system as shown in Figure 12.52.
(a) Use the Routh-Hurwitz array (Appendix B.2.1) to find the range of K_A for control system stability.
(b) Use *MATLAB* **rlocus** function to obtain the root-locus plot.
(c) The amplifier gain is set to $K_A = 40$. Find the system closed-loop transfer function, and use *MATLAB* to obtain the step response.
(d) Construct the *SIMULINK* block diagram and obtain the step response.

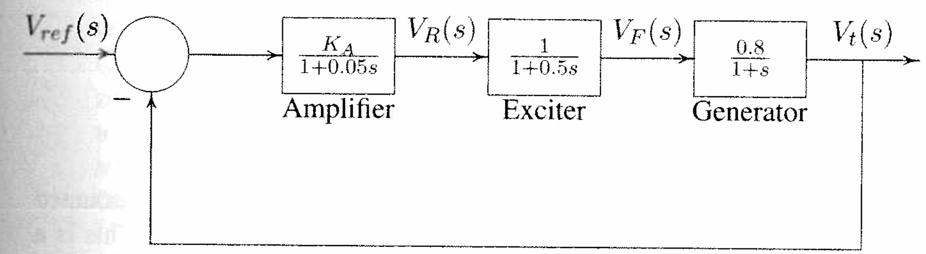

FIGURE 12.52
AVR system of Problem 12.9.

12.10. A rate feedback stabilizer is added to the AVR system of Problem 12.9 as shown in Figure 12.53. The stabilizer time constant is $\tau_F = 0.04$ second, and the derivative gain is adjusted to $K_F = 0.1$.
(a) Find the system closed-loop transfer function, and use *MATLAB* to obtain the step response.
(b) Construct the *SIMULINK* model, and obtain the step response.

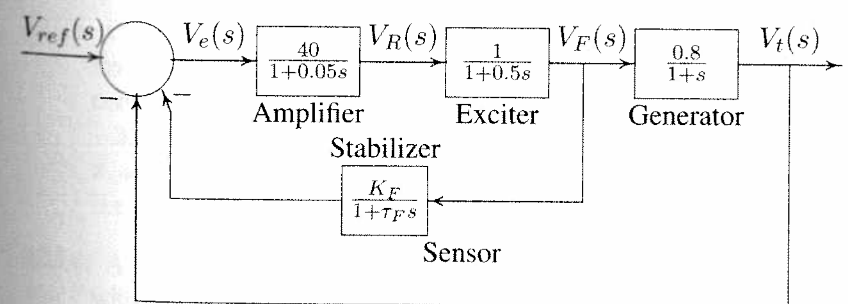

FIGURE 12.53
AVR system with rate feedback for Problem 12.10.

12.11. A PID controller is added in the forward path of the AVR system of Problem 12.9 as shown in Figure 12.54. Construct the *SIMULINK* model. Set the proportional gain K_P to 2.0, and adjust K_I and K_D until a step response with a minimum overshoot and a very small settling time is obtained (suggested values $K_P = 1$, $K_I = 0.15$, and $K_D = 0.17$).

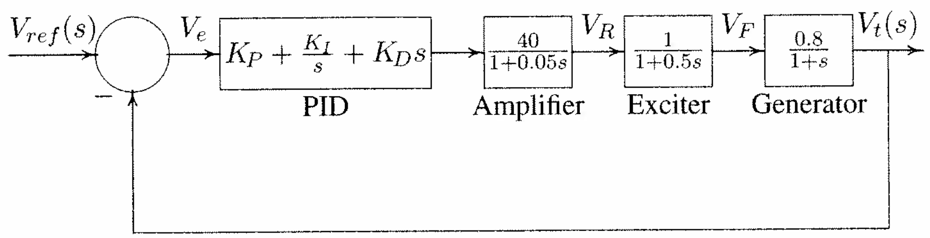

FIGURE 12.54
AVR system with PID controller for Problem 12.11.

12.12. Figure 12.55 shows an inverted pendulum of length L and mass m mounted on a cart of mass M, by means of a force u applied to the cart. This is a model of the attitude control of a space booster on takeoff. The differential equations describing the motion of the system is obtained by summing the forces on the pendulum, which result in the following nonlinear equations.

$$(M + m)\ddot{x} + mL\cos\theta\,\ddot{\theta} = mL\sin\theta\,\dot{\theta}^2 + u$$
$$mL\cos\theta\,\ddot{x} + mL^2\ddot{\theta} = mgL\sin\theta$$

(a) Linearize the above equations in the neighborhood of the zero initial

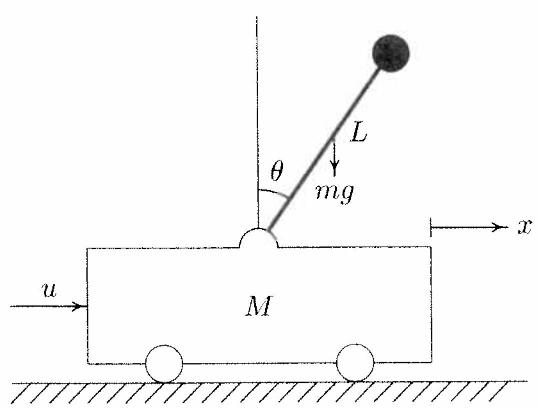

FIGURE 12.55
Inverted pendulum on a cart.

states. Hint: Substitute θ for $\sin\theta$, 1 for $\cos\theta$ and 0 for $\dot{\theta}^2$. With the state variables defined as $x_1 = \theta$, $x_2 = \dot{\theta}$, $x_3 = x$, and $x_4 = \dot{x}$, show that the linearized state equation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M+m}{ML}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m}{M}g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{ML} \\ 0 \\ \frac{1}{M} \end{bmatrix} u$$

Assume $M = 4$ kg, $m = 0.2$ kg, $L = 0.5$ m, and $g = 9.81$ m/s^2.
(b) Use the *MATLAB* function **eig(A)** to find the roots of the system characteristic equation.
(c) Define C as the identity matrix of rank 4, i.e., C = eye(4) and D = zeros(4, 1). Use the *MATLAB* function [y, x] = initial($\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{X_0}, \mathbf{t}$) to simulate the system for 20 seconds in response to an initial condition offset of $\theta(0) = 0.1$ rad, and $x(0) = 0.1$ m (i.e., $x_0 = [0.1 \quad 0 \quad 0.1 \quad 0]$). Obtain a plot of θ and x, and comment on the stability of the system.
(d) You may have found that the inverted pendulum is unstable, that is, it will fall over unless a suitable control force via state feedback is used. The purpose is to design a control system such that for a small initial disturbance the pendulum can be brought back to the vertical position ($\theta = 0$), and the cart can be brought back to the reference position ($x = 0$). A simple method is to use a state feedback gain to place the compensated closed-loop poles in the left-half of the s-plane. Use the custom made function **[K, Af] = placepol(A, B, C, P)** and design a state feedback controller to place the compensated closed-loop poles at $-2 \pm j0.5$, -4, and -5. Simulate the system for 20 seconds in response to the same initial condition offset. Obtain a plot of θ, x, and $u = -Kx'$.

12.13. Construct the *SIMULINK* block diagram for the linearized model of the inverted pendulum described in Problem 12.12 (a) with the state feedback controller. Assume the state feedback gains are $K_1 = -170.367$, $K_2 = -38.054$, $K_3 = -17.3293$, and $K_4 = -24.1081$. Obtain the response for θ, x, and u for the initial condition offset of $\theta(0) = 0.1$ rad and $x(0) = 0.1$m (i.e., $x_0 = [0.1 \quad 0 \quad 0.1 \quad 0]$).

12.14. A classical problem in control systems is to find the optimal control law which will transfer the system from its initial state to the final state, such that a given performance index is minimized.
(a) Design an LQR state feedback for the linearized model of the inverted pendulum described in Problem 12.12, and find the optimal feedback gain vector to minimize the performance index

$$\mathbf{J} = \int_0^\infty \left(10{x_1}^2 + 10{x_2}^2 + 5{x_3}^2 + +5{x_4}^2 + 0.2u^2\right) dt$$

The admissible states and control values are unconstrained.
(b) Define C as the identity matrix of rank 4, i.e., $C = \text{eye}(4)$ and $D = \text{zeros}(4,1)$. Use the *MATLAB* function **[K, P] = lqr2 (A, B, Q, R)** to design a state feedback controller in response to an initial condition offset of $\theta(0) = 0.1$ rad and $x(0) = 0.1$m (i.e., $x_0 = [0.1 \quad 0 \quad 0.1 \quad 0]$). Use the *MATLAB* function $[\mathbf{y}, \mathbf{x}] = \text{initial}(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{X_0}, \mathbf{t})$ to simulate the system for 20 seconds. Obtain a plot of θ, x, and the control law $u = -kx'$.

12.15. Construct the *SIMULINK* block diagram for the linearized model of the inverted pendulum described in Problem 12.12(a) using the *SIMULINK* LQR model. Obtain the response for θ, x, and u for the initial condition offset described in Problem 12.14.

12.16. Obtain the state variable representation of the LFC system of Problem 12.4 with one input ΔP_L, and perform the following analysis.
(a) Use the *MATLAB* **step** function to obtain the frequency deviation step response for a sudden load change of $\Delta P_L = 0.2$ per unit.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (a).
(c) Use **placepol(A, B, C, p)** function to place the compensated closed-loop pole at $-4 \pm j6$ and -4. Obtain the frequency deviation step response of the compensated system.
(d) Construct the *SIMULINK* block diagram and obtain the frequency deviation response for the condition in part (c).

12.17. Design a LQR state feedback for the system described in Problem 12.16.
(a) Find the optimal feedback gain vector to minimize the performance index

$$\mathbf{J} = \int_0^\infty \left(40{x_1}^2 + 20{x_2}^2 + 10{x_3}^2 + 0.2u^2\right) dt$$

The admissible states and control values are unconstrained. Obtain the frequency deviation step response for a sudden load change of $\Delta P_L = 0.2$ per unit.
(b) Construct the *SIMULINK* block diagram and obtain the frequency deviation step response for the condition in part (a).

APPENDIX A

INTRODUCTION TO *MATLAB*

MATLAB, developed by Math Works Inc., is a software package for high performance numerical computation and visualization. The combination of analysis capabilities, flexibility, reliability, and powerful graphics makes *MATLAB* the premier software package for electrical engineers.

MATLAB provides an interactive environment with hundreds of reliable and accurate built-in mathematical functions. These functions provide solutions to a broad range of mathematical problems including matrix algebra, complex arithmetic, linear systems, differential equations, signal processing, optimization, nonlinear systems, and many other types of scientific computations. The most important feature of *MATLAB* is its programming capability, which is very easy to learn and to use, and which allows user-developed functions. It also allows access to Fortran algorithms and C codes by means of external interfaces. There are several optional toolboxes written for special applications such as signal processing, control systems design, system identification, statistics, neural networks, fuzzy logic, symbolic computations, and others. *MATLAB* has been enhanced by the very powerful *SIMULINK* program. *SIMULINK* is a graphical mouse-driven program for the simulation of dynamic systems. *SIMULINK* enables students to simulate linear, as well as nonlinear, systems easily and efficiently.