

Impedance

The impedance Z of a resistance R in series with a reactance X is:

$$Z = R + jX$$

Rectangular and polar forms of impedance Z :

$$Z = R + jX = (R^2 + X^2)^{1/2} \angle \tan^{-1}(X/R) = |Z| \angle \phi = |Z| \cos \phi + j|Z| \sin \phi$$

Addition of impedances Z_1 and Z_2 :

$$Z_1 + Z_2 = (R_1 + jX_1) + (R_2 + jX_2) = (R_1 + R_2) + j(X_1 + X_2)$$

Subtraction of impedances Z_1 and Z_2 :

$$Z_1 - Z_2 = (R_1 + jX_1) - (R_2 + jX_2) = (R_1 - R_2) + j(X_1 - X_2)$$

Multiplication of impedances Z_1 and Z_2 :

$$Z_1 * Z_2 = |Z_1| \angle \phi_1 * |Z_2| \angle \phi_2 = (|Z_1| * |Z_2|) \angle (\phi_1 + \phi_2)$$

Division of impedances Z_1 and Z_2 :

$$Z_1 / Z_2 = |Z_1| \angle \phi_1 / |Z_2| \angle \phi_2 = (|Z_1| / |Z_2|) \angle (\phi_1 - \phi_2)$$

In summary:

- use the rectangular form for addition and subtraction,
- use the polar form for multiplication and division.

Admittance

An impedance Z comprising a resistance R in series with a reactance X can be converted to an admittance Y comprising a conductance G in parallel with a susceptance B :

$$Y = Z^{-1} = 1 / (R + jX) = (R - jX) / (R^2 + X^2) = R / (R^2 + X^2) - jX / (R^2 + X^2) = G - jB$$

$$G = R / (R^2 + X^2) = R / |Z|^2$$

$$B = X / (R^2 + X^2) = X / |Z|^2$$

Using the polar form of impedance Z :

$$Y = 1 / |Z| \angle \phi = |Z|^{-1} \angle -\phi = |Y| \angle -\phi = |Y| \cos \phi - j|Y| \sin \phi$$

Conversely, an admittance Y comprising a conductance G in parallel with a susceptance B can be converted to an impedance Z comprising a resistance R in series with a reactance X :

$$Z = Y^{-1} = 1 / (G - jB) = (G + jB) / (G^2 + B^2) = G / (G^2 + B^2) + jB / (G^2 + B^2) = R + jX$$

$$R = G / (G^2 + B^2) = G / |Y|^2$$

$$X = B / (G^2 + B^2) = B / |Y|^2$$

Using the polar form of admittance Y :

$$Z = 1 / |Y| \angle -\phi = |Y|^{-1} \angle \phi = |Z| \angle \phi = |Z| \cos \phi + j|Z| \sin \phi$$

The total impedance Z_S of impedances Z_1, Z_2, Z_3, \dots connected in series is:

$$Z_S = Z_1 + Z_2 + Z_3 + \dots$$

The total admittance Y_P of admittances Y_1, Y_2, Y_3, \dots connected in parallel is:

$$Y_P = Y_1 + Y_2 + Y_3 + \dots$$

In summary:

- use impedances when operating on series circuits,
- use admittances when operating on parallel circuits.

Reactance

Inductive Reactance

The inductive reactance X_L of an inductance L at angular frequency ω and frequency f is:

$$X_L = \omega L = 2\pi fL$$

For a sinusoidal current i of amplitude I and angular frequency ω :

$$i = I \sin \omega t$$

If sinusoidal current i is passed through an inductance L , the voltage e across the inductance is:

$$e = L di/dt = \omega LI \cos \omega t = X_L I \cos \omega t$$

The current through an inductance lags the voltage across it by 90° .

Capacitive Reactance

The capacitive reactance X_C of a capacitance C at angular frequency ω and frequency f is:

$$X_C = 1 / \omega C = 1 / 2\pi fC$$

For a sinusoidal voltage v of amplitude V and angular frequency ω :

$$v = V \sin \omega t$$

If sinusoidal voltage v is applied across a capacitance C , the current i through the capacitance is:

$$i = C dv/dt = \omega CV \cos \omega t = V \cos \omega t / X_C$$

The current through a capacitance leads the voltage across it by 90° .

Resonance

Series Resonance

A series circuit comprising an inductance L , a resistance R and a capacitance C has an impedance Z_S of:

$$Z_S = R + j(X_L - X_C)$$

where $X_L = \omega L$ and $X_C = 1 / \omega C$

At resonance, the imaginary part of Z_S is zero:

$$X_C = X_L$$

$$Z_{Sr} = R$$

$$\omega_r = (1 / LC)^{1/2} = 2\pi f_r$$

Parallel resonance

A parallel circuit comprising an inductance L with a series resistance R , connected in parallel with a capacitance C , has an admittance Y_P of:

$$Y_P = 1 / (R + jX_L) + 1 / (-jX_C) = (R / (R^2 + X_L^2)) - j(X_L / (R^2 + X_L^2)) - 1 / X_C$$

where $X_L = \omega L$ and $X_C = 1 / \omega C$

At resonance, the imaginary part of Y_P is zero:

$$X_C = (R^2 + X_L^2) / X_L = X_L + R^2 / X_L = X_L(1 + R^2 / X_L^2)$$

$$Z_{Pr} = Y_{Pr}^{-1} = (R^2 + X_L^2) / R = X_L X_C / R = L / CR$$

$$\omega_r = (1 / LC - R^2 / L^2)^{1/2} = 2\pi f_r$$

Note that for the same values of L , R and C , the parallel resonance frequency is lower than the series resonance frequency, but if the ratio R / L is small then the parallel resonance frequency is close to the series resonance frequency.

Reactive Loads and Power Factor

Resistance and Series Reactance

The impedance Z of a reactive load comprising resistance R and series reactance X is:

$$Z = R + jX = |Z|\angle\phi$$

Converting to the equivalent admittance Y :

$$Y = 1 / Z = 1 / (R + jX) = (R - jX) / (R^2 + X^2) = R / |Z|^2 - jX / |Z|^2$$

When a voltage V (taken as reference) is applied across the reactive load Z , the current I is:

$$I = VY = V(R / |Z|^2 - jX / |Z|^2) = VR / |Z|^2 - jVX / |Z|^2 = I_P - jI_Q$$

The active current I_P and the reactive current I_Q are:

$$I_P = VR / |Z|^2 = |I|\cos\phi$$

$$I_Q = VX / |Z|^2 = |I|\sin\phi$$

The apparent power S , active power P and reactive power Q are:

$$S = V|I| = V^2 / |Z| = |I|^2|Z|$$

$$P = VI_P = I_P^2|Z|^2 / R = V^2R / |Z|^2 = |I|^2R$$

$$Q = VI_Q = I_Q^2|Z|^2 / X = V^2X / |Z|^2 = |I|^2X$$

The power factor $\cos\phi$ and reactive factor $\sin\phi$ are:

$$\cos\phi = I_P / |I| = P / S = R / |Z|$$

$$\sin\phi = I_Q / |I| = Q / S = X / |Z|$$

Resistance and Shunt Reactance

The impedance Z of a reactive load comprising resistance R and shunt reactance X is found from:

$$1 / Z = 1 / R + 1 / jX$$

Converting to the equivalent admittance Y comprising conductance G and shunt susceptance B :

$$Y = 1 / Z = 1 / R - j / X = G - jB = |Y|\angle-\phi$$

When a voltage V (taken as reference) is applied across the reactive load Y , the current I is:

$$I = VY = V(G - jB) = VG - jVB = I_P - jI_Q$$

The active current I_P and the reactive current I_Q are:

$$I_P = VG = V / R = |I|\cos\phi$$

$$I_Q = VB = V / X = |I|\sin\phi$$

The apparent power S , active power P and reactive power Q are:

$$S = V|I| = |I|^2 / |Y| = V^2|Y|$$

$$P = VI_P = I_P^2 / G = |I|^2G / |Y|^2 = V^2G$$

$$Q = VI_Q = I_Q^2 / B = |I|^2B / |Y|^2 = V^2B$$

The power factor $\cos\phi$ and reactive factor $\sin\phi$ are:

$$\cos\phi = I_P / |I| = P / S = G / |Y|$$

$$\sin\phi = I_Q / |I| = Q / S = B / |Y|$$

Complex Power

When a voltage \mathbf{V} causes a current \mathbf{I} to flow through a reactive load \mathbf{Z} , the complex power \mathbf{S} is:
 $\mathbf{S} = \mathbf{V}\mathbf{I}^*$ where \mathbf{I}^* is the conjugate of the complex current \mathbf{I} .

Inductive Load

$$\mathbf{Z} = \mathbf{R} + j\mathbf{X}_L$$

$$\mathbf{I} = \mathbf{I}_P - j\mathbf{I}_Q$$

$$\cos\phi = \mathbf{R} / |\mathbf{Z}| \text{ (lagging)}$$

$$\mathbf{I}^* = \mathbf{I}_P + j\mathbf{I}_Q$$

$$\mathbf{S} = \mathbf{P} + j\mathbf{Q}$$

An inductive load is a sink of lagging VARs (a source of leading VARs).

Capacitive Load

$$\mathbf{Z} = \mathbf{R} - j\mathbf{X}_C$$

$$\mathbf{I} = \mathbf{I}_P + j\mathbf{I}_Q$$

$$\cos\phi = \mathbf{R} / |\mathbf{Z}| \text{ (leading)}$$

$$\mathbf{I}^* = \mathbf{I}_P - j\mathbf{I}_Q$$

$$\mathbf{S} = \mathbf{P} - j\mathbf{Q}$$

A capacitive load is a source of lagging VARs (a sink of leading VARs).

Three Phase Power

For a balanced **star** connected load with line voltage \mathbf{V}_{line} and line current \mathbf{I}_{line} :

$$\mathbf{V}_{\text{star}} = \mathbf{V}_{\text{line}} / \sqrt{3}$$

$$\mathbf{I}_{\text{star}} = \mathbf{I}_{\text{line}}$$

$$\mathbf{Z}_{\text{star}} = \mathbf{V}_{\text{star}} / \mathbf{I}_{\text{star}} = \mathbf{V}_{\text{line}} / \sqrt{3}\mathbf{I}_{\text{line}}$$

$$\mathbf{S}_{\text{star}} = 3\mathbf{V}_{\text{star}}\mathbf{I}_{\text{star}} = \sqrt{3}\mathbf{V}_{\text{line}}\mathbf{I}_{\text{line}} = \mathbf{V}_{\text{line}}^2 / \mathbf{Z}_{\text{star}} = 3\mathbf{I}_{\text{line}}^2\mathbf{Z}_{\text{star}}$$

For a balanced **delta** connected load with line voltage \mathbf{V}_{line} and line current \mathbf{I}_{line} :

$$\mathbf{V}_{\text{delta}} = \mathbf{V}_{\text{line}}$$

$$\mathbf{I}_{\text{delta}} = \mathbf{I}_{\text{line}} / \sqrt{3}$$

$$\mathbf{Z}_{\text{delta}} = \mathbf{V}_{\text{delta}} / \mathbf{I}_{\text{delta}} = \sqrt{3}\mathbf{V}_{\text{line}} / \mathbf{I}_{\text{line}}$$

$$\mathbf{S}_{\text{delta}} = 3\mathbf{V}_{\text{delta}}\mathbf{I}_{\text{delta}} = \sqrt{3}\mathbf{V}_{\text{line}}\mathbf{I}_{\text{line}} = 3\mathbf{V}_{\text{line}}^2 / \mathbf{Z}_{\text{delta}} = \mathbf{I}_{\text{line}}^2\mathbf{Z}_{\text{delta}}$$

The apparent power \mathbf{S} , active power \mathbf{P} and reactive power \mathbf{Q} are related by:

$$\mathbf{S}^2 = \mathbf{P}^2 + \mathbf{Q}^2$$

$$\mathbf{P} = \mathbf{S}\cos\phi$$

$$\mathbf{Q} = \mathbf{S}\sin\phi$$

where $\cos\phi$ is the power factor and $\sin\phi$ is the reactive factor

Note that for equivalence between balanced star and delta connected loads:

$$\mathbf{Z}_{\text{delta}} = 3\mathbf{Z}_{\text{star}}$$

Per-unit System

For each system parameter, per-unit value is equal to the actual value divided by a base value:

$$\mathbf{E}_{\text{pu}} = \mathbf{E} / \mathbf{E}_{\text{base}}$$

$$\mathbf{I}_{\text{pu}} = \mathbf{I} / \mathbf{I}_{\text{base}}$$

$$\mathbf{Z}_{\text{pu}} = \mathbf{Z} / \mathbf{Z}_{\text{base}}$$

Select rated values as base values, usually rated power in MVA and rated phase voltage in kV:

$$\mathbf{S}_{\text{base}} = \mathbf{S}_{\text{rated}} = \sqrt{3}\mathbf{E}_{\text{line}}\mathbf{I}_{\text{line}}$$

$$\mathbf{E}_{\text{base}} = \mathbf{E}_{\text{phase}} = \mathbf{E}_{\text{line}} / \sqrt{3}$$

The base values for line current in kA and per-phase star impedance in Ohms/phase are:

$$\mathbf{I}_{\text{base}} = \mathbf{S}_{\text{base}} / 3\mathbf{E}_{\text{base}} (= \mathbf{S}_{\text{base}} / \sqrt{3}\mathbf{E}_{\text{line}})$$

$$\mathbf{Z}_{\text{base}} = \mathbf{E}_{\text{base}} / \mathbf{I}_{\text{base}} = 3\mathbf{E}_{\text{base}}^2 / \mathbf{S}_{\text{base}} (= \mathbf{E}_{\text{line}}^2 / \mathbf{S}_{\text{base}})$$

Note that selecting the base values for any two of \mathbf{S}_{base} , \mathbf{E}_{base} , \mathbf{I}_{base} or \mathbf{Z}_{base} fixes the base values of all four. Note also that Ohm's Law is satisfied by each of the sets of actual, base and per-unit values for voltage, current and impedance.

Transformers

The primary and secondary MVA ratings of a transformer are equal, but the voltages and currents in the primary (subscript 1) and the secondary (subscript 2) are usually different:

$$\sqrt{3}E_{1\text{line}}I_{1\text{line}} = S = \sqrt{3}E_{2\text{line}}I_{2\text{line}}$$

Converting to base (per-phase star) values:

$$3E_{1\text{base}}I_{1\text{base}} = S_{\text{base}} = 3E_{2\text{base}}I_{2\text{base}}$$

$$E_{1\text{base}} / E_{2\text{base}} = I_{2\text{base}} / I_{1\text{base}}$$

$$Z_{1\text{base}} / Z_{2\text{base}} = (E_{1\text{base}} / E_{2\text{base}})^2$$

The impedance $Z_{21\text{pu}}$ referred to the primary side, equivalent to an impedance $Z_{2\text{pu}}$ on the secondary side, is:

$$Z_{21\text{pu}} = Z_{2\text{pu}}(E_{1\text{base}} / E_{2\text{base}})^2$$

The impedance $Z_{12\text{pu}}$ referred to the secondary side, equivalent to an impedance $Z_{1\text{pu}}$ on the primary side, is:

$$Z_{12\text{pu}} = Z_{1\text{pu}}(E_{2\text{base}} / E_{1\text{base}})^2$$

Note that per-unit and percentage values are related by:

$$Z_{\text{pu}} = Z_{\%} / 100$$

Symmetrical Components

In any three phase system, the line currents I_a , I_b and I_c may be expressed as the phasor sum of:

- a set of balanced positive phase sequence currents I_{a1} , I_{b1} and I_{c1} (phase sequence a-b-c),
- a set of balanced negative phase sequence currents I_{a2} , I_{b2} and I_{c2} (phase sequence a-c-b),
- a set of identical zero phase sequence currents I_{a0} , I_{b0} and I_{c0} (cophasal, no phase sequence).

The positive, negative and zero sequence currents are calculated from the line currents using:

$$I_{a1} = (I_a + hI_b + h^2I_c) / 3$$

$$I_{a2} = (I_a + h^2I_b + hI_c) / 3$$

$$I_{a0} = (I_a + I_b + I_c) / 3$$

The positive, negative and zero sequence currents are combined to give the line currents using:

$$I_a = I_{a1} + I_{a2} + I_{a0}$$

$$I_b = I_{b1} + I_{b2} + I_{b0} = h^2I_{a1} + hI_{a2} + I_{a0}$$

$$I_c = I_{c1} + I_{c2} + I_{c0} = hI_{a1} + h^2I_{a2} + I_{a0}$$

The residual current I_r is equal to the total zero sequence current:

$$I_r = I_{a0} + I_{b0} + I_{c0} = 3I_{a0} = I_a + I_b + I_c = I_e$$

which is measured using three current transformers with parallel connected secondaries.

I_e is the earth fault current of the system.

Similarly, for phase-to-earth voltages V_{ae} , V_{be} and V_{ce} , the residual voltage V_r is equal to the total zero sequence voltage:

$$V_r = V_{a0} + V_{b0} + V_{c0} = 3V_{a0} = V_{ae} + V_{be} + V_{ce} = 3V_{ne}$$

which is measured using an earthed-star / open-delta connected voltage transformer.

V_{ne} is the neutral displacement voltage of the system.

The h-operator

The h-operator ($1\angle 120^\circ$) is the complex cube root of unity:

$$h = -1 / 2 + j\sqrt{3} / 2 = 1\angle 120^\circ = 1\angle -240^\circ$$

$$h^2 = -1 / 2 - j\sqrt{3} / 2 = 1\angle 240^\circ = 1\angle -120^\circ$$

Some useful properties of h are:

$$1 + h + h^2 = 0$$

$$h + h^2 = -1 = 1\angle 180^\circ$$

$$h - h^2 = j\sqrt{3} = \sqrt{3}\angle 90^\circ$$

$$h^2 - h = -j\sqrt{3} = \sqrt{3}\angle -90^\circ$$

Fault Calculations

The different types of short-circuit fault which occur on a power system are:

- single phase to earth,
- double phase,
- double phase to earth,
- three phase,
- three phase to earth.

For each type of short-circuit fault occurring on an unloaded system:

- the first column states the phase voltage and line current conditions at the fault,
- the second column states the phase 'a' sequence current and voltage conditions at the fault,
- the third column provides formulae for the phase 'a' sequence currents at the fault,
- the fourth column provides formulae for the fault current and the resulting line currents.

By convention, the faulted phases are selected for fault symmetry with respect to reference phase 'a'.

I_f = fault current

I_e = earth fault current

E_a = normal phase voltage at the fault location

Z_1 = positive phase sequence network impedance to the fault

Z_2 = negative phase sequence network impedance to the fault

Z_0 = zero phase sequence network impedance to the fault

Single phase to earth - fault from phase 'a' to earth:

$V_a = 0$ $I_b = I_c = 0$ $I_f = I_a = I_e$	$I_{a1} = I_{a2} = I_{a0} = I_a / 3$ $V_{a1} + V_{a2} + V_{a0} = 0$	$I_{a1} = E_a / (Z_1 + Z_2 + Z_0)$ $I_{a2} = I_{a1}$ $I_{a0} = I_{a1}$	$I_f = 3I_{a0} = 3E_a / (Z_1 + Z_2 + Z_0) = I_e$ $I_a = I_f = 3E_a / (Z_1 + Z_2 + Z_0)$
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Double phase - fault from phase 'b' to phase 'c':

$V_b = V_c$ $I_a = 0$ $I_f = I_b = -I_c$	$I_{a1} + I_{a2} = 0$ $I_{a0} = 0$ $V_{a1} = V_{a2}$	$I_{a1} = E_a / (Z_1 + Z_2)$ $I_{a2} = -I_{a1}$ $I_{a0} = 0$	$I_f = -j\sqrt{3}I_{a1} = -j\sqrt{3}E_a / (Z_1 + Z_2)$ $I_b = I_f = -j\sqrt{3}E_a / (Z_1 + Z_2)$ $I_c = -I_f = j\sqrt{3}E_a / (Z_1 + Z_2)$
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Double phase to earth - fault from phase 'b' to phase 'c' to earth:

$V_b = V_c = 0$ $I_a = 0$ $I_f = I_b + I_c = I_e$	$I_{a1} + I_{a2} + I_{a0} = 0$ $V_{a1} = V_{a2} = V_{a0}$	$I_{a1} = E_a / Z_{net}$ $I_{a2} = -I_{a1}Z_0 / (Z_2 + Z_0)$ $I_{a0} = -I_{a1}Z_2 / (Z_2 + Z_0)$	$I_f = 3I_{a0} = -3E_aZ_2 / \Sigma_{zz} = I_e$ $I_b = I_f / 2 - j\sqrt{3}E_a(Z_2 / 2 + Z_0) / \Sigma_{zz}$ $I_c = I_f / 2 + j\sqrt{3}E_a(Z_2 / 2 + Z_0) / \Sigma_{zz}$
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$Z_{net} = Z_1 + Z_2Z_0 / (Z_2 + Z_0)$ and $\Sigma_{zz} = Z_1Z_2 + Z_2Z_0 + Z_0Z_1 = (Z_2 + Z_0)Z_{net}$

Three phase (and three phase to earth) - fault from phase 'a' to phase 'b' to phase 'c' (to earth):

$V_a = V_b = V_c (= 0)$ $I_a + I_b + I_c = 0 (= I_e)$ $I_f = I_a = hI_b = h^2I_c$	$V_{a0} = V_a (= 0)$ $V_{a1} = V_{a2} = 0$	$I_{a1} = E_a / Z_1$ $I_{a2} = 0$ $I_{a0} = 0$	$I_f = I_{a1} = E_a / Z_1 = I_a$ $I_b = E_b / Z_1$ $I_c = E_c / Z_1$
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Note that the single phase fault current is greater than the three phase fault current if Z_0 is less than $(2Z_1 - Z_2)$.

The values of Z_1 , Z_2 and Z_0 are each determined from the respective positive, negative and zero sequence impedance networks by network reduction to a single impedance.

Note that if the system is earthed through an impedance Z_n (carrying current $3I_0$) then an impedance $3Z_n$ (carrying current I_0) must be included in the zero sequence impedance network.

Three Phase Fault Level

The symmetrical three phase short-circuit current I_{sc} of a power system with no-load line and phase voltages E_{line} and E_{phase} and source impedance Z_S per-phase star is:

$$I_{sc} = E_{phase} / |Z_S| = E_{line} / \sqrt{3}|Z_S|$$

The three phase fault level S_{sc} of the power system is:

$$S_{sc} = 3I_{sc}^2|Z_S| = 3E_{phase}I_{sc} = 3E_{phase}^2 / |Z_S| = E_{line}^2 / |Z_S|$$

Note that if the X / R ratio of the source impedance Z_S (comprising resistance R_S and reactance X_S) is sufficiently large, $|Z_S| \approx X_S$.

Power Factor Correction

If an inductive load with an active power demand P has an uncorrected power factor of $\cos\phi_1$ lagging, and is required to have a corrected power factor of $\cos\phi_2$ lagging, the uncorrected and corrected reactive power demands, Q_1 and Q_2 , are:

$$Q_1 = P \tan\phi_1$$

$$Q_2 = P \tan\phi_2$$

$$\text{where } \tan\phi_n = (1 / \cos^2\phi_n - 1)^{1/2}$$

The leading (capacitive) reactive power demand Q_C which must be connected across the load is:

$$Q_C = Q_1 - Q_2 = P (\tan\phi_1 - \tan\phi_2)$$

The uncorrected and corrected apparent power demands, S_1 and S_2 , are related by:

$$S_1 \cos\phi_1 = P = S_2 \cos\phi_2$$

Comparing corrected and uncorrected load currents and apparent power demands:

$$I_2 / I_1 = S_2 / S_1 = \cos\phi_1 / \cos\phi_2$$

If the load is required to have a corrected power factor of unity, Q_2 is zero and:

$$Q_C = Q_1 = P \tan\phi_1$$

$$I_2 / I_1 = S_2 / S_1 = \cos\phi_1 = P / S_1$$

Shunt Capacitors

For star-connected shunt capacitors each of capacitance C_{star} on a three phase system of line voltage V_{line} and frequency f , the leading reactive power demand $Q_{C\text{star}}$ and the leading reactive line current I_{line} are:

$$Q_{C\text{star}} = V_{\text{line}}^2 / X_{C\text{star}} = 2\pi f C_{\text{star}} V_{\text{line}}^2$$

$$I_{\text{line}} = Q_{C\text{star}} / \sqrt{3} V_{\text{line}} = V_{\text{line}} / \sqrt{3} X_{C\text{star}}$$

$$C_{\text{star}} = Q_{C\text{star}} / 2\pi f V_{\text{line}}^2$$

For delta-connected shunt capacitors each of capacitance C_{delta} on a three phase system of line voltage V_{line} and frequency f , the leading reactive power demand $Q_{C\text{delta}}$ and the leading reactive line current I_{line} are:

$$Q_{C\text{delta}} = 3V_{\text{line}}^2 / X_{C\text{delta}} = 6\pi f C_{\text{delta}} V_{\text{line}}^2$$

$$I_{\text{line}} = Q_{C\text{delta}} / \sqrt{3} V_{\text{line}} = \sqrt{3} V_{\text{line}} / X_{C\text{delta}}$$

$$C_{\text{delta}} = Q_{C\text{delta}} / 6\pi f V_{\text{line}}^2$$

Note that for the same leading reactive power Q_C :

$$X_{C\text{delta}} = 3X_{C\text{star}}$$

$$C_{\text{delta}} = C_{\text{star}} / 3$$

Reactors

Shunt Reactors

For star-connected shunt reactors each of inductance L_{star} on a three phase system of line voltage V_{line} and frequency f , the lagging reactive power demand $Q_{L\text{star}}$ and the lagging reactive line current I_{line} are:

$$Q_{L\text{star}} = V_{\text{line}}^2 / X_{L\text{star}} = V_{\text{line}}^2 / 2\pi f L_{\text{star}}$$

$$I_{\text{line}} = Q_{L\text{star}} / \sqrt{3} V_{\text{line}} = V_{\text{line}} / \sqrt{3} X_{L\text{star}}$$

$$L_{\text{star}} = V_{\text{line}}^2 / 2\pi f Q_{L\text{star}}$$

For delta-connected shunt reactors each of inductance L_{delta} on a three phase system of line voltage V_{line} and frequency f , the lagging reactive power demand $Q_{L\text{delta}}$ and the lagging reactive line current I_{line} are:

$$Q_{L\text{delta}} = 3V_{\text{line}}^2 / X_{L\text{delta}} = 3V_{\text{line}}^2 / 2\pi f L_{\text{delta}}$$

$$I_{\text{line}} = Q_{L\text{delta}} / \sqrt{3} V_{\text{line}} = \sqrt{3} V_{\text{line}} / X_{L\text{delta}}$$

$$L_{\text{delta}} = 3V_{\text{line}}^2 / 2\pi f Q_{L\text{delta}}$$

Note that for the same lagging reactive power Q_L :

$$X_{L\text{delta}} = 3X_{L\text{star}}$$

$$L_{\text{delta}} = 3L_{\text{star}}$$

Series Reactors

For series line reactors each of inductance L_{series} carrying line current I_{line} on a three phase system of frequency f , the voltage drop V_{drop} across each line reactor and the total lagging reactive power demand $Q_{L\text{series}}$ of the set of three line reactors are:

$$V_{\text{drop}} = I_{\text{line}} X_{L\text{series}} = 2\pi f L_{\text{series}} I_{\text{line}}$$

$$Q_{L\text{series}} = 3V_{\text{drop}}^2 / X_{L\text{series}} = 3V_{\text{drop}} I_{\text{line}} = 3I_{\text{line}}^2 X_{L\text{series}} = 6\pi f L_{\text{series}} I_{\text{line}}^2$$

$$L_{\text{series}} = Q_{L\text{series}} / 6\pi f I_{\text{line}}^2$$

Note that the apparent power rating S_{rating} of the set of three line reactors is based on the line voltage V_{line} and not the voltage drop V_{drop} :

$$S_{\text{rating}} = \sqrt{3} V_{\text{line}} I_{\text{line}}$$

Harmonic Resonance

If a node in a power system operating at frequency f has an inductive source reactance X_L per phase and has power factor correction with a capacitive reactance X_C per phase, the source inductance L and the correction capacitance C are:

$$L = X_L / \omega$$

$$C = 1 / \omega X_C$$

where $\omega = 2\pi f$

The series resonance angular frequency ω_r of an inductance L with a capacitance C is:

$$\omega_r = (1 / LC)^{1/2} = \omega(X_C / X_L)^{1/2}$$

The three phase fault level S_{sc} at the node for no-load phase voltage E and source impedance Z per-phase star is:

$$S_{sc} = 3E^2 / |Z| = 3E^2 / |R + jX_L|$$

If the ratio X_L / R of the source impedance Z is sufficiently large, $|Z| \approx X_L$ so that:

$$S_{sc} \approx 3E^2 / X_L$$

The reactive power rating Q_C of the power factor correction capacitors for a capacitive reactance X_C per phase at phase voltage E is:

$$Q_C = 3E^2 / X_C$$

The harmonic number f_r / f of the series resonance of X_L with X_C is:

$$f_r / f = \omega_r / \omega = (X_C / X_L)^{1/2} \approx (S_{sc} / Q_C)^{1/2}$$

Note that the ratio X_L / X_C which results in a harmonic number f_r / f is:

$$X_L / X_C = 1 / (f_r / f)^2$$

so for f_r / f to be equal to the geometric mean of the third and fifth harmonics:

$$f_r / f = \sqrt{15} = 3.873$$

$$X_L / X_C = 1 / 15 = 0.067$$

Dielectric Dissipation Factor

If an alternating voltage V of frequency f is applied across an insulation system comprising capacitance C and equivalent series loss resistance R_s , then the voltage V_R across R_s and the voltage V_C across C due to the resulting current I are:

$$V_R = IR_s$$

$$V_C = IX_C$$

$$V = (V_R^2 + V_C^2)^{1/2}$$

The dielectric dissipation factor of the insulation system is the tangent of the dielectric loss angle δ between V_C and V :

$$\tan\delta = V_R / V_C = R_s / X_C = 2\pi f C R_s$$

$$R_s = X_C \tan\delta = \tan\delta / 2\pi f C$$

Note that an increase in the dielectric losses of a insulation system (from an increase in the series loss resistance R_s) results in an increase in $\tan\delta$. Note also that $\tan\delta$ increases with frequency.

The dielectric power loss P is related to the capacitive reactive power Q_C by:

$$P = I^2 R_s = I^2 X_C \tan\delta = Q_C \tan\delta$$

The power factor of the insulation system is the cosine of the phase angle ϕ between V_R and V :

$$\cos\phi = V_R / V$$

so that δ and ϕ are related by:

$$\delta + \phi = 90^\circ$$

$\tan\delta$ and $\cos\phi$ are related by:

$$\tan\delta = 1 / \tan\phi = \cos\phi / \sin\phi = \cos\phi / (1 - \cos^2\phi)^{1/2}$$

so that when $\cos\phi$ is close to zero, $\tan\delta \approx \cos\phi$

Note that the series loss resistance R_s is not related to the shunt leakage resistance of the insulation system (which is measured using direct current).

Notation

The library uses the [symbol font](#) for some of the notation and formulae. If the symbols for the letters 'alpha beta delta' do not appear here [α β δ] then the symbol font needs to be installed before all notation and formulae will be displayed correctly.

C	capacitance	[farads, F]	Q	charge	[coulombs, C]
E	voltage source	[volts, V]	q	instantaneous Q	[coulombs, C]
e	instantaneous E	[volts, V]	R	resistance	[ohms, Ω]
G	conductance	[siemens, S]	T	time constant	[seconds, s]
I	current	[amps, A]	t	instantaneous time	[seconds, s]
i	instantaneous I	[amps, A]	V	voltage drop	[volts, V]
k	coefficient	[number]	v	instantaneous V	[volts, V]
L	inductance	[henrys, H]	W	energy	[joules, J]
M	mutual inductance	[henrys, H]	Φ	magnetic flux	[webers, Wb]
N	number of turns	[number]	Ψ	magnetic linkage	[webers, Wb]
P	power	[watts, W]	ψ	instantaneous Ψ	[webers, Wb]

Resistance

The resistance **R** of a circuit is equal to the applied direct voltage **E** divided by the resulting steady current **I**:
 $R = E / I$

Resistances in Series

When resistances **R**₁, **R**₂, **R**₃, ... are connected in series, the total resistance **R**_S is:
 $R_S = R_1 + R_2 + R_3 + \dots$

Voltage Division by Series Resistances

When a total voltage **E**_S is applied across series connected resistances **R**₁ and **R**₂, the current **I**_S which flows through the series circuit is:

$$I_S = E_S / R_S = E_S / (R_1 + R_2)$$

The voltages **V**₁ and **V**₂ which appear across the respective resistances **R**₁ and **R**₂ are:

$$V_1 = I_S R_1 = E_S R_1 / R_S = E_S R_1 / (R_1 + R_2)$$

$$V_2 = I_S R_2 = E_S R_2 / R_S = E_S R_2 / (R_1 + R_2)$$

In general terms, for resistances **R**₁, **R**₂, **R**₃, ... connected in series:

$$I_S = E_S / R_S = E_S / (R_1 + R_2 + R_3 + \dots)$$

$$V_n = I_S R_n = E_S R_n / R_S = E_S R_n / (R_1 + R_2 + R_3 + \dots)$$

Note that the highest voltage drop appears across the highest resistance.

Resistances in Parallel

When resistances **R**₁, **R**₂, **R**₃, ... are connected in parallel, the total resistance **R**_P is:

$$1 / R_P = 1 / R_1 + 1 / R_2 + 1 / R_3 + \dots$$

Alternatively, when conductances **G**₁, **G**₂, **G**₃, ... are connected in parallel, the total conductance **G**_P is:

$$G_P = G_1 + G_2 + G_3 + \dots$$

$$\text{where } G_n = 1 / R_n$$

For two resistances **R**₁ and **R**₂ connected in parallel, the total resistance **R**_P is:

$$R_P = R_1 R_2 / (R_1 + R_2)$$

$$R_P = \text{product} / \text{sum}$$

The resistance **R**₂ to be connected in parallel with resistance **R**₁ to give a total resistance **R**_P is:

$$R_2 = R_1 R_P / (R_1 - R_P)$$

$$R_2 = \text{product} / \text{difference}$$

Current Division by Parallel Resistances

When a total current I_P is passed through parallel connected resistances R_1 and R_2 , the voltage V_P which appears across the parallel circuit is:

$$V_P = I_P R_P = I_P R_1 R_2 / (R_1 + R_2)$$

The currents I_1 and I_2 which pass through the respective resistances R_1 and R_2 are:

$$I_1 = V_P / R_1 = I_P R_P / R_1 = I_P R_2 / (R_1 + R_2)$$

$$I_2 = V_P / R_2 = I_P R_P / R_2 = I_P R_1 / (R_1 + R_2)$$

In general terms, for resistances R_1, R_2, R_3, \dots (with conductances G_1, G_2, G_3, \dots) connected in parallel:

$$V_P = I_P R_P = I_P / G_P = I_P / (G_1 + G_2 + G_3 + \dots)$$

$$I_n = V_P / R_n = V_P G_n = I_P G_n / G_P = I_P G_n / (G_1 + G_2 + G_3 + \dots)$$

where $G_n = 1 / R_n$

Note that the highest current passes through the highest conductance (with the lowest resistance).

Capacitance

When a voltage is applied to a circuit containing capacitance, current flows to accumulate charge in the capacitance:

$$Q = \int i dt = CV$$

Alternatively, by differentiation with respect to time:

$$dq/dt = i = C dv/dt$$

Note that the rate of change of voltage has a polarity which opposes the flow of current.

The capacitance C of a circuit is equal to the charge divided by the voltage:

$$C = Q / V = \int i dt / V$$

Alternatively, the capacitance C of a circuit is equal to the charging current divided by the rate of change of voltage:

$$C = i / dv/dt = dq/dt / dv/dt = dq/dv$$

Capacitances in Series

When capacitances C_1, C_2, C_3, \dots are connected in series, the total capacitance C_S is:

$$1 / C_S = 1 / C_1 + 1 / C_2 + 1 / C_3 + \dots$$

For two capacitances C_1 and C_2 connected in series, the total capacitance C_S is:

$$C_S = C_1 C_2 / (C_1 + C_2)$$

C_S = product / sum

Voltage Division by Series Capacitances

When a total voltage E_S is applied to series connected capacitances C_1 and C_2 , the charge Q_S which accumulates in the series circuit is:

$$Q_S = \int i_S dt = E_S C_S = E_S C_1 C_2 / (C_1 + C_2)$$

The voltages V_1 and V_2 which appear across the respective capacitances C_1 and C_2 are:

$$V_1 = \int i_S dt / C_1 = E_S C_S / C_1 = E_S C_2 / (C_1 + C_2)$$

$$V_2 = \int i_S dt / C_2 = E_S C_S / C_2 = E_S C_1 / (C_1 + C_2)$$

In general terms, for capacitances C_1, C_2, C_3, \dots connected in series:

$$Q_S = \int i_S dt = E_S C_S = E_S / (1 / C_S) = E_S / (1 / C_1 + 1 / C_2 + 1 / C_3 + \dots)$$

$$V_n = \int i_S dt / C_n = E_S C_S / C_n = E_S / C_n (1 / C_S) = E_S / C_n (1 / C_1 + 1 / C_2 + 1 / C_3 + \dots)$$

Note that the highest voltage appears across the lowest capacitance.

Capacitances in Parallel

When capacitances C_1, C_2, C_3, \dots are connected in parallel, the total capacitance C_P is:

$$C_P = C_1 + C_2 + C_3 + \dots$$

Charge Division by Parallel Capacitances

When a voltage E_P is applied to parallel connected capacitances C_1 and C_2 , the charge Q_P which accumulates in the parallel circuit is:

$$Q_P = \int i_P dt = E_P C_P = E_P (C_1 + C_2)$$

The charges Q_1 and Q_2 which accumulate in the respective capacitances C_1 and C_2 are:

$$Q_1 = \int i_1 dt = E_P C_1 = Q_P C_1 / C_P = Q_P C_1 / (C_1 + C_2)$$

$$Q_2 = \int i_2 dt = E_P C_2 = Q_P C_2 / C_P = Q_P C_2 / (C_1 + C_2)$$

In general terms, for capacitances C_1, C_2, C_3, \dots connected in parallel:

$$Q_P = \int i_P dt = E_P C_P = E_P (C_1 + C_2 + C_3 + \dots)$$

$$Q_n = \int i_n dt = E_P C_n = Q_P C_n / C_P = Q_P C_n / (C_1 + C_2 + C_3 + \dots)$$

Note that the highest charge accumulates in the highest capacitance.

Inductance

When the current changes in a circuit containing inductance, the magnetic linkage changes and induces a voltage in the inductance:

$$d\psi/dt = e = L di/dt$$

Note that the induced voltage has a polarity which opposes the rate of change of current.

Alternatively, by integration with respect to time:

$$\Psi = \int e dt = LI$$

The inductance L of a circuit is equal to the induced voltage divided by the rate of change of current:

$$L = e / di/dt = d\psi/dt / di/dt = d\psi/di$$

Alternatively, the inductance L of a circuit is equal to the magnetic linkage divided by the current:

$$L = \Psi / I$$

Note that the magnetic linkage Ψ is equal to the product of the number of turns N and the magnetic flux Φ :

$$\Psi = N\Phi = LI$$

Mutual Inductance

The mutual inductance M of two coupled inductances L_1 and L_2 is equal to the mutually induced voltage in one inductance divided by the rate of change of current in the other inductance:

$$M = E_{2m} / (di_1/dt)$$

$$M = E_{1m} / (di_2/dt)$$

If the self induced voltages of the inductances L_1 and L_2 are respectively E_{1s} and E_{2s} for the same rates of change of the current that produced the mutually induced voltages E_{1m} and E_{2m} , then:

$$M = (E_{2m} / E_{1s}) L_1$$

$$M = (E_{1m} / E_{2s}) L_2$$

Combining these two equations:

$$M = (E_{1m} E_{2m} / E_{1s} E_{2s})^{1/2} (L_1 L_2)^{1/2} = k_M (L_1 L_2)^{1/2}$$

where k_M is the mutual coupling coefficient of the two inductances L_1 and L_2 .

If the coupling between the two inductances L_1 and L_2 is perfect, then the mutual inductance M is:

$$M = (L_1 L_2)^{1/2}$$

Inductances in Series

When uncoupled inductances L_1, L_2, L_3, \dots are connected in series, the total inductance L_S is:

$$L_S = L_1 + L_2 + L_3 + \dots$$

When two coupled inductances L_1 and L_2 with mutual inductance M are connected in series, the total inductance L_S is:

$$L_S = L_1 + L_2 \pm 2M$$

The plus or minus sign indicates that the coupling is either additive or subtractive, depending on the connection polarity.

Inductances in Parallel

When uncoupled inductances L_1, L_2, L_3, \dots are connected in parallel, the total inductance L_P is:
 $1 / L_P = 1 / L_1 + 1 / L_2 + 1 / L_3 + \dots$

Time Constants

Capacitance and resistance

The time constant of a capacitance C and a resistance R is equal to CR , and represents the time to change the voltage on the capacitance from zero to E at a constant charging current E / R (which produces a rate of change of voltage E / CR across the capacitance).

Similarly, the time constant CR represents the time to change the charge on the capacitance from zero to CE at a constant charging current E / R (which produces a rate of change of voltage E / CR across the capacitance).

If a voltage E is applied to a series circuit comprising a discharged capacitance C and a resistance R , then after time t the current i , the voltage v_R across the resistance, the voltage v_C across the capacitance and the charge q_C on the capacitance are:

$$i = (E / R)e^{-t/CR}$$

$$v_R = iR = Ee^{-t/CR}$$

$$v_C = E - v_R = E(1 - e^{-t/CR})$$

$$q_C = Cv_C = CE(1 - e^{-t/CR})$$

If a capacitance C charged to voltage V is discharged through a resistance R , then after time t the current i , the voltage v_R across the resistance, the voltage v_C across the capacitance and the charge q_C on the capacitance are:

$$i = (V / R)e^{-t/CR}$$

$$v_R = iR = Ve^{-t/CR}$$

$$v_C = v_R = Ve^{-t/CR}$$

$$q_C = Cv_C = CVe^{-t/CR}$$

Inductance and resistance

The time constant of an inductance L and a resistance R is equal to L / R , and represents the time to change the current in the inductance from zero to E / R at a constant rate of change of current E / L (which produces an induced voltage E across the inductance).

If a voltage E is applied to a series circuit comprising an inductance L and a resistance R , then after time t the current i , the voltage v_R across the resistance, the voltage v_L across the inductance and the magnetic linkage ψ_L in the inductance are:

$$i = (E / R)(1 - e^{-tR/L})$$

$$v_R = iR = E(1 - e^{-tR/L})$$

$$v_L = E - v_R = Ee^{-tR/L}$$

$$\psi_L = Li = (LE / R)(1 - e^{-tR/L})$$

If an inductance L carrying a current I is discharged through a resistance R , then after time t the current i , the voltage v_R across the resistance, the voltage v_L across the inductance and the magnetic linkage ψ_L in the inductance are:

$$i = Ie^{-tR/L}$$

$$v_R = iR = IR e^{-tR/L}$$

$$v_L = v_R = IR e^{-tR/L}$$

$$\psi_L = Li = LI e^{-tR/L}$$

Rise Time and Fall Time

The rise time (or fall time) of a change is defined as the transition time between the 10% and 90% levels of the total change, so for an exponential rise (or fall) of time constant T , the rise time (or fall time) t_{10-90} is:

$$t_{10-90} = (\ln 0.9 - \ln 0.1)T \approx 2.2T$$

The half time of a change is defined as the transition time between the initial and 50% levels of the total change, so for an exponential change of time constant T , the half time t_{50} is :

$$t_{50} = (\ln 1.0 - \ln 0.5)T \approx 0.69T$$

Note that for an exponential change of time constant T :

- over time interval T , a rise changes by a factor $1 - e^{-1}$ (≈ 0.63) of the remaining change,
- over time interval T , a fall changes by a factor e^{-1} (≈ 0.37) of the remaining change,
- after time interval $3T$, less than 5% of the total change remains,
- after time interval $5T$, less than 1% of the total change remains.

Power

The power P dissipated by a resistance R carrying a current I with a voltage drop V is:

$$P = V^2 / R = VI = I^2R$$

Similarly, the power P dissipated by a conductance G carrying a current I with a voltage drop V is:

$$P = V^2G = VI = I^2 / G$$

The power P transferred by a capacitance C holding a changing voltage V with charge Q is:

$$P = VI = CV(dv/dt) = Q(dv/dt) = Q(dq/dt) / C$$

The power P transferred by an inductance L carrying a changing current I with magnetic linkage Ψ is:

$$P = VI = LI(di/dt) = \Psi(di/dt) = \Psi(d\psi/dt) / L$$

Energy

The energy W consumed over time t due to power P dissipated in a resistance R carrying a current I with a voltage drop V is:

$$W = Pt = V^2t / R = VI t = I^2tR$$

Similarly, the energy W consumed over time t due to power P dissipated in a conductance G carrying a current I with a voltage drop V is:

$$W = Pt = V^2tG = VI t = I^2t / G$$

The energy W stored in a capacitance C holding voltage V with charge Q is:

$$W = CV^2 / 2 = QV / 2 = Q^2 / 2C$$

The energy W stored in an inductance L carrying current I with magnetic linkage Ψ is:

$$W = LI^2 / 2 = \Psi I / 2 = \Psi^2 / 2L$$

Batteries

If a battery of open-circuit voltage E_B has a loaded voltage V_L when supplying load current I_L , the battery internal resistance R_B is:

$$R_B = (E_B - V_L) / I_L$$

The load voltage V_L and load current I_L for a load resistance R_L are:

$$V_L = I_L R_L = E_B - I_L R_B = E_B R_L / (R_B + R_L)$$

$$I_L = V_L / R_L = (E_B - V_L) / R_B = E_B / (R_B + R_L)$$

The battery short-circuit current I_{sc} is:

$$I_{sc} = E_B / R_B = E_B I_L / (E_B - V_L)$$

Voltmeter Multiplier

The resistance R_S to be connected in series with a voltmeter of full scale voltage V_V and full scale current drain I_V to increase the full scale voltage to V is:

$$R_S = (V - V_V) / I_V$$

The power P dissipated by the resistance R_S with voltage drop $(V - V_V)$ carrying current I_V is:

$$P = (V - V_V)^2 / R_S = (V - V_V) I_V = I_V^2 R_S$$

Ammeter Shunt

The resistance R_P to be connected in parallel with an ammeter of full scale current I_A and full scale voltage drop V_A to increase the full scale current to I is:

$$R_P = V_A / (I - I_A)$$

The power P dissipated by the resistance R_P with voltage drop V_A carrying current $(I - I_A)$ is:

$$P = V_A^2 / R_P = V_A (I - I_A) = (I - I_A)^2 R_P$$