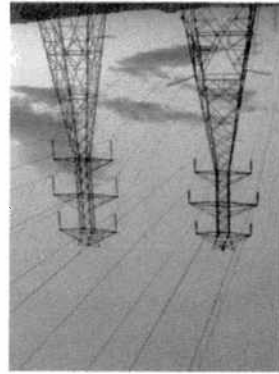


TK 1001 S23 2010

**POWER  
SYSTEM  
ANALYSIS**  
THIRD EDITION



Hadi Saadat  
Professor Emeritus of  
Electrical Engineering

PSA Publishing  
AP <http://www.psapublishing.com/>

All rights reserved: No part of this book may be reproduced or distributed in any form or by any means, or stored in a data base retrieval system, without prior written permission of the author.

Notice of Liability: The author and the publisher of this book have used their best efforts in preparing this book and the accompanying Power System Toolbox. Although the developed *MATLAB* programs and functions have been tested with care no warranty, expressed or implied, is made by the author and the publisher of this book as to the accuracy and functioning of the programs and related program materials. The author shall have no liability to any person or entity with respect to any loss or damage caused or alleged to be caused directly or indirectly by the instruction contained in this book or by the *MATLAB* programs and functions in the Power System Toolbox.

Trademark Acknowledgments: *MATLAB*® is a registered trademark of The MathWorks, Inc. Further information about *MATLAB* and other related products—including *SIMULINK*, and Control System Toolbox may be obtained from: <http://www.mathworks.com/>

Typeset by the author with the L<sup>A</sup>T<sub>E</sub>X Documentation System.  
All drawings and graphs designed and embedded in L<sup>A</sup>T<sub>E</sub>X by the author.  
Cover designed by the author.

Front cover photos: Transmission Tower—“Courtesy of DOE/NREL, Credit - Warren Gretz.” Wind Turbine—“Courtesy of DOE/NREL, Credit - Invenery LLC.” Abengoa Solar power plant (PS20). To this date, Abengoa Solar located in Seville, Spain is the world’s largest solar power plant—“Courtesy of Abengoa Solar.”

Printed by Victor Graphics, Inc. in the United States of America.

PSA Publishings: <http://www.psapublishings.com>  
ISBN: 978-0-9845438-0-9

Library of Congress Control Number: 2010906755

1	1.1	INTRODUCTION	1
2	1.2	ELECTRIC INDUSTRY STRUCTURE	2
5	1.3	ENERGY RESOURCES FOR ELECTRICITY GENERATION	5
8	1.4	FOSSIL FUEL POWER PLANTS	8
8	1.4.1	COAL-FIRED POWER PLANTS	8
10	1.4.2	GAS TURBINE POWER PLANTS	10
11	1.4.3	COMBINED-CYCLE POWER PLANTS	11
11	1.4.4	NUCLEAR POWER PLANTS	11
15	1.5	ELECTRIC POWER GENERATION FROM RENEWABLE ENERGY SOURCES	15
14	1.6	HYDROELECTRIC POWER PLANTS	14
18	1.6.1	RUN-OF-THE-RIVER POWER PLANTS	18
18	1.6.2	PUMPED-STORAGE HYDRO POWER PLANTS	18
19	1.7	SOLAR POWER	19
20	1.7.1	PARABOLIC TROUGH PARABOLIC DISH CONCENTRATORS (DISH STR-LING)	20
21	1.7.2	PARABOLIC DISH CONCENTRATORS (DISH STR-LING)	21
23	1.7.3	SOLAR TOWER	23
24	1.7.4	PHOTOVOLTAIC ELECTRIC POWER PLANTS	24
26	1.8	WIND POWER PLANTS	26
26	1.8.1	TYPE OF WIND TURBINES	26
27	1.8.2	WIND POWER	27
28	1.8.3	FIXED-SPEED WIND TURBINE	28
29	1.8.4	VARIABLE-SPEED WIND TURBINE	29
31	1.9	GEOTHERMAL POWER	31

101	3.4	SALIENT-POLE SYNCHRONOUS GENERATORS
103	3.5	POWER TRANSFORMER
103	3.6	EQUIVALENT CIRCUIT OF A TRANSFORMER
107	3.7	DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS
109	3.8	TRANSFORMER PERFORMANCE
113	3.9	THREE-PHASE TRANSFORMER CONNECTIONS
115	3.9.1	THE PER-PHASE MODEL OF A THREE-PHASE TRANSFORMER
116	3.10	AUTOTRANSFORMERS
120	3.10.1	AUTOTRANSFORMER MODEL
120	3.11	THREE-WINDING TRANSFORMERS
121	3.11.1	THREE-WINDING TRANSFORMER MODEL
122	3.12	VOLTAGE CONTROL OF TRANSFORMERS
122	3.12.1	TAP CHANGING TRANSFORMERS
125	3.12.2	REGULATING TRANSFORMERS OR BOOSTERS
127	3.13	THE PER-UNIT SYSTEM
129	3.14	CHANGE OF BASE
141	4	TRANSMISSION LINE PARAMETERS
141	4.1	INTRODUCTION
142	4.2	OVERHEAD TRANSMISSION LINES
144	4.3	LINE RESISTANCE
145	4.4	INDUCTANCE OF A SINGLE CONDUCTOR
146	4.4.1	INTERNAL INDUCTANCE
147	4.4.2	INDUCTANCE DUE TO EXTERNAL FLUX LINKAGE
148	4.5	INDUCTANCE OF SINGLE-PHASE LINES
149	4.6	FLUX LINKAGE IN TERMS OF SELF- AND MUTUAL INDUCTANCES
151	4.7	INDUCTANCE OF THREE-PHASE TRANSMISSION LINES
151	4.7.1	SYMMETRICAL SPACING
152	4.7.2	ASYMMETRICAL SPACING
153	4.7.3	TRANSPOSE LINE
154	4.8	INDUCTANCE OF BUNDLED OR STRANDED CONDUCTORS
157	4.8.1	<i>GMR</i> OF BUNDLED CONDUCTORS
158	4.9	INDUCTANCE OF THREE-PHASE DOUBLE-CIRCUIT LINES
159	4.10	LINE CAPACITANCE
160	4.11	CAPACITANCE OF SINGLE-PHASE LINES

34	1.10	BIOMASS POWER PLANTS
34	1.11	TIDAL POWER PLANTS
35	1.11.1	TIDAL ENERGY SYSTEMS
37	1.11.2	TIDAL WAVE SYSTEMS
38	1.12	FUEL CELL
39	1.12.1	PHOSPHORIC ACID FUEL CELL
40	1.12.2	MOLTEN CARBONATE FUEL CELL
41	1.13	MODERN POWER SYSTEM
41	1.13.1	GENERATION
42	1.13.2	TRANSMISSION AND SUBTRANSMISSION
42	1.13.3	DISTRIBUTION
44	1.13.4	LOADS
47	1.14	SMART GRID
48	1.15	ROLE OF POWER ELECTRONICS IN MODERN POWER SYSTEMS
50	1.16	SYSTEM PROTECTION
50	1.17	ENERGY CONTROL CENTER
50	1.18	COMPUTER ANALYSIS
53	2	BASIC PRINCIPLES
53	2.1	INTRODUCTION
54	2.2	POWER IN SINGLE-PHASE AC CIRCUITS
58	2.3	COMPLEX POWER
60	2.4	THE COMPLEX POWER BALANCE
62	2.5	POWER FACTOR CORRECTION
65	2.6	COMPLEX POWER FLOW
69	2.7	BALANCED THREE-PHASE CIRCUITS
71	2.8	Y-CONNECTED LOADS
73	2.9	Δ-CONNECTED LOADS
74	2.10	Δ-Y TRANSFORMATION
75	2.11	PER-PHASE ANALYSIS
76	2.12	BALANCED THREE-PHASE POWER
87	3	GENERATOR AND TRANSFORMER MODELS; THE PER-UNIT SYSTEM
87	3.1	INTRODUCTION
88	3.2	SYNCHRONOUS GENERATORS
88	3.2.1	GENERATOR MODEL
88	3.3	STEADY-STATE CHARACTERISTICS—
95		CYLINDRICAL ROTOR
95	3.3.1	POWER FACTOR CONTROL
96	3.3.2	POWER ANGLE CHARACTERISTICS

4.12	POTENTIAL DIFFERENCE IN A	162
4.13	MULTICONDUCTOR CONFIGURATION	163
4.14	CAPACITANCE OF THREE-PHASE LINES	165
4.15	EFFECT OF BUNDLING	165
4.16	CAPACITANCE OF THREE-PHASE	165
4.17	DOUBLE-CIRCUIT LINES	166
4.18	EFFECT OF EARTH ON THE CAPACITANCE	172
4.19	MAGNETIC FIELD INDUCTION	174
4.20	ELECTROSTATIC INDUCTION	174
4.21	CORONA	174
<b>5 LINE MODEL AND PERFORMANCE</b>		
5.1	INTRODUCTION	181
5.2	SHORT LINE MODEL	181
5.3	MEDIUM LINE MODEL	182
5.4	LONG LINE MODEL	186
5.5	VOLTAGE AND CURRENT WAVES	190
5.6	SURGE IMPEDANCE LOADING	195
5.7	COMPLEX POWER FLOW	198
5.8	THROUGH TRANSMISSION LINES	200
5.9	POWER TRANSMISSION CAPABILITY	202
5.9.1	LINE COMPENSATION	204
5.9.2	SHUNT REACTORS	204
5.9.3	SHUNT CAPACITOR COMPENSATION	207
5.10	SERIES CAPACITOR COMPENSATION	207
5.11	LINE PERFORMANCE PROGRAM	210
<b>6 POWER FLOW ANALYSIS</b>		
6.1	INTRODUCTION	228
6.2	BUS ADMITTANCE MATRIX	228
6.3	SOLUTION OF NONLINEAR	229
6.4	ALGEBRAIC EQUATIONS	234
6.4.1	GAUSS-SEIDEL METHOD	234
6.4.2	NEWTON-RAPHSON METHOD	239
6.4.3	POWER FLOW SOLUTION	247
6.4.4	POWER FLOW EQUATION	247
6.5	GAUSS-SEIDEL POWER FLOW SOLUTION	248
6.6	LINE FLOWS AND LOSSES	251
6.7	TAP CHANGING TRANSFORMERS	259
6.8	POWER FLOW PROGRAMS	261
6.9	DATA PREPARATION	262
6.10	NEWTON-RAPHSON POWER FLOW SOLUTION	271
<b>7 OPTIMAL DISPATCH OF GENERATION</b>		
7.1	FAST DECOUPLED POWER FLOW SOLUTION	279
7.2	INTRODUCTION	296
7.2.1	NONLINEAR FUNCTION OPTIMIZATION	297
7.2.2	CONSTRAINED PARAMETER OPTIMIZATION:	299
7.3	EQUALITY CONSTRAINTS	303
7.4	OPERATING COST OF A THERMAL PLANT	306
7.5	ECONOMIC DISPATCH NEGLECTING	307
7.6	LOSSES AND NO GENERATOR LIMITS	315
7.7	ECONOMIC DISPATCH INCLUDING LOSSES	318
7.8	DERIVATION OF LOSS FORMULA	328
<b>8 SYNCHRONOUS MACHINE TRANSIENT ANALYSIS</b>		
8.1	INTRODUCTION	353
8.2	TRANSIENT PHENOMENA	353
8.3	SYNCHRONOUS MACHINE TRANSIENTS	354
8.3.1	INDUCTANCES OF SALIENT-POLE MACHINES	357
8.4	THE PARK TRANSFORMATION	359
8.5	BALANCED THREE-PHASE SHORT CIRCUIT	360
8.6	UNBALANCED SHORT CIRCUITS	364
8.6.1	LINE-TO-LINE SHORT CIRCUIT	369
8.6.2	LINE-TO-GROUND SHORT CIRCUIT	369
8.7	SIMPLIFIED MODELS OF SYNCHRONOUS	372
8.8	MACHINES FOR TRANSIENT ANALYSES	374
8.8.1	DC COMPONENTS OF STATOR CURRENTS	374
8.8.2	DETERMINATION OF TRANSIENT CONSTANTS	379
8.8.3	EFFECT OF LOAD CURRENT	381
8.9	INTRODUCTION	386
8.10	BALANCED FAULT	392
9.1	BALANCED THREE-PHASE FAULT	392
9.2	SHORT-CIRCUIT CAPACITY (SCC)	393
9.3	SYSTEMATIC FAULT ANALYSIS	401
9.4	USING BUS IMPEDANCE MATRIX	402
9.5	ALGORITHM FOR FORMATION	408
9.6	OF THE BUS IMPEDANCE MATRIX	420
9.7	ZBUILD AND SYMFAULT PROGRAMS	420

531	11.6 APPLICATION TO THREE-PHASE FAULT
540	11.7 NUMERICAL SOLUTION OF NONLINEAR EQUATION
543	11.8 NUMERICAL SOLUTION OF THE SWING EQUATION
550	11.9 MULTIMACHINE SYSTEMS 11.10 MULTIMACHINE
553	TRANSIENT STABILITY
566	<b>12 POWER SYSTEM CONTROL</b>
566	12.1 INTRODUCTION
567	12.2 BASIC GENERATOR CONTROL LOOPS
567	12.3 LOAD FREQUENCY CONTROL
568	12.3.1 GENERATOR MODEL
569	12.3.2 LOAD MODEL
570	12.3.3 PRIME MOVER MODEL
571	12.3.4 GOVERNOR MODEL
581	12.4 AUTOMATIC GENERATION CONTROL
581	12.4.1 AGC IN A SINGLE AREA SYSTEM
584	12.4.2 AGC IN THE MULTAREA SYSTEM
588	12.4.3 THE-LINE BIAS CONTROL
593	12.5 AGC WITH OPTIMAL DISPATCH OF GENERATION
594	12.6 REACTIVE POWER AND VOLTAGE CONTROL
594	12.6.1 AMPLIFIER MODEL
595	12.6.2 EXCITER MODEL
596	12.6.3 GENERATOR MODEL
596	12.6.4 SENSOR MODEL
601	12.6.5 EXCITATION SYSTEM STABILIZER — RATE FEEDBACK
601	12.6.6 EXCITATION SYSTEM STABILIZER — PID CONTROLLER
603	12.7 AGC INCLUDING EXCITATION SYSTEM
605	12.8 INTRODUCING MODERN CONTROL APPLICATION
606	12.8.1 POLE-PLACEMENT DESIGN
608	12.8.2 OPTIMAL CONTROL DESIGN
615	<b>A INTRODUCTION TO MATLAB</b>
626	A.1 INSTALLING THE TEXT TOOLBOX
626	A.2 RUNNING MATLAB
628	A.3 VARIABLES
629	A.4 OUTPUT FORMAT

438	<b>10 SYMMETRICAL COMPONENTS AND UNBALANCED FAULT</b>
438	10.1 INTRODUCTION
439	10.2 FUNDAMENTALS OF SYMMETRICAL COMPONENTS
445	10.3 SEQUENCE IMPEDANCES
445	10.3.1 SEQUENCE IMPEDANCES OF Y-CONNECTED LOADS
446	10.3.2 SEQUENCE IMPEDANCES OF TRANSMISSION LINES
448	10.3.3 SEQUENCE IMPEDANCES OF SYNCHRONOUS MACHINE
449	10.3.4 SEQUENCE IMPEDANCES OF TRANSFORMER
450	10.4 SEQUENCE NETWORKS OF A LOADED GENERATOR
457	10.5 SINGLE LINE-TO-GROUND FAULT
460	10.6 LINE-TO-LINE FAULT
462	10.7 DOUBLE LINE-TO-GROUND FAULT
464	10.8 UNBALANCED FAULT ANALYSIS
471	10.8.1 SINGLE LINE-TO-GROUND FAULT USING $Z_{bus}$
471	10.8.2 LINE-TO-LINE FAULT USING $Z_{bus}$
472	10.8.3 DOUBLE LINE-TO-GROUND FAULT USING $Z_{bus}$
473	10.8.4 BUS VOLTAGES AND LINE CURRENTS DURING FAULT
473	10.9 UNBALANCED FAULT PROGRAMS
481	<b>11 STABILITY</b>
499	11.1 INTRODUCTION
499	11.2 SWING EQUATION
500	11.3 SYNCHRONOUS MACHINE MODELS FOR STABILITY STUDIES
503	11.3.1 SYNCHRONOUS MACHINE MODEL INCLUDING SALIENCY
506	11.4 STEADY-STATE STABILITY — SMALL DISTURBANCES
510	11.5 TRANSIENT STABILITY — EQUAL-AREA CRITERION
525	11.5.1 APPLICATION TO SUDDEN INCREASE IN POWER INPUT
527	

631	A.5 CHARACTER STRING
632	A.6 VECTOR OPERATIONS
635	A.7 ELEMENTARY MATRIX OPERATIONS
638	A.7.1 UTILITY MATRICES
638	A.7.2 EIGENVALUES
639	A.8 COMPLEX NUMBERS
641	A.9 POLYNOMIAL ROOTS
641	AND CHARACTERISTIC POLYNOMIAL
643	A.9.1 PRODUCT AND DIVISION OF POLYNOMIALS
643	A.9.2 POLYNOMIAL CURVE FITTING
644	A.9.3 POLYNOMIAL EVALUATION
644	A.9.4 PARTIAL-FRACTION EXPANSION
645	A.10 GRAPHICS
648	A.11 GRAPHICS HARD COPY
654	A.12 THREE-DIMENSIONAL PLOTS
655	A.13 HANDLE GRAPHICS
658	A.14 LOOPS AND LOGICAL STATEMENTS
665	A.15 SOLUTION OF DIFFERENTIAL EQUATIONS
670	A.16 NONLINEAR SYSTEMS
673	A.17 SIMULATION DIAGRAM
673	A.18 INTRODUCTION TO SIMULINK
674	A.18.1 SIMULATION PARAMETERS AND SOLVER
675	A.18.2 THE SIMULATION PARAMETERS DIALOG BOX
676	A.18.3 BLOCK DIAGRAM CONSTRUCTION
682	A.18.4 USING THE TO WORKSPACE BLOCK
683	A.18.5 LINEAR STATE-SPACE
683	MODEL FROM SIMULINK DIAGRAM
685	A.18.6 SUBSYSTEMS AND MASKING
687	B.1 THE CONTROL PROBLEM
688	B.2 STABILITY
689	B.2.1 THE ROUTH-HURWITZ
689	STABILITY CRITERION
690	B.2.2 ROOT-LOCUS METHOD
691	B.3 STEADY-STATE ERROR
693	B.4 STEP RESPONSE
694	B.5 ROOT-LOCUS DESIGN
694	B.5.1 GAIN FACTOR COMPENSATION
695	OR P CONTROLLER
696	B.5.2 PHASE-LEAD DESIGN

**B REVIEW OF FEEDBACK CONTROL SYSTEMS**

697	B.5.3 PHASE-LAG DESIGN
698	B.5.4 PID DESIGN
698	B.5.5 PD CONTROLLER
699	B.5.6 PI CONTROLLER
699	B.5.7 PID CONTROLLER
706	B.6 FREQUENCY RESPONSE
706	B.6.1 BODE PLOT
707	B.6.2 POLAR PLOT
707	B.6.3 RELATIVE STABILITY
708	B.6.4 GAIN AND PHASE MARGINS
709	B.6.5 NYQUIST STABILITY CRITERION
709	B.6.6 SIMPLIFIED NYQUIST CRITERION
710	B.6.7 CLOSED-LOOP FREQUENCY RESPONSE
711	B.6.8 FREQUENCY RESPONSE DESIGN
714	B.7 Control System Toolbox LTI Models and LTI Viewer
714	B.7.1 LTI Models
714	B.7.2 The LTI Viewer

**C POWER SYSTEM TOOLBOX**

In addition, *SIMULINK* provides a highly interactive environment for simulation of both linear and nonlinear dynamic systems. Both programs are integrated into discussions and problems. I developed a power system toolbox containing a set of M-files to help in typical power system analysis. In fact, all the examples and figures in this book have been generated by *MATLAB* functions and the use of this toolbox. The power system toolbox allows the student to analyze and design power systems without having to do detailed programming. Some of the programs, such as power flow, optimization, short-circuit, and stability analysis, were originally developed for a mainframe computer when I worked for power system consulting firms many years ago. These programs have been refined and modularized for interactive use with *MATLAB* for many problems related to the operation and analysis of power systems. These software modules are versatile, allowing some of the typical problems to be solved by several methods, thus enabling students to investigate alternative solution techniques. Furthermore, the software modules are structured in such a way that the user may mix them for other power system analyses.

This book has more than 140 illustrative examples that use *MATLAB* to assist in the analysis of power systems. Each example illustrates a specific concept and usually contains a script of the *MATLAB* commands used for the model creation and computation. Some examples are quite elaborate in order to bring the practical world closer. The *MATLAB* M-files on the accompanying CD-Rom can be copied to the user's computer and used to solve all the examples. The scripts can also be utilized with modifications as the foundation for solving the end-of-chapter problems.

The book is organized into 12 chapters and three appendices. Each chapter begins with an introduction describing the topics students will encounter. In the third edition, **Chapter 1**, titled "The Power System and Electric Power Generation," is an entirely new chapter. This chapter presents an overview of the electric power system and the electric industry structure. The chapter also describes the restructuring, deregulation of electric utilities, and recent progress in technology. Students are introduced to various energy resources for electricity generation, including fossil fuel, nuclear fuel, renewable energy sources and their environmental impacts. Various methods for electricity generation are covered. These include the description, main components, and operation of coal-fired power plants, nuclear power plants, gas turbine power plants, combined cycle power plants, and electric power generation from renewable energy sources, such as hydroelectric power plants, thermal solar power plants, photovoltaic power plants, wind power plants, geothermal power plants, biomass power plants, tidal power plants, and fuel cells. Included is a discussion of transmission, distribution networks, and the smart grid that conveys the energy from the primary source to the load areas. **Chapter 2** re-studies much of this material. However, this specialized topic of networks may

This book is intended for use by senior undergraduate or graduate electrical engineering students studying power system analysis and design or as a reference for practicing engineers. As a reference, the book is written with self-study in mind. The text has grown out of many years of teaching the subject material to students in electrical engineering at various universities, including Michigan Technological University and Milwaukee School of Engineering.

Prerequisites for students using this text are physics and mathematics through differential equations and a circuit course. A background in electric machines is desirable, but not essential. Other required background materials, including *MATLAB* and an introduction to control systems, are provided in the appendices.

In recent years, the analysis and design of power systems have been affected dramatically by the widespread use of personal computers. Personal computers have become so powerful and advanced that they can be used easily to perform steady-state and transient analysis of large interconnected power systems. Modern personal computers' ability to provide information, ask questions, and react to responses has enabled engineering educators to integrate computers into the curriculum. One of the difficulties of teaching power system analysis courses is not having a real system with which to experiment in the laboratory. Therefore, this book is written to supplement the teaching of power system analysis with a computer-simulated system. I developed many programs for power system analysis, giving students a valuable tool that allows them to spend more time on analysis and design of practical systems and less on programming, thereby enhancing the learning process. The book also provides a basis for further exploration of more advanced topics in power system analysis.

*MATLAB* is a matrix-based software package, which makes it ideal for power system analysis. *MATLAB*, with its extensive numerical resources, can be used to obtain numerical solutions that involve various types of vector-matrix operations.

havior of a one-machine system due to a small disturbance is investigated, and the analytical solution of this linearized model is obtained. *MATLAB* and *SIMULINK* are used conveniently to simulate the system, and the model is extended to multimachine systems. Next, the transient stability using equal area criteria is discussed, and the result is represented graphically, providing physical insight into the dynamic behavior of the machine. An introduction to nonlinear differential equations and their numerical solutions is given. *MATLAB* is used to obtain the numerical solution of the swing equation of a one-machine system. Simulation is also obtained using the *SIMULINK* toolbox. A program compatible with the power flow programs is developed for the transient stability analysis of the multimachine systems. **Chapter 12** is concerned with power system control and develops some of the control schemes required to operate the power system in the steady state. Simple models of the essential components used in control systems are presented. The automatic voltage regulator (AVR) and the load frequency control (LFC) are discussed. The automatic generation control (AGC) in single-area and multiarea systems, including tie-line power control, are analyzed. For each case, the responses to the real power demand are obtained. The generator responses with the AVR and various compensators, such as rate feedback and *Proportional Integral Derivative* (PID) controllers, are obtained. Both AGC and AVR systems are illustrated by several examples, and the responses are obtained using *MATLAB*. These analyses are supplemented by constructing the *SIMULINK* block diagram, which provides a highly interactive environment for simulation. Some basic materials of modern control theory are discussed, including the pole-placement state feedback design and the optimal controller designs using the linear quadratic regulator based on the *Riccati* equation. These modern techniques are then applied for simulation of the LFC systems.

**Appendix A** is a self-study *MATLAB* and *SIMULINK* tutorial focused on power and control systems and coordinated with the text. **Appendix B** includes a brief introduction to the fundamentals of control systems and is suitable for students without a background in control systems. **Appendix C** lists all functions, script files, and chapter examples. Answers to problems are given at the end of the book. The instructor's manual for this text contains the worked-out solutions for all of the book's problems.

The material in the text is designed to be fully covered in a two-semester undergraduate course sequence. The organization is flexible, allowing instructors to select the material that best suits the requirements of a one-quarter or a one-semester course. In a one-semester course, the first six chapters, which form the basis for power system analysis, should be covered. The material in Chapter 2 contains power concepts and three-phase systems, which are usually covered in circuit courses. This chapter can be excluded if the students are well prepared, or it can be used for review. Also, for students with electrical machinery background, Chapter

not be included in circuit theory courses, and the review here will reinforce these electrical power systems. **Chapter 3** addresses the steady-state presentation and modeling of synchronous machines and transformers. Also, the per unit system is presented, followed by the one-line diagram representation of the network.

**Chapter 4** discusses the parameters of a multicircuit transmission line. These parameters are computed for the balanced system on a per phase basis. **Chapter 5** thoroughly covers transmission line modeling and the performance and compensation of the transmission lines. This chapter provides the concepts and tools necessary for the preliminary transmission line design. **Chapter 6** presents a comprehensive coverage of the power flow solution of an interconnected power system during normal operation. First, the commonly used iterative techniques for the solution of nonlinear algebraic equation are discussed. Then several approaches to the solution of power flow are described. These techniques are applied to the solution of practical systems using the developed software modules.

**Chapter 7** covers some essential classical optimization of continuous functions and their application to optimal dispatch of generation. The programs developed here are designed to work in synergy with the power flow programs. **Chapter 8** deals with synchronous machine transient analysis. The voltage equations of the synchronous machine are first developed. These nonlinear equations are transformed into linear differential equations using Park's transformation. Analytical solution of the transformed equations can be obtained by the Laplace transform technique. However, *MATLAB* is used with ease to simulate the nonlinear differential equations of the synchronous machine directly in time-domain in matrix form for all modes of operation. Thus students can observe the dynamic response of the synchronous machine during short circuits and appreciate the significance and consequence of the change of machine parameters. The ultimate objective of this chapter is to develop simple network models of the synchronous generator for power system fault analysis and transient stability studies.

**Chapter 9** covers balanced fault analysis. The bus impedance matrix by the *building algorithms* is formulated and employed for the systematic computation of bus voltages and line currents during faults. **Chapter 10** discusses methods of symmetrical components that resolve the problem of an unbalanced circuit into a solution of a number of balanced circuits. Included are graphical displays of the symmetrical components transformation and some applications. The method is applied to the unbalanced fault, which once again allows the treatment of the problem on simple per phase basis. Algorithms have been developed to simulate different types of unbalanced faults. The software modules developed for unbalanced faults include single line-to-ground fault, line-to-line fault, and double line-to-ground fault.

**Chapter 11** covers power system stability problems. First, the dynamic be-



learn the theory and problem-solving skills for power system analysis. The assignment of computer exercises in parallel with manual calculations can help students to develop a stronger intuition and a deeper understanding of power system analysis.

The second edition was published in 2002, the major improvement being the inclusion of several Graphical User Interface (GUI) programs that are interactive and menu-driven so students do not need to perform the laborious calculations required to solve meaningful problems, thereby enhancing the learning process. The Power System Toolbox supplied with the book was revised and expanded, and Appendix A was extended and revised to include the more recent *MATLAB* developments and extended graphical facilities.

Due to the continuing increase of renewable energy generation units and distributed generation, many professors who are using the second edition have strongly suggested the addition of a chapter on electric power generation. To accomplish this, in this third edition, Chapter 1 is revised comprehensively to include energy resources and their environmental impacts. It covers various fossil-fuel power plants as well as all modern power plants using renewable energy sources. Also, this chapter includes discussion of the emergence of the smart grid and the role of power electronics in modern power systems. In this edition, programs, functions, and *SIMULINK* diagrams have been updated to run on the latest version of *MATLAB* (V7.10, R210a). All *MATLAB* files are downward compatible with the earlier versions, except the GUI programs, which have been designed with version 6.1 and will not run on the earlier versions.

I would like to thank many readers from all over the world for their helpful comments, which have enhanced this edition.

Hadi Sadat

Professor Emeritus of Electrical Engineering  
at Milwaukee School of Engineering  
May 2010

3 might be omitted. After the above coverage, additional material from the remaining chapters may then be appropriate, depending on the syllabus requirements and the individual preferences. One choice is to cover Chapter 7 (optimal dispatch of generation); another choice is Chapter 9 (balanced fault). The generator reactances required in Chapter 9 may be covered briefly from Section 8.7 without covering Chapter 8 in its entirety.

After reading the book, students should have a good perspective of power system analysis and an active knowledge of various numerical techniques that can be applied to the solution of large interconnected power systems. Students should find *MATLAB* helpful in learning the material in the text, particularly in solving the problems at the end of each chapter.

I would like to express my appreciation and thanks to the following reviewers for their many helpful comments and suggestions: Professor Max D. Anderson, University of Missouri-Rolla; Professor Miroslav Begovic, Georgia Institute of Technology; Professor Karen L. Butler, Texas A&M University; Professor Kevin A. Clements, Worcester Polytechnic Institute; Professor Maritesa L. Crow, University of Missouri-Rolla; Professor Malik Eibulok, University of Akron; Professor A. A. El-Keib, University of Alabama; Professor F. P. Emad, University of Maryland; Professor L. L. Grigsby, Auburn University; Professor Kwang Y. Lee, Pennsylvania State University; Professor M. A. Pai, University of Illinois-Urbana; Professor E. K. Stanek, University of Missouri-Rolla.

My sincere thanks to Lynn Kallas, who proofread the early version of the first edition. Special thanks goes to the staff of McGraw-Hill: Lynn Cox, the editor, for her constant encouragement, Nina Kreiden, editorial coordinator, for her support, and Eve Strock, senior project manager, for her attention to detail during all phases of editing and production of the first edition. Special thanks goes to Mary Coman of McGraw-Hill for publication of the second edition.

I wish to express my thanks to the Electrical Engineering and Computer Science Department of Milwaukee School of Engineering and to Professor Ray Palmer, chairman of the department, for giving me the opportunity to prepare the material for the first edition.

Last, but not least, I thank my wife Jila and my children, Dana, Fred, and Cameron, who were a constant and active source of support throughout the endeavor.

New to this Edition

The first edition of *Power System Analysis*, published in 1999, was one of the first texts on power systems that integrated *MATLAB* and *SIMULINK* accompanied by a Power System Toolbox for easily performing large-scale practical systems analysis and simulation. My goal has been to provide powerful software tools for students to

# THE POWER SYSTEM AND ELECTRIC POWER GENERATION

---

## CHAPTER I

---

Electric energy is the most popular form of energy, because it can be transported easily at high efficiency and reasonable cost.

The first electric network in the United States was established by Thomas Edison in 1882 at the Pearl Street Station in New York City. The station supplied DC power for lighting the lower Manhattan area. The power was generated by DC generators and distributed by underground cables. In the same year the first waterwheel-driven generator was installed in Appleton, Wisconsin. Within a few years many companies were established to produce energy for lighting - all operating under Edison's patents. Because of the excessive power loss,  $RI^2$  at low voltage, Edison's companies could deliver energy only a short distance from their stations.

With the invention of the transformer (William Stanley, 1885) to raise the level of AC voltage for transmission and distribution, and the invention of the induction motor (Nikola Tesla, 1888) to replace the DC motors, the advantages of the AC system became apparent and made the AC system prevalent. Another advantage of the AC system is that due to the lack of commutators in the AC generators, more power can be produced conveniently at higher voltages.

utility generation and capacity.

- Publicly owned utilities, which are nonprofit state and local government agencies and include Municipal, Public Power Districts, and Irrigation Districts, accounting for 10.7 percent.

- Federally owned utilities, such as the Tennessee Valley Authority, Bonneville Power Administration, and U.S. Army Corps of Engineers; they account for 8.2 percent.

- Cooperatively owned utilities, owned by rural farmers and communities, which provide service mostly to members; they account for 3.1 percent.

Nonutilities, which generate power for their own use and/or for sale in wholesale power markets, accounts for about 11.9 percent. The above information is obtained from the Energy Information Administration, Share of Utility and Nonutility Nameplate Capacity by Ownership Category for 1998.<sup>1</sup> Today, Investor-Owned Utilities still produce most of the U.S. electrical power, but this is gradually changing as the independent power producers are a growing segment in the industry.

The transmission system of electric utilities in the United States and Canada is interconnected into a large power grid known as the North American Power Systems Interconnection. The power grid has evolved into three major separated areas. These are:

- The Eastern Interconnection, which includes all the eastern and central states. The Eastern Interconnection is tied to both of the other major interconnections via high voltage DC transmission facilities and also has ties to the northern Canada system.
- The Western Interconnection, comprised of Western North America, from the Rocky Mountains to the Pacific coast. It is tied to the Eastern Interconnection at six points, and also has ties to systems in northern Canada and Northwestern Mexico.

- The Texas Interconnection, which includes most of the state of Texas. It is tied to the Eastern Interconnection at two points, and also has ties to systems in Mexico.

Each area consists of several neighboring utilities which operate jointly to schedule generation in a cost-effective manner. These electrically separate areas import and export power to each other but are not synchronized electrically. Two separate levels of regulation currently regulate the United States' electric system.

<sup>1</sup>[http://www.eia.doe.gov/cneaf/electricity/chg-stru-upd/elec/chapter3\\_2.html](http://www.eia.doe.gov/cneaf/electricity/chg-stru-upd/elec/chapter3_2.html)

The first single-phase AC system in the United States was at Oregon City, where power was generated by two 300 hp waterwheel turbines and transmitted at 4 kV to Portland. Southern California Edison Company installed the first three-phase system at 2.3 kV in 1893. Many electric companies were developed throughout the country. In the beginning, individual companies were operating at different frequencies, ranging from 25 Hz to 133 Hz. But, as the need for interconnection and parallel operation became evident, a standard frequency of 60 Hz was adopted throughout the U.S. and Canada. Most European countries selected the 50-Hz system. Transmission voltages have since risen steadily, and the extra high voltage (EHV) in commercial use of 765 kV was first put into operation in the United States in 1969.

For transmitting power over very long distances it may be more economical to convert the EHV AC to EHV DC, transmit the power over two lines, and invert it back to AC at the other end. Studies show that it is advantageous to consider DC lines when the transmission distance is 500 km or more. DC lines have no reactance and, for the same conductor size, are capable of transferring more power than AC lines. DC transmission is especially advantageous when two remotely located large systems are to be connected. The DC transmission line acts as an asynchronous link between the two rigid systems, eliminating the instability problem inherent in the AC links. The main disadvantages of the DC link are that the production of harmonics requires filtering, and that a large amount of reactive power compensation is required at both ends of the line. The first  $\pm 400$ -kV DC line in the United States was the Pacific Intertie, spanning 850 miles from Oregon to California, built in 1970.

The entire continental United States is interconnected by an overall network called the *power grid*. A small part of the network is federally and municipally owned, but the bulk is privately owned. The system is divided into several geographical regions called *power pools*. In an interconnected system, fewer generators are required as a reserve for peak load and spinning reserve. Also, interconnection makes the energy generation and transmission more economical and reliable, since power can readily be transferred from one area to others. At times, it may be cheaper for a company to buy bulk power from neighboring utilities than to produce it in one of its older plants.

## 1.2 ELECTRIC INDUSTRY STRUCTURE

Generation of electricity in the United States is performed by two types of companies—utilities and nonutilities. Utilities are further classified into four subcategories:

- Investor-owned utilities (IOUs), which account for about 66.1 percent of all

### 1.3 ENERGY RESOURCES FOR ELECTRICITY GENERATION

Sources of energy to generate electricity are fossil fuels, uranium, water, wind, solar, geothermal, biomass, fuel cell, and, occasionally, oil. Each system has advantages and disadvantages, but many of the systems pose environmental concerns. With today's emphasis on environmental consideration and conservation of fossil fuels, many alternative sources are being considered for employing the untapped energy sources of the sun and the earth for generation of power. These energy sources are inexhaustible and are known as *renewable energy sources*. These include energy from water, wind, the sun, geothermal sources, and biomass sources such as energy crops. Renewable energy sources have the potential to provide electricity to homes and businesses without causing air pollution or depleting nonrenewable resources. Due to extensive efforts to reduce the greenhouse gas emissions in recent years, the production of electric energy from renewable sources has become the fastest-growing source of electricity generation, and several utilities have abandoned plans for coal plants. Undoubtedly, if this trend continues, it will leave a good portion of dirty coals in the mines within two decades. With the advent of electric utility deregulation, renewable energy sources are being utilized in the development of small power stations, which are directly connected to the consumers, as well as large scale utility power plants.

According to the U.S. Energy Information Administration Existing Generating Capacity,<sup>2</sup> the total installed electric generating capacity in 2008 was about 1,010,171 MW. The estimated United States population in 2008 was 304,060,000. That is,

$$\text{Installed capacity per capita} = \frac{1,010,171 \times 10^6}{304,06 \times 10^6} = 3,322 \text{ W}$$

Assuming uniform distribution, this is equivalent to 4.45 horsepower (HP) per person. In 2008, the annual kWh consumption in the United States was about  $4,119 \times 10^9$  kWh. The latest figures for the total generation shares by energy source for the United States, published by the U.S. Energy Information Administration (EIA) and released in March 15, 2010 in its "Electric Monthly Report,"<sup>3</sup> are presented in Figure 1.1. This shows that the nonrenewable fossil fuels, namely coal, natural gas, nuclear, and oil, accounts for about 89.2 percent of the United States' requirements for electricity generation in 2009. Approximately 44.7 percent is generated from coal, 20.2 percent from nuclear, 23.3 percent from natural gas, and 1 percent from oil.

The combustion of coal produces carbon dioxide, sulfur dioxide, nitrogen oxide, and ozone as well as fine particles which are then released into the atmosphere.

<sup>2</sup><http://www.eia.doe.gov/cneaf/electricity/epa/epatl1.html>  
<sup>3</sup><http://www.eia.doe.gov/cneaf/electricity/cpm/epm-sum.html>

One is the Federal Energy Regulatory Commission (FERC), which regulates the price of wholesale electricity, service terms, and conditions. The other is the Securities and Exchange Commission (SEC), which regulates the business structure of electric utilities. Under provisions of the U.S. Energy Policy Act of 2005, FERC has designated the North American Electric Reliability Corporation (NERC) to be responsible for maintaining system standards and reliability. NERC works cooperatively with every provider and distributor of power to ensure reliability. NERC coordinates its efforts with FERC as well as other organizations such as the Edison Electric Institute (EII).

The electric power industry in the United States has undergone fundamental changes in the last two decades. The generation business is rapidly becoming market-driven. This is a major change for an industry where power generation was once dominated by large, vertically integrated monopolies. The implementation of open transmission access has resulted in many new companies that produce and sell power wholesale. These new companies are in direct competition with the traditional electric utilities. As of 2008, retail electricity customers in 14 states can now choose their electricity company, and the retail price of electricity is determined through a competitive bidding processes. However, retail competition has resulted in higher prices for residential customers in several states. The transition to retail competition has not been smooth and uniform. In particular, the flaws in the partially deregulated California energy system allowed independent producers to manipulate prices, and other companies, such as Enron, preyed on California's energy crisis by gaming the market, artificially creating shortages and using sham trades to drive up prices astronomically.

In the future, utilities may possibly be divided into power generation, transmission, and retail segments. Generating utilities would sell directly to customers instead of to local distributors. This would eliminate the monopoly that distributors currently have. The distributors would sell their services as electricity distributors instead of being a retailer of electricity itself. The retail structure of power distribution would resemble the current structure of the telephone communication industry. Extensive efforts are being made to create a more competitive environment for electricity markets in order to promote greater efficiency. Thus, the power industry faces many new problems, with one of the highest priority issues being reliability—that is, bringing a steady, uninterrupted power supply to all electricity consumers. The restructuring and deregulation of electric utilities, together with recent progress in technology, introduce unprecedented challenges and opportunities for power systems research and open up new opportunities to young power engineers.

(sequestration) technologies. Renewable Portfolio Standard (RPS) and Renewable Fuel Standard (RFS) policies adopted by many states are efforts to mitigate carbon dioxide emissions (USDOE/EIA 2008). RFS requires electric power facilities to use a minimum percentage of their fuel from renewable sources, and RFS requires blending of renewable fuels (ethanol) with gasoline at specified minimum levels. The efficiency of these systems can be improved by cogeneration (combined heat and power) methods. Process steam can be extracted from steam turbines. Waste heat produced by thermal generating stations can be used for space heating of nearby buildings. By combining electric power production and heating, less fuel is consumed, thereby reducing the environmental effects compared with separate heat and power systems. Multiple technologies for carbon dioxide capture are available; most of them can be classified into three main groups:

- Post-combustion:  $CO_2$  capture from the flue gas after combustion of the fossil fuel.
- Pre-combustion: Removal of  $CO_2$  from the fossil fuel prior to combustion.
- Oxy-fuel: Combustion of fossil fuel with pure oxygen rather than air.

Post-combustion involves capturing the carbon dioxide produced by the combustion of coal immediately before it enters the stack and storing it underground. Post-combustion capture technology can be added to existing coal or gas power plants without modifying the original power plant. Pre-combustion is a process where carbon in the fuel is separated or removed before the combustion process. Instead of coal or natural gas being burned in a combustion plant, the fuel can be converted to hydrogen and  $CO_2$  prior to combustion. The  $CO_2$  can then be captured and stored, while the hydrogen is combusted to produce power. Pre-combustion capture technology is applicable only to new fossil fuel power plants because the capture process requires strong integration with the combustion process.

Oxy-fuel combustion with  $CO_2$  capture is very similar to post-combustion  $CO_2$  capture. The main difference is that the combustion is carried out with pure oxygen instead of air. As a result the flue gas contains mainly  $CO_2$  and water vapor, which can be easily separated. The challenge is that it is expensive to produce pure oxygen.

Nuclear and most renewable energy sources do not have direct  $CO_2$  emissions. Biomass fuels do emit  $CO_2$  upon combustion; however, the carbon is re-absorbed by growing plants over the lifecycle of biofuel production. To reduce greenhouse gas emissions, many states have adopted a Renewable Portfolio Standard policy, requiring electric power facilities to reach a specified minimum percentage of their fuels from renewable resources by a certain date (USDOE/EIA

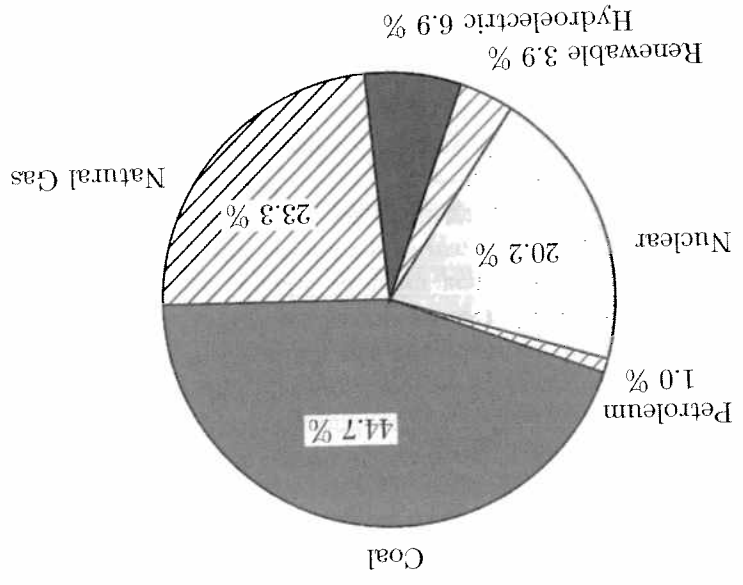


FIGURE 1.1 Net generation shares by energy source, year-to-date through December, 2009.

An increase in the earth's levels of atmospheric carbon dioxide increases the greenhouse effect and contributes to global climate change. Sulfur dioxide and nitrogen oxides contribute to smog and acid rain. Coal also contains traces of toxic heavy elements such as mercury, arsenic, and dilute radioactive material. Burning them in very large quantities releases this material into the environment and leads to low levels of radioactive contamination, the levels of which are, ironically, higher than that of a nuclear power station. Coal has the highest carbon intensity among fossil fuels, resulting in coal-fired plants having the highest output rate of  $CO_2$  per kWh. In 2007, the national average output rate for coal-fired electricity generation was 2.16 pounds  $CO_2$  per kWh. Natural gas is the least carbon-intensive fossil fuel, and the output rate for  $CO_2$  from natural gas-fired plants in 2007 was 1.01 pounds  $CO_2$  per kWh. Renewed interest in environmental issues has raised concerns about electric power plants burning coal, resulting in the application of cleaner coal technologies to reduce the emission of mono-nitrogen oxides, sulfur dioxide, and carbon dioxide. Buildup of human-related greenhouse gases has become an issue of concern because of its negative effects on the environment, human health, and economic well-being of all the people of the world. The Intergovernmental Panel on Climate Change (IPCC) has set the goal of reducing carbon dioxide to 350 ppm (parts per million) by 2025. Nations and the international community as a whole are trying to bring the rate of increase in emissions of greenhouse gases under control, or to mitigate the effects of such emissions through carbon capture and storage

2007). Presently, 30 states (including the District of Columbia) have adopted a mandatory Renewable Portfolio Standard (RPS), and four states have voluntary goals. The mandated goals for renewable resource use ranges from 4 percent in 2009 in Massachusetts to 25 percent in 2025 in Illinois, Minnesota, and Oregon (USDOE/EIA 2007). It is assumed that the use of renewable energy resources will reduce greenhouse gas emissions. Renewable Portfolio Standards may bring economic and health benefits such as job creation and cleaner air, in addition to reducing greenhouse gas emissions (FEW Center 2008).

The source of the mechanical power, commonly known as the *prime mover*, is steam turbines, whose energy comes from the burning of coal, gas and nuclear fuel, gas turbines, hydraulic turbines at waterfalls, or, occasionally, internal combustion engines burning oil. Today many other technologies are used to generate electricity, using wind energy, solar energy, geothermal energy, and biomass.

#### 1.4 FOSSIL FUEL POWER PLANTS

Most electricity today is generated by burning coal and producing steam, which is then used to drive a steam turbine that, in turn, drives an electrical generator. Steam turbines operate at relatively high speeds of 3600, or 1800 rpm for 60 Hz operation. The generators to which they are coupled are cylindrical rotor, two-pole for 3600 rpm or four-pole for 1800 rpm. The following sections describe various types of fossil fuel power plants and power plants using renewable sources of energy.

##### 1.4.1 COAL-FIRED POWER PLANTS

In a coal-fired plant, the burning of pulverized coal, natural gas, or oil in a huge boiler produces high-pressure and high-temperature steam. The steam flows through a series of turbines, where part of the heat energy is converted to mechanical energy that spins a coupled generator. The generator converts the mechanical energy into electrical energy. A simplified diagram of a conventional coal-fired steam generator is shown in Figure 1.2. In coal-fired power plants, the delivered coal is crushed into small pieces and then transported to storage silos. The coal from storage silos is fed into a pulverizer that grinds the crushed coal into powder and mixes it with primary combustion air. The pulverized coal is fed to the boiler by means of a conveyor belt. The coal is burnt in the boiler, and the heat produced is used to produce steam at high temperature and pressure. The steam produced in the boiler is dried and superheated by the flue gases on the way to the chimney. Superheated steam from the superheater flows through a control valve into the high-pressure turbine (HP). The control valve regulates the steam flow in accordance with the power output needed from the plant. The exhaust steam from the high pressure turbine returns to

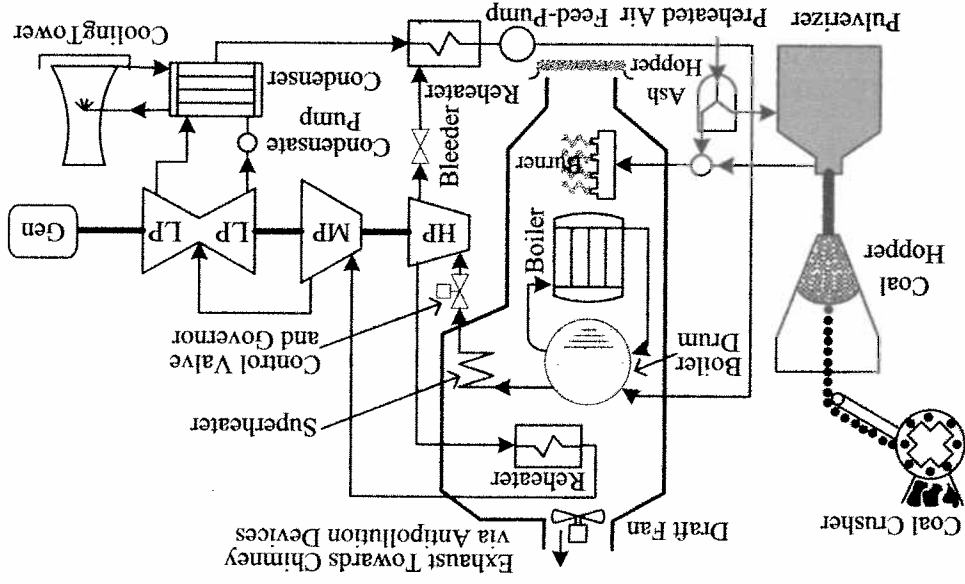


FIGURE 1.2 Simplified diagram of a conventional coal-fired steam generator.

A steam turbine power plant operates on a Rankine cycle, and its efficiency is governed by the laws of thermodynamics and is fundamentally limited by the ratio of the absolute temperatures of the steam at turbine input and output. A low output temperature results in a large amount of heat loss at the condenser. Due to the chimney. Other antipollution apparatus, and they are exhausted into the atmosphere through carries the gases and other products of combustion towards stack scrubbers and water before it passes through the condenser tubes. Finally, an induced draft-fan these installations, the water is filtered to remove debris and aquatic life from the ing towers. Some power plants use river water or lake water as cooling water. In water from the condenser through natural draft, forced draft, or induced draft cool- the circulating cooling water in the condenser is removed by pumping the warm a feed water pump and the boiler to start the process over. The heat absorbed by exhaust steam from the turbines is fed to a condenser, which condenses the steam back into water. A condensate pump drives the water through a reheater towards the turbines is used to heat the deaerator and the boiler feedwater preheaters. The the low pressure turbines flows into the condenser. A small fraction of steam from the medium-pressure turbine (MP). The exhaust steam from the medium-pressure turbine flows directly into the low-pressure turbines (LP), and the exhaust steam from the steam generator's reheating tubes, where it is reheated before it flows into the

additional heat losses at other parts of the plant, the overall efficiency is quite low. In practice, the steam power plant consists of several other components and stages to achieve higher thermal efficiencies. With higher quality coals, such as lignite coal and bituminous coal, the most advanced steam power plants have efficiencies approaching 43 to 45 percent.

Coal-fired power plants are major emitters of carbon dioxide, which has become an issue of concern because of its negative effects on the environment and human health. If limits on carbon dioxide emissions are imposed via a carbon tax or a cap-and-trade system, the operating cost of fossil-fuel-based power plants would increase. Electric utilities are already backing away from coal and turning to clean, renewable sources of energy, such as wind, solar, and geothermal.

#### 1.4.2 GAS TURBINE POWER PLANTS

Gas turbine plants operate on the Brayton cycle. They use a compressor to compress the inlet air. The compressed air is preheated in a regenerator by using heat from the exhaust gas. The hot and high pressure air is then admitted to the combustion chamber produces hot gases from the combustion of natural gas. The hot combustion gas is then passed to the gas turbine where it expands and does the mechanical work. The gas turbine drives a coupled generator, which converts the mechanical energy into electrical energy. The schematic diagram of a simple gas turbine power plant is shown in Figure 1.3.

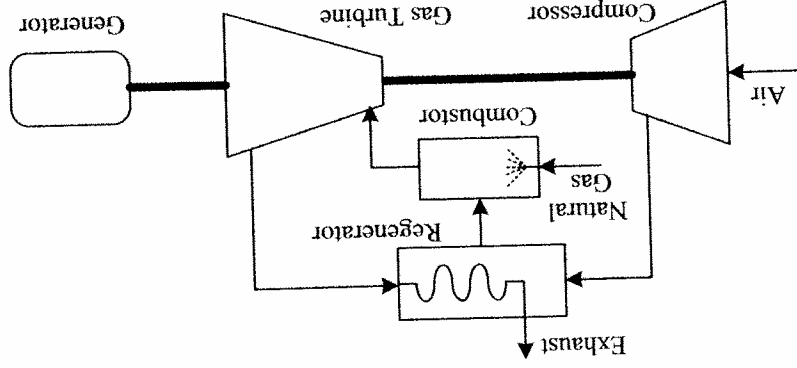


FIGURE 1.3

Schematic diagram of a simple gas turbine power plant.

Before the turbine is started, the compressor is started by a motor mounted on the same shaft and powered by batteries. Once the unit starts, the external power is not needed as the turbine itself supplies the necessary power to the compressor. The compressor typically uses about 50 percent of the power produced by the turbine.

The gas turbine power plant is simple in design and requires a smaller investment and operating cost as compared to steam power stations. It requires much less water compared to steam power plants, and there are no standby losses. Modern large gas turbine plants have 35–40 percent thermal efficiency. In addition to using the exhaust gas to heat air from the compressor, various methods have been used to improve the efficiency of gas turbines, mainly by increasing turbine entry gas temperatures, increasing the efficiency and capability of the compressor, and using materials with improved heat-resisting properties. The most efficient turbines have reached 46 percent efficiency.

A gas turbine power plant can supply the grid with electric power within minutes after start-up. They are usually used during peak demand and for standby plants for hydro-electric stations, and they operate very cost-effectively for these short periods of time. In areas with a shortage of base load, a gas turbine power plant may operate continuously. This type of power station is usually used in conjunction with the heat recovery system generator (HRSG) for a combined cycle or cogeneration power plant. The main difference in these plants is that in a combined cycle power plant, the steam generated in the HRSG is used for the production of power, while in a cogeneration plant, the steam can be used for heating and industrial processes as well as power production. Both methods are useful in increasing the overall efficiency of the generating plant. The next section describes the simple combined-cycle power plant.

#### 1.4.3 COMBINED-CYCLE POWER PLANTS

The combined-cycle power plant consists of a gas turbine (Brayton cycle) in conjunction with a steam turbine (Rankine cycle). In the gas turbine, the output temperature of the flue gas is quite high, in the range of 900°F to 1200°F. This is high enough to generate steam by passing it through a heat recovery steam generator (HRSG), which is used as input heat to the steam turbine power plant. Combined-cycle power plants have become a popular generation scheme in recent years. Typically, the gas turbine produces about 65 percent of the power, and the steam turbine produces about 35 percent. A large set would be a 400 MW gas turbine coupled to a 200 MW steam turbine, giving 600 MW. A typical power station might be comprised of between two and six such sets. The overall thermal efficiency of combined-cycle plants built today is more than 60 percent. The schematic diagram of a combined-cycle power plant is shown in Figure 1.4.

#### 1.4.4 NUCLEAR POWER PLANTS

Nuclear power accounts for 20 percent of all power generation in the United States. Nuclear power plant is a steam power plant, except that the boiler is replaced by a clear power plant without emitting any significant pollution or greenhouse gases into the air. A nu-

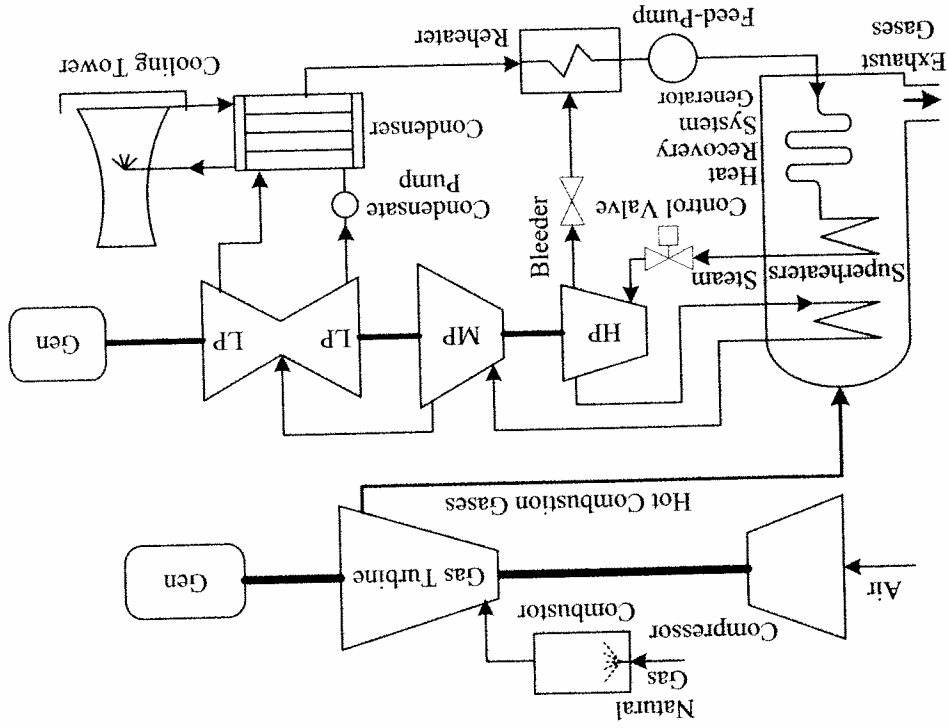


FIGURE 14 Schematic of a combined-cycle power plant.

nuclear reactor. There are many different types of reactors, using different fuels and coolants. Two types of light water reactors commonly used in a nuclear power plant are the *boiling-water reactor (BWR)*<sup>4</sup> and the *pressurized-water reactor (PWR)*.<sup>5</sup> In a BWR, water boils inside the reactor itself, and the steam is used directly to drive the turbine. In a PWR, the superheated water in the primary cooling loop is used to transfer heat energy to a secondary loop for the creation of steam. Approximately two-thirds of the reactors in the United States are pressurized-water reactors, and one-third are boiling-water reactors.

A schematic diagram of a pressurized-water reactor is shown in Figure 1.5. The reactor, the pressurizer, and the heat exchanger, known as the *steam generator*, are placed inside a dome-shaped containment building made of extremely thick concrete and dense steel. The reactor core in which nuclear fission chain reactions are initiated and controlled is placed in a heavy steel, pressurized vessel. The reac-

<sup>4</sup><http://www.eia.doe.gov/cneaf/nuclear/page/nuc reactors/bwr.html>  
<sup>5</sup><http://www.eia.doe.gov/cneaf/nuclear/page/nuc reactors/pwr.html>

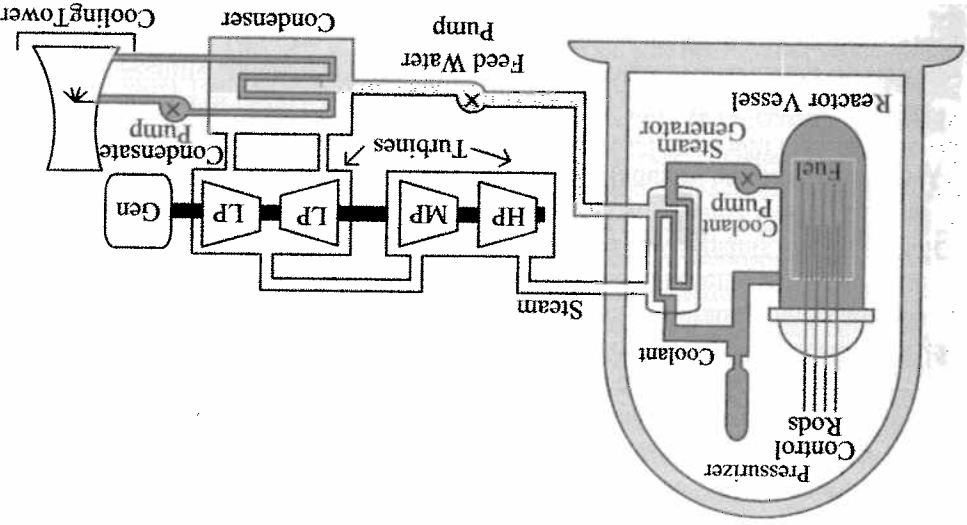


FIGURE 1.5 Schematic diagram of a pressurized water reactor.

tor core contains the fuel rods and assemblies, the control rods, the moderator, and the coolant. The fuel consists of uranium oxide pellets sealed in long metal tubes. The tubes are grouped into large bundles called *fuel assemblies*. Control rods that are made of very efficient neutron absorbers are placed among the fuel assemblies. The control rods control the speed of the nuclear chain reaction, by sliding up and down between the fuel assemblies and regulating the generator power output. Another component of the reactor is the moderator. The moderator serves to slow down the neutrons released from fission so that they cause more fission. In light water reactors, the water moderator functions also as a primary coolant. Outside the core are the turbines, the heat exchanger, and part of the cooling system.

During the operation, the coolant water circulates through the reactor core and absorbs the heat produced by the fission process. A pressurizer is used in order to prevent the generation of steam in the primary loop. The hot water is pumped from the reactor vessel to a steam generator, where the heat is transferred to a secondary water loop. The reactor coolant pumps keep the coolant circulating back to the reactor vessel to collect more heat and deliver it to the steam generator.

The pressure in the secondary loop is kept sufficiently low, allowing the water to boil and produce steam. The steam runs the turbine generator to produce electricity. The exhaust steam from the turbines is fed to a condenser, which condenses the steam back into water. A condensate pump drives the water towards the steam generator to start the process over. Capital costs of nuclear power plants are



greater than those for coal-fired power plants, but they are cost competitive, even with decommissioning and waste disposal costs. The efficiency of nuclear power plants is limited to around 33 percent, because water can be heated only to a certain temperature, and only a certain amount of heat can be taken out of water.

The U.S. electricity demand is projected to increase by 50 percent over the next 25 years. Therefore, new base-load electric power-generating plants are needed to meet this demand. Since governmental regulations will eventually be adopted to mitigate the carbon dioxide emissions from coal-fired power plants, the larger-scale solutions will have to come from nuclear fuel, as it can provide a large amount of power without emitting carbon dioxide. The nuclear industry has developed several advanced reactor designs which can recycle nuclear fuel. This will significantly reduce the amount of radioactive waste to be disposed. Also, the researchers at MIT<sup>6</sup> have successfully shown that replacing the water coolant with a nanofluid will significantly increase the efficiency and improve the plant safety of nuclear power plants. This will make nuclear power a sustainable energy source that reduces the carbon emissions and increases energy independence of the United States.

### 1.5 ELECTRIC POWER GENERATION FROM RENEWABLE ENERGY SOURCES

Water energy has been the most widely used form of renewable energy for the production of electricity. With today's emphasis on environmental considerations and conservation of fossil fuels, other renewable resources are being used to employ the energy sources of the sun and the earth for electricity generation. Some of these resources that represent a viable alternative to fossil fuels are solar power, wind power, geothermal, biomass, and tidal power. These resources, especially solar power and wind power, have the capability to produce sustainable energy indefinitely with no direct emission of pollutant and greenhouse gases. Power plants using these renewable sources of energy are described in the following sections. The aspiration for bulk generation of power in the future is nuclear fusion. If nuclear fusion is harnessed economically, it would provide clean energy from an abundant source of fuel, namely water.

Hydropower is considered to be a renewable energy source because it uses the continuous flow of water without using up the water resource. It is also nonpolluting, since it does not rely on burning fossil fuels. Hydropower is currently the leading

<sup>6</sup><http://web.mit.edu/erc/spolights/nano-nuclear.html>

renewable energy source in the United States. In 2009, it accounted for about 63 percent of all other renewable energy sources, such as wind, solar, and biomass. Reclamation<sup>7</sup> is the nation's second largest producer of hydroelectric power, with 58 hydroelectric power plants and 194 generating units in operation and an installed capacity of 14,693 MW. Almost all suitable sites for dams have already been developed, so there is not much scope for further growth in water power. However, there are numerous areas where research can lead to increases in the efficiency and reliability of hydroelectric plants and decreases in maintenance costs. Presently, wind and solar energy are growing at a rapid rate, and in a near future they will be the major sources of renewable energy for production of electric power.

The hydroelectric power plants usually require a dam to store water, a penstock for delivering the falling water, electric generators, a valve house which contains the main sluice valves, automatic isolating valves, and related control equipments. Also, a surge tank is located just before the valve house to protect the penstock from a pressure surge, called *water hammer*, in case the turbine gates are suddenly closed. In addition to electric energy production, most dams in the United States are built for other uses, including recreation, irrigation, flood control, and public water supply. A schematic diagram of a hydroelectric power plant is shown in Figure 1.6.

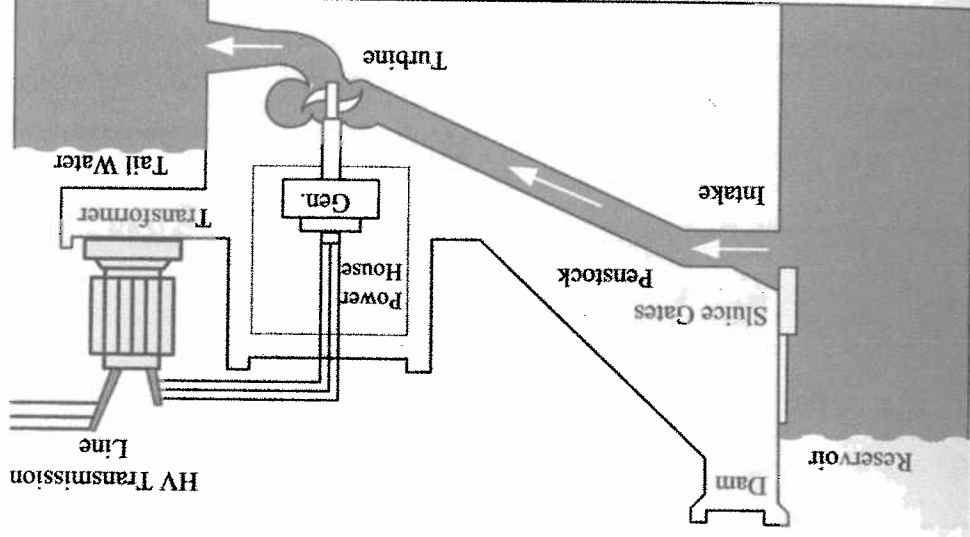


FIGURE 1.6 Schematic diagram of a hydroelectric power plant.

The water from the dam is led to the water turbine through the penstock, and

<sup>7</sup>[http://www.usbr.gov/power/data/role\\_rpl.html#power](http://www.usbr.gov/power/data/role_rpl.html#power)

The potential energy of the elevated water is transformed into kinetic energy. The water turbine converts hydraulic energy into mechanical energy, and the generator converts mechanical energy into electrical energy. After passing through the turbine, the water reenters the river on the downstream side of the dam. The most significant operating characteristics of hydropower plants are rapid start-up and loading, long life, and low operating and maintenance costs. Hydraulic turbines, particularly those operating with a low pressure, operate at low speed. Their generators are usually salient-type rotor with many poles. To maintain the generator voltage frequency constant, the turbine must spin the generator at a constant speed given by

$$n = \frac{120f}{p} \quad (1.1)$$

where  $f$  is the generated voltage frequency and  $p$  is the number of poles of the generator.

Elaborate control schemes are used to regulate the flow of water in order to keep the turbine speed constant.

The potential energy of the water in the reservoir is proportional to the mass of water and the difference in height between the water impoundment and the water outflow. This height difference is called the *head* or *effective head*. That is,  $P.E. = mgh$ . The mass of water is its volume times its density. Therefore,  $P.E. = volume \times \rho gh$  and the available hydro power becomes

$$P_w = \frac{P.E.}{t} = \frac{volume}{t} \rho gh \quad (1.2)$$

or

$$P_w = q\rho gh \quad W \quad (1.3)$$

$q$  = rate of flow of water in  $m^3/s$   
 $h$  = effective head of water in m  
 $\rho$  = density of water  $\approx 1000 \text{ kg/m}^3$   
 $g$  = acceleration of gravity =  $9.81 \text{ m/s}^2$

Given  $\rho = 1000$ , the available hydro power  $P$  in kW is given by

$$P = 9.81qh \quad \text{kW} \quad (1.4)$$

If  $\eta$  is the overall efficiency of the hydropower plant, the electrical power output in kW is

$$P_o = 9.81qh\eta \quad \text{kW} \quad (1.5)$$

where  $\eta = \eta_p \eta_g$

$\eta_p$  = penstock efficiency,  $\eta_t$  = turbine efficiency,  $\eta_g$  = generator efficiency

There are three basic types of hydraulic turbines. Propeller or axial turbines are used for low heads (10 to 100 ft). A Kaplan turbine is a propeller turbine with variable-pitch blades that can be adjusted to give high efficiency during light loads. The Francis turbine is one of the more common radial turbines used for medium heads (15 to 1500 ft). Fixed-blade propeller turbines and Francis turbines have relatively low efficiencies at light loads. The Pelton wheel is an impulse hydraulic turbine that is normally used for heads above 150 ft to very high heads. In general, the hydraulic turbine efficiency during normal operation is between 80 and 94 percent, and the generator efficiency is from 95 to 99 percent.

The Three Gorges hydroelectric power plant in China is the largest development of its kind in operation in the world. Presently, the installed capacity is 19,600 MW. When completed by the year 2011, the total electric generating capacity will be 22,400 MW. The largest installation in North America is at La Grande on James Bay in Canada with a total installed capacity of 7,326 MW. The largest installation in the United States is at the Grand Coulee Dam on the Columbia River in the state of Washington, with a total capacity of about 6,500 MW. A panoramic view of the Grand Coulee dam is shown in Figure 1.7. Powerhouse number three is located at the lower left side of the dam.

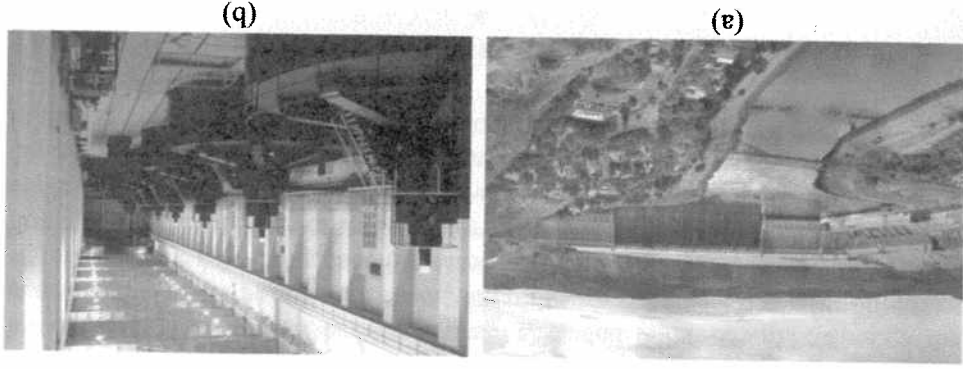


FIGURE 1.7

(a) Grand Coulee Dam, "Courtesy of Wikimedia Commons, Credit - Gregg M. Erickson,"

(b) Right Powerhouse Generators, "Courtesy of US Bureau of Reclamation, Credit - C. Hubbard,"

### Example 1.1

A large hydroelectric power plant has a head of 116 m and an average flow of  $3100 \text{ m}^3/s$  (about that of Grand Coulee). Assume the following efficiencies: penstock efficiency  $\eta_p=97$  percent, turbine efficiency  $\eta_t=77$  percent, and the generator efficiency  $\eta_g=95$  percent.

In pumped storage hydro, when the water is pumped uphill, more power is consumed than is generated when water is released back into the lower reservoir. However, because the hydroelectric plant can be started up and brought to full power within a few minutes, it can supply the temporary peak in demand. This procedure can effectively flatten variation in load, thus improving the overall economy and stability of the power grid. Pumped-storage hydro plants are becoming more important due to the increasing use of wind and solar power generation.

## 1.7 SOLAR POWER

Solar power makes use of the abundant energy of sunlight, and it has the potential to meet a significant portion of the future energy demands in an environmentally clean and cost-effective way. Solar energy can be converted directly into electricity by photovoltaic (PV), using the semiconductor materials in solar panels. A more economical method is using concentrating solar power (CSP) technologies. CSP technologies use mirrors to reflect and concentrate sunlight onto receivers that heat a working fluid to high temperatures. The resulting heat energy is used to power a steam turbine, driving a generator. Solar concentrators come in three main designs: parabolic troughs, parabolic dishes, and central receivers. Tracking techniques are required in these systems to ensure that the maximum amount of sunlight enters the concentrating system. In all of these systems, a working fluid is heated by the concentrated sunlight and is then used for power generation or energy storage.

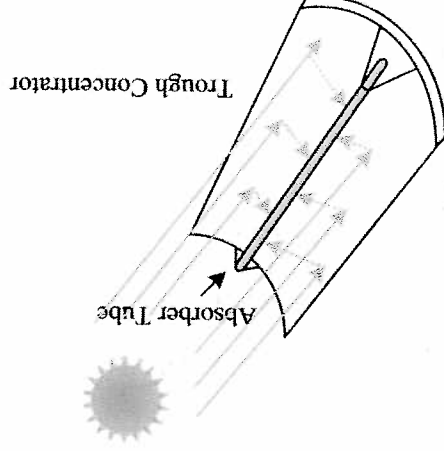


FIGURE 1.8 Schematic of a parabolic trough concentrator.

- (a) Calculate the generated electric power.  
 (b) Assuming the average household in America uses 10,960 kWh (including all transmission and distribution losses), approximately how many homes are supplied by this hydropower plant?

(a) From (1.5) the generated electric power is

$$P = (9.81)(3100)(116)(0.97)(0.77)(0.95) = 2,503,080 \text{ kW} = 2,503 \text{ MW}$$

(b) The annual energy production is

$$W = Pt = 2,503,080 \times 24 \times 365 = 21,926 \times 10^9 \text{ kWh}$$

Number of homes supplied is  $\frac{10,960}{W} = 2 \times 10^6 = 2$  million homes.

## 1.6.1 RUN-OF-THE-RIVER POWER PLANTS

Hydroelectric plants with no reservoir capacity are called *run-of-the-river plants*. These plants use the natural flow of rivers to capture the kinetic energy carried by water. Typically water is taken from a river and is led downhill through the penstock at a lower point to the power station's turbine to generate electricity. The water leaves the power plant and is returned to the river without altering the existing flow or water levels. Power stations on rivers with great seasonal fluctuations can experience significant fluctuations in power output and would require impounding the river to provide a steady water flow through the turbines. Such a system is relatively cheap and has very little environmental impact.

## 1.6.2 PUMPED-STORAGE HYDRO POWER PLANTS

Pumped-storage hydro power production is a means of saving surplus electricity during off-peak times as the potential energy of the elevated water. A typical pumped-storage development is composed of two reservoirs situated at two different elevations. At times of low electric demand, the synchronous machine is operated as a motor from the grid, which turns the hydro turbine in the reverse direction as a pump, and the water is transferred from the lower reservoir to the upper reservoir. When operating in this mode, the synchronous motor is usually overexcited, supplying reactive power<sup>8</sup> to the grid. When there is higher demand, water is released back into the lower reservoir through the turbine to generate power to supply the grid. This type of project is especially productive when there is a large difference in elevation between the upper and the lower reservoirs.

<sup>8</sup>Active and reactive powers are described in section 2.1-2.4.

1.7.1 PARABOLIC TROUGHS

A parabolic trough system consists of many long parallel rows of curved mirrors that concentrate light onto a receiver pipe positioned along the reflector's focal line, as shown in Figure 1.8. The troughs follow the trajectory of the sun by rotating along their axis to ensure that the maximum amount of sunlight enters the concentrating system. The concentrated solar radiation heats up a fluid circulating in the pipes, typically synthetic oil or molten salt, to temperatures of up to 750°F. The hot oil is pumped to heat exchangers to generate steam, which is used to drive a conventional steam turbine generator. A schematic diagram of a solar power plant using parabolic trough concentrators is shown in Figure 1.9.

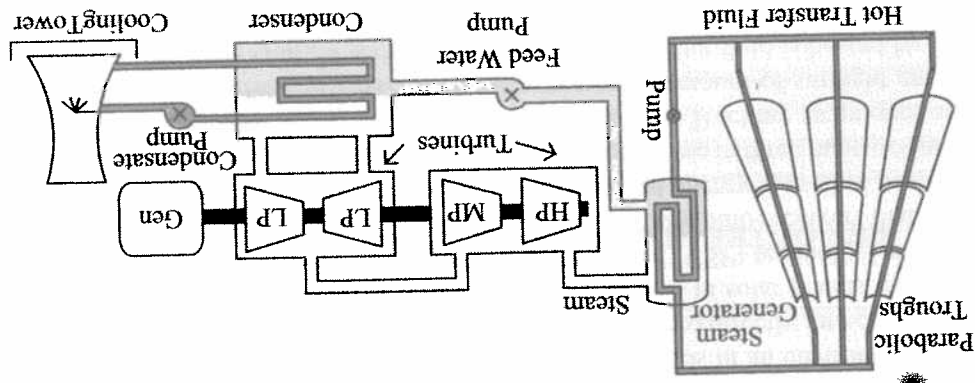


FIGURE 1.9 Schematic diagram of a solar power plant with parabolic trough concentrators.

Solar power is intermittent and is not available overnight; therefore, some solar power plants are designed to operate as hybrid solar/fossil plants. As hybrids, they have the capability to generate electricity during periods of low solar radiation. The new parabolic trough plants use molten salt for the heat transfer medium, which is cheaper and safer than oil. Also, because salts are an effective storage medium, the spare solar power is used in the form of heated molten salt in storage tanks, for use during periods when solar power is not available. This makes the CSP technology truly dispatchable. One of the largest parabolic trough power plants is the Solar Energy Generating Stations (SEGS) in California's Mojave Desert. It consists of nine solar power plants that have a combined capacity of 354 MW. Over the past 20 years these plants have delivered power with a high degree of reliability, and they continue to operate well in the Mojave Desert. A new solar project based on advanced parabolic trough technology and thermal storage using molten salts is being built

by Abengoa Solar, Inc., a Spanish firm under contract with Arizona Public Service (APS). This plant, called *Solana* (which means "a sunny place" in Spanish) Generating Station, has a capacity of 280 MW and is scheduled to go on-line in 2010. It is claimed that the plant will have the capacity to supply clean power to 70,000 homes and will eliminate around 400,000 tons of carbon dioxide. On May 22, 2009, APS and Starwood Energy Group announced plans for a 290 MW concentrating solar plant to be built in the Harquahala Valley, Arizona. The project will use the same parabolic trough technology with molten salt storage. The plant, scheduled for completion in 2013, will have the capacity to supply clean power to 73,000 homes. The largest solar thermal parabolic trough technology project proposed in the United States is the 553 megawatt Mojave Solar Park in the Mojave Desert, to be built by the Israeli company Solel for Pacific Gas and Electric in order to meet California's renewable energy laws, which require 20 percent of the power provided to be from renewable sources. Construction is scheduled to begin in 2009 and is due for completion in 2011.

1.7.2 PARABOLIC DISH CONCENTRATORS (DISH STRLING)

A parabolic dish concentrator consists of a parabolic dish-shaped mirror that reflects solar radiation onto a receiver located at the focal point of the dish. The dish structure is designed to track the sun on two axes, allowing the capture of solar energy at its highest density. A schematic of a parabolic dish concentrator is shown in Figure 1.10.

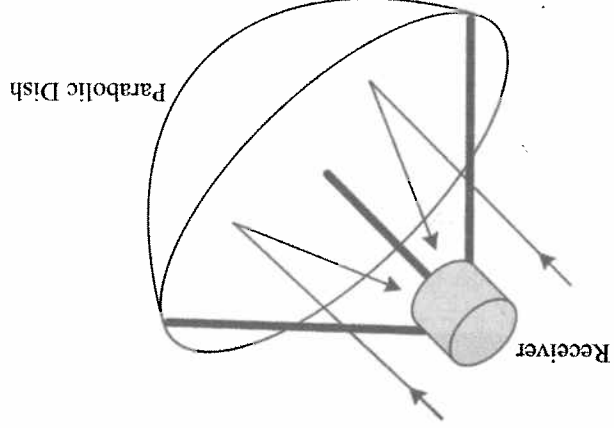


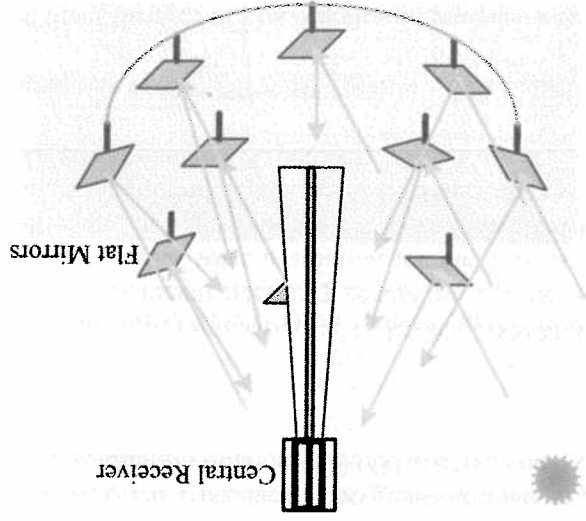
FIGURE 1.10 Schematic of a parabolic dish concentrator.

The solar dish concentration ratio is much higher than the solar trough, typ-

This system consists of a field of thousands of flat mirrors, called *heliostats*, shown

### 1.7.3 SOLAR TOWER

in Figure 1.12.



**FIGURE 1.12**  
Schematic of a solar tower.

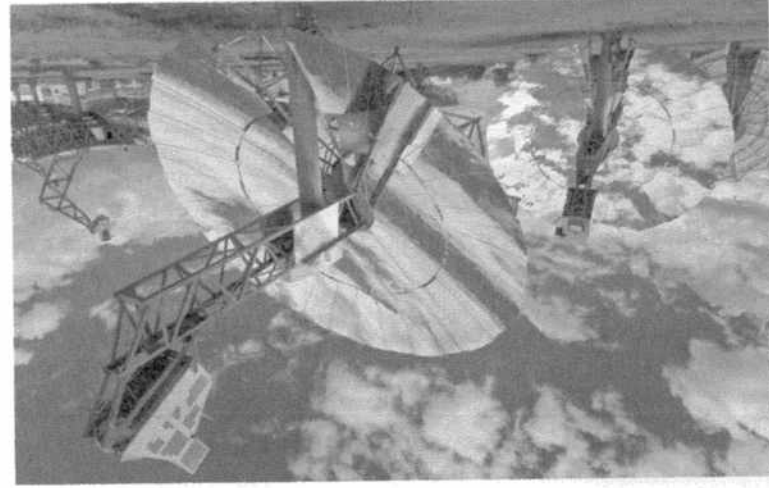
The computer-controlled mirrors track the sun and reflect the sunlight onto a central receiver mounted on top of a tower. A working fluid such as synthetic oil or molten salt circulates in the receiver, where it is heated to over 1300°F. The heated fluid is pumped to heat exchangers to generate steam, which is used to drive a conventional steam turbine generator located at the foot of the tower.

Among the earliest solar power towers were the 10 MW Solar One and Solar Two projects in the Mojave Desert. Solar One operated successfully from 1982 to 1988. Solar One was upgraded to Solar Two, which operated from 1996 to 1999. Solar Two's main objective, which was successfully met during the project, was to demonstrate advanced molten-salt power tower technology. On May 13, 2009, Pacific Gas and Electric Company (PG&E) announced that it had entered into a series of contracts with BrightSource Energy, Inc. for a record total of 1,310 MW of solar thermal power. This is considered one of the world's biggest solar projects. This project is to proceed in seven phases. The first of these solar power plants, sized at 110 MW and located in Ivanpah, California, is contracted to begin operation in 2012. Each plant will consist of thousands of computer-controlled heliostats that track the sun and reflect the solar energy onto a water boiler located on a central-ized power tower. The boiler heats the water inside to more than 1000°F, creating

One of the first commercial-scale Dish Stirling systems will be built near Phoenix, Arizona by Tessera Solar International. The 1.5 MW plant consists of sixty 25 kW units known as the *SunCatcher*, as shown in Figure 1.11.

The thermal receiver absorbs the solar energy and converts it to heat that is delivered to a Stirling engine, which is attached to the receiver. The engine is coupled to an electric generator to generate electricity. Similar to the parabolic trough, a concentrator can be made up of multiple mirrors that approximate a parabolic dish to reflect the solar energy at a central focal point.

The *SunCatcher* dish is formed into a parabolic shape using multiple arrays of curved glass mirrors. Each unit is designed to track the sun continuously and to reflect the solar energy onto a Power Conversion Unit (PCU) positioned at the focal point of the dish. The focused solar thermal energy heats up the hydrogen gas in Stirling engine. The engine then drives a generator to produce electricity. The main advantage of the solar dish technology is that the Stirling system requires no water for heating or cooling. The project will serve as a precursor to the deployment of much larger commercial projects to be developed by Tessera Solar and Stirling Energy Systems, including a 27 MW solar project for CPS Energy in West Texas and a 500 MW solar project for Southern California Edison.



**FIGURE 1.11**  
Dish Stirling systems – SunCatchers by Tessera Solar. "Courtesy of Sandia National Laboratories."

superheated steam. The steam is directly transported to a steam turbine generator to produce electricity. The high efficiency turbine uses air rather than water to cool the steam. Therefore, very little make-up water is required for the closed cycle, which can be a major advantage over other CSP technologies. Upon completion, all seven projects are expected to produce 3,666 gigawatt-hours of power each year, equal to the annual consumption of about 530,000 average homes.

### Example 1.2

From the Energy Information Administration's, October 2009 Monthly Energy Review, Table 7.2a,<sup>9</sup> the total annual electricity generation in 2008 was 4,110,259 × 10<sup>6</sup> kWh. The average annual solar irradiation for the U.S. is approximately 5 kWh/m<sup>2</sup>/day. If all electricity generation were to be generated by solar thermal power plants, estimate the minimum area needed. Assume the overall efficiency of all the solar thermal power plants is 20 percent.

The energy use per day is  $4,110,259 \times 10^6 / 365 = 1.126 \times 10^{10}$  kWh/day  
 The area required is  $(1.126 \times 10^{10}) / (5 \times 0.2) = 1.126 \times 10^{10} \text{ m}^2 = 4,347.9 \text{ mi}^2$ .  
 This is roughly ten times the area of Los Angeles. In practice, a much larger area is required to arrange heliostats in the solar field with adequate spacing. Additionally, more area is required for the necessary infrastructure.

## 1.7.4 PHOTOVOLTAIC ELECTRIC POWER PLANTS

Photovoltaic technology is the process that converts sunlight directly into electricity using semiconductor materials. When sunlight hits the surface of these materials, the solar energy causes the release of electrons. The cells are designed to channel the electrons into an electric field that flows in one specific direction and creates an electric potential. Photovoltaic (PV) cells with only one junction absorb a portion of the light spectrum. Large sets of PV cells are connected together and placed under non-reflective glass to form a flat panel PV. To give a desired electrical power output, a number of panels are connected together to form a PV array. Flat panel PVs can use both direct sunlight and the diffuse sunlight that is reflected from clouds. They are used for small applications, typically on rooftops. PV cells generate DC electricity, which varies by the amount of sunlight falling onto the modules. Inverters are used to convert the DC to a regulated AC. Solar installations in recent years are growing both for newly constructed homes and for pre-existing ones, using special inverters that are wired into the power grid. With the advent of a smart grid and monitoring, when more solar energy is collected than used, the energy returned could provide revenue for the user. With governments offering sev-

<sup>9</sup><http://www.eia.doe.gov/emeu/mert/elect.html>

eral tax incentives for non-polluting power, solar energy is becoming a more economically viable option. PV cells using multi-junction devices can achieve higher total conversion efficiency because they can convert a much larger spectrum of light to electricity. In addition, sunlight can be concentrated with mirrors or lenses so that smaller photocells can be used efficiently. Concentrating PV (CPV) uses large mirrors or lenses to concentrate and focus the direct sunlight onto a string of cells, resulting in a significant increase in the output power. The CPV structure is designed to track the sun on two axes, allowing the capture of solar energy at its highest density. The CPV is used for large electric power generation. A high-concentration photovoltaic system using optical lenses with a two-axis tracking mechanism made by Amonix for Arizona Public Service is shown in Figure 1.13.

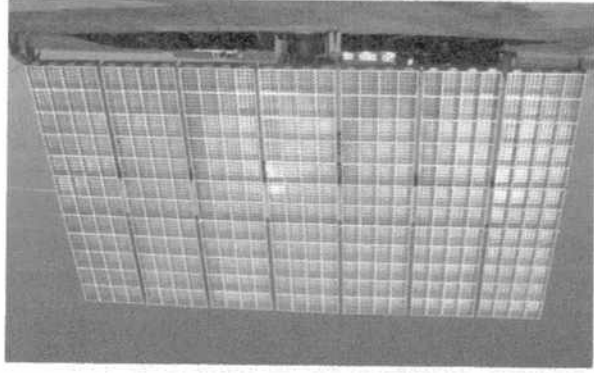


FIGURE 1.13

An Amonix high-concentration photovoltaic system. "Courtesy of DOE/NREL."

In recent years several photovoltaic power plants have been built and are in operation in several European countries, notably in Spain, which has 33 power plants with a total capacity of 650 MW, Germany with 8 power plants with a total capacity of 150 MW, and Portugal with two plants totaling 57 MW. Presently only three photovoltaic power plants are in operation in the United States. These are Nellis Solar Power Plant (14 MW), located within Nellis Air Force Base in Nevada, Alamosa photovoltaic power plant (8.2 MW), located in San Luis Valley, south central Colorado, and Semptra Generation PV Plant (12.6 MW) in Southern Nevada. Many large scale photovoltaic power plants are being developed and planned worldwide. There are eight large photovoltaic power plants being planned or under construction in the United States with a total capacity of 1,766.5 MW. The largest planned PV power plant (600 MW), called *Rancho Cielo*, will be built by Signet Solar in Belen, New Mexico. The next largest ones are a 550 MW plant to be built by First Solar, and a 250 MW plant to be built by Sun Power in Carrizo Plain, California for Pacific Gas and Electric. Also, Pacific Gas and Electric Com-

pany has entered into a contract with AV Solar Ranch, a subsidiary of NextLight Renewable Power, for a 230 MW solar photovoltaic power plant.

## 1.8 WIND POWER PLANTS

Since ancient times, people have harnessed wind energy to pump water or grind grain. Wind machines were used in Persia as early as 200 B.C. to grind grain, and the first practical windmills were built in Sistan, Iran in the 7th century. Wind-mills were used in 14th-century Holland to pump water. The first known functional electricity-generating windmill was a battery charging machine installed in 1887 by James Blyth in Scotland. The first use of a large windmill to generate electricity was in Cleveland, Ohio, in 1888, by Charles Brush. European countries, particularly Denmark, Germany, and France, continued the developments of large-scale wind turbines from the first quarter of the 20th century. Today, Denmark, Spain, the United States, and Germany are the top four worldwide suppliers in the wind turbine market.

Over the past decade, wind power has been the fastest growing form of generation in the United States and in other parts of the world. Like solar, wind is intermittent and is highly dependent upon weather and location. Since solar power and wind power can complement each other as energy sources, a hybrid solar-wind power system may be used for base-load generation. Such a system would be a viable alternative to fossil fuels.

### 1.8.1 TYPE OF WIND TURBINES

Modern wind turbines come in two types: Vertical axis, or "egg-beater" style, and horizontal axis, or "propeller" style. Vertical axis turbines are particularly suited to small wind power applications because they have a small environmental impact and make no noise. The horizontal axis turbine's blades rotate in a vertical plane about a horizontal axis, and the turbine is dynamically rotated on its tower by computer-controlled motors to face the wind. Most modern utility-scale turbines are horizontal-axis turbines. Two kinds of horizontal wind turbines commonly used for electric power generation are the fixed-speed and the variable-speed turbines. The rotor of modern wind turbines typically has three blades, which converts the energy in the wind to rotational shaft energy. The center of the rotor is connected to the turbine shaft, which turns a generator, usually through a gearbox. Larger wind turbines are often grouped together in the same location known as a *wind farm* to provide power to the electrical grid. Shown in Figure 1.14 is the Dillon Wind Farm consisting of 45 MHI 1.0 Mw horizontal wind turbines located in Palm Springs, California.

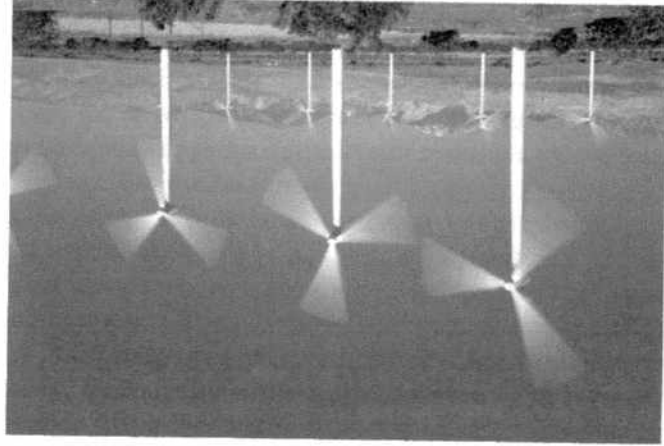


FIGURE 1.14

Dillon Wind Farm, "Courtesy of DOE/NREL, Credit - Iberdrola Renewables."

### 1.8.2 WIND POWER

Wind turbines convert the kinetic energy from the wind into mechanical energy, which is then used to drive a generator that converts this energy into electricity. From Newton's second law, the kinetic energy of a "packet" of air with mass  $m$  (kg) moving at a speed  $v$  (m/s) is

$$K.E. = \frac{1}{2}mv^2 \quad (1.6)$$

The mass of air passing through an area is the product of the area  $A$ , air density  $\rho$ , speed  $v$ , and the time  $t$ , or

$$m = A\rho vt \quad (1.7)$$

Substituting in for  $m$  in (1.6), we have

$$K.E. = \frac{1}{2}A\rho v^3 t$$

Since power is energy per unit time, the theoretical wind power is given by

$$P_w = \frac{K.E.}{t} = \frac{1}{2}A\rho v^3 \quad \text{W} \quad (1.8)$$

The blade of the turbine captures only part of the available wind energy, and the actual power extracted by a wind turbine is given by

$$P = C_p P_w = C_p \left( \frac{1}{2} A \rho v^3 \right) \quad \text{W} \quad (1.9)$$

Where  $C_p$  is the coefficient of performance or power coefficient known as *Betz Limit*. The German engineer Betz showed that the maximum power extracted from an air stream is 16/27 or 0.59 of the theoretical available power. The area of a horizontal axis wind turbine is given by  $A = (\pi/4)D^2$ , where  $D$  is the rotor diameter in m. Therefore, power extracted from wind is

$$P = \frac{8}{\pi} C_p \rho D^2 v^3 \quad \text{W} \quad (1.10)$$

Considering the mechanical losses of the gearbox, turbine blades, and the losses in the generator, and the inclusion of the corresponding efficiencies ( $\eta_g, \eta_b, \eta_g$ ), the net output power becomes

$$P_o = (\eta_g \eta_b \eta_g) \left( \frac{8}{\pi} C_p \rho D^2 v^3 \right) \quad \text{W} \quad (1.11)$$

Modern wind turbines operate with a power coefficient of about  $C_p \approx 0.4$ . The air density varies with the height and depends on the pressure and temperature. The air density at sea level, 1 atm (14.7 psi) and 60°F is 1.225 kg/m<sup>3</sup>. Equation (1.10) shows that the power is proportional to the cube of wind speed and to the square of the blade diameter. Therefore, when the wind speed doubles, the power increases by a factor of eight, and when the blade diameter is doubled, the power increases by a factor of four.

### Example 1.3

A very large horizontal wind turbine is mounted with its hub at 135 m and has a rotor diameter of 126 m. The turbine operates in an area with an average wind velocity of 12.5 m/s at 135 m altitude and an air density of 1.18 at 70°F. The turbine power coefficient is 0.46. The generator is directly coupled to the turbine without a gearbox. Assume the following efficiencies: turbine blade efficiency,  $\eta_b=94.6$  percent, and generator efficiency,  $\eta_g=96$  percent. Determine the power output.

From (1.11), we have

$$P = 0.946 \times 0.96 \times \frac{8}{\pi} \times 0.46 \times 1.18 \times 126^2 \times 12.5^3 = 6.0 \times 10^6 \quad \text{W} = 6 \text{ MW}$$

### 1.8.3 FIXED-SPEED WIND TURBINE

In a fixed-speed turbine, the machine is typically a squirrel cage induction generator. A gearbox connects the low-speed (15 to 60 rpm) turbine rotor to the generator, and rotor and steps up the speed to about 900 rpm for an 8-pole, 60 Hz generator, or to 1800 rpm for a 4-pole, 60 Hz generator. Turbine blades are turned, or pitched, out of the wind to control the rotor speed and keep the rotor from turning in winds

that are too high or too low to produce electricity. Privately owned isolated wind generators can turn at whatever speeds their owners choose. For operation with a grid system, the stator winding of a squirrel cage induction generator is directly connected to the grid via a transformer. The frequency of the grid determines the rotational speed of the generator. Thus, the wind turbine must run at constant speed as dictated by the grid frequency (60 Hz or 50 Hz) and the number of poles of the generator. The disadvantages of induction generators are high starting currents, which usually are smoothed by a thyristor controller, and their demand for reactive power, which can be compensated by installing shunt capacitors in the stator circuit. Also, a wound-rotor induction generator is used for direct connection to the grid. In this system, the three-phase external resistances connected to the rotor slip ring can be adjusted to control the speed over a limited range.

### 1.8.4 VARIABLE-SPEED WIND TURBINE

Due to advances in the electronic inverter systems, most new wind turbines operate at variable speeds, which allow a more efficient capture of wind. Wind turbines operating at variable speeds would produce a variable voltage and frequency output. Thus, the generator is decoupled from the grid, and a suitable power electronic interface is used that converts the generator output to the correct grid frequency and voltage. Several types of generators are used for variable speed turbines: doubly-fed wound-rotor induction generators, where only a part of the power production is fed through the power electronic converter, squirrel-cage induction generators and synchronous generators, where total power production must be fed through the power electronic interface.

**Doubly-fed wound-rotor induction generator** — Doubly-fed induction generators (DFIG) are the most common technology used by the wind turbine industry due to their flexible rotor speed with respect to the constant stator frequency. Figure 1.15 presents an overview of the DFIG system. The generator is connected to the turbine through a gearbox to adapt the low rotating speed of the wind turbine to the generator speed. The stator of the generator is directly connected to the grid via a transformer. The rotor windings are connected to the grid via slip rings and back-to-back voltage source converters and a transformer. With this arrangement, the energy is delivered to the power grid from both the stator and the rotor. Hence, this system is called “*doubly-fed*.” The power electronic converters enable DFIG to operate at optimal rotor speed and to maximize power generation by controlling the active and reactive power injected into the grid at constant voltage frequency.

When a fault occurs in the external grid, a voltage dip occurs at the AC output terminals, which results in excessively high current through the stator and the rotor. The converters must be protected against such a high current. The protection of



the converter is usually achieved by short circuiting the generator rotor through a so-called *crowbar* in the rotor circuit. During this time, no power can flow through the rotor. Wind farms supplying power to the grid are required to ride-through the voltage dip, that is, to have the ability of the fault ride-through without disconnection during grid faults. The DFIG would contribute to a limited extent fault ride-through.

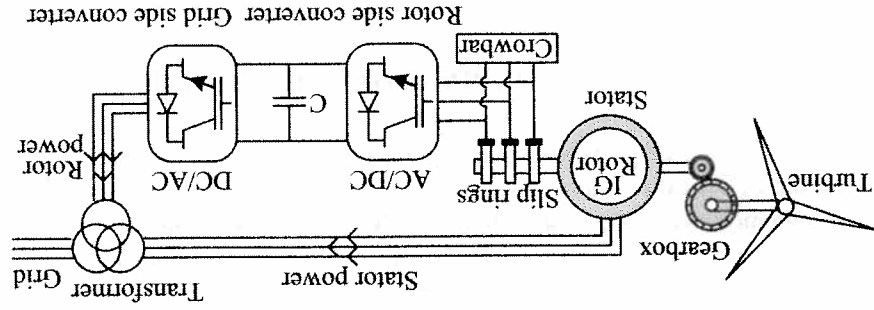


FIGURE 1.15 An overview of the variable speed WT with DFIG.

**Synchronous generator** — In systems with a synchronous generator, the rotor winding is excited by a DC source. The rotor is connected to the turbine, which allows variable speed operation over a wide range. The AC generated by the synchronous generator is first rectified into direct current and then inverted back to AC at standard grid frequency (50 Hz or 60 Hz), before feeding it into the grid. This system can provide reactive power, voltage and frequency regulation and, with recent developments, fault ride-through. Power ramp regulation is also being provided with machines of this type. Figure 1.16 presents an overview of the system with a synchronous generator.

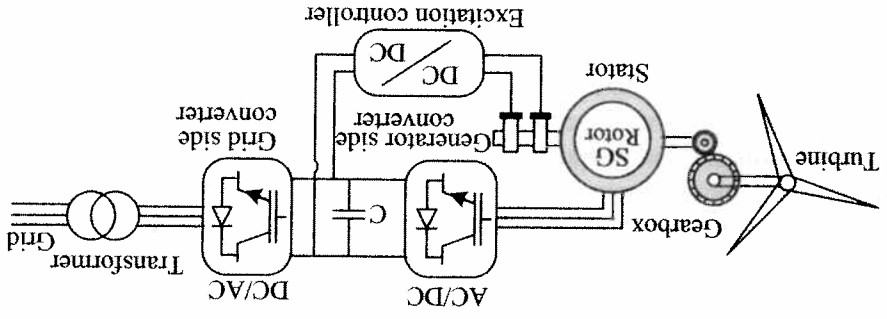
The gearboxes in some of the latest systems have been eliminated. The size of wind turbines varies widely. Small turbines used to power a single home or business may have a capacity of less than 100 kW. Some large, commercial-sized turbines may have a capacity of over 5 MW.

As of 2009, the world's largest wind turbine is the E-126 from German wind turbine manufacturer Enercon GmbH. This turbine has a rotor diameter of 126 m (413 feet), the unit is officially rated at 6 MW, and it is claimed to be capable of producing up to 7 MW. The rotor will turn at 12 rpm, and the generator is directly coupled to the turbine and connects to the grid through a power electronic converter. The manufacturers of very large wind turbines are now using newer, permanent magnet synchronous generators without a gearbox, combined with a full power converter. Such systems operate with the highest efficiency at low speeds,

and have the best fault ride-through capability. They also have a very low maintenance due to lack of slip rings, rotor excitation, and gearbox.

According to the Global Wind Energy Council,<sup>10</sup> over the past ten years, global wind power capacity has continued to grow at an average cumulative rate of over 30 percent, and, at the end of 2008, over 120,791 MW of wind capacity was installed worldwide. As of 2008, the United States ranks first in the world in wind power capacity, followed by Germany, Spain, and China. Denmark ranks ninth in the world in wind power capacity, but generates about 20 percent of its electricity from wind. The latest figure released by American Wind Energy Association in June of 2009 puts the United States' total installed wind power capacity at 29,440 MW, with another 5,866 MW under construction. Thirty-five states have wind generated electricity.<sup>11</sup> The top six states with the most wind power generation are Texas (8,361 MW), Iowa (3,043 MW), California (2,787 MW), Minnesota (1,805 MW), Washington (1,575), and Oregon (1,508 MW). In 2008, the U.S. Department of Energy announced that wind power could provide 20 percent of U.S. electricity by 2030.

FIGURE 1.16 An overview of the variable speed WT with SG.



## 1.9 GEOTHERMAL POWER

Geothermal energy is derived from heat within the earth, usually in the form of underground steam or hot water. The sources of geothermal energy are due to the molten rocks that are found beneath the surface of the earth known as *magma*. Most magma remains below the earth's crust and heats the surrounding rocks and subterranean water. Some of this water comes all the way up to the surface through faults

<sup>10</sup><http://www.gwec.net/index.php?id=13>  
<sup>11</sup><http://www.gwec.org/projects/Default.aspx>

and cracks in the earth as *hot springs* or *geysers*. When this rising hot water and steam is tapped in permeable rocks under a layer of impermeable rocks, *geothermal reservoirs* are formed. The thermal energy of these reservoirs can be taken from different depths through wells hundreds to thousands of feet deep. It can be utilized for space heating and industrial process applications, but most commonly it is used for producing base-load electric power generation. Geothermal energy is considered a renewable energy source because heat is continuously produced inside the earth. Geothermal power is cost effective, and environmentally friendly, with very low carbon emission, and, unlike solar and wind, geothermal power is immune from weather changes. However, there have been concerns that the pressurized water forced into the rock generates micro-earthquakes. It can also interact with existing deep faults, potentially causing larger temblors.

There are three types of geothermal power plants: dry-steam, flash-steam, and binary-cycle plants. The type selected depends on the temperatures and pressures of the geothermal reservoir. Typically two wells are constructed: a production well and an injection well.<sup>12</sup>

**Dry-Steam Power Plant** — Dry-steam power plant systems were the first type of geothermal power generation plants built. They use the steam from the geothermal reservoir as it comes from the production well. The steam is piped directly to a turbine, which drives a generator that produces electricity. The steam is condensed and pumped down into the injection well to sustain production. The largest complex of geothermal power plants in the world is The Geysers north of San Francisco, California. It consists of 15 power plants with a total capacity of 727 MW. The Geysers project is located in an active seismic zone, and the increase in earthquake activity since the project began has created opposition among some area residents and environmental groups.

**Flash-Steam Power Plant** — Geothermal reservoirs that contain mostly hot water above 350°F (176°C) are used in flash power plants. The hot water from the production well is depressurized or “flashed” into steam which can then be used to drive the turbine. Steam exhausted from the steam turbine is condensed in a condenser cooled by cold water from a cooling tower and is used to provide make-up water for the cooling tower. Hot water not flashed into steam is returned to the geothermal reservoir through the injection well. Both dry-steam and flash-steam power plants emit minute amounts of gases such as carbon dioxide, nitric oxide, and sulfur. This type of plant is the most common type of geothermal power generation plant.

**Binary-Cycle Power Plant** — When the geothermal reservoir temperature is not

<sup>12</sup> <http://www.geo-energy.org/basics.aspx>

high enough (between 250°F to 350°F) to flash steam, the hot water is passed through a heat exchanger. In this system, known as *binary-cycle* plant, the heat is recovered by a secondary fluid with a lower boiling point than water. The secondary fluid flashes to vapor, which, like steam, drives the turbines. The vapor is then condensed and circulated back to the heat exchanger. The cooled geothermal fluid is returned to the geothermal reservoir through the injection well. In a binary-cycle plant, the electricity can be generated from more common reservoirs with lower temperatures. Also, because the water from the geothermal reservoir never comes in contact with the turbine/generator unit, no gases are emitted to the atmosphere. For these reasons, binary-cycle power plants are the fastest growing geothermal power plants. A schematic of a binary-cycle power plant is depicted in Figure 1.17.

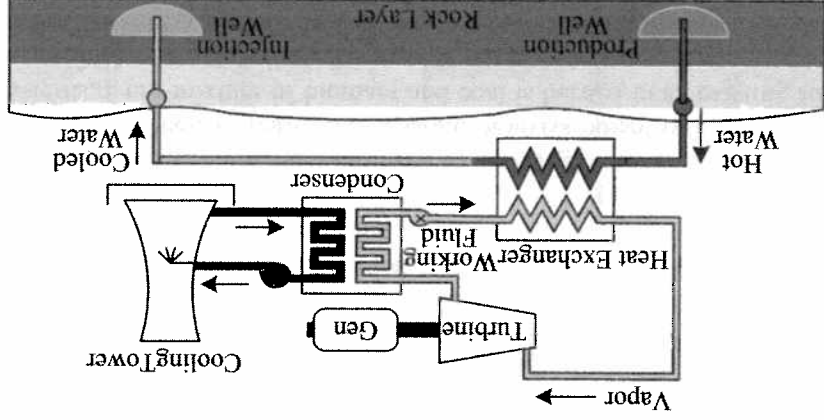


FIGURE 1.17 Schematic of a binary-cycle power plant.

The United States is the world leader in online capacity of electricity from geothermal energy. In the U.S., the geothermal reservoirs of steam and hot water are mostly located in the western states, Alaska, and Hawaii. However, geothermal energy can be tapped almost anywhere. According to the Geothermal Energy Association, as of March 2009, the United States has a total installed geothermal capacity of 3,040.27 MW.

President Obama has allocated \$350 million from the American Reinvestment and Recovery Act to expand and accelerate the development, deployment, and use of geothermal energy throughout the United States. However, a high-profile geothermal project in California that was launched in the fall of 2009, with millions of dollars in taxpayer funding, was permanently halted in December of 2009 in response to concerns that the project is causing an increase in regional earthquake activity.

## 1.10 BIOMASS POWER PLANTS

Biomass is renewable organic material which contains stored energy from the sun. There is a wide variety of biomass energy resources, including wood, plants, agricultural residues, and aquatic vegetation, and animal wastes. Biomass can also be converted into fuels like ethanol, biodiesel, and methane. Combustion of biomass produces heat that can be used for various purposes. Since biomass is part of the carbon cycle, its combustion causes no net increase in carbon dioxide emissions to the atmosphere, helping to reduce greenhouse gas emissions from fossil fuels. In addition, utilization of the gases found in landfills in a controlled environment for producing energy is a good disposal method for waste materials that can otherwise be hazardous to the environment and human health.

When biomass is used to generate electricity, it is called *biomass power* or *biopower*. There are several biopower technologies, including direct firing, co-firing, gasification, anaerobic digestion, and combined power and process heating. Virtually all biomass power plants use direct combustion, operating on a conventional steam-cycle. In this process, biomass is burned in a boiler to make steam. The steam then turns a turbine, which is connected to a generator that produces electricity.<sup>13</sup>

Several different techniques are used for co-firing, among them blended feed systems, separate feed systems, and separate biomass boiler systems. In blended feed systems, a pre-mixture of biomass and coal is burned in an existing furnace. In separate feed systems, boilers are retrofitted with a separate feed system for the biomass; in this arrangement, boilers can fire biomass when there is ample supply of biomass, and switch back to coal when biomass supplies are low. Finally, separate biomass boiler systems allow for separate steam supplies to be produced. More advanced approaches include combined power and heat, where in addition to electricity generation, steam is also used for manufacturing processes and/or building heat. The use of biomass for power generation is rapidly increasing worldwide, and, in 2009, the United States remained the world leader in geothermal power development with a total installed capacity of 14,000 MW. By the end of 2010, the total biomass installed capacity in the U.S. may reach 22,000 MW.

## 1.11 TIDAL POWER PLANTS

Tidal power is generated by capturing the energy of tides caused by the gravitational pull of the moon and sun on the world's oceans. The effect of the moon is about twice that of the sun due to its much closer position to the earth. As a result, the tide closely follows the moon during its rotation around the earth, causing

<sup>13</sup><http://www.eere.energy.gov/de/biomass-power.html>

the movement of a huge amount of water twice daily. Tidal energy is one of the most abundant clean forms of renewable energy with no greenhouse gas or other pollutions. There are basically two methods of extracting energy from tidal flows:

- Extracting the potential energy of tides moving in vertical direction, known as *tidal energy systems*.
- Extracting the kinetic energy of tidal motion in the horizontal direction, known as *tidal stream systems* or *tidal wave systems*.

## 1.11.1 TIDAL ENERGY SYSTEMS

In this system a barrage (a type of dam) is built across a river estuary in order to make use of the relative differences in the height of water between high tides and low tides, as shown in Figure 1.18.

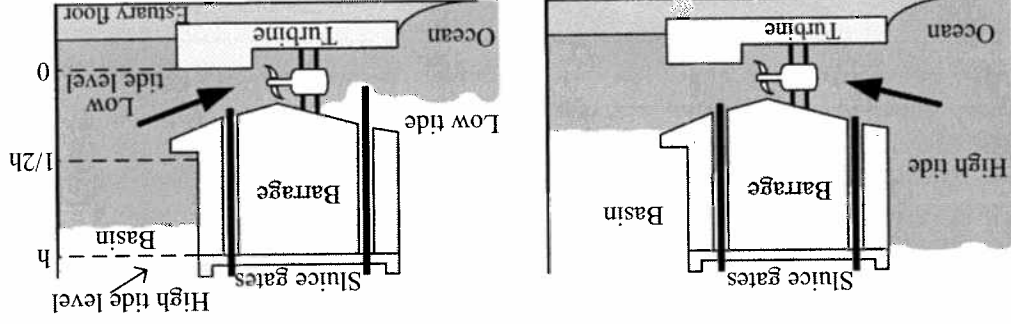


FIGURE 1.18 Simplified diagram of a tidal barrage.

Sluice gates on the barrage are opened to allow the reservoir behind the dam to be filled during the high tide. During the low tide, the gates are closed and the water behind the dam is released through the turbine, just as in a regular hydroelectric power plant. The turbine turns the generator to produce electricity. An alternative method is a two-way generation scheme which generates power during both incoming and outgoing tides. The amount of energy generated from a tidal barrage is determined by the difference in height between a high tide and a low tide, namely the tidal range. The mass of water moving through the tidal range is the volume times the water density  $\rho$ , that is,

$$m = \rho Ah$$

(1.12)

where  $A$  is the area of the tidal basin in  $m^2$ , and  $h$  is the range in  $m$ .

As water exits the bay, the level of the reservoir decreases; therefore, the height of the center of gravity is  $\frac{1}{2}h$ , so the released energy is  $mg(\frac{1}{2}h) = \frac{1}{2}pgAh^2$ . Hence the average tidal power in Watts is

$$P = \frac{pgAh^2}{2T} \tag{1.13}$$

where  $T$  = time interval between tides or tidal period, which is approximately 12 hours,  $g = 9.81 \text{ m/s}^2$

25 minutes

$\rho$  = density of seawater  $\approx 1025 \text{ kg/m}^3$

Taking into account the blade capacity factor or power coefficient  $C_p$ , turbine efficiency  $\eta$ , and the generator efficiency  $\eta_g$ , the net power output is

$$P_o = \eta_g \eta C_p \frac{pgAh^2}{2T} \text{ W} \tag{1.14}$$

For systems designed to generate power during both incoming and outgoing tides, the potential energy of (1.14) is doubled. The total average tidal power for a plant using both tidal directions is

$$P_{o(2)} = \eta_g \eta C_p \frac{J}{pgAh^2} \text{ W} \tag{1.15}$$

The blade capacity factor or power coefficient is in the range of 20 - 35 percent.

The first large-scale tidal barrage power plant in the world was built in 1966 at La Rance in France. It generates 240 MW, using 24 low-head Kaplan turbines, with an annual production of 600 GWh, powering approximately 240,000 homes. Only two other tidal barrage plants operate worldwide, a 24MW plant on the Bay of Fundy in Nova Scotia, Canada, and a 0.5 MW plant in Kislaya Guba, Russia. A tidal power plant under construction in Ansan, South Korea, utilizes the seawater at high tide when it comes to Sihwa artificial lake, made by a tide embankment. The plant will be equipped with 10 turbine generators for a total capacity of 254 MW. Also, South Korea plans to build the world's largest tidal power station, 812 MW, on Ganghwa Island by the year 2014. As of 2009, a plan to build a large-scale tidal barrage across the river from Brean Down in England to Cardiff in Wales is under consideration.

**Example 1.4**

A tidal barrage is constructed in an area where the average range between low tide and high tide levels is 8.5 m and the seawater density is  $1025 \text{ kg/m}^3$ . The area of the tidal basin is  $22 \text{ km}^2$ , and the blade capacity factor is 33 percent. The tidal period is about 12.5 hours. Determine the average power output of the tidal plant, assuming turbine efficiency is 90.5 percent and the generator efficiency is 94.4 percent.

Tidal period =  $12.5 \times 60 \times 60 = 45 \times 10^3 \text{ s}$ , and from (1.14), we have

$$P_o = (0.944 \times 0.905 \times 0.33) \frac{1025 \times 9.81 \times 22 \times 10^6 \times 8.5^2}{2 \times 45 \times 10^3} = 50 \times 10^6 \text{ W} = 50 \text{ MW}$$

**1.11.2 TIDAL WAVE SYSTEMS**

Tidal wave systems or tidal stream systems capture the kinetic energy of the moving waves of the ocean and convert it directly to mechanical power, similar to a wind turbine, without interrupting the natural flow. Tidal stream systems are relatively new; they have a much lower cost since they do not require barrage, and they have very little environmental impacts compared to barrage, which can have a huge impact on the environment. Thus, tidal stream systems are preferred to tidal energy systems. Suitable sites for tidal stream systems must have a speed of at least 2 knots (1 m/s).

Four types of turbines are used in tidal stream systems: horizontal-axis, vertical-axis, Venturi turbines, and oscillating devices using aerofoil, which are pushed sideways by the flow. Tidal turbines are mounted in rows on the seabed or suspended from a floating platform. The horizontal-axis propeller turbines are similar to wind turbines and are governed by the same equations as described in section 1.8.2. From (1.10), the power available from a stream of water through a turbine is

$$P = \frac{8}{\pi} C_p \rho D^2 v^3 \tag{1.16}$$

where

$\rho$  = density of seawater

$D$  = rotor diameter in m

$v$  = the stream velocity in m/s

$C_p$  = the coefficient of performance or power coefficient (0.3 - 0.59 Betz Limit)

Considering the mechanical losses of the gearbox, turbine blades, and the losses in the generator, and the inclusion of the corresponding efficiencies ( $\eta_{gb}$ ,  $\eta_g$ ), the net output power becomes

$$P_o = (\eta_{gb} \eta_g) \frac{8}{\pi} C_p \rho D^2 v^3 \tag{1.17}$$

The density of water is 830 times greater than the density of air; this means that tidal currents can provide a much higher power density than wind. The world's first commercial prototype tidal stream generator, SeaGen, was installed on August 20, 2006 in Northern Ireland's Strangford Lough. It generates 1.2 Megawatts of energy to power approximately 1000 houses.

#### Example 1.5

A tidal stream turbine-generator, consisting of a twin rotor, gearbox, and generator, is designed as an integral unit. Rotors are located on either side of a single mono-pile, and each has a rotor diameter of 16 m (similar to SeaGen unit). The seawater density is  $1025 \text{ kg/m}^3$ . Determine the unit power output when the tidal current is  $2.78 \text{ m/s}$ . Assume the following efficiencies: gearbox efficiency 94 percent, turbine efficiency 92 percent, generator efficiency 95 percent, and a power coefficient of 33 percent.

From (1.17) for two rotors, the power output is

$$P_o = 2(0.94 \times 0.92 \times 0.95) \frac{\pi}{8} (0.33 \times 1025 \times 16^2 \times 2.78^3) = 1.2 \times 10^6 \text{ W} = 1.2 \text{ MW}$$

There are currently no tidal power plants in the United States, but there exists a large potential for wave power systems in the Pacific Northwest of the United States. For this reason, about 30 tidal wave projects are expected to start operation to determine the cost effectiveness of tidal power.

### 1.12 FUEL CELL

A fuel cell is an electrochemical device that converts the chemical energy of a fuel directly into electrical energy. Like batteries, fuel cells contain two electrodes, an anode and a cathode, separated by an electrolyte that serves as an ion conductor. The fuel reactant, typically hydrogen or carbon monoxide, is supplied from an external source. A battery, in contrast, has a finite storage of energy before it needs to be recharged.

Since fuel cells produce electricity directly, they have a much higher efficiency compared to the electricity generated by the electromechanical conversion process. In addition, because no combustion is involved, fuel cells do not emit carbon dioxide, sulfur dioxide, nitrogen oxide, or particular matter. A typical fuel cell has a DC voltage of about 0.7 volts; many individual cells are grouped in series and parallel connections to obtain the desired voltage and power. They require power conditioning units, including a DC-to-DC converter to convert the unregulated DC output of the fuel cells to a high voltage DC source, and a DC-to-AC inverter for

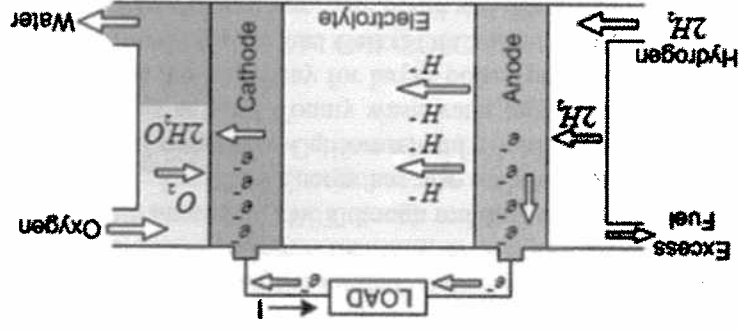


FIGURE 1.19  
Phosphoric Acid Fuel Cell.

<sup>14</sup><http://www.fuelcells.org/basics/how.html>

AC load applications and grid connection. In general, fuel cells are classified by the type of electrolytes. Six major types of fuel cells are listed below:

- Direct Methanol Fuel Cell (DMFC)
- Polymer Electrolyte Fuel Cell (PEFC)
- Alkaline Fuel Cell (AFC)
- Phosphoric Acid Fuel Cell (PAFC)
- Molten Carbonate Fuel Cell (MCFC)
- Solid Oxide Fuel Cell (SOFC)

The above listing is in the order of approximate operating temperature, ranging from  $60^\circ\text{C}$  for DMFC to  $1,000^\circ\text{C}$  for SOFC. They all work on the same principle, although each one operates slightly differently from the others. They are used in numerous applications, including spacecraft, transportation, industry, portable power applications requiring quiet operations, power generation/cogeneration, and distributed generation. Two fuel cells most widely used in the commercial sector, industry, and electric utilities are described below.<sup>14</sup>

#### 1.12.1 PHOSPHORIC ACID FUEL CELL

In Phosphoric Acid Fuel Cells (PAFCs) the electrolyte is liquid phosphoric acid, which is stored in a Teflon-bonded silicon carbide matrix and uses porous carbon electrodes containing a platinum catalyst, as shown in Figure 1.19.

In a PAFC, hydrogen fuel is fed into the anode, and oxygen (or air) enters the fuel cell through the cathode. Helped by the catalyst on the anode, hydrogen atoms are split into electrons and protons, which take different paths to the cathode. The proton passes through the electrolyte membrane. Electrons flow through an external circuit to be utilized before returning to the cathode. Oxygen entering the cathode combines with the protons and electrons coming from the load to form water. Water vapor and heat are released as byproducts of this reaction. The PAFC were among the first stationary applications: approximately 300 units, mostly 200 kW, are installed in the United States, Europe, and Japan by United Technologies Corporation (UTC). Typical installations include buildings, hotels, and hospitals. UTC is supplying 12 newer 400 kW (called *Pure Cell Model 400*) units, totaling 4.8 megawatts of power for the Freedom Tower and three other new towers under construction at the World Trade Center site in New York. The PAFC electrical efficiency is about 36 to 40 percent. The PAFC plant also produces heat for domestic hot water and space heating. When operating in cogeneration applications, the overall efficiency is approximately 85 percent.

### 1.12.2 MOLTEN CARBONATE FUEL CELL

Molten carbonate fuel cells use a carbonate salt mixture electrolyte (usually sodium or lithium) that is heated to about 600 to 700°C. At these temperatures, the salt turns into a molten state, providing ionic conduction between the electrodes. The electrodes are made with relatively inexpensive nickel catalysts. Hydrogen is obtained by internal reforming of hydrocarbon-based fuels, such as natural gas, biogas, synthesis gas, methane, and propane. The high exhaust temperature of MCFCs makes them suitable for large-scale combined-cycle multi-megawatt applications. The efficiency can approach 60 percent when it is used to generate electricity and up to 85 percent with a combined cycle. Fuel Cell Energy Incorporated has installed many MCFCs units in the United States. Typical installations include wastewater facilities, hotels, universities, and grid support. Most of these are about 250 kW, although multiple units have been combined for larger installations. Fuel Cell Energy has also installed larger units, including a 2-MW pilot unit in Santa Clara, California, and a 1-MW power plant fueled by wastewater digester gas at King County wastewater treatment facility in Renton, Washington. Programs are underway for larger power plants with outputs of 5 to 10 MW. Finally, the Solid Oxide Fuel Cell (SOFC) is suitable for electric utilities and large distributed generation. For more detail and description of all other types of fuel cells and their applications, see Fuel Cell Technology at the U.S. Department of Energy Website.<sup>15</sup>

<sup>15</sup>Fuel Cell Technology: <http://www1.eere.energy.gov/hydrogenandfuelcells/fuelcells/fc-types.html>

## 1.13 MODERN POWER SYSTEM

The power system of today is a complex interconnected network, as shown in Figure 1.20 (page 43). A power system can be subdivided into four major parts:

- Generation
- Transmission and Subtransmission
- Distribution
- Loads

### 1.13.1 GENERATION

**Generators** — One of the essential components of power systems is the three-phase AC generator, known as the synchronous generator or alternator. Synchronous generators have two synchronously rotating fields: one field is produced by the rotor driven at synchronous speed and excited by DC current, and the other field is produced in the stator windings by the three-phase armature currents. The DC current for the rotor windings is provided by excitation systems. In the older units, the exciters are DC generators mounted on the same shaft, providing excitation through slip rings. Today's systems use AC generators with rotating rectifiers, known as *brushless excitation systems*. The generator excitation system maintains generator voltage and controls the reactive power flow. Because they lack the commutator, AC generators can generate high power at high voltage, typically 30 kV. In a power plant, the size of generators can vary from 50 MW to 1500 MW. Steam turbines operate at relatively high speeds of 3600 or 1800 rpm for 60 Hz operation. The generators to which they are coupled are cylindrical rotor, two-pole for 3600 rpm or four-pole for 1800 rpm operation. Hydraulic turbines, particularly those operating with a low pressure, operate at low speed. Their generators are usually a salient-type rotor with many poles. In a power station, several generators are operated in parallel in the power grid to provide the total power needed. They are connected at a common point called a *bus*.

**Transformers** — Another major component of a power system is the transformer. It transfers power with very high efficiency from one level of voltage to another level. The power transferred to the secondary is almost the same as the primary, except for losses in the transformer, and the product  $VI$  on the secondary side is approximately the same as the primary side. Therefore, using a step-up transformer of turns ratio  $a$  will reduce the secondary current by a ratio of  $1/a$ . This will reduce losses in the line, which makes possible the transmission of power over

long distances. The insulation requirements and other practical design problems limit the generated voltage to low values, usually 30 kV. Thus, step-up transformers are used for transmission of power. At the receiving end of the transmission lines, step-down transformers are used to reduce the voltage to suitable values for distribution or utilization. In a modern utility system, the power may undergo four or five transformations between generator and ultimate user.

1.13.2 TRANSMISSION AND SUBTRANSMISSION

The purpose of an overhead transmission network is to transfer electric energy from generating units at various locations to the distribution system, which ultimately supplies the load. Transmission lines also interconnect neighboring utilities, which permits not only economic dispatch of power within regions during normal conditions, but also the transfer of power between regions during emergencies. Standard transmission voltages are established in the United States by the American National Standards Institute (ANSI). Transmission voltage lines operating at more than 60 kV are standardized at 69 kV, 115 kV, 138 kV, 161 kV, 230 kV, 345 kV, 500 kV, and 765 kV line-to-line. Transmission voltages above 230 kV are usually referred to as extra-high voltage (EHV). Figure 1.20 shows an elementary diagram of a transmission and distribution system. High voltage transmission lines are terminated in substations, which are called *high-voltage substations*, receiving circuits in primary substations. The function of some substations is switching circuits in and out of service; they are referred to as switching stations. At the primary substations, the voltage is stepped down to a value more suitable for the next part of the journey toward the load. Very large industrial customers may be served from the transmission system. The portion of the transmission system that connects the high-voltage substations through step-down transformers to the distribution substations is called the *subtransmission network*. There is no clear delineation between transmission and subtransmission voltage levels. Typically, the subtransmission voltage level ranges from 69 to 138 kV. Some large industrial customers may be served from the subtransmission system. Capacitor banks and reactor banks are usually installed in the substations for maintaining the transmission line voltage.

1.13.3 DISTRIBUTION

The distribution system is that part which connects the distribution substations to the consumers' service-entrance equipment. The primary distribution lines are usually in the range of 4 to 34.5 kV and supply the load in a well-defined geographical area. Some small industrial customers are served directly by the primary feeders. The secondary distribution network reduces the voltage for utilization by commercial and residential consumers. Lines and cables not exceeding a few hundred feet in length then deliver power to the individual consumers. In the U.S., the secondary

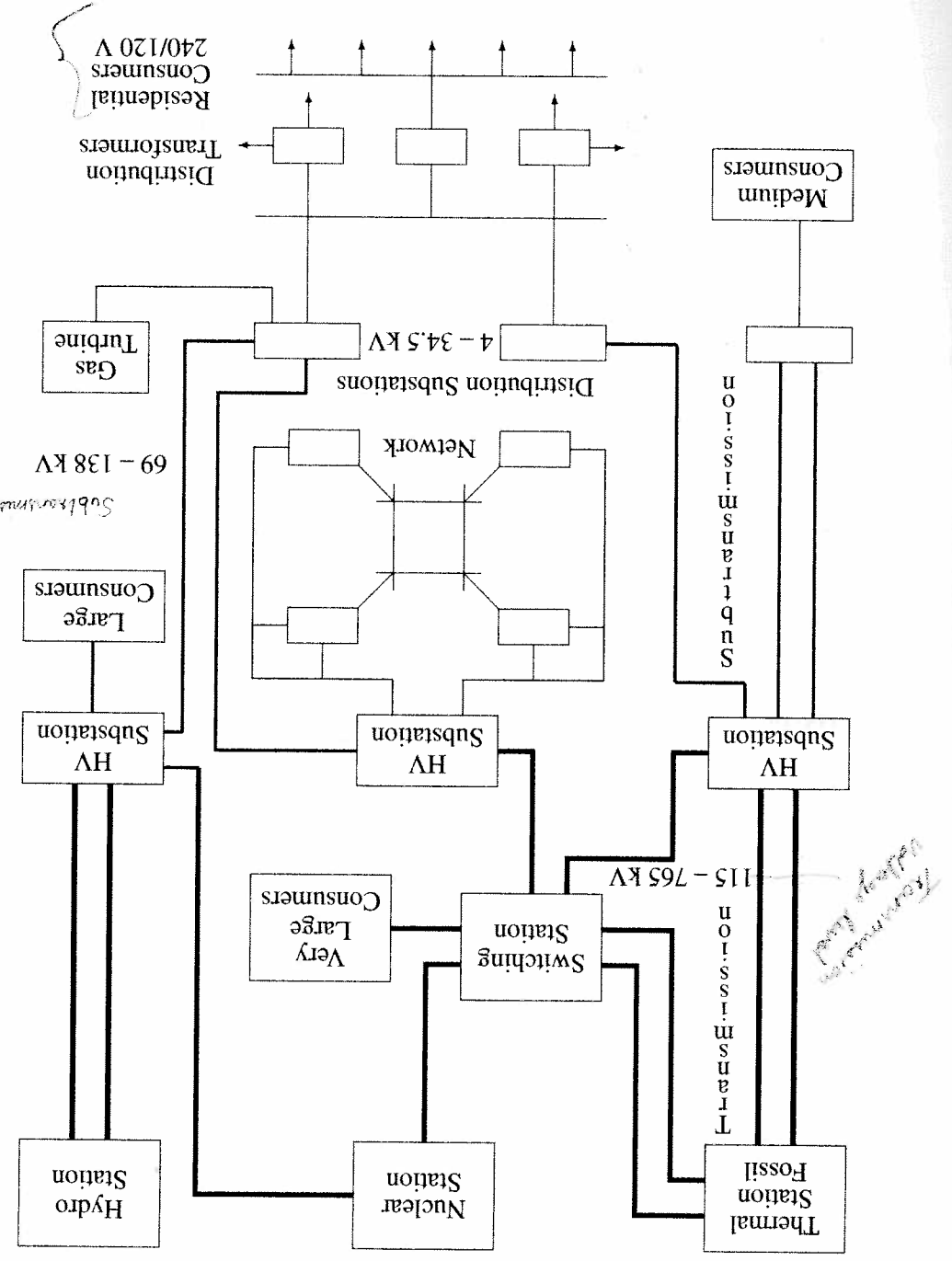


FIGURE 1.20 Basic components of a power system.

distribution serves most of the customers at levels of 240/120 V, single-phase, three-wire; 208Y/120 V, three-phase, four-wire; or 480Y/277 V, three-phase, four-wire. The power for a typical home is derived from a transformer that reduces the primary feeder voltage to 240/120 V using a three-wire line. Distribution systems are both overhead and underground. The growth of underground distribution has been extremely rapid, and as much as 70 percent of new residential construction is served underground.

### 1.13.4 LOADS

Loads of power systems are divided into industrial, commercial, and residential. Very large industrial loads may be served from the transmission system. Large industrial loads are served directly from the subtransmission network, and small industrial loads are served from the primary distribution network. The industrial loads are composite loads, and induction motors form a high proportion of these loads. These composite loads are functions of voltage and frequency and form a major part of the system load. Commercial and residential loads consist largely of lighting, heating, and cooling. These loads are independent of frequency and consume negligibly small reactive power.

The real power of loads is expressed in terms of kilowatts or megawatts. The magnitude of load varies throughout the day, and power must be available to consumers on demand.

The daily-load curve of a utility is a composite of demands made by various classes of users. The greatest value of load during a 24-hr period is called the *peak or maximum demand*. Smaller peaking generators may be commissioned to meet the peak load that occurs for only a few hours. In order to assess the usefulness of the generating plant, the *load factor* is defined. The load factor is the ratio of average load over a designated period of time to the peak load occurring in that period. Load factors may be given for a day, a month, or a year. The yearly or annual load factor is the most useful since a year represents a full cycle of time. The daily load factor is

$$\text{Daily L.F.} = \frac{\text{average load}}{\text{peak load}} \quad (1.18)$$

Multiplying the numerator and denominator of (1.18) by a time period of 24 hr, we have

$$\text{Daily L.F.} = \frac{\text{average load} \times 24 \text{ hr}}{\text{peak load} \times 24 \text{ hr}} = \frac{\text{energy consumed during 24 hr}}{\text{peak load} \times 24 \text{ hr}} \quad (1.19)$$

The annual load factor is

$$\text{Annual L.F.} = \frac{\text{total annual energy}}{\text{peak load} \times 8760 \text{ hr}} \quad (1.20)$$

Generally there is diversity in the peak load between different classes of loads, which improves the overall system load factor. In order for a power plant to operate economically, it must have a high system load factor. Today's typical system load factors are in the range of 55 to 70 percent.

There are a few other factors used by utilities. *Utilization factor* is the ratio of maximum demand to the installed capacity, and *plant factor* is the ratio of annual energy generation to the plant capacity  $\times 8760$  hr. These factors indicate how well the system capacity is utilized and operated.

A *MATLAB* function **barcycle(data)** is developed which obtains a plot of the load cycle for a given interval. The demand interval and the load must be defined by the variable **data** in a three-column matrix. The first two columns are the demand interval, and the third column is the load value. The demand interval may be minutes, hours, or months, in ascending order. Hourly intervals must be expressed in military time.

### Example 1.6 (chp1ex6)

The daily load on a power system varies, as shown in Table 1.2. Use the **barcycle** function to obtain a plot of the daily load curve. Using the given data, compute the average load and the daily load factor (Figure 1.21).

Table 1.2 Daily System Load

Interval, hr	Load, MW
12 A.M. - 2 A.M.	6
2 - 6	5
6 - 9	10
9 - 12 P.M.	15
12 P.M. - 2 P.M.	12
2 - 4	14
4 - 6	16
6 - 8	18
8 - 10	16
10 - 11	12
11 - 12 A.M.	6

The following commands

```
data = [ 0 2 6
        2 6 5
        6 9 10
        9 12 15
```



## 1.14 SMART GRID

The United States' electric grid infrastructure was constructed more than fifty years ago. The grid has grown to a complex spider web of power lines and aging networks and systems with obsolete technology and outdated communications. Today, with the rise in demand for renewable energy sources and the requirements for lower carbon emissions, the over-burdened grid is being pushed to its limits. Addition of new transmission lines alone is not sufficient to improve system reliability and security. What is needed is the modernization of the grid by installing analog and digital sensors, electronic switches, smart energy meters, advanced communication, and data acquisition systems and interactive software with real-time control that optimize the operation of the entire electrical system and make more efficient use of the grid assets. An electric grid enabled with such a capability is known as the *smart grid*.

A smart grid would integrate the renewable energy sources such as wind and solar with conventional power plants in an intelligent and coordinated manner that not only would increase reliability and service continuity, but would effectively reduce energy consumption and significantly reduce the carbon emissions. According to the United States Department of Energy's Smart Grid System Report, dated July 2009,<sup>16</sup> a smart grid must have the following characteristics:

- **Enable Informed Participation by Customers** — The Smart Grid will give consumers information, control, and options that enable them to become active participants in the grid. Well-informed customers will modify consumption based on the balancing of their demands and resources with the electric system's capability to meet those demands

- **Accommodate All Generation and Storage Options** — The Smart Grid will integrate all types of electrical generation and storage systems, including small-scale power plants that serve their local loads, known as *Distributed Generation*, with a simplified interconnection process analogous to "plug-and-play".

- **Enable New Products, Services, and Markets** — The Smart Grid will enable market participation, allowing buyers and sellers to bid on their energy resources through the supply and demand interactions of markets and real time price quotes.

- **Provide the Power Quality for the Range of Needs** — The Smart Grid will enable utilities to balance load sensitivity with power quality, and consumers will have the option of purchasing varying grades of power quality

<sup>16</sup>[http://www.oe.energy.gov/DocumentsandMedia/CGSRMain\\_090707\\_lowres.pdf](http://www.oe.energy.gov/DocumentsandMedia/CGSRMain_090707_lowres.pdf)

```

P = data(:,3);
Dt = data(:,1) - data(:,2); % Column array of demand interval
W = P'*Dt; % Total energy, area under the curve
Pavg = W/sum(Dt) % Average load
Peak = max(P) % Peak load
LF = Pavg/Peak*100 % Percent load factor
barcycle(data)
xlabel('Time, hr'), ylabel('P, MW')

```

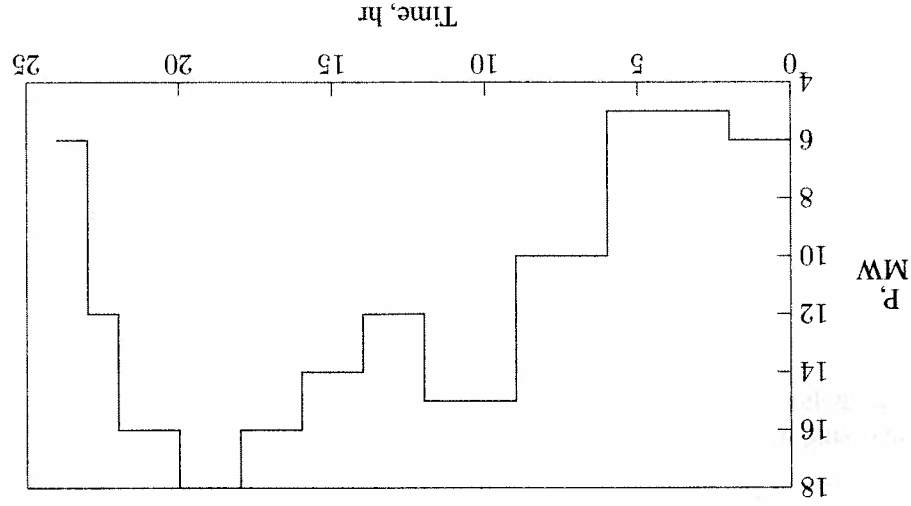


FIGURE 1.21 Daily load cycle for Example 1.1.

Pavg = 11.5417  
Peak = 18  
LF = 64.12

at different prices. Also, irregularities caused by certain consumer loads will be buffered to prevent propagation.

- **Optimize Asset Utilization & Operating Efficiency** — The Smart Grid optimizes the utilization of existing and new assets, will improve load factors, and lower system losses in order to maximize the operational efficiency and reduce the cost. Advanced sensing and robust communications will allow early problem detection, preventive maintenance, and corrective action.

- **Operate Resiliently to Disturbances, Attacks, and Natural Disasters** — The Smart Grid operates resiliently, that is, it has the ability to withstand and recover from disturbances in a self-healing manner to prevent or mitigate power outages and to maintain reliability, stability, and service continuity. The Smart Grid will operate resiliently against attack and natural disaster. It incorporates new technology and higher cyber security, covering the entire electric system, reducing physical and cyber vulnerabilities and enabling a rapid recovery from disruptions.

The American Recovery and Reinvestment Act of 2009 (ARRA), signed into law in February of 2009, has allotted \$4.5 billion to the United States Department of Energy (DOE), Office of Electricity Delivery and Energy Reliability. These funds are specifically allocated for Smart Grid grants to transform the nation's aging electricity grid, enhance security of U.S. energy infrastructure, and ensure reliable electricity delivery to meet growing demand. A significant share of the funds will be used to support these programs through a competitive grant process. In September of 2009, the DOE awarded \$44 million to state public utility commissions and \$100 million in available funding for smart grid workforce training programs. The five state public utility commissions with the largest awards were California Public Utilities Commission, Public Utility Commission of Texas, New York State Public Service Commission, Florida Public Service Commission, and Illinois Commerce Commission.

Already there are many smart grid pilots and projects in several states. Many utilities have started the first phases of their Smart Grid plan and have provided their customers with smart metering systems. These projects have shown significant enhancement in grid reliability, reduction of outages, and lower consumers' electric bills.

## 1.15 ROLE OF POWER ELECTRONICS IN MODERN POWER SYSTEMS

With the continuing increase of renewable energy generation units and distributed generation, and with the emergence of the Smart Grid, there has been a tremendous

increase in the use of power electronics, now integral components of modern power systems. Power electronics and other static controllers provide essential functionality through application of the Flexible AC Transmission Systems, called FACTS. FACTS is a new technology based on power electronics that aims to improve the power transfer capability of AC transmission systems and enhance the quality, reliability, and stability of large interconnected power systems. Several FACTS-devices, such as dynamic and static VAR (volt ampere reactive) compensators, power converters, and other power electronic modules, are utilized for the following purposes:

- To interconnect renewable energy sources such as wind power plants, concentrated solar plants and distributed generation.
- To interact with the grid on a real-time basis to improve the power transfer capability of AC transmission systems, improve efficiency, and prevent blackouts.
- To enhance the quality, reliability, and stability of large interconnected power systems.
- To provide advanced control functions for power flow and VAR/voltage regulation, where it is needed on the grid.
- To manage and optimize the energy usage in microgrids,<sup>17</sup> and to allow microgrids to be separated from the main grid in the event of a sustained disturbance.
- To transfer bulk power via high-voltage DC transmission systems over long distances.
- To allow small local storage and high volume centralized storage.
- To provide customers with advanced technology tools and information to take an active role in the supply of electricity.

With the wide spread application of electronic devices in all aspects of power systems, EB students majoring in the field of power must learn about these devices and related analytical tools required for the operation of the Smart Grid. Therefore, in addition to the basic power electronics course, it is crucial to introduce a course in the EB undergraduate curriculum on the application of power electronics in modern power systems for students majoring in the field of power.

<sup>17</sup>Cluster of interconnected distributed generators, the associated loads and storage units that cooperate with each other.

## 1.16 SYSTEM PROTECTION

In addition to generators, transformers, and transmission lines, other devices are required for the satisfactory operation and protection of a power system. Some of the protective devices directly connected to the circuits are called *switchgear*. They include instrument transformers, circuit breakers, disconnect switches, fuses, and lightning arresters. These devices are necessary to deenergize either for normal operation or in the occurrence of faults. The associated control equipment and protective relays are placed on *switchboard* in control houses.

## 1.17 ENERGY CONTROL CENTER

For reliable and economical operation of the power system, it is necessary to monitor the entire system in a control center. The modern control center of today is called the *energy control center* (ECC). Energy control centers are equipped with on-line computers performing all signal processing through the remote acquisition system. Computers work in a hierarchical structure to coordinate different functional requirements in normal as well as emergency conditions. Every energy control center contains a control console, which consists of a visual display unit (VDU), keyboard, and light pen. Computers may give alarms as advance warnings to the operators (dispatchers) when deviation from the normal state occurs. The dispatcher makes judgments and decisions and executes them with the aid of a computer. Simulation tools and software packages written in high-level language are implemented for efficient operation and reliable control of the system. This is referred to as SCADA, an acronym for "supervisory control and data acquisition."

## 1.18 COMPUTER ANALYSIS

For a power system to be practical, it must be safe, reliable, and economical. Thus, many analyses must be performed to design and operate an electrical system. However, before going into system analysis, we have to model all components of electrical power systems. Therefore, in this text, after reviewing the concepts of power and three-phase circuits, we will calculate the parameters of a multi-circuit transmission line. Then, we will model the transmission line and look at the performance of the transmission line. Since transformers and generators are a part of the system, we will model these devices. Design of a power system, its operation, and expansion requires much analysis. This text presents methods of power system analysis with the aid of a personal computer and the use of *MATLAB*. The *MATLAB* environment permits a nearly direct transition from mathematical expression to simulation. Some of the basic analysis covered in this text are:

Many *MATLAB* functions are developed for the above studies, allowing students to concentrate on analysis and design of practical systems and spend less time on programming.

## PROBLEMS

**1.1.** A hydroelectric power plant has a head of 85 ft, and an average flow of 3000 ft<sup>3</sup>/s. If the overall efficiency is 69.5 percent, determine the generated electric power.

**1.2.** A hydroelectric power plant has eight generators rated at 64 MW each under a 20 m head, and two generators rated at 42 MW each under 15 m head. Each turbine discharges water at a rate of 400 m<sup>3</sup>/s. Determine the overall efficiency of the hydroelectric power plant.

**1.3.** A 1.2 MW wind turbine has a rotor diameter of 64 m. The turbine operates in an area with an average wind velocity of 7.5 m/s and the air density of 1.18 kg/m<sup>3</sup>. The turbine power coefficient including the unit efficiency is 42.8 percent.

(a) Determine the power output and the annual energy produced.

(b) The total installed cost is \$1.2 million, and its annual cost is based on the equivalent of a 20-year, 5 percent loan to cover the capital cost. The annual Debt Payment (DP) is determined from  $DP = CRF \times$  Total installed cost. The CRF is Capital Recovery Factor evaluated from

$$CRF = \frac{i(1+i)^n}{(1+i)^n - 1}$$

2.1 INTRODUCTION

The concept of power is of central importance in electrical power systems and is the main topic of this chapter. The typical student will already have studied much of this material, and the review here will serve to reinforce the power concepts encountered in the electric circuit theory.

In this chapter, the flow of energy in an ac circuit is investigated. By using various trigonometric identities, the instantaneous power  $p(t)$  is resolved into two components. A plot of these components is obtained using *MATLAB* to observe that ac networks not only consume energy at an average rate, but also borrow and return energy to its sources. This leads to the basic definitions of average power  $P$  and reactive power  $Q$ . The volt-ampere  $S$ , which is a mathematical formulation based on the phasor forms of voltage and current, is introduced. Then the complex power balance is demonstrated, and the transmission inefficiencies caused by loads with low power factors are discussed and demonstrated by means of several examples.

Next, the transmission of complex power between two voltage sources is considered, and the dependency of real power on the voltage phase angle and the dependency of reactive power on voltage magnitude is established. *MATLAB* is used conveniently to demonstrate this idea graphically.

Finally, the balanced three-phase circuit is examined. An important property of a balanced three-phase system is that it delivers constant power. That is, the

where  $i$  is the interest rate in decimal form, and  $n$  is number of years. If the annual operation and maintenance cost is 1 percent of the capital cost, determine the cost of electricity in cents/kWh.

1.4. A tidal energy system has an average tidal range of 5 m and a basin area of 21.5 km<sup>2</sup>. The system is designed to generate power during both incoming and outgoing tides. The seawater density is 1025 kg/m<sup>3</sup>, and the tidal period is about 12.5 hours.

- (a) Determine the maximum average tidal power at the site.
- (b) If the overall efficiency of the plant is 30 percent, find the annual electric energy produced.

1.5. The demand estimation is the starting point for planning the future electric power supply. The consistency of demand growth over the years has led to numerous attempts to fit mathematical curves to this trend. One of the simplest curves is

$$P = P_0 e^{a(t-t_0)}$$

where  $a$  is the average per unit growth rate,  $P$  is the demand in year  $t$ , and  $P_0$  is the given demand at year  $t_0$ . Assume that the peak power demand in the United States in 1984 is 480 GW with an average growth rate of 3.4 percent. Using *MATLAB*, plot the predicated peak demand in GW from 1984 to 1999. Estimate the peak power demand for the year 1999.

1.6. In a certain country, the energy consumption is expected to double in 10 years. Assuming a simple exponential growth given by

$$P = P_0 e^{at}$$

calculate the growth rate  $a$ .

1.7. The annual load of a substation is given in the following table. During each month, the power is assumed constant at an average value. Using *MATLAB* and the *barcycle* function, obtain a plot of the annual load curve. Write the necessary statements to find the average load and the annual load factor.

Annual System Load		
Interval, month	Load, MW	Interval, month
January	8	July
February	6	August
March	4	September
April	2	October
May	6	November
June	12	December
		8



pulsates with twice the frequency and has an average value of zero. This component accounts for power oscillating into and out of the load because of its reactive element (inductive or capacitive). The amplitude of this pulsating power is called *reactive power* and is designated by  $Q$ .

$$Q = |V||I| \sin \theta \quad (2.9)$$

Both  $P$  and  $Q$  have the same dimension. However, in order to distinguish between the real and the reactive power, the term "var" is used for the reactive power (var is an acronym for the phrase "volt-ampere reactive"). For an inductive load, current is lagging the voltage,  $\theta = (\theta_v - \theta_i) > 0$  and  $Q$  is positive; whereas, for a capacitive load, current is leading the voltage,  $\theta = (\theta_v - \theta_i) < 0$  and  $Q$  is negative. A careful study of Equations (2.6) and (2.8) reveals the following characteristics of the instantaneous power.

- For a pure resistor, the impedance angle is zero and the power factor is unity (UPF), so that the apparent and real power are equal. The electric energy is transformed into thermal energy.
- If the circuit is purely inductive, the current lags the voltage by  $90^\circ$  and the average power is zero. Therefore, in a purely inductive circuit, there is no transformation of energy from electrical to non-electrical form. The instantaneous power at the terminal of a purely inductive circuit oscillates between the circuit and the source. When  $p(t)$  is positive, energy is being stored in the magnetic field associated with the inductive elements, and when  $p(t)$  is negative, energy is being extracted from the magnetic fields of the inductive elements.

- If the load is purely capacitive, the current leads the voltage by  $90^\circ$ , and the average power is zero, so there is no transformation of energy from electrical to non-electrical form. In a purely capacitive circuit, the power oscillates between the source and the electric field associated with the capacitive elements.

### Example 2.1 (chp2ex1, chp2ex1gui)

The supply voltage in Figure 2.1 is given by  $v(t) = 100 \cos \omega t$  and the load is inductive with impedance  $Z = 1.25 \angle 60^\circ \Omega$ . Determine the expression for the instantaneous current  $i(t)$  and the instantaneous power  $p(t)$ . Use *MATLAB* to plot  $i(t)$ ,  $v(t)$ ,  $p_R(t)$ , and  $p_X(t)$  over an interval of 0 to  $2\pi$ .

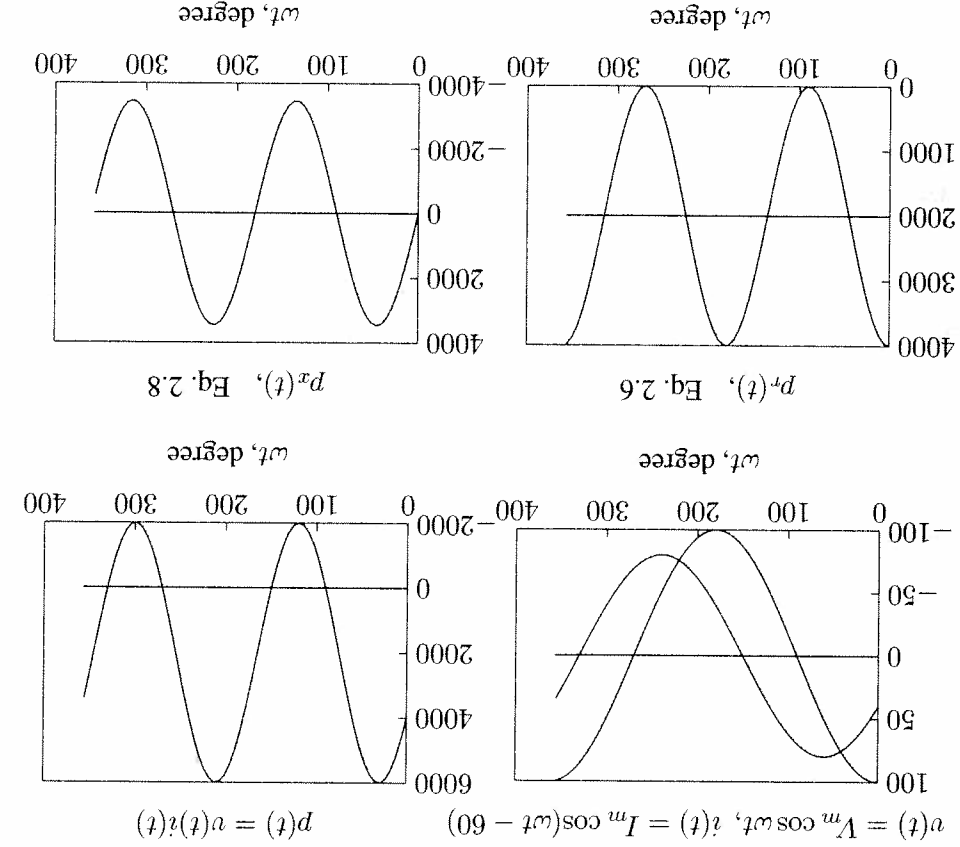
$$I_{max} = \frac{100 \angle 0^\circ}{1.25 \angle 60^\circ} = 80 \angle -60^\circ \text{ A}$$


FIGURE 2.2

Instantaneous current, voltage, power, Eqs. 2.6 and 2.8.

therefore

$$i(t) = 80 \cos(\omega t - 60^\circ) \text{ A}$$

$$p(t) = v(t)i(t) = 8000 \cos \omega t \cos(\omega t - 60^\circ) \text{ W}$$

The following statements are used to plot the above instantaneous quantities and the instantaneous terms given by (2.6) and (2.8).

```
Vm = 100; theta = 0; % Voltage amplitude and phase angle
Z = 1.25; gama = 60; % Impedance magnitude and phase angle
theta = theta - gama; % Current phase angle in degree
theta = (theta - theta)*pi/180; % Degree to radian
Im = Vm/Z; % Current amplitude
wt = 0:0.05:2*pi; % wt from 0 to 2*pi
v = Vm*cos(wt); % Instantaneous voltage
```

```

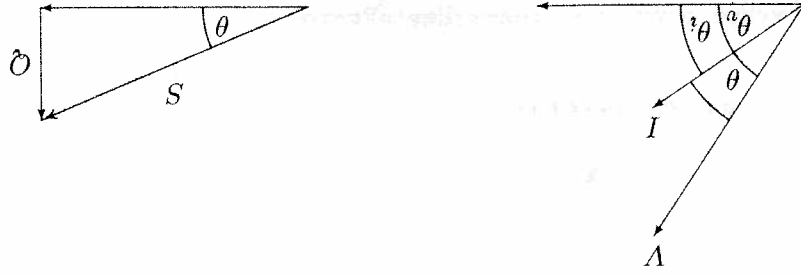
i = Im*cos(wt + theta*pi/180); % Instantaneous current
p = v.*i; % Instantaneous power
V = Vm/sqrt(2); I = Im/sqrt(2); % rms voltage and current
P = V*I*cos(theta); % Average power
Q = V*I*sin(theta); % Reactive power
S = P + j*Q % Complex power
pr = P*(1 + cos(2*(wt + theta))); % Eq. (2.6)
px = Q*sin(2*(wt + theta)); % Eq. (2.8)
PP = P*ones(1, length(wt)); %Average power of length w for plot
xline = zeros(1, length(wt)); %generates a zero vector
wt=180/pi*wt; % converting radian to degree
subplot(2,2,1), plot(wt, v, wt, i, wt, xline), grid
title(['v(t)=Vm coswt, i(t)=Im cos(wt+ num2str(theta)), '])
xlabel('wt, degree', subplot(2,2,2), plot(wt, p, wt, xline))
title('p(t)=v(t) i(t)', xlabel('wt, degree'), grid
subplot(2,2,3), plot(wt, pr, wt, px, wt, xline), grid
title('pr(t) Eq. 2.6', xlabel('wt, degree')
subplot(2,2,4), plot(wt, px, wt, xline), grid
title('px(t) Eq. 2.8', xlabel('wt, degree'), subplot(111)

```

The rms voltage phasor of (2.1) and the rms current phasor of (2.2) shown in Figure 2.3 are

$$V = |V| \angle \theta_v \text{ and } I = |I| \angle \theta_i$$

The term  $VI^*$  results in



**FIGURE 2.3** Phasor diagram and power triangle for an inductive load (lagging PF).

$$VI^* = |V||I| \angle \theta_v - \theta_i = |V||I| \angle \theta$$

## 2.3 COMPLEX POWER

Run the new GUI program (`chp2ex1gui`). This allows you to see instantly the effect of changing load from inductive to resistive and capacitive on the instantaneous power  $p(t)$ ,  $pR(t)$ , and  $pX(t)$ .

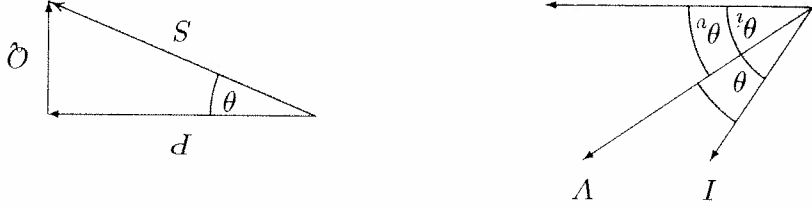
$$= |V||I| \cos \theta + j|V||I| \sin \theta$$

The above equation defines a complex quantity where its real part is the average (real) power  $P$  and its imaginary part is the reactive power  $Q$ . Thus, the complex power designated by  $S$  is given by

$$S = VI^* = P + jQ \quad (2.10)$$

The magnitude of  $S$ ,  $|S| = \sqrt{P^2 + Q^2}$ , is the apparent power; its unit is volt-amperes and the larger units are kVA or MVA. Apparent power gives a direct indication of heating and is used as a rating unit of power equipment. Apparent power has practical significance for an electric utility company since a utility company must supply both average and apparent power to consumers.

The reactive power  $Q$  is positive when the phase angle  $\theta$  between voltage and current (impedance angle) is positive (i.e., when the load impedance is inductive, and  $I$  lags  $V$ ).  $Q$  is negative when  $\theta$  is negative (i.e., when the load impedance is capacitive and  $I$  leads  $V$ ) as shown in Figure 2.4. In working with Equation (2.10) it is convenient to think of  $P$ ,  $Q$ , and  $S$  as forming the sides of a right triangle as shown in Figures 2.3 and 2.4.



**FIGURE 2.4** Phasor diagram and power triangle for a capacitive load (leading PF).

If the load impedance is  $Z$  then

$$V = ZI \quad (2.11)$$

substituting for  $V$  into (2.10) yields

$$S = VI^* = ZII^* = R|I|^2 + jX|I|^2 \quad (2.12)$$

From (2.12) it is evident that complex power  $S$  and impedance  $Z$  have the same angle. Because the power triangle and the impedance triangle are similar triangles, the impedance angle is sometimes called the *power angle*. Similarly, substituting for  $I$  from (2.11) into (2.10) yields

$$S = VI^* = \frac{Z^*}{|V|^2} = \frac{Z^*}{|V|^2} \quad (2.13)$$

From (2.13), the impedance of the complex power  $S$  is given by

$$Z = \frac{S^*}{|V|^2} \quad (2.14)$$

## 2.4 THE COMPLEX POWER BALANCE

From the conservation of energy, it is clear that real power supplied by the source is equal to the sum of real powers absorbed by the load. At the same time, a balance between the reactive power must be maintained. Thus the total complex power delivered to the loads in parallel is the sum of the complex powers delivered to each. Proof of this is as follows:

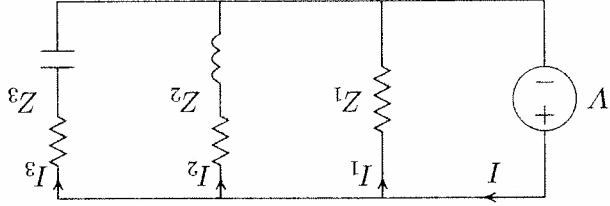


FIGURE 2.5

Three loads in parallel.

For the three loads shown in Figure 2.5, the total complex power is given by

$$S = VI^* = V[I_1^* + I_2^* + I_3^*] = VI_1^* + VI_2^* + VI_3^* \quad (2.15)$$

### Example 2.2 (chp2ex2)

In the above circuit  $V = 1200\angle 0^\circ$  V,  $Z_1 = 60 + j0 \Omega$ ,  $Z_2 = 6 + j12 \Omega$  and  $Z_3 = 30 - j30 \Omega$ . Find the power absorbed by each load and the total complex

power.

$$I_1 = \frac{1200\angle 0^\circ}{60\angle 0^\circ} = 20 + j0 \text{ A}$$

$$I_2 = \frac{1200\angle 0^\circ}{6 + j12} = 40 - j80 \text{ A}$$

$$I_3 = \frac{1200\angle 0^\circ}{30 - j30} = 20 + j20 \text{ A}$$

and

$$I = I_1 + I_2 + I_3 = (20 + j0) + (40 - j80) + (20 + j20) = 80 - j60 = 100\angle -36.87^\circ \text{ A}$$

first finding the total current.

Alternatively, the sum of complex power delivered to the load can be obtained by

$$S = S_1 + S_2 + S_3 = 96,000 \text{ W} + j72,000 \text{ var}$$

The total load complex power adds up to

$$S_1 = VI_1^* = 1200\angle 0^\circ(20 - j0) = 24,000 \text{ W} + j0 \text{ var}$$

$$S_2 = VI_2^* = 1200\angle 0^\circ(40 + j80) = 48,000 \text{ W} + j96,000 \text{ var}$$

$$S_3 = VI_3^* = 1200\angle 0^\circ(20 - j20) = 24,000 \text{ W} - j24,000 \text{ var}$$

A final insight is contained in Figure 2.6, which shows the current phasor diagram and the complex power representation.

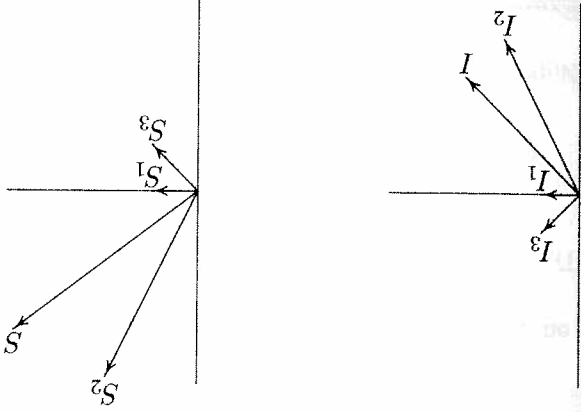


FIGURE 2.6

Current phasor diagram and power plane diagram.

The complex powers may also be obtained directly from (2.14)

$$S_1 = \frac{|V|^2 Z_1^*}{(1200)^2} = \frac{60}{(1200)^2} = 24,000 \text{ W} + j0$$

$$S_2 = \frac{|V|^2 Z_2^*}{(1200)^2} = \frac{6 - j12}{(1200)^2} = 48,000 \text{ W} + j96,000 \text{ var}$$

$$S_3 = \frac{|V|^2 Z_3^*}{(1200)^2} = \frac{30 + j30}{(1200)^2} = 24,000 \text{ W} - j24,000 \text{ var}$$



## 2.5 POWER FACTOR CORRECTION

It can be seen from (2.7) that the apparent power will be larger than  $P$  if the power factor is less than 1. Thus the current  $I$  that must be supplied will be larger for  $PF < 1$  than it would be for  $PF = 1$ , even though the average power  $P$  supplied is the same in either case. A larger current cannot be supplied without additional cost to the utility company. Thus, it is in the power company's (and its customer's) best interest that major loads on the system have power factors as close to 1 as possible. In order to maintain the power factor close to unity, power companies install banks of capacitors throughout the network as needed. They also impose an additional charge to industrial consumers who operate at low power factors. Since industrial loads are inductive and have low lagging power factors, it is beneficial to install capacitors to improve the power factor. This consideration is not important for residential and small commercial customers because their power factors are close to unity.

### Example 2.3 (chp2ex3)

Two loads  $Z_1 = 100 + j0 \Omega$  and  $Z_2 = 10 + j20 \Omega$  are connected across a 200-V rms, 60-Hz source as shown in Figure 2.7.

(a) Find the total real and reactive power, the power factor at the source, and the total current.

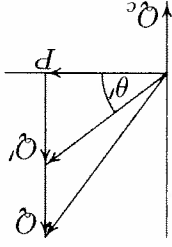
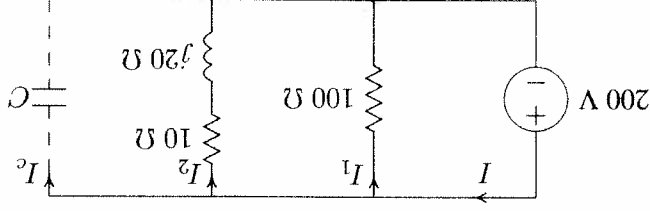


FIGURE 2.7

Circuit for Example 2.3 and the power triangle.

$$I_1 = \frac{100}{200\angle 0^\circ} = 270^\circ \text{ A}$$

$$I_2 = \frac{10 + j20}{200\angle 0^\circ} = 4 - j8 \text{ A}$$

$$S_1 = VI_1^* = 200\angle 0^\circ(2 - j0) = 400 \text{ W} + j0 \text{ var}$$

$$S_2 = VI_2^* = 200\angle 0^\circ(4 + j8) = 800 \text{ W} + j1600 \text{ var}$$

## 2.5 POWER FACTOR CORRECTION

It can be seen from (2.7) that the apparent power will be larger than  $P$  if the power factor is less than 1. Thus the current  $I$  that must be supplied will be larger for  $PF < 1$  than it would be for  $PF = 1$ , even though the average power  $P$  supplied is the same in either case. A larger current cannot be supplied without additional cost to the utility company. Thus, it is in the power company's (and its customer's) best interest that major loads on the system have power factors as close to 1 as possible. In order to maintain the power factor close to unity, power companies install banks of capacitors throughout the network as needed. They also impose an additional charge to industrial consumers who operate at low power factors. Since industrial loads are inductive and have low lagging power factors, it is beneficial to install capacitors to improve the power factor. This consideration is not important for residential and small commercial customers because their power factors are close to unity.

### Example 2.3 (chp2ex3)

Two loads  $Z_1 = 100 + j0 \Omega$  and  $Z_2 = 10 + j20 \Omega$  are connected across a 200-V rms, 60-Hz source as shown in Figure 2.7.

(a) Find the total real and reactive power, the power factor at the source, and the total current.

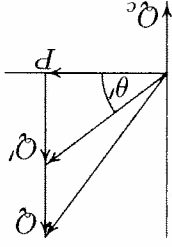
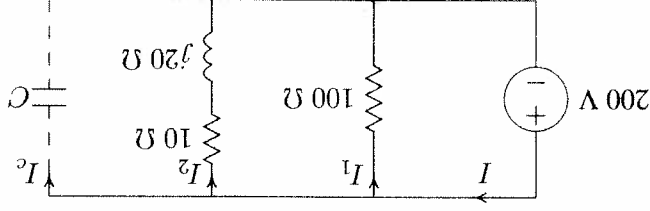


FIGURE 2.7

Circuit for Example 2.3 and the power triangle.

$$I_1 = \frac{100}{200\angle 0^\circ} = 270^\circ \text{ A}$$

$$I_2 = \frac{10 + j20}{200\angle 0^\circ} = 4 - j8 \text{ A}$$

$$S_1 = VI_1^* = 200\angle 0^\circ(2 - j0) = 400 \text{ W} + j0 \text{ var}$$

$$S_2 = VI_2^* = 200\angle 0^\circ(4 + j8) = 800 \text{ W} + j1600 \text{ var}$$

Total apparent power and current are

$$S = P + jQ = 1200 + j1600 = 2000\angle 53.13^\circ \text{ VA}$$

$$I = \frac{S^*}{V^*} = \frac{2000\angle -53.13^\circ}{200\angle 0^\circ} = 10\angle -53.13^\circ \text{ A}$$

Power factor at the source is

$$PF = \cos(53.13^\circ) = 0.6 \text{ lagging}$$

(b) Find the capacitance of the capacitor connected across the loads to improve the overall power factor to 0.8 lagging.

Total real power  $P = 1200 \text{ W}$  at the new power factor 0.8 lagging. Therefore

$$\theta = \cos^{-1}(0.8) = 36.87^\circ$$

$$Q' = P \tan \theta = 1200 \tan(36.87^\circ) = 900 \text{ var}$$

$$Q_c = 1600 - 900 = 700 \text{ var}$$

$$Z_c = \frac{|V|^2}{S_c^*} = \frac{(200)^2}{j700} = -j57.14 \Omega$$

$$C = \frac{10^6}{2\pi(60)(57.14)} = 46.42 \mu\text{F}$$

The total power and the new current are

$$S' = 1200 + j900 = 1500\angle 36.87^\circ$$

$$I' = \frac{S'^*}{V^*} = \frac{1500\angle -36.87^\circ}{200\angle 0^\circ} = 7.5\angle -36.87^\circ$$

Note the reduction in the supply current from 10 A to 7.5 A.

### Example 2.4 (chp2ex4)

Three loads are connected in parallel across a 1400-V rms, 60-Hz single-phase supply as shown in Figure 2.8.

Load 1: Inductive load, 125 kVA at 0.28 power factor.

Load 2: Capacitive load, 10 kW and 40 kvar.

Load 3: Resistive load of 15 kW.

(a) Find the total kW, kvar, kVA, and the supply power factor.

Total real power  $P = 60 \text{ kW}$  at the new power factor of 0.8 lagging results in the new reactive power  $Q'$ .

$$\theta' = \cos^{-1}(0.8) = 36.87^\circ$$

$$Q' = 60 \tan(36.87^\circ) = 45 \text{ kvar}$$

Therefore, the required capacitor kvar is

$$Q_c = 80 - 45 = 35 \text{ kvar}$$

and

$$X_c = \frac{|V|^2}{|S_c^*|} = \frac{1400^2}{j35,000} = -j56 \Omega$$

$$C = \frac{10^6}{2\pi(60)(56)} = 47.37 \mu\text{F}$$

and the new current is

$$I' = \frac{V^*}{S'^*} = \frac{1400\angle 0^\circ}{60,000 - j45,000} = 53.57\angle -36.87^\circ \text{ A}$$

Note the reduction in the supply current from 71.43 A to 53.57 A.

## 2.6 COMPLEX POWER FLOW

Consider two ideal voltage sources connected by a line of impedance  $Z = R + jX \Omega$  as shown in Figure 2.9.

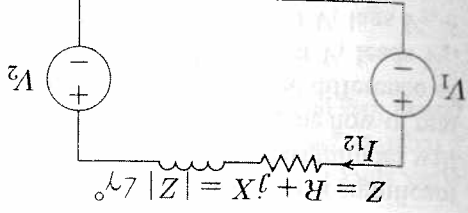


FIGURE 2.9 Two interconnected voltage sources.

Let the phasor voltage be  $V_1 = |V_1|\angle\delta_1$  and  $V_2 = |V_2|\angle\delta_2$ . For the assumed direction of current

$$I_{12} = \frac{|V_1|\angle\delta_1 - |V_2|\angle\delta_2}{|Z|\angle\gamma} = \frac{|V_1|\angle\delta_1 - |V_2|\angle\delta_2}{|Z|\angle\gamma}$$

An inductive load has a lagging power factor, the capacitive load has a leading power factor, and the resistive load has a unity power factor.

For Load 1:

$$\theta_1 = \cos^{-1}(0.28) = 73.74^\circ \text{ lagging}$$

The load complex powers are

$$S_1 = 125\angle 73.74^\circ \text{ kVA} = 35 \text{ kW} + j120 \text{ kvar}$$

$$S_2 = 10 \text{ kW} - j40 \text{ kvar}$$

$$S_3 = 15 \text{ kW} + j0 \text{ kvar}$$

The total apparent power is

$$\begin{aligned} S &= P + jQ = S_1 + S_2 + S_3 \\ &= (35 + j120) + (10 - j40) + (15 + j0) \\ &= 60 \text{ kW} + j80 \text{ kvar} = 100\angle 53.13^\circ \text{ kVA} \end{aligned}$$

The total current is

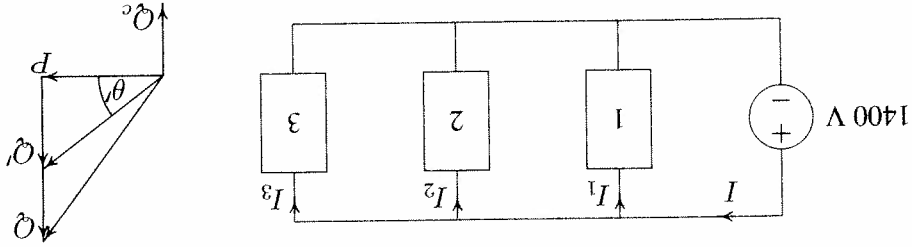
$$I = \frac{S^*}{V^*} = \frac{100,000\angle -53.13^\circ}{1400\angle 0^\circ} = 71.43\angle -53.13^\circ \text{ A}$$

The supply power factor is

$$PF = \cos(53.13^\circ) = 0.6 \text{ lagging}$$

(b) A capacitor of negligible resistance is connected in parallel with the above loads to improve the power factor to 0.8 lagging. Determine the kvar rating of this capacitor and the capacitance in  $\mu\text{F}$ .

FIGURE 2.8 Circuit for Example 2.4.



3. For maintaining transient stability, the power system is usually operated with small load angle  $\delta$ . Also, from (2.19) the reactive power flow is determined by the magnitude difference of terminal voltages, (i.e.,  $Q \propto |V_1| - |V_2|$ ).

### Example 2.5 (chp2ex5)

Two voltage sources  $V_1 = 120\angle -5^\circ$  V and  $V_2 = 100\angle 0^\circ$  V are connected by a short line of impedance  $Z = 1 + j7\ \Omega$  as shown in Figure 2.9. Determine the real and reactive power supplied or received by each source and the power loss in the line.

$$I_{12} = \frac{120\angle -5^\circ - 100\angle 0^\circ}{1 + j7} = 3.135\angle -110.02^\circ \text{ A}$$

$$I_{21} = \frac{100\angle 0^\circ - 120\angle -5^\circ}{1 + j7} = 3.135\angle 69.98^\circ \text{ A}$$

$$S_{12} = V_1 I_{12}^* = 376.2\angle 105.02^\circ = -97.5 \text{ W} + j363.3 \text{ var}$$

$$S_{21} = V_2 I_{21}^* = 313.5\angle -69.98^\circ = 107.3 \text{ W} - j294.5 \text{ var}$$

Line loss is given by

$$S_L = S_1 + S_2 = 9.8 \text{ W} + j68.8 \text{ var}$$

From the above results, since  $P_1$  is negative and  $P_2$  is positive, source 1 receives 97.5 W, and source 2 generates 107.3 W and the real power loss in the line is 9.8 W. The real power loss in the line can be checked by

$$P_L = R|I_{12}|^2 = (1)(3.135)^2 = 9.8 \text{ W}$$

Also, since  $Q_1$  is positive and  $Q_2$  is negative, source 1 delivers 363.3 var and source 2 receives 294.5 var, and the reactive power loss in the line is 68.8 var. The reactive power loss in the line can be checked by

$$Q_L = X|I_{12}|^2 = (7)(3.135)^2 = 68.8 \text{ var}$$

### Example 2.6 (chp2ex6)

This example concerns the direction of power flow between two voltage sources. Write a *MATLAB* program for the system of Example 2.5 such that the phase angle of source 1 is changed from its initial value by  $\pm 30^\circ$  in steps of  $5^\circ$ . Voltage magnitudes of the two sources and the voltage phase angle of source 2 is to be kept constant. Compute the complex power for each source and the line loss. Tabulate the real power and plot  $P_1$ ,  $P_2$ , and  $P_L$  versus voltage phase angle  $\delta$ . The following commands

The complex power  $S_{12}$  is given by

$$S_{12} = V_1 I_{12}^* = |V_1| \angle \delta_1 [ |V_2| \angle \gamma - \delta_1 - \delta_2 ] = \frac{|V_1|^2}{|V_1||V_2|} \angle \gamma + \delta_1 - \delta_2$$

Thus, the real and reactive power at the sending end are

$$P_{12} = \frac{|V_1|^2}{|V_1||V_2|} \cos(\gamma + \delta_1 - \delta_2) \quad (2.16)$$

$$Q_{12} = \frac{|V_1|^2}{|V_1||V_2|} \sin(\gamma + \delta_1 - \delta_2) \quad (2.17)$$

Power system transmission lines have small resistance compared to the reactance. Assuming  $R = 0$  (i.e.,  $Z = X \angle 90^\circ$ ), the above equations become

$$P_{12} = \frac{X}{|V_1||V_2|} \sin(\delta_1 - \delta_2) \quad (2.18)$$

$$Q_{12} = \frac{X}{|V_1||V_2|} [ |V_1| - |V_2| \cos(\delta_1 - \delta_2) ] \quad (2.19)$$

Since  $R = 0$ , there are no transmission line losses and the real power sent equals the real power received.

From the above results, for a typical power system with small  $R/X$  ratio, the following important observations are made :

1. Equation (2.18) shows that small changes in  $\delta_1$  or  $\delta_2$  will have a significant effect on the real power flow, while small changes in voltage magnitudes will not have appreciable effect on the real power flow. Therefore, the flow of real power on a transmission line is governed mainly by the angle difference of the terminal voltages (i.e.,  $P_{12} \propto \sin \delta$ ), where  $\delta = \delta_1 - \delta_2$ . If  $V_1$  leads  $V_2$ ,  $\delta$  is positive and the real power flows from node 1 to node 2. If  $V_1$  lags  $V_2$ ,  $\delta$  is negative and power flows from node 2 to node 1.

2. Assuming  $R = 0$ , the theoretical maximum power (static transmission capacity) occurs when  $\delta = 90^\circ$  and the maximum power transfer is given by

$$P_{max} = \frac{X}{|V_1||V_2|} \quad (2.20)$$

In Chapter 3 we learn that increasing  $\delta$  beyond the static transmission capacity will result in loss of synchronism between the two machines.

Delta 1	P-1	P-2	P-L
-35.0000	-872.2049	967.0119	94.8070
-30.0000	-759.8461	832.1539	72.3078
-25.0000	-639.5125	692.4848	52.9723
-20.0000	-512.1201	549.0676	36.9475
-15.0000	-378.6382	402.9938	24.3556
-10.0000	-240.0828	255.3751	15.2923
-5.0000	-97.5084	107.3349	9.8265
0	48.0000	-40.0000	8.0000

```
Source # 1 Voltage Mag. = 120
Source # 1 Phase Angle = -5
Source # 2 Voltage Mag. = 100
Source # 2 Phase Angle = 0
Line Resistance = 1
Line Reactance = 7
```

```

E1 = input('Source # 1 Voltage Mag. = ');
a1 = input('Source # 1 Phase Angle = ');
E2 = input('Source # 2 Voltage Mag. = ');
a2 = input('Source # 2 Phase Angle = ');
R = input('Line Resistance = ');
X = input('Line Reactance = ');
Z = R + j*X;
a1 = (-30+a1:5:30+a1)'; % Change a1 by +/- 30, col. array
a1r = a1*pi/180; % Convert degree to radian
k = length(a1);
a2 = ones(k,1)*a2; % Create col. array of same length for a2
a2r = a2*pi/180; % Convert degree to radian
V1 = E1.*cos(a1r) + j*E1.*sin(a1r);
V2 = E2.*cos(a2r) + j*E2.*sin(a2r);
I12 = (V1 - V2)./Z; I21=-I12;
S1 = V1.*conj(I12); P1 = real(S1); Q1 = imag(S1);
S2 = V2.*conj(I21); P2 = real(S2); Q2 = imag(S2);
SL = S1+S2;
PL = real(SL); QL = imag(SL);
Result1 = [a1, P1, P2, PL];
disp(' Delta 1 P-1 P-2 P-L ');
disp(Result1)
plot(a1, P1, a1, P2, a1, PL)
xlabel('Source #1 Voltage Phase Angle')
ylabel('P, Watts');
text(-26, -550, 'P1'), text(-26, 600, 'P2'),
text(-26, 100, 'PL');
result in

```

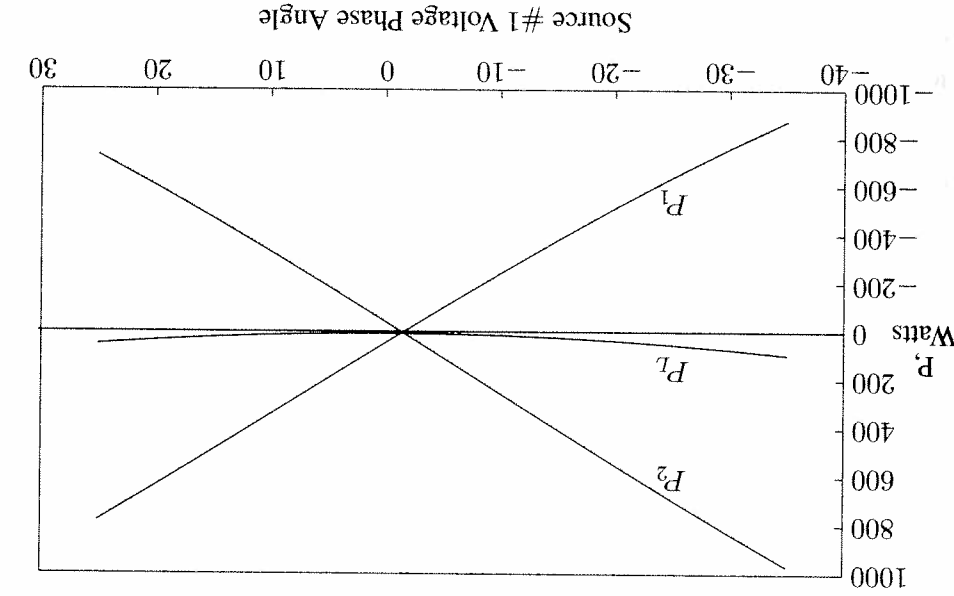


FIGURE 2.10 Real power versus voltage phase angle  $\delta$ .

Examination of Figure 2.10 shows that the flow of real power along the interconnection is determined by the angle difference of the terminal voltages. Problem 2.9 requires the development of a similar program for demonstrating the dependency of reactive power on the magnitude difference of terminal voltages. Run the new GUI program (**chp2ex6gui**). This allows you to see instantly the effect of sweeping voltage phase angle  $\delta_1$  on the direction of real power flow, and the effect of sweeping voltage magnitude  $V_1$  on the direction of the reactive power flow.

## 2.7 BALANCED THREE-PHASE CIRCUITS

The generation, transmission and distribution of electric power is accomplished by means of three-phase circuits. At the generating station, three sinusoidal voltages are generated having the same amplitude but displaced in phase by  $120^\circ$ . This is

called a *balanced source*. If the generated voltages reach their peak values in the sequential order ABC, the generator is said to have a *positive phase sequence*, shown in Figure 2.11(a). If the phase order is ACB, the generator is said to have a *negative phase sequence*, as shown in Figure 2.11(b).

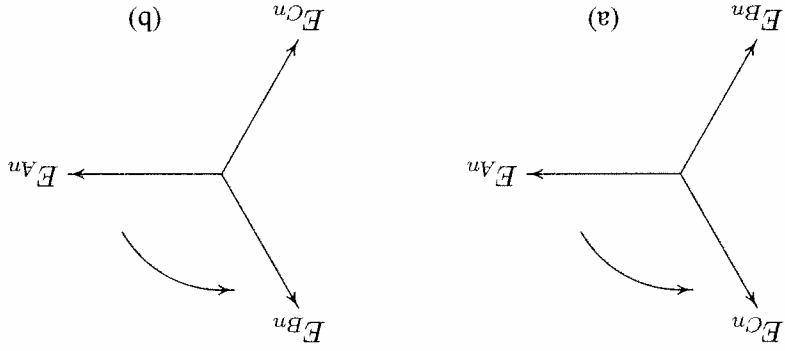


FIGURE 2.11 (a) Positive, or ABC, phase sequence. (b) Negative, or ACB, phase sequence.

In a three-phase system, the instantaneous power delivered to the external loads is constant rather than pulsating as it is in a single-phase circuit. Also, three-phase motors, having constant torque, start and run much better than single-phase motors. This feature of three-phase power, coupled with the inherent efficiency of its transmission compared to single-phase (less wire for the same delivered power), accounts for its universal use.

A power system has Y-connected generators and usually includes both  $\Delta$ - and Y-connected loads. Generators are rarely  $\Delta$ -connected, because if the voltages are not perfectly balanced, there will be a net voltage, and consequently a circulating current, around the  $\Delta$ . Also, the phase voltages are lower in the Y-connected generator, and thus less insulation is required. Figure 2.12 shows a Y-connected generator supplying balanced Y-connected loads through a three-phase line. Assuming a positive phase sequence (phase order ABC) the generated voltages are:

$$\begin{aligned} E_{An} &= |E_p| \angle 0^\circ \\ E_{Bn} &= |E_p| \angle -120^\circ \\ E_{Cn} &= |E_p| \angle -240^\circ \end{aligned} \quad (2.21)$$

In power systems, great care is taken to ensure that the loads of transmission lines are balanced. For balanced loads, the terminal voltages of the generator  $V_{An}$ ,  $V_{Bn}$  and  $V_{Cn}$  and the phase voltages  $V_{an}$ ,  $V_{bn}$  and  $V_{cn}$  at the load terminals are balanced. For "phase A," these are given by

$$\begin{aligned} V_{An} &= E_{An} - Z_G I_a \\ V_{an} &= V_{An} - Z_L I_a \end{aligned} \quad (2.22)$$

$$V_{cn} = V_{An} - Z_L I_c \quad (2.23)$$

## 2.8 Y-CONNECTED LOADS

To find the relationship between the line voltages (line-to-line voltages) and the phase voltages (line-to-neutral voltages), we assume a positive, or ABC, sequence. We arbitrarily choose the line-to-neutral voltage of the a-phase as the reference, thus

$$\begin{aligned} V_{an} &= |V_p| \angle 0^\circ \\ V_{bn} &= |V_p| \angle -120^\circ \\ V_{cn} &= |V_p| \angle -240^\circ \end{aligned} \quad (2.24)$$

where  $|V_p|$  represents the magnitude of the phase voltage (line-to-neutral voltage). The line voltages at the load terminals in terms of the phase voltages are found by the application of Kirchhoff's voltage law

$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} = |V_p| (170^\circ - 17^\circ - 120^\circ) = \sqrt{3} |V_p| \angle 73^\circ \\ V_{bc} &= V_{bn} - V_{cn} = |V_p| (17^\circ - 120^\circ - 17^\circ - 240^\circ) = \sqrt{3} |V_p| \angle -90^\circ \\ V_{ca} &= V_{cn} - V_{an} = |V_p| (17^\circ - 240^\circ - 17^\circ - 120^\circ) = \sqrt{3} |V_p| \angle 150^\circ \end{aligned} \quad (2.25)$$

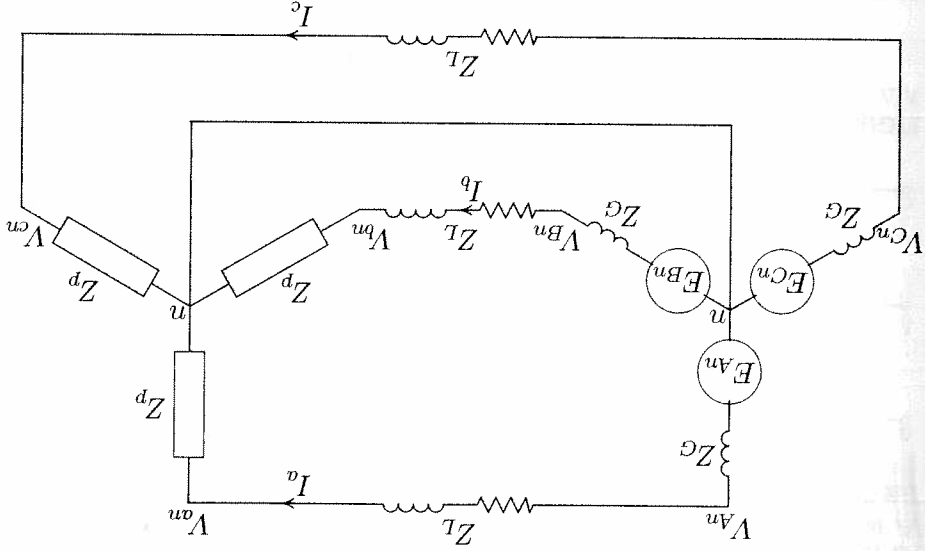


FIGURE 2.12 A Y-connected generator supplying a Y-connected load.

The voltage phasor diagram of the Y-connected loads of Figure 2.12 is shown in Figure 2.13. The relationship between the line voltages and phase voltages is demonstrated graphically.

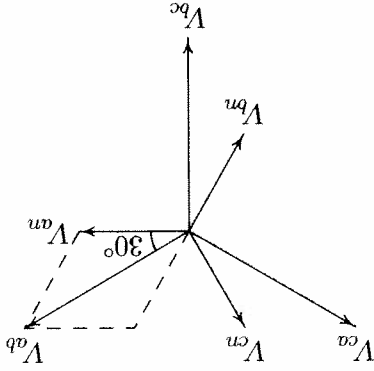


FIGURE 2.13 Phasor diagram showing phase and line voltages.

If the rms value of any of the line voltages is denoted by  $V_L$ , then one of the important characteristics of the Y-connected three-phase load may be expressed as

$$V_L = \sqrt{3} |V_p| \angle 30^\circ \quad (2.26)$$

Thus in the case of Y-connected loads, the magnitude of the line voltage is  $\sqrt{3}$  times the magnitude of the phase voltage, and for a positive phase sequence, the set of line voltages leads the set of phase voltages by  $30^\circ$ .

The three-phase currents in Figure 2.12 also possess three-phase symmetry and are given by

$$\begin{aligned} I_a &= \frac{V_{an}}{Z_p} = |I_p| \angle -\theta \\ I_b &= \frac{V_{bn}}{Z_p} = |I_p| \angle -120^\circ - \theta \\ I_c &= \frac{V_{cn}}{Z_p} = |I_p| \angle -240^\circ - \theta \end{aligned} \quad (2.27)$$

where  $\theta$  is the impedance phase angle. The currents in lines are also the phase currents (the current carried by the phase impedances). Thus

$$I_L = I_p \quad (2.28)$$

## 2.9 Δ-CONNECTED LOADS

A balanced  $\Delta$ -connected load (with equal phase impedances) is shown in Figure 2.14.

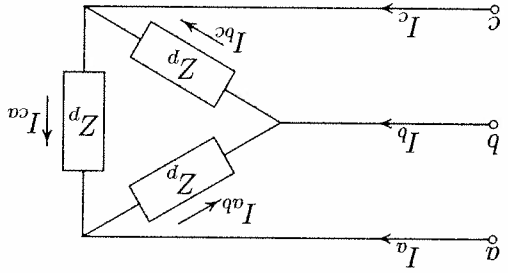


FIGURE 2.14 A  $\Delta$ -connected load.

It is clear from the inspection of the circuit that the line voltages are the same as phase voltages.

$$V_L = V_p \quad (2.29)$$

Consider the phasor diagram shown in Figure 2.15, where the phase current  $I_{ab}$  is arbitrarily chosen as reference. We have

$$\begin{aligned} I_{ab} &= |I_p| \angle 0^\circ \\ I_{bc} &= |I_p| \angle -120^\circ \\ I_{ca} &= |I_p| \angle -240^\circ \end{aligned} \quad (2.30)$$

where  $|I_p|$  represents the magnitude of the phase current.

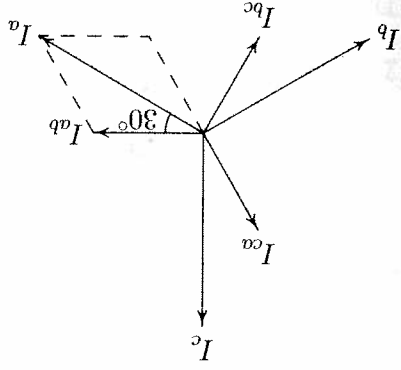


FIGURE 2.15 Phasor diagram showing phase and line currents.

The relationship between phase and line currents can be obtained by applying Kirchhoff's current law at the corners of  $\Delta$ .

$$\begin{aligned} I_a &= I_{ab} - I_{ca} = |I^p|(\angle 170^\circ - \angle -17^\circ - 240^\circ) = \sqrt{3}|I^p|\angle -30^\circ \\ I_b &= I_{bc} - I_{ab} = |I^p|(\angle 17^\circ - \angle 120^\circ - \angle 170^\circ) = \sqrt{3}|I^p|\angle -150^\circ \\ I_c &= I_{ca} - I_{bc} = |I^p|(\angle 17^\circ - \angle 240^\circ - \angle -17^\circ - 120^\circ) = \sqrt{3}|I^p|\angle 90^\circ \end{aligned} \quad (2.31)$$

The relationship between the line currents and phase currents is demonstrated graphically in Figure 2.15.

If the rms of any of the line currents is denoted by  $I_L$ , then one of the important characteristics of the  $\Delta$ -connected three-phase load may be expressed as

$$I_L = \sqrt{3}|I^p|\angle -30^\circ \quad (2.32)$$

Thus in the case of  $\Delta$ -connected loads, the magnitude of the line current is  $\sqrt{3}$  times the magnitude of the phase current, and with positive phase sequence, the set of line currents lags the set of phase currents by  $30^\circ$ .

## 2.10 $\Delta$ -Y TRANSFORMATION

For analyzing network problems, it is convenient to replace the  $\Delta$ -connected circuit with an equivalent Y-connected circuit. Consider the fictitious Y-connected circuit of  $Z_Y$   $\Omega$ /phase which is equivalent to a balanced  $\Delta$ -connected circuit of  $Z_\Delta$   $\Omega$ /phase, as shown in Figure 2.16.

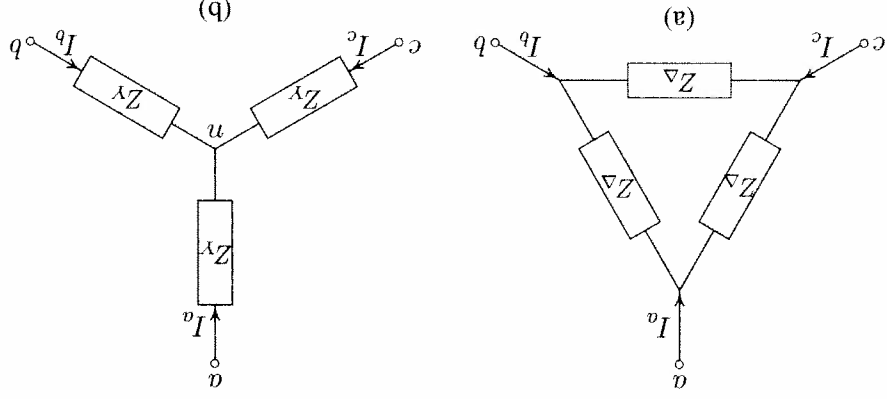


FIGURE 2.16

(a)  $\Delta$  to (b) Y-connection.

For the  $\Delta$ -connected circuit, the phase current  $I_a$  is given by

$$I_a = \frac{Z_\Delta}{V_{ab} + Z_\Delta} + \frac{Z_\Delta}{V_{ac}} = \frac{Z_\Delta}{V_{ab} + V_{ac}} \quad (2.33)$$

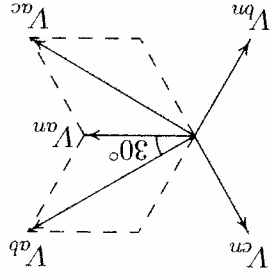


FIGURE 2.17

Phasor diagram showing phase and line voltages.

$$V_{ab} + V_{ac} = \sqrt{3}|V_{an}|\angle 30^\circ + \sqrt{3}|V_{an}|\angle -30^\circ \quad (2.34)$$

(2.35)

$$= 3V_{an}$$

or

$$V_{an} = \frac{Z_\Delta}{3}I_a \quad (2.36)$$

Now, for the Y-connected circuit, we have

(2.37)

$$V_{an} = Z_Y I_a \quad (2.37)$$

Thus, from (2.36) and (2.37), we find that

(2.38)

$$Z_Y = \frac{Z_\Delta}{3} \quad (2.38)$$

## 2.11 PER-PHASE ANALYSIS

The current in the neutral of the balanced Y-connected loads shown in Figure 2.12

$$I_n = I_a + I_b + I_c = 0 \quad (2.39)$$

is given by

Since the neutral carries no current, a neutral wire of any impedance may be placed by any other impedance, including a short circuit and an open circuit. The return line may not actually exist, but regardless, a line of zero impedance is included between the two neutral points. The balanced power system problems are then solved on a "per-phase" basis. It is understood that the other two phases carry identical currents except for the phase shift.

We may then look at only one phase, say "phase A," consisting of the source  $V_{an}$  in series with  $Z_L$  and  $Z_p$ , as shown in Figure 2.18. The neutral is taken as datum and usually a single-subscript notation is used for phase voltages.

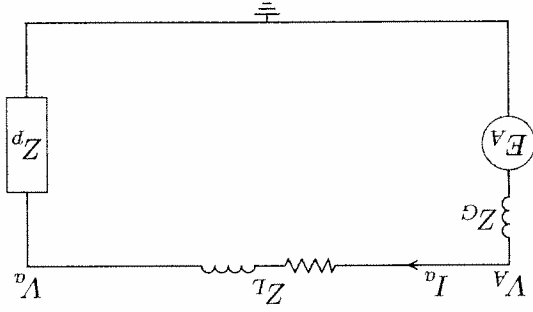


FIGURE 2.18

Single-phase circuit for per-phase analysis.

If the load in a three-phase circuit is connected in a  $\Delta$ , it can be transformed into a Y by using the  $\Delta$ -to-Y transformation. When the load is balanced, the impedance of each leg of the Y is one-third the impedance of each leg of the  $\Delta$ , as given by (2.38), and the circuit is modeled by the single-phase equivalent circuit.

## 2.12 BALANCED THREE-PHASE POWER

Consider a balanced three-phase source supplying a balanced Y- or  $\Delta$ -connected load with the following instantaneous voltages

$$\begin{aligned} v_{an} &= \sqrt{2}|V_p| \cos(\omega t + \theta_a) \\ v_{bn} &= \sqrt{2}|V_p| \cos(\omega t + \theta_b - 120^\circ) \\ v_{cn} &= \sqrt{2}|V_p| \cos(\omega t + \theta_b - 240^\circ) \end{aligned} \quad (2.40)$$

$$\begin{aligned} i_a &= \sqrt{2}|I_p| \cos(\omega t + \theta_i) \\ i_b &= \sqrt{2}|I_p| \cos(\omega t + \theta_i - 120^\circ) \\ i_c &= \sqrt{2}|I_p| \cos(\omega t + \theta_i - 240^\circ) \end{aligned} \quad (2.41)$$

For a balanced load the phase currents are

$$p_{3\phi} = v_{an}i_a + v_{bn}i_b + v_{cn}i_c \quad (2.42)$$

Substituting for the instantaneous voltages and currents from (2.40) and (2.41) into

(2.42)

$$\begin{aligned} p_{3\phi} &= 2|V_p||I_p| \cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \\ &+ 2|V_p||I_p| \cos(\omega t + \theta_v - 120^\circ) \cos(\omega t + \theta_i - 120^\circ) \\ &+ 2|V_p||I_p| \cos(\omega t + \theta_v - 240^\circ) \cos(\omega t + \theta_i - 240^\circ) \end{aligned}$$

Using the trigonometric identity (2.4)

$$\begin{aligned} p_{3\phi} &= |V_p||I_p| [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \\ &+ |V_p||I_p| [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 240^\circ)] \\ &+ |V_p||I_p| [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 480^\circ)] \end{aligned} \quad (2.43)$$

The three double frequency cosine terms in (2.43) are out of phase with each other by  $120^\circ$  and add up to zero, and the three-phase instantaneous power is

$$P_{3\phi} = 3|V_p||I_p| \cos \theta \quad (2.44)$$

$\theta = \theta_v - \theta_i$  is the angle between phase voltage and phase current or the impedance angle.

Note that although the power in each phase is pulsating, the total instantaneous power is constant and equal to three times the real power in each phase. Indeed, this constant power is the main advantage of the three-phase system over the single-phase system. Since the power in each phase is pulsating, the power, then, its made up of the real power and the reactive power. In order to obtain formula symmetry between real and reactive powers, the concept of complex or apparent power ( $S$ ) is extended to three-phase systems by defining the three-phase reactive power as

$$Q_{3\phi} = 3|V_p||I_p| \sin \theta \quad (2.45)$$

Thus, the complex three-phase power is

$$S_{3\phi} = P_{3\phi} + jQ_{3\phi} \quad (2.46)$$

or

$$S_{3\phi} = 3V_p I_p^* \quad (2.47)$$

Equations (2.44) and (2.45) are sometimes expressed in terms of the rms magnitude of the line voltage and the rms magnitude of the line current. In a Y-connected load the phase voltage  $|V_p| = |V_L|/\sqrt{3}$  and the phase current  $I_p = I_L$ .



In the  $\Delta$ -connection  $V_p = V_L$  and  $|I_p| = |I_L|/\sqrt{3}$ . Substituting for the phase voltage and phase currents in (2.44) and (2.45), the real and reactive powers for either connection are given by

$$P_{3\phi} = \sqrt{3}|V_L||I_L|\cos\theta \quad (2.48)$$

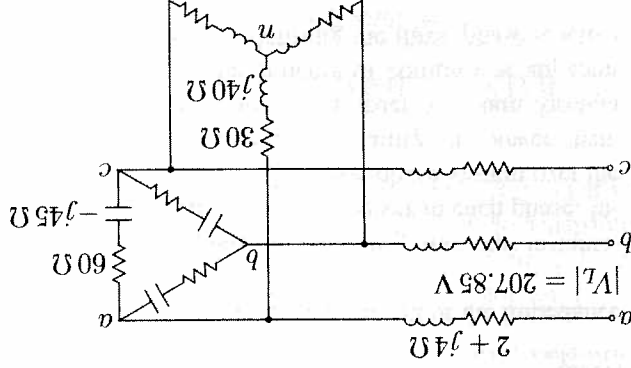
$$Q_{3\phi} = \sqrt{3}|V_L||I_L|\sin\theta \quad (2.49)$$

A comparison of the last two expressions with (2.44) and (2.45) shows that the equation for the power in a three-phase system is the same for either a Y or a  $\Delta$  connection when the power is expressed in terms of line quantities.

When using (2.48) and (2.49) to calculate the total real and reactive power, remember that  $\theta$  is the phase angle between the phase voltage and the phase current. As in the case of single-phase systems for the computation of power, it is best to use the complex power expression in terms of phase quantities given by (2.47). The rated power is customarily given for the three-phase and rated voltage is the line-to-line voltage. Thus, in using the per-phase equivalent circuit, care must be taken to use per-phase voltage by dividing the rated voltage by  $\sqrt{3}$ .

### Example 2.7 (chp2ex7)

A three-phase line has an impedance of  $2 + j4 \Omega$  as shown in Figure 2.19.



The line feeds two balanced three-phase loads that are connected in parallel. The first load is Y-connected and has an impedance of  $30 + j40 \Omega$  per phase. The second load is  $\Delta$ -connected and has an impedance of  $60 - j45 \Omega$ . The line is energized at the sending end from a three-phase balanced supply of line voltage 207.85 V. Taking the phase voltage  $V_a$  as reference, determine:

- (a) The current, real power, and reactive power drawn from the supply.
- (b) The line voltage at the combined loads.

FIGURE 2.19 Three-phase circuit diagram for Example 2.7.

- (c) The current per phase in each load.
- (d) The total real and reactive powers in each load and the line.

(a) The  $\Delta$ -connected load is transformed into an equivalent Y. The impedance per phase of the equivalent Y is

$$Z_2 = \frac{60 - j45}{3} = 20 - j15 \Omega$$

The phase voltage is

$$V_1 = \frac{207.85}{\sqrt{3}} = 120 \text{ V}$$

The single-phase equivalent circuit is shown in Figure 2.20.

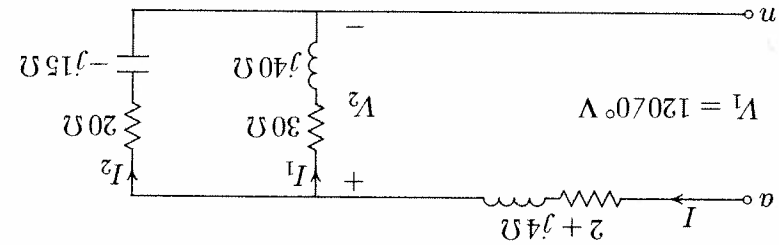


FIGURE 2.20 Single-phase equivalent circuit for Example 2.7.

The total impedance is

$$Z = 2 + j4 + \frac{(30 + j40)(20 - j15)}{(30 + j40) + (20 - j15)} = 2 + j4 + 22 - j4 = 24 \Omega$$

With the phase voltage  $V_{an}$  as reference, the current in phase  $a$  is

$$I = \frac{V_1}{Z} = \frac{120 \angle 0^\circ}{24} = 5 \text{ A}$$

The three-phase power supplied is

$$S = 3V_1I^* = 3(120 \angle 0^\circ)(5 \angle 0^\circ) = 1800 \text{ W}$$

(b) The phase voltage at the load terminal is

$$V_2 = 120 \angle 0^\circ - (2 + j4)(5 \angle 0^\circ) = 110 - j20 = 111.87 \angle -10.3^\circ \text{ V}$$

The line voltage at the load terminal is

$$V_{ab} = \sqrt{3} \angle 30^\circ V_2 = \sqrt{3} (111.8) \angle 19.7^\circ = 193.64 \angle 19.7^\circ \text{ V}$$

(c) The current per phase in the Y-connected load and in the equivalent Y of the  $\Delta$  load is

$$\begin{aligned} \frac{V_2}{I_1} = \frac{Z_1}{30 + j40} &= 1 - j2 = 2.236 \angle -63.4^\circ \text{ A} \\ \frac{V_2}{I_2} = \frac{Z_2}{110 - j20} &= 4 + j2 = 4.472 \angle 26.56^\circ \text{ A} \\ \frac{V_2}{I_3} = \frac{Z_3}{20 - j15} &= 4 + j2 = 4.472 \angle 26.56^\circ \text{ A} \end{aligned}$$

The phase current in the original  $\Delta$ -connected load, i.e.,  $I_{ab}$  is given by

$$I_{ab} = \frac{I_2}{\sqrt{3} \angle -30^\circ} = \frac{4.472 \angle 26.56^\circ}{\sqrt{3} \angle -30^\circ} = 2.582 \angle 56.56^\circ \text{ A}$$

(d) The three-phase power absorbed by each load is

$$\begin{aligned} S_1 &= 3V_2 I_1^* = 3(111.8 \angle -10.3^\circ)(2.236 \angle 63.4^\circ) = 450 \text{ W} + j600 \text{ var} \\ S_2 &= 3V_2 I_2^* = 3(111.8 \angle -10.3^\circ)(4.472 \angle -26.56^\circ) = 1200 \text{ W} - j900 \text{ var} \end{aligned}$$

The three-phase power absorbed by the line is

$$S_L = 3(R_L + jX_L)|I|^2 = 3(2 + j4)(5)^2 = 150 \text{ W} + j300 \text{ var}$$

It is clear that the sum of load powers and line losses is equal to the power delivered from the supply, i.e.,

$$S_1 + S_2 + S_L = (450 + j600) + (1200 - j900) + (150 + j300) = 1800 \text{ W} + j0 \text{ var}$$

### Example 2.8 (chp2ex8)

A three-phase line has an impedance of  $0.4 + j2.7 \Omega$  per phase. The line feeds two balanced three-phase loads that are connected in parallel. The first load is absorbing  $560.1 \text{ kVA}$  at  $0.707$  power factor lagging. The second load absorbs  $132 \text{ kW}$  at unity power factor. The line-to-line voltage at the load end of the line is  $3810.5 \text{ V}$ . Determine:

(a) The magnitude of the line voltage at the source end of the line.

(b) Total real and reactive power loss in the line.

(c) Real power and reactive power supplied at the sending end of the line.

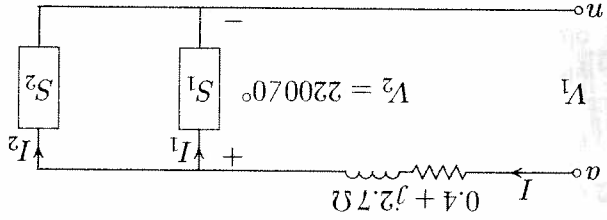


FIGURE 2.21

Single-phase equivalent diagram for Example 2.8.

(a) The phase voltage at the load terminals is

$$V_2 = \frac{\sqrt{3}}{3810.5} = 2200 \text{ V}$$

The single-phase equivalent circuit is shown in Figure 2.21.

The total complex power is

$$S_{R(3\phi)} = 560.1(0.707 + j0.707) + 132 = 528 + j396$$

$$= 660 \angle 36.87^\circ \text{ kVA}$$

With the phase voltage  $V_2$  as reference, the current in the line is

$$I = \frac{S_{R(3\phi)}^*}{3V_2^*} = \frac{660,000 \angle -36.87^\circ}{3(2200 \angle 0^\circ)} = 100 \angle -36.87^\circ \text{ A}$$

The phase voltage at the sending end is

$$V_1 = 2200 \angle 0^\circ + (0.4 + j2.7)100 \angle -36.87^\circ = 2401.7 \angle 4.58^\circ \text{ V}$$

The magnitude of the line voltage at the sending end of the line is

$$|V_{1L}| = \sqrt{3}|V_1| = \sqrt{3}(2401.7) = 4160 \text{ V}$$

(b) The three-phase power loss in the line is

$$S_{L(3\phi)} = 3R|I|^2 + j3X|I|^2 = 3(0.4)(100)^2 + j3(2.7)(100)^2$$

$$= 12 \text{ kW} + j81 \text{ kvar}$$

(c) The three-phase sending power is

$$S_{S(3\phi)} = 3V_1 I^* = 3(2401.7 \angle 4.58^\circ)(100 \angle -36.87^\circ) = 540 \text{ kW} + j477 \text{ kvar}$$

It is clear that the sum of load powers and the line losses is equal to the power delivered from the supply, i.e.,

$$S_{S(3\phi)} = S_{R(3\phi)} + S_{L(3\phi)} = (528 + j396) + (12 + j81) = 540 \text{ kW} + j477 \text{ kvar}$$

**PROBLEMS**

2.1. Modify the program in Example 2.1 such that the following quantities can be entered by the user:  
 The peak amplitude  $V_m$ , and the phase angle  $\theta_v$  of the sinusoidal supply  $v(t) = V_m \cos(\omega t + \theta_v)$ . The impedance magnitude  $Z$ , and the phase angle  $\gamma$  of the load.  
 The program should produce plots for  $i(t)$ ,  $v(t)$ ,  $p(t)$ ,  $p_r(t)$ , and  $p_x(t)$ , similar to Example 2.1. Run the program for  $V_m = 100$  V,  $\theta_v = 0$  and the following loads:

- An inductive load,  $Z = 1.25 \angle 60^\circ \Omega$
- A capacitive load,  $Z = 2.0 \angle -30^\circ \Omega$
- A resistive load,  $Z = 2.5 \angle 0^\circ \Omega$

(a) From  $p_r(t)$  and  $p_x(t)$  plots, estimate the real and reactive power for each load. Draw a conclusion regarding the sign of reactive power for inductive and capacitive loads.  
 (b) Using phasor values of current and voltage, calculate the real and reactive power for each load and compare with the results obtained from the curves.  
 (c) If the above loads are all connected across the same power supply, determine the total real and reactive power taken from the supply.

2.2. A single-phase load is supplied with a sinusoidal voltage

$$v(t) = 200 \cos(377t)$$

The resulting instantaneous power is

$$p(t) = 800 + 1000 \cos(754t - 36.87^\circ)$$

(a) Find the complex power supplied to the load.  
 (b) Find the instantaneous current  $i(t)$  and the rms value of the current supplied to the load.  
 (c) Find the load impedance.

(d) Use *MATLAB* to plot  $v(t)$ ,  $p(t)$ , and  $i(t)$  over a range of 0 to 16.67 ms in steps of 0.1 ms. From the current plot, estimate the peak amplitude, phase angle and the angular frequency of the current, and verify the results obtained in part (b). Note in *MATLAB* the command for array or element-by-element division is ./.

2.3. An inductive load consisting of  $R$  and  $X$  in series feeding from a 2400-V rms supply absorbs 288 kW at a lagging power factor of 0.8. Determine  $R$  and  $X$ .

2.4. An inductive load consisting of  $R$  and  $X$  in parallel feeding from a 2400-V rms supply absorbs 288 kW at a lagging power factor of 0.8. Determine  $R$  and  $X$ .

2.5. Two loads connected in parallel are supplied from a single-phase 240-V rms source. The two loads draw a total real power of 400 kW at a power factor of 0.8 lagging. One of the loads draws 120 kW at a power factor of 0.96 leading. Find the complex power of the other load.

2.6. The load shown in Figure 2.22 consists of a resistance  $R$  in parallel with a capacitor of reactance  $X$ . The load is fed from a single-phase supply through a line of impedance  $8.4 + j11.2 \Omega$ . The rms voltage at the load terminal is 1200  $\angle 0^\circ$  V rms, and the load is taking 30 kVA at 0.8 power factor leading.  
 (a) Find the values of  $R$  and  $X$ .  
 (b) Determine the supply voltage  $V$ .

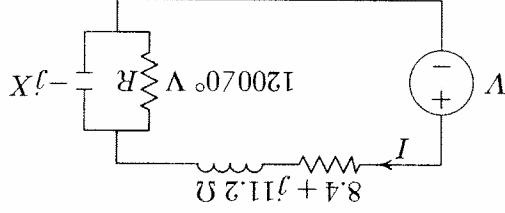


FIGURE 2.22  
 Circuit for Problem 2.6.

2.7. Two impedances,  $Z_1 = 0.8 + j5.6 \Omega$  and  $Z_2 = 8 - j16 \Omega$ , and a single-phase motor are connected in parallel across a 200-V rms, 60-Hz supply as shown in Figure 2.23. The motor draws 5 kVA at 0.8 power factor lagging.

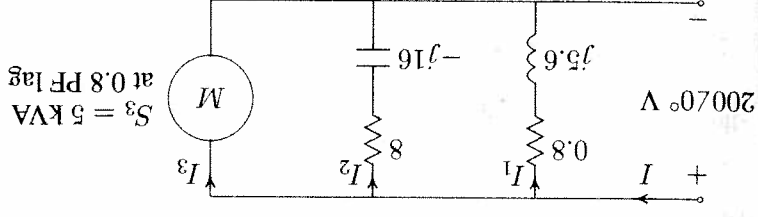


FIGURE 2.23  
 Circuit for Problem 2.7.

- (a) Find the complex powers  $S_1$ ,  $S_2$  for the two impedances, and  $S_3$  for the motor.  
 (b) Determine the total power taken from the supply, the supply current, and the overall power factor.  
 (c) A capacitor is connected in parallel with the loads. Find the kvar and the capacitance in  $\mu\text{F}$  to improve the overall power factor to unity. What is the new line current?

Two single-phase ideal voltage sources are connected by a line of impedance of  $0.7 + j2.4 \Omega$  as shown in Figure 2.24.  $V_1 = 500\angle 16.26^\circ \text{ V}$  and  $V_2 = 585\angle 70^\circ \text{ V}$ . Find the complex power for each machine and determine whether they are delivering or receiving real and reactive power. Also, find the real and the reactive power loss in the line.

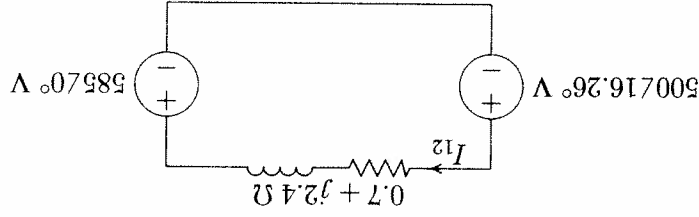


FIGURE 2.24

Circuit for Problem 2.8.

- 2.9. Write a *MATLAB* program for the system of Example 2.6 such that the voltage magnitude of source 1 is changed from 75 percent to 100 percent of the given value in steps of 1 V. The voltage magnitude of source 2 and the phase angles of the two sources is to be kept constant. Compute the complex power for each source and the line loss. Tabulate the reactive powers and plot  $Q_1$ ,  $Q_2$ , and  $Q_T$  versus voltage magnitude  $|V_1|$ . From the results, show that the flow of reactive power along the interconnection is determined by the magnitude difference of the terminal voltages.

- 2.10. A balanced three-phase source with the following instantaneous phase voltages

$$\begin{aligned} v_{an} &= 2500 \cos(\omega t) \\ v_{bn} &= 2500 \cos(\omega t - 120^\circ) \\ v_{cn} &= 2500 \cos(\omega t - 240^\circ) \end{aligned}$$

- supplies a balanced Y-connected load of impedance  $Z = 250\angle 36.87^\circ \Omega$  per phase.  
 (a) Using *MATLAB*, plot the instantaneous powers  $p_a$ ,  $p_b$ ,  $p_c$  and their sum versus  $\omega t$  over a range of  $0:0.05:2\pi$  on the same graph. Comment on the nature of the instantaneous power in each phase and the total three-phase real power.  
 (b) Use (2.44) to verify the total power obtained in part (a).

- 2.11. A 4157-V rms, three-phase supply is applied to a balanced Y-connected three-phase load consisting of three identical impedances of  $48\angle 36.87^\circ \Omega$ . Taking the phase to neutral voltage  $V_{an}$  as reference, calculate  
 (a) The phasor currents in each line.  
 (b) The total active and reactive power supplied to the load.  
 2.12. Repeat Problem 2.11 with the same three-phase impedances arranged in a  $\Delta$  connection. Take  $V_{ab}$  as reference.

- 2.13. A balanced delta connected load of  $15 + j18 \Omega$  per phase is connected at the end of a three-phase line as shown in Figure 2.25. The line impedance is  $1 + j2 \Omega$  per phase. The line is supplied from a three-phase source with a line-to-line voltage of 207.85 V rms. Taking  $V_{an}$  as reference, determine the following:  
 (a) Current in phase a.  
 (b) Total complex power supplied from the source.  
 (c) Magnitude of the line-to-line voltage at the load terminal.

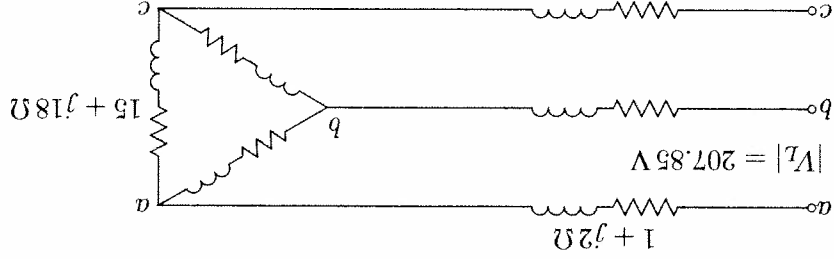


FIGURE 2.25

Circuit for Problem 2.13.

- 2.14. Three parallel three-phase loads are supplied from a 207.85-V rms, 60-Hz three-phase supply. The loads are as follows:

Load 1: A 15 hp motor operating at full-load, 93.25 percent efficiency, and 0.6 lagging power factor.

GENERATOR AND  
TRANSFORMER MODELS;  
THE PER-UNIT SYSTEM

Before the power systems network can be modeled, it must first be modeled. The three-phase balanced system is represented on a per-phase basis, which was described in Section 2.10. The single-phase representation is also used for unbalanced systems by means of symmetrical components which is treated in a later chapter. In this chapter we deal with the balanced system, where transmission lines are represented by the  $\pi$  model as described in Chapter 4. Other essential components of a power system are generators and transformers; their theory and construction are discussed in standard electric machine textbooks. In this chapter, we represent simple models of generators and transformers for steady-state balanced operation. Next we review the one-line diagram of a power system showing generators, transformers, transmission lines, capacitors, reactors, and loads. The diagram is usually limited to major transmission systems. As a rule, distribution circuits and small loads are not shown in detail but are taken into account merely as lumped loads on substation busses.

3.1 INTRODUCTION

- Load 2: A balanced resistive load that draws a total of 6 kW.  
Load 3: A Y-connected capacitor bank with a total rating of 16 kvar.
- (a) What is the total system kW, kvar, power factor, and the supply current per phase?  
(b) What is the system power factor and the supply current per phase when the resistive load and induction motor are operating but the capacitor bank is switched off?

2.15. Three loads are connected in parallel across a 12.47 kV three-phase supply.

- Load 1: Inductive load, 60 kW and 660 kvar.  
Load 2: Capacitive load, 240 kW at 0.8 power factor.  
Load 3: Resistive load of 60 kW.

(a) Find the total complex power, power factor, and the supply current.  
(b) A Y-connected capacitor bank is connected in parallel with the loads.

Find the total kvar and the capacitance per phase in  $\mu\text{F}$  to improve the overall power factor to 0.8 lagging. What is the new line current?

2.16. A balanced  $\Delta$ -connected load consisting of a pure resistances of  $18 \Omega$  per phase is in parallel with a purely resistive balanced Y-connected load of  $12 \Omega$  per phase as shown in Figure 2.26. The combination is connected to a three-phase balanced supply of 346.41-V rms (line-to-line) via a three-phase line having an inductive reactance of  $j3 \Omega$  per phase. Taking the phase voltage  $V_{an}$  as reference, determine

- (a) The current, real power, and reactive power drawn from the supply.  
(b) The line-to-neutral and the line-to-line voltage of phase  $a$  at the combined load terminals.

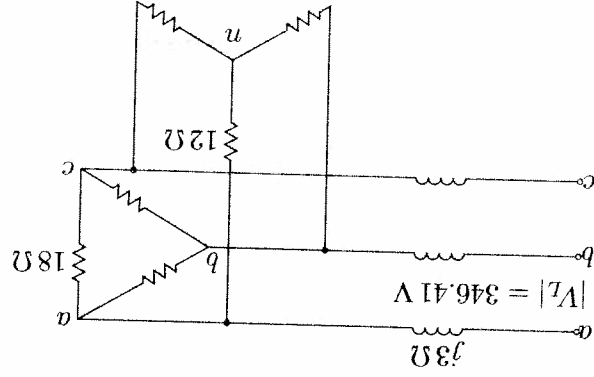


FIGURE 2.26  
Circuit for Problem 2.16.