CHAPTER 7

OPTIMAL DISPATCH OF GENERATION

7.1 INTRODUCTION

The formulation of power flow problem and its solutions were discussed in Chapter 6. One type of bus in the power flow was the voltage-controlled bus, where real power generation and voltage magnitude were specified. The power flow solution provided the voltage phase angle and the reactive power generation. In a practical power system, the power plants are not located at the same distance from the center of loads and their fuel costs are different. Also, under normal operating conditions, the generation capacity is more than the total load demand and losses. Thus, there are many options for scheduling generation. In an interconnected power system, the objective is to find the real and reactive power scheduling of each power plant in such a way as to minimize the operating cost. This means that the generator's real and reactive power are allowed to vary within certain limits so as to meet a particular load demand with minimum fuel cost. This is called the *optimal power flow* (OPF) problem. The OPF is used to optimize the power flow solution of large scale power system. This is done by minimizing selected objective functions while maintaining an acceptable system performance in terms of generator capability limits and the output of the compensating devices. The objective functions, also

known as *cost functions*, may present economic costs, system security, or other objectives. Efficient reactive power planning enhances economic operation as well as system security. The OPF has been studied by many researchers and many algorithms using different objective functions and methods have been presented [11, 12, 22, 23, 40, 42, 54, 78].

In this chapter, we will limit our analysis to the economic dispatch of real power generation. The classical optimization of continuous functions is introduced. The application of constraints to optimization problems is presented. Following this, the incremental production cost of generation is introduced. The economic dispatch of generation for minimization of the total operating cost with transmission losses neglected is obtained. Next, the transmission loss formula is derived and the economic dispatch of generation based on the loss formula is obtained. A program named **bloss** is developed for the evaluation of the transmission loss **B** coefficients which can be used following any one of the power flow programs **lfgauss**, **lfnewton**, or **decouple** discussed in Chapter 6. Also, a general program called **dispatch** is developed for the optimal scheduling of real power generation and can be used in conjunction with the **bloss** program.

7.2 NONLINEAR FUNCTION OPTIMIZATION

Unconstrained Parameter Optimization

Nonlinear function optimization is an important tool in computer-aided design and is part of a broader class of optimization called *nonlinear programming*. The underlying theory and the computational methods are discussed in many books. The basic goal is the minimization of some nonlinear objective cost function subject to nonlinear equality and inequality constraints.

The mathematical tools that are used to solve unconstrained parameter optimization problems come directly from multivariable calculus. The necessary condition to minimize the cost function

$$f(x_1, x_2, \ldots, x_n) \tag{7.1}$$

is obtained by setting derivative of f with respect to the variables equal to zero, i.e.,

$$\frac{\partial f}{\partial x_i} = 0 \qquad i = 1, \ldots, n \tag{7.2}$$

or

$$\nabla f = 0 \tag{7.3}$$

where

$$\nabla f = (\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}) \tag{7.4}$$

which is known as the *gradient vector*. The terms associated with second derivatives is given by

$$H = \frac{\partial^2 f}{\partial x_i \, \partial x_j} \tag{7.5}$$

The above equation results in a symmetric matrix called the *Hessian matrix* of the function.

Once the derivative of f is vanished at local extrema $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$, for f to have a relative minimum, the Hessian matrix evaluated at $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ must be a positive definite matrix. This condition requires that all the eigenvalues of the Hessian matrix evaluated at $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ be positive.

In summary, the unconstrained minimum of a function is found by setting its partial derivatives (with respect to the parameters that may be varied) equal to zero and solving for the parameter values. Among the sets of parameter values obtained, those at which the matrix of second partial derivatives of the cost function is positive definite are local minima. If there is a single local minimum, it is also the global minimum; otherwise, the cost function must be evaluated at each of the local minima to determine which one is the global minimum.

Example 7.1 (chp7ex1)

Find the minimum of

$$f(x_1, x_2, \ldots, x_n) = x_1^2 + 2x_2^2 + 3x_3^2 + x_1x_2 + x_2x_3 - 8x_1 - 16x_2 - 32x_3 + 110$$

Equating the first derivatives to zero, results in

$$\frac{\partial f}{\partial x_1} = 2x_1 + x_2 - 8 = 0$$
$$\frac{\partial f}{\partial x_2} = x_1 + 4x_2 + x_3 - 16 = 0$$
$$\frac{\partial f}{\partial x_3} = x_2 + 6x_3 - 32 = 0$$

or

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 32 \end{bmatrix}$$

The solution of the above linear simultaneous equation is readily obtained (in *MATLAB* use $X = A \backslash B$) and is given by $(\hat{x}_1, \hat{x}_2, \hat{x}_3) = (3, 2, 5)$. The function evaluated at this point is $f(3, 2, 5) = 2$. To see if this point is a minimum, we evaluate the second derivatives and form the Hessian matrix

$$H(\hat{X}) = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 6 \end{bmatrix}$$

Using the *MATLAB* function **eig(H)**, the eigenvalues are found to be 1.55, 4.0 and 6.45, which are all positive. Thus, the Hessian matrix is a positive definite matrix and (3, 2, 5) is a minimum point.

7.2.1 CONSTRAINED PARAMETER OPTIMIZATION: EQUALITY CONSTRAINTS

This type of problem arises when there are functional dependencies among the parameters to be chosen. The problem is to minimize the cost function

$$f(x_1, x_2, \ldots, x_n) \tag{7.6}$$

subject to the equality constraints

$$g_i(x_1, x_2, \ldots, x_n) = 0 \qquad i = 1, 2, \ldots, k \tag{7.7}$$

Such problems may be solved by the *Lagrange multiplier* method. This provides an augmented cost function by introducing k-vector λ of undetermined quantities. The unconstrained cost function becomes

$$\mathcal{L} = f + \sum_{i=1}^{k} \lambda_i g_i \tag{7.8}$$

The resulting necessary conditions for constrained local minima of $\mathcal{L}$ are the following:

$$\frac{\partial \mathcal{L}}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{i=1}^{k} \lambda_i \frac{\partial g_i}{\partial x_i} = 0 \tag{7.9}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = g_i = 0 \tag{7.10}$$

Note that Equation (7.10) is simply the original constraints.

Example 7.2 (chp7ex2)

Use the Lagrange multiplier method for solving constrained parameter optimizations to determine the minimum distance from origin of the xy plane to a circle described by

$$(x-8)^2 + (y-6)^2 = 25$$

The minimum distance is obtained by minimization of the distance square, given by

$$f(x,y) = x^2 + y^2$$

The *MATLAB* **plot** command is used to plot the circle as shown in Figure 7.1.

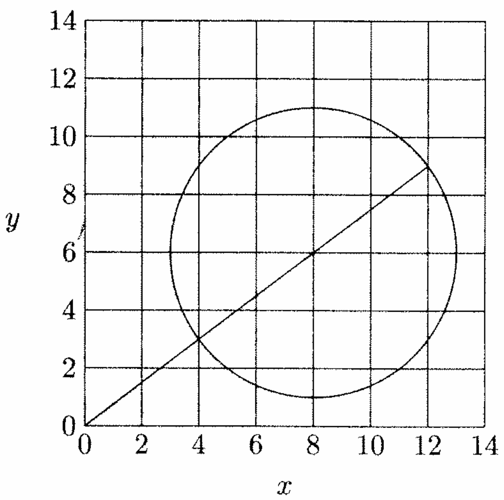


FIGURE 7.1
Constraint function of Example 7.2

From this graph, clearly the minimum distance is 5, located at point (4, 3).

Now let us use Lagrange multiplier to minimize $f(x, y)$ subject to the constraint described by the circle equation. Forming the Lagrange function, we obtain

$$\mathcal{L} = x^2 + y^2 + \lambda[(x-8)^2 + (y-6)^2 - 25]$$

The necessary conditions for extrema are

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + \lambda(2x - 16) = 0 \quad \text{or} \quad 2x(\lambda+1) = 16\lambda$$

$$\frac{\partial \mathcal{L}}{\partial y} = 2y + \lambda(2y - 12) = 0 \quad \text{or} \quad 2y(\lambda+1) = 12\lambda$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = (x-8)^2 + (y-6)^2 - 25 = 0$$

The solution of the above three equations will provide optimal points. In this problem, a direct solution can be obtained as follows:

Eliminating λ from the first two equations results in

$$y = \frac{3}{4}x$$

Substituting for y in the third equation yields

$$\frac{25}{16}x^2 - 25x + 75 = 0$$

The solutions of the above quadratic equations are $x = 4$ and $x = 12$. Thus, the corresponding extrema are at points (4, 3) with $\lambda = 1$, and (12, 9) with $\lambda = -3$. From Figure 7.1, it is clear that the minimum distance is at point (4, 3) and the maximum distance is at point (12, 9). To distinguish these points, the second derivatives are obtained and the Hessian matrices evaluated at these points are formed. The matrix with positive eigenvalues is a positive definite matrix and the parameters correspond to the minimum point.

In many problems, a direct solution is not possible and the above equations are solved iteratively. Many iterative schemes are available. The simplest search method is to assume a value for λ and compute Δf. If Δf is zero, the estimated λ corresponds to the optimum solution. If not, depending on the sign of Δf, λ is increased or decreased, and another solution is obtained. With two solutions, a better value of λ is obtained by extrapolation and the process is continued until Δf is within a specified accuracy. A significantly superior method applicable to continuous functions is the Newton-Raphson method. One way to apply the Newton-Raphson method to the problem at hand is as follows: From the first two equations, x and y are found. These are

$$x = \frac{8\lambda}{\lambda+1}$$
$$y = \frac{6\lambda}{\lambda+1}$$

Substituting into the third equation results in

$$f(\lambda) = \frac{100\lambda^2}{(\lambda+1)^2} - \frac{200\lambda}{\lambda+1} + 75 = 0$$

This is a nonlinear equation in terms of λ and can be solved by the Newton-Raphson method. The Newton-Raphson method is a successive approximation procedure based on an initial estimate of the unknown and the use of Taylor's series expansion (see Chapter 6 for more details). For a one-dimensional case,

$$\Delta\lambda^{(k)} = \frac{-\Delta f(\lambda)^k}{(\frac{df}{d\lambda})^k} \tag{7.11}$$

and

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)} \tag{7.12}$$

Starting with an estimated value of λ, a new value is found in the direction of steepest decent (negative gradient). The process is repeated in the direction of negative gradient until $\Delta f(\lambda)$ is less than a specified accuracy. This algorithm is known as the *gradient method*. For the above function, the gradient is

$$\frac{df(\lambda)}{d\lambda} = \frac{200\lambda}{(\lambda+1)^3} - \frac{200}{(\lambda+1)^2} = \frac{-200}{(\lambda+1)^3}$$

The following commands show the procedure for the solution of the given equation by the Newton-Raphson method.

```
iter = 0;                              % Iteration counter
Df = 10;               % Error in Df is set to a high value
Lambda = input('Enter estimated value of Lambda = ');
fprintf('\n ')
disp(['    Iter      Df         J     DLambda    Lambda' ...
'       x           y'])
while abs(Df)  >= 0.0001             % Test for convergence
iter = iter + 1;                        % No. of iterations
x = 8*Lambda/(Lambda + 1);
y = 6*Lambda/(Lambda + 1);
Df = (x- 8)^2 + (y - 6)^2 - 25;                % Residual
J = -200/(Lambda + 1)^3;
Delambda =-Df/J;                      % Change in variable
disp([iter, Df, J, Delambda, Lambda, x, y])
Lambda = Lambda + Delambda;          % Successive solution
end
```

When the program is run, the user is prompted to enter the initial estimate for λ. Using a value of $\lambda = 0.4$, the result is

```
Enter estimated value of Lambda = 0.4
```


Iter	Δf	J	$\Delta\lambda$	λ	x	y
1	26.0240	-72.8863	0.3570	0.4000	2.2857	1.7134
2	7.3934	-36.8735	0.2005	0.7570	3.4468	2.5851
3	1.0972	-26.6637	0.0411	0.9575	3.9132	2.9349
4	0.0337	-25.0505	0.0013	0.9987	3.9973	2.9980
5	0.0000	-25.0001	0.0000	1.0000	4.0000	3.0000

After five iterations, the solution converges to $\lambda = 1.0$, $x = 4$, and $y = 3$, corresponding to the minimum length. If the program is run with an initial estimate of -2, the solution converges to $\lambda = -3$, $x = 12$, $y = 9$, which corresponds to the maximum length.

7.2.2 CONSTRAINT PARAMETER OPTIMIZATION: INEQUALITY CONSTRAINTS

Practical optimization problems contain inequality constraints as well as equality constraints. The problem is to minimize the cost function

$$f(x_1, x_2, \ldots, x_n) \tag{7.13}$$

subject to the equality constraints

$$g_i(x_1, x_2, \ldots, x_n) = 0 \qquad i = 1, 2, \ldots, k \tag{7.14}$$

and the inequality constraints

$$u_j(x_1, x_2, \ldots, x_n) \le 0 \qquad i = 1, 2, \ldots, m \tag{7.15}$$

The Lagrange multiplier is extended to include the inequality constraints by introducing m-vector μ of undetermined quantities. The unconstrained cost function becomes

$$\mathcal{L} = f + \sum_{i=1}^{k} \lambda_i g_i + \sum_{j=1}^{m} \mu_j u_j \tag{7.16}$$

The resulting necessary conditions for constrained local minima of $\mathcal{L}$ are the following:

$$\frac{\partial \mathcal{L}}{\partial x_i} = 0 \quad i = 1, \ldots, n \tag{7.17}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_i} = g_i = 0 \quad i = 1, \ldots, k \tag{7.18}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_j} = u_j \le 0 \quad j = 1, \ldots, m \tag{7.19}$$

$$\mu_j u_j = 0 \;\;\&\;\; \mu_j > 0 \quad j = 1, \ldots, m \tag{7.20}$$

Note that Equation (7.18) is simply the original equality constraints. Suppose $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ is a relative minimum. The inequality constraints in (7.19) is said to be inactive if strict inequality holds at $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n)$ and $\mu_j = 0$. On the other hand, when strict equality holds, the constraint is active at this point, (i.e., if the constraint $\mu_j u_j(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n) = 0$ and $\mu_j > 0$. This is known as the *Kuhn-Tucker* necessary condition.

Example 7.3 (chp7ex3)

Solve Example 7.2 with an additional inequality constraint defined below. The problem is to find the minimum value of the function

$$f(x, y) = x^2 + y^2$$

subject to one equality constraint

$$g(x, y) = (x - 8)^2 + (y - 6)^2 - 25 = 0$$

and one inequality constraint,

$$u(x, y) = 2x + y \geq 12$$

The unconstrained cost function from (7.16) is

$$\mathcal{L} = x^2 + y^2 + \lambda[(x - 8)^2 + (y - 6)^2 - 25] + \mu(2x + y - 12)$$

The resulting necessary conditions for constrained local minima of $\mathcal{L}$ are

$$\frac{\partial \mathcal{L}}{\partial x} = 2x + 2\lambda(x - 8) + 2\mu = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = 2y + 2\lambda(y - 6) + \mu = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = (x - 8)^2 + (y - 6)^2 - 25 = 0$$
$$\frac{\partial \mathcal{L}}{\partial \mu} = 2x + y - 12 = 0$$

Eliminating μ from the first two equations result in

$$(2x - 4y)(1 + \lambda) + 8\lambda = 0$$

From the fourth condition, we have

$$y = 12 - 2x$$

Substituting for y in the above equation, yields

$$x = \frac{4\lambda + 4.8}{1 + \lambda}$$

Now substituting for x in the previous equation, we get

$$y = \frac{4\lambda + 2.4}{1 + \lambda}$$

Substituting for x and y in the third condition (equality constraint) results in an equation in terms of λ

$$\left(\frac{4\lambda + 4.8}{1 + \lambda} - 8\right)^2 + \left(\frac{4\lambda + 2.4}{1 + \lambda} - 6\right)^2 - 25 = 0$$

from which we have the following equation

$$\lambda^2 + 2\lambda + 0.36 = 0$$

Roots of the above equation are $\lambda = -0.2$ and $\lambda = -1.8$. Substituting for these values of λ in the expression for x and y, the corresponding extrema are

$$(x, y) = (5, 2) \quad \text{for} \quad \lambda = -0.2, \quad \mu = -5.6$$
$$(x, y) = (3, 6) \quad \text{for} \quad \lambda = -1.8, \quad \mu = -12$$

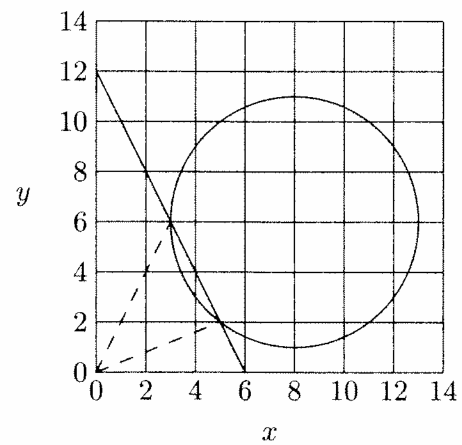


FIGURE 7.2
Constraint functions of Example 7.3.

The minimum distance from the cost function is 5.385, located at point $(5, 2)$, and the maximum distance is 6.71 located at point $(3, 6)$.

Adding the inequality constraint $2x + y \geq 12$ to the graphs in Figure 7.1, the solution is verified graphically as shown in Figure 7.2.

7.3 OPERATING COST OF A THERMAL PLANT

The factors influencing power generation at minimum cost are operating efficiencies of generators, fuel cost, and transmission losses. The most efficient generator in the system does not guarantee minimum cost as it may be located in an area where fuel cost is high. Also, if the plant is located far from the load center, transmission losses may be considerably higher and hence the plant may be overly uneconomical. Hence, the problem is to determine the generation of different plants such that the total operating cost is minimum. The operating cost plays an important role in the economic scheduling and are discussed here.

The input to the thermal plant is generally measured in Btu/h, and the output is measured in MW. A simplified input-output curve of a thermal unit known as *heat-rate* curve is given in Figure 7.3(a). Converting the ordinate of heat-rate

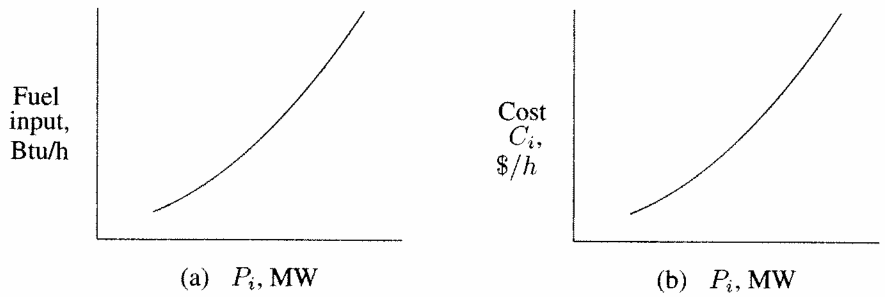


FIGURE 7.3
(a) Heat-rate curve. (b) Fuel-cost curve.

curve from Btu/h to $/h results in the *fuel-cost* curve shown in Figure 7.3(b). In all practical cases, the fuel cost of generator i can be represented as a quadratic function of real power generation

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \tag{7.21}$$

An important characteristic is obtained by plotting the derivative of the fuel-cost curve versus the real power. This is known as the *incremental fuel-cost* curve shown in Figure 7.4.

$$\frac{dC_i}{dP_i} = 2\gamma_i P_i + \beta_i \tag{7.22}$$

The incremental fuel-cost curve is a measure of how costly it will be to produce the next increment of power. The total operating cost includes the fuel cost, and the cost of labor, supplies and maintenance. These costs are assumed to be a fixed percentage of the fuel cost and are generally included in the incremental fuel-cost curve.

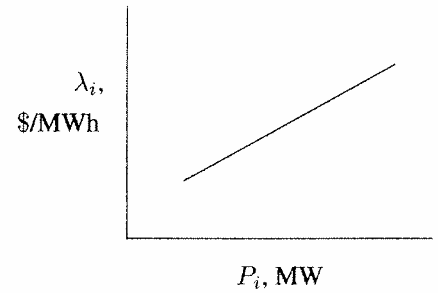


FIGURE 7.4
Typical incremental fuel-cost curve.

7.4 ECONOMIC DISPATCH NEGLECTING LOSSES AND NO GENERATOR LIMITS

The simplest economic dispatch problem is the case when transmission line losses are neglected. That is, the problem model does not consider system configuration and line impedances. In essence, the model assumes that the system is only one bus with all generation and loads connected to it as shown schematically in Figure 7.5.

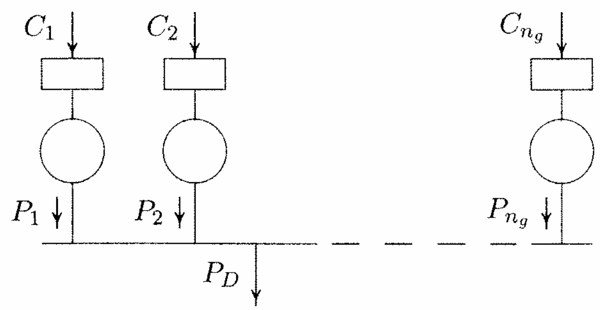


FIGURE 7.5
Plants connected to a common bus.

Since transmission losses are neglected, the total demand P_D is the sum of all generation. A cost function C_i is assumed to be known for each plant. The problem is to find the real power generation for each plant such that the objective function (i.e., total production cost) as defined by the equation

$$C_t = \sum_{i=1}^{n_g} C_i$$

$$= \sum_{i=1}^{n} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \tag{7.23}$$

is minimum, subject to the constraint

$$\sum_{i=1}^{n_g} P_i = P_D \tag{7.24}$$

where C_t is the total production cost, C_i is the production cost of ith plant, P_i is the generation of ith plant, P_D is the total load demand, and n_g is the total number of dispatchable generating plants.

A typical approach is to augment the constraints into objective function by using the Lagrange multipliers

$$\mathcal{L} = C_t + \lambda \left(P_D - \sum_{i=1}^{n_g} P_i \right) \tag{7.25}$$

The minimum of this unconstrained function is found at the point where the partials of the function to its variables are zero.

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \tag{7.26}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \tag{7.27}$$

First condition, given by (7.26), results in

$$\frac{\partial C_t}{\partial P_i} + \lambda(0-1) = 0$$

Since

$$C_t = C_1 + C_2 + \cdots + C_{n_g}$$

then

$$\frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda$$

and therefore the condition for optimum dispatch is

$$\frac{dC_i}{dP_i} = \lambda \qquad i = 1, \ldots, n_g \tag{7.28}$$

or

$$\beta_i + 2\gamma_i P_i = \lambda \tag{7.29}$$

Second condition, given by (7.27), results in

$$\sum_{i=1}^{n_g} P_i = P_D \tag{7.30}$$

Equation (7.30) is precisely the equality constraint that was to be imposed. In summary, when losses are neglected with no generator limits, for most economic operation, all plants must operate at equal incremental production cost while satisfying the equality constraint given by (7.30). In order to find the solution, (7.29) is solved for P_i

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i} \tag{7.31}$$

The relations given by (7.31) are known as the *coordination equations*. They are functions of λ. An analytical solution can be obtained for λ by substituting for P_i in (7.30), i.e.,

$$\sum_{i=1}^{n_g} \frac{\lambda - \beta_i}{2\gamma_i} = P_D \tag{7.32}$$

or

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_g} \frac{1}{2\gamma_i}} \tag{7.33}$$

The value of λ found from (7.33) is substituted in (7.31) to obtain the optimal scheduling of generation.

The solution for economic dispatch neglecting losses was found analytically. However when losses are considered the resulting equations as seen in Section 7.6 are nonlinear and must be solved iteratively. Thus, an iterative procedure is introduced here and (7.31) is solved iteratively. In an iterative search technique, starting with two values of λ, a better value of λ is obtained by extrapolation, and the process is continued until ΔP_i is within a specified accuracy. However, as mentioned earlier, a rapid solution is obtained by the use of the gradient method. To do this, (7.32) is written as

$$f(\lambda) = P_D \tag{7.34}$$

Expanding the left-hand side of the above equation in Taylor's series about an operating point $\lambda^{(k)}$, and neglecting the higher-order terms results in

$$f(\lambda)^{(k)} + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} = P_D \tag{7.35}$$

or

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} = \frac{\Delta P^{(k)}}{\sum\left(\frac{dP_i}{d\lambda}\right)^{(k)}} \tag{7.36}$$

or

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\sum \frac{1}{2\gamma_i}} \tag{7.37}$$

and therefore,

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)} \tag{7.38}$$

where

$$\Delta P^{(k)} = P_D - \sum_{i=1}^{n_g} P_i^{(k)} \tag{7.39}$$

The process is continued until $\Delta P^{(k)}$ is less than a specified accuracy.

Example 7.4 (chp7ex4)

The fuel-cost functions for three thermal plants in \$/h are given by

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2$$
$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$
$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

where P_1, P_2, and P_3 are in MW. The total load, P_D, is 800 MW. Neglecting line losses and generator limits, find the optimal dispatch and the total cost in \$/h
(a) by analytical method using (7.33)
(b) by graphical demonstration.
(c) by iterative technique using the gradient method.

(a) From (7.33), λ is found to be

$$\lambda = \frac{800 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \frac{800 + 1443.0555}{263.8889} = 8.5 \quad \$/\text{MWh}$$

Substituting for λ in the coordination equation, given by (7.31), the optimal dispatch is

$$P_1 = \frac{8.5 - 5.3}{2(0.004)} = 400.0000$$
$$P_2 = \frac{8.5 - 5.5}{2(0.006)} = 250.0000$$
$$P_3 = \frac{8.5 - 5.8}{2(0.009)} = 150.0000$$

(b) From (7.28), the necessary conditions for optimal dispatch are

$$\frac{dC_1}{dP_1} = 5.3 + 0.008P_1 = \lambda$$
$$\frac{dC_2}{dP_2} = 5.5 + 0.012P_2 = \lambda$$
$$\frac{dC_3}{dP_3} = 5.8 + 0.018P_3 = \lambda$$

subject to

$$P_1 + P_2 + P_3 = P_D$$

To demonstrate the concept of equal incremental cost for optimal dispatch, we can use *MATLAB* **plot** command to plot the incremental cost of each plant on the same graph as shown in Figure 7.6. To obtain a solution, various values of λ could be tried until one is found which produces $\sum P_i = P_D$. For each λ, if $\sum P_i < P_D$, we increase λ otherwise, if $\sum P_i > P_D$, we reduce λ. Therefore, the horizontal dashed-line shown in the graph is moved up or down until at the optimum point $\hat{\lambda}$, $\sum P_i = P_D$. For this example, with $P_D = 800$ MW, the optimal dispatch is $P_1 = 400$, $P_2 = 250$, and $P_3 = 150$ at $\hat{\lambda} = 8.5$ \$/MWh.

(c) For the numerical solution using the gradient method, assume the initial value of $\lambda^{(1)} = 6.0$. From coordination equations, given by (7.31), P_1, P_2, and P_3 are

$$P_1^{(1)} = \frac{6.0 - 5.3}{2(0.004)} = 87.5000$$
$$P_2^{(1)} = \frac{6.0 - 5.5}{2(0.006)} = 41.6667$$
$$P_3^{(1)} = \frac{6.0 - 5.8}{2(0.009)} = 11.1111$$

Since $P_D = 800$ MW, the error ΔP from (7.39) is

$$\Delta P^{(1)} = 800 - (87.5 + 41.6667 + 11.1111) = 659.7222$$

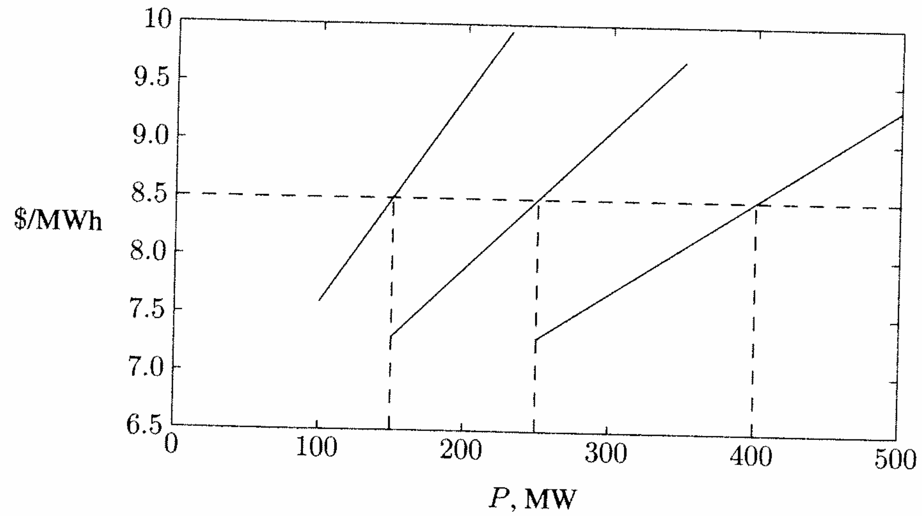


FIGURE 7.6
Illustrating the concept of equal incremental cost production cost.

From (7.37)

$$\Delta\lambda^{(1)} = \frac{659.7222}{\frac{1}{2(0.004)} + \frac{1}{2(0.006)} + \frac{1}{2(0.009)}} = \frac{659.7222}{263.8888} = 2.5$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 6.0 + 2.5 = 8.5$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{8.5 - 5.3}{2(0.004)} = 400.0000$$
$$P_2^{(2)} = \frac{8.5 - 5.5}{2(0.006)} = 250.0000$$
$$P_3^{(2)} = \frac{8.5 - 5.8}{2(0.009)} = 150.0000$$

and

$$\Delta P^{(2)} = 800 - (400 + 250 + 150) = 0.0$$

Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iterations. Therefore, the optimal dispatch are

$$P_1 = 400 \quad \text{MW}$$

$$P_2 = 250 \quad \text{MW}$$
$$P_3 = 150 \quad \text{MW}$$
$$\hat{\lambda} = 8.5 \quad \$/\text{MWh}$$

and the total fuel cost is

$$C_t = 500 + 5.3(400) + 0.004(400)^2 + 400 + 5.5(250) + 0.006(250)^2$$
$$+200 + 5.8(150) + 0.009(150)^2 = 6,682.5 \quad \$/\text{h}$$

To demonstrate the above method, the following simple program is written for Example 7.4.

```
alpha =[500; 400; 200];
beta = [5.3; 5.5; 5.8]; gamma=[.004; .006; .009];
PD=800;
DelP = 10;          % Error in DelP is set to a high value
lambda = input('Enter estimated value of Lambda = ');
fprintf(' ')
disp(['   Lambda      P1       P2        P3        DP'...
 '     grad      Delambda'])
iter = 0;                              % Iteration counter
while abs(DelP)  >= 0.001            % Test for convergence
iter = iter + 1;                       % No. of iterations
P = (lambda - beta)./(2*gamma);    % Coordination equation
DelP =PD - sum(P);                              % Residual
J = sum(ones(length(gamma), 1)./(2*gamma));% Gradient sum
Delambda = DelP/J;                    % Change in variable
disp([lambda, P(1), P(2), P(3), DelP, J, Delambda])
lambda = lambda + Delambda;          % Successive solution
end
totalcost = sum(alpha + beta.*P + gamma.*P.^2)
```

When the program is run, the result is

```
Enter estimated value of Lambda = 6

Lambda     P1      P2       P3        DP      grad    Delambda
6.0000  87.500  41.6667  11.1111  659.7222  263.8889  2.500
8.5000 400.000 250.0000 150.0000    0.0000  263.8889  0.000

   totalcost =

         6682.5
```

A general program called **dispatch** is developed for the optimal dispatch problem. The program returns the system λ, the optimal dispatch generation vector P, and the total cost. The following reserved variables are required by the **dispatch** program:

Pdt This reserved name must be used to specify the total load in MW. If **Pdt** is not specified the user is prompted to input the total load. If **dispatch** is used following any of the power flow programs, the total load is automatically passed by the power flow program.

cost This reserved name must be used to specify the cost function coefficients. The coefficients are arranged in the *MATLAB* matrix format. Each row contains the coefficients of the cost function in ascending powers of P.

mwlimits This name is reserved for the generator's real power limits and are discussed in Section 7.5. This entry is specified in matrix form with the first column representing the minimum value and the second column representing the maximum value. If **mwlimits** is not specified, the program obtains the optimal dispatch of generation with no limits.

B B0 B00 These names are reserved for the loss formula coefficient matrices and are discussed in Section 7.6. If these variables are not specified, optimal dispatch of generation is obtained neglecting losses.

The total generation cost of a thermal power system can be obtained with the aid of the **gencost** command. This program can be used following any of the power flow programs or the **dispatch** program, provided cost function matrix is defined.

Example 7.5 (chp7ex5)

Neglecting generator limits and system losses, use **dispatch** program to obtain the optimal dispatch of generation for thermal plants specified in Example 7.4.

We use the following command:

```
cost = [500   5.3   0.004
        400   5.5   0.006
        200   5.8   0.009];
Pdt = 800;
dispatch
gencost
```


The result is

```
Incremental cost of delivered power(system lambda) = 8.5$/MWh
Optimal Dispatch of Generation:

      400.0000
      250.0000
      150.0000
   Total generation cost = 6682.50 $/h
```

7.5 ECONOMIC DISPATCH NEGLECTING LOSSES AND INCLUDING GENERATOR LIMITS

The power output of any generator should not exceed its rating nor should it be below that necessary for stable boiler operation. Thus, the generations are restricted to lie within given minimum and maximum limits. The problem is to find the real power generation for each plant such that the objective function (i.e., total production cost) as defined by (7.23) is minimum, subject to the constraint given by (7.24) and the inequality constraints given by

$$P_{i(min)} \leq P_i \leq P_{i(max)} \qquad i = 1, \ldots, n_g \tag{7.40}$$

Where $P_{i(min)}$ and $P_{i(max)}$ are the minimum and maximum generating limits respectively for plant i.

The Kuhn-Tucker conditions complement the Lagrangian conditions to include the inequality constraints as additional terms. The necessary conditions for the optimal dispatch with losses neglected becomes

$$\begin{aligned}
\frac{dC_i}{dP_i} &= \lambda \qquad \text{for} \quad P_{i(min)} < P_i < P_{i(max)} \\
\frac{dC_i}{dP_i} &\leq \lambda \qquad \text{for} \quad P_i = P_{i(max)} \\
\frac{dC_i}{dP_i} &\geq \lambda \qquad \text{for} \quad P_i = P_{i(min)}
\end{aligned} \tag{7.41}$$

The numerical solution is the same as before. That is, for an estimated λ, P_i are found from the coordination Equation (7.31) and iteration is continued until $\sum P_i = P_D$. As soon as any plant reaches a maximum or minimum, the plant becomes pegged at the limit. In effect, the plant output becomes a constant, and only the unviolated plants must operate at equal incremental cost.

Example 7.6 (chp7ex6)

Find the optimal dispatch and the total cost in \$/h for the thermal plants of Example 7.4 when the total load is 975 MW with the following generator limits (in MW):

$$200 \le P_1 \le 450$$
$$150 \le P_2 \le 350$$
$$100 \le P_3 \le 225$$

Assume the initial value of $\lambda^{(1)} = 6.0$. From coordination equations given by (7.31), P_1, P_2, and P_3 are

$$P_1^{(1)} = \frac{6.0 - 5.3}{2(0.004)} = 87.5000$$
$$P_2^{(1)} = \frac{6.0 - 5.5}{2(0.006)} = 41.6667$$
$$P_3^{(1)} = \frac{6.0 - 5.8}{2(0.009)} = 11.1111$$

Since $P_D = 975$ MW, the error ΔP from (7.39) is

$$\Delta P^{(1)} = 975 - (87.5 + 41.6667 + 11.1111) = 834.7222$$

From (7.37)

$$\Delta\lambda^{(1)} = \frac{834.7222}{\frac{1}{2(0.004)} + \frac{1}{2(0.006)} + \frac{1}{2(0.009)}} = \frac{834.7222}{263.8888} = 3.1632$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 6.0 + 3.1632 = 9.1632$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{9.1632 - 5.3}{2(0.004)} = 482.8947$$
$$P_2^{(2)} = \frac{9.1632 - 5.5}{2(0.006)} = 305.2632$$
$$P_3^{(2)} = \frac{9.1632 - 5.8}{2(0.009)} = 186.8421$$

and

$$\Delta P^{(2)} = 975 - (482.8947 + 305.2632 + 186.8421) = 0.0$$

Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iterations. However, P_1 exceeds its upper limit. Thus, this plant is pegged at its upper limit. Hence $P_1 = 450$ and is kept constant at this value. Thus, the new imbalance in power is

$$\Delta P^{(2)} = 975 - (450 + 305.2632 + 186.8421) = 32.8947$$

From (7.37)

$$\Delta\lambda^{(2)} = \frac{32.8947}{\frac{1}{2(0.006)} + \frac{1}{2(0.009)}} = \frac{32.8947}{138.8889} = 0.2368$$

Therefore, the new value of λ is

$$\lambda^{(3)} = 9.1632 + 0.2368 = 9.4$$

For the third iteration, we have

$$P_1^{(3)} = 450$$
$$P_2^{(3)} = \frac{9.4 - 5.5}{2(0.006)} = 325$$
$$P_3^{(3)} = \frac{9.4 - 5.8}{2(0.009)} = 200$$

and

$$\Delta P^{(3)} = 975 - (450 + 325 + 200) = 0.0$$

$\Delta P^{(3)} = 0$, and the equality constraint is met and P_2 and P_3 are within their limits. Thus, the optimal dispatch is

$$P_1 = 450 \quad \text{MW}$$
$$P_2 = 325 \quad \text{MW}$$
$$P_3 = 200 \quad \text{MW}$$
$$\hat{\lambda} = 9.4 \quad \$/\text{MWh}$$

and the total fuel cost is

$$C_t = 500 + 5.3(450) + 0.004(450)^2 + 400 + 5.5(325) + 0.006(325)^2$$
$$+200 + 5.8(200) + 0.009(200)^2 = 8,236.25 \quad \$/\text{h}$$

The following commands can be used to obtain the optimal dispatch of generation including generator limits.

```
cost = [500    5.3    0.004
        400    5.5    0.006
        200    5.8    0.009];
mwlimits=[200  450
          150  350
          100  225];
Pdt = 975;
dispatch
gencost
```

The result is

```
Incremental cost of delivered power(system lambda) = 9.4$/MWh
Optimal Dispatch of Generation:

     450
     325
     200

Total generation cost = 8236.25 $/h
```

7.6 ECONOMIC DISPATCH INCLUDING LOSSES

When transmission distances are very small and load density is very high, transmission losses may be neglected and the optimal dispatch of generation is achieved with all plants operating at equal incremental production cost. However, in a large interconnected network where power is transmitted over long distances with low load density areas, transmission losses are a major factor and affect the optimum dispatch of generation. One common practice for including the effect of transmission losses is to express the total transmission loss as a quadratic function of the generator power outputs. The simplest quadratic form is

$$P_L = \sum_{i=1}^{n_g}\sum_{j=1}^{n_g} P_i B_{ij} P_j \tag{7.42}$$

A more general formula containing a linear term and a constant term, referred to as *Kron's loss formula*, is

$$P_L = \sum_{i=1}^{n_g}\sum_{j=1}^{n_g} P_i B_{ij} P_j + \sum_{i=1}^{n_g} B_{0i} P_i + B_{00} \tag{7.43}$$

The coefficients B_{ij} are called *loss coefficients* or *B-coefficients*. B-coefficients are assumed constant, and reasonable accuracy can be expected provided the actual operating conditions are close to the base case where the B-constants were computed. There are various ways of arriving at a loss equation. A method for obtaining these B-coefficients is presented in Section 7.7.

The economic dispatching problem is to minimize the overall generating cost C_i, which is the function of plant output

$$\begin{aligned} C_t &= \sum_{i=1}^{n_g} C_i \\ &= \sum_{i=1}^{n} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \end{aligned} \tag{7.44}$$

subject to the constraint that generation should equal total demands plus losses, i.e.,

$$\sum_{i=1}^{n_g} P_i = P_D + P_L \tag{7.45}$$

satisfying the inequality constraints, expressed as follows:

$$P_{i(min)} \le P_i \le P_{i(max)} \qquad i = 1, \ldots, n_g \tag{7.46}$$

where $P_{i(min)}$ and $P_{i(max)}$ are the minimum and maximum generating limits, respectively, for plant i.

Using the Lagrange multiplier and adding additional terms to include the inequality constraints, we obtain

$$\begin{aligned} \mathcal{L} = C_t + \lambda(P_D + P_L - \sum_{i=1}^{n_g} P_i) + \sum_{i=1}^{n_g} \mu_{i(max)}(P_i - P_{i(max)}) + \\ \sum_{i=1}^{n_g} \mu_{i(min)}(P_i - P_{i(min)}) \end{aligned} \tag{7.47}$$

The constraints should be understood to mean the $\mu_{i(max)} = 0$ when $P_i < P_{i(max)}$ and that $\mu_{i(min)} = 0$ when $P_i > P_{i(min)}$. In other words, if the constraint is not violated, its associated μ variable is zero and the corresponding term in (7.47) does not exist. The constraint only becomes active when violated. The minimum of this unconstrained function is found at the point where the partials of the function to its variables are zero.

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \tag{7.48}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \tag{7.49}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{i(max)}} = P_i - P_{i(max)} = 0 \tag{7.50}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{i(min)}} = P_i - P_{i(min)} = 0 \tag{7.51}$$

Equations (7.50) and (7.51) imply that P_i should not be allowed to go beyond its limit, and when P_i is within its limits $\mu_{i(min)} = \mu_{i(max)} = 0$ and the Kuhn-Tucker function becomes the same as the Lagrangian one. First condition, given by (7.48), results in

$$\frac{\partial C_t}{\partial P_i} + \lambda(0 + \frac{\partial P_L}{\partial P_i} - 1) = 0$$

Since

$$C_t = C_1 + C_2 + \cdots + C_{n_g}$$

then

$$\frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i}$$

and therefore the condition for optimum dispatch is

$$\frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \qquad i = 1, \ldots, n_g \tag{7.52}$$

The term $\frac{\partial P_L}{\partial P_i}$ is known as the incremental transmission loss. Second condition, given by (7.49), results in

$$\sum_{i=1}^{n_g} P_i = P_D + P_L \tag{7.53}$$

Equation (7.53) is precisely the equality constraint that was to be imposed.

Classically, Equation (7.52) is rearranged as

$$\left(\frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \right) \frac{dC_i}{dP_i} = \lambda \qquad i = 1, \ldots, n_g \tag{7.54}$$

or

$$L_i \frac{dC_i}{dP_i} = \lambda \qquad i = 1, \ldots, n_g \tag{7.55}$$

where L_i is known as the *penalty factor* of plant i and is given by

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \tag{7.56}$$

Hence, the effect of transmission loss is to introduce a penalty factor with a value that depends on the location of the plant. Equation (7.55) shows that the minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants.

The incremental production cost is given by (7.22), and the incremental transmission loss is obtained from the loss formula (7.43) which yields

$$\frac{\partial P_L}{\partial P_i} = 2 \sum_{j=1}^{n_g} B_{ij} P_j + B_{0i} \tag{7.57}$$

Substituting the expression for the incremental production cost and the incremental transmission loss in (7.52) results in

$$\beta_i + 2\gamma_i P_i + 2\lambda \sum_{j=1}^{n_g} B_{ij} P_j + B_{0i}\lambda = \lambda$$

or

$$\left(\frac{\gamma_i}{\lambda} + B_{ii} \right) P_i + \sum_{\substack{j=1 \\ j \neq i}}^{n_g} B_{ij} P_j = \frac{1}{2} \left(1 - B_{0i} - \frac{\beta_i}{\lambda} \right) \tag{7.58}$$

Extending (7.58) to all plants results in the following linear equations in matrix form

$$\begin{bmatrix} \frac{\gamma_1}{\lambda} + B_{11} & B_{12} & \cdots & B_{1n_g} \\ B_{21} & \frac{\gamma_2}{\lambda} + B_{22} & \cdots & B_{2n_g} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & \frac{\gamma_{n_g}}{\lambda} + B_{n_gn_g} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{n_g} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 - B_{01} - \frac{\beta_1}{\lambda} \\ 1 - B_{02} - \frac{\beta_2}{\lambda} \\ \vdots \\ 1 - B_{0n_g} - \frac{\beta_{n_g}}{\lambda} \end{bmatrix} \tag{7.59}$$

or in short form

$$\boldsymbol{EP} = \boldsymbol{D} \tag{7.60}$$

To find the optimal dispatch for an estimated value of $\lambda^{(1)}$, the simultaneous linear equation given by (7.60) is solved. In *MATLAB* use the command $\mathbf{P} = \mathbf{E} \backslash \mathbf{D}$.

Then the iterative process is continued using the gradient method. To do this, from (7.58), P_i at the kth iteration is expressed as

$$P_i^{(k)} = \frac{\lambda^{(k)}(1 - B_{0i}) - \beta_i - 2\lambda^{(k)} \sum_{j \neq i} B_{ij} P_j^{(k)}}{2(\gamma_i + \lambda^{(k)} B_{ii})} \tag{7.61}$$

Substituting for P_i from (7.61) in (7.53) results in

$$\sum_{i=1}^{n_g} \frac{\lambda^{(k)}(1 - B_{0i}) - \beta_i - 2\lambda^{(k)} \sum_{j \neq i} B_{ij} P_j^{(k)}}{2(\gamma_i + \lambda^{(k)} B_{ii})} = P_D + P_L^{(k)} \tag{7.62}$$

or

$$f(\lambda)^{(k)} = P_D + P_L^{(k)} \tag{7.63}$$

Expanding the left-hand side of the above equation in Taylor's series about an operating point $\lambda^{(k)}$, and neglecting the higher-order terms results in

$$f(\lambda)^{(k)} + \left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} \Delta\lambda^{(k)} = P_D + P_L^{(k)} \tag{7.64}$$

or

$$\Delta\lambda^{(k)} = \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}} = \frac{\Delta P^{(k)}}{\sum \left(\frac{dP_i}{d\lambda}\right)^{(k)}} \tag{7.65}$$

where

$$\sum_{i=1}^{n_g} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(k)} = \sum_{i=1}^{n_g} \frac{\gamma_i(1 - B_{0i}) + B_{ii}\beta_i - 2\gamma_i \sum_{j \neq i} B_{ij} P_j^{(k)}}{2(\gamma_i + \lambda^{(k)} B_{ii})^2} \tag{7.66}$$

and therefore,

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)} \tag{7.67}$$

where

$$\Delta P^{(k)} = P_D + P_L^{(k)} - \sum_{i=1}^{n_g} P_i^{(k)} \tag{7.68}$$

The process is continued until $\Delta P^{(k)}$ is less than a specified accuracy.

If an approximate loss formula expressed by

$$P_L = \sum_{i=1}^{n_g} B_{ii} P_i^2 \tag{7.69}$$

is used, $B_{ij} = 0$, $B_{00} = 0$, and solution of the simultaneous equation given by (7.61) reduces to the following simple expression

$$P_i^{(k)} = \frac{\lambda^{(k)} - \beta_i}{2(\gamma_i + \lambda^{(k)} B_{ii})} \tag{7.70}$$

and (7.66) reduces to

$$\sum_{i=1}^{n_g} \left(\frac{\partial P_i}{\partial \lambda}\right)^{(k)} = \sum_{i=1}^{n_g} \frac{\gamma_i + B_{ii}\beta_i}{2(\gamma_i + \lambda^{(k)} B_{ii})^2} \tag{7.71}$$

Example 7.7 (chp7ex7)

The fuel cost in \$/h of three thermal plants of a power system are

$$C_1 = 200 + 7.0P_1 + 0.008P_1^2 \quad \$/h$$
$$C_2 = 180 + 6.3P_2 + 0.009P_2^2 \quad \$/h$$
$$C_3 = 140 + 6.8P_3 + 0.007P_3^2 \quad \$/h$$

where P_1, P_2, and P_3 are in MW. Plant outputs are subject to the following limits

$$10 \text{ MW} \leq P_1 \leq 85 \text{ MW}$$
$$10 \text{ MW} \leq P_2 \leq 80 \text{ MW}$$
$$10 \text{ MW} \leq P_3 \leq 70 \text{ MW}$$

For this problem, assume the real power loss is given by the simplified expression

$$P_{L(pu)} = 0.0218P_{1(pu)}^2 + 0.0228P_{2(pu)}^2 + 0.0179P_{3(pu)}^2$$

where the loss coefficients are specified in per unit on a 100-MVA base. Determine the optimal dispatch of generation when the total system load is 150 MW.

In the cost function P_i is expressed in MW. Therefore, the real power loss in terms of MW generation is

$$P_L = \left[0.0218\left(\frac{P_1}{100}\right)^2 + 0.0228\left(\frac{P_2}{100}\right)^2 + 0.0179\left(\frac{P_3}{100}\right)^2\right] \times 100 \quad \text{MW}$$
$$= 0.000218P_1^2 + 0.000228P_2^2 + 0.000179P_3^2 \quad \text{MW}$$

For the numerical solution using the gradient method, assume the initial value of $\lambda^{(1)} = 8.0$. From coordination equations, given by (7.70), $P_1^{(1)}$, $P_2^{(1)}$, and $P_3^{(1)}$ are

$$P_1^{(1)} = \frac{8.0 - 7.0}{2(0.008 + 8.0 \times 0.000218)} = 51.3136 \quad \text{MW}$$
$$P_2^{(1)} = \frac{8.0 - 6.3}{2(0.009 + 8.0 \times 0.000228)} = 78.5292 \quad \text{MW}$$
$$P_3^{(1)} = \frac{8.0 - 6.8}{2(0.007 + 8.0 \times 0.000179)} = 71.1575 \quad \text{MW}$$

The real power loss is

$$P_L^{(1)} = 0.000218(51.3136)^2 + 0.000228(78.5292)^2 + 0.000179(71.1575)^2 = 2.886$$

Since $P_D = 150$ MW, the error $\Delta P^{(1)}$ from (7.68) is

$$\Delta P^{(1)} = 150 + 2.8864 - (51.3136 + 78.5292 + 71.1575) = -48.1139$$

From (7.71)

$$\sum_{i=1}^{3}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(1)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 8.0 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 8.0 \times 0.000228)^2}$$
$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 8.0 \times 0.000179)^2} = 152.4924$$

From (7.65)

$$\Delta\lambda^{(1)} = \frac{-48.1139}{152.4924} = -0.31552$$

Therefore, the new value of λ is

$$\lambda^{(2)} = 8.0 - 0.31552 = 7.6845$$

Continuing the process, for the second iteration, we have

$$P_1^{(2)} = \frac{7.6845 - 7.0}{2(0.008 + 7.6845 \times 0.000218)} = 35.3728 \quad \text{MW}$$
$$P_2^{(2)} = \frac{7.6845 - 6.3}{2(0.009 + 7.6845 \times 0.000228)} = 64.3821 \quad \text{MW}$$
$$P_3^{(2)} = \frac{7.6845 - 6.8}{2(0.007 + 7.6845 \times 0.000179)} = 52.8015 \quad \text{MW}$$

The real power loss is

$$P_L^{(2)} = 0.000218(35.3728)^2 + 0.000228(64.3821)^2 + 0.000179(52.8015)^2 = 1.717$$

Since $P_D = 150$ MW, the error $\Delta P^{(2)}$ from (7.68) is

$$\Delta P^{(2)} = 150 + 1.7169 - (35.3728 + 64.3821 + 52.8015) = -0.8395$$

From (7.71)

$$\sum_{i=1}^{3}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(2)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 7.684 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 7.684 \times 0.000228)^2}$$
$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 7.6845 \times 0.000179)^2} = 154.588$$

From (7.65)

$$\Delta\lambda^{(2)} = \frac{-0.8395}{154.588} = -0.005431$$

Therefore, the new value of λ is

$$\lambda^{(3)} = 7.6845 - 0.005431 = 7.679$$

For the third iteration, we have

$$P_1^{(3)} = \frac{7.679 - 7.0}{2(0.008 + 7.679 \times 0.000218)} = 35.0965 \quad \text{MW}$$
$$P_2^{(3)} = \frac{7.679 - 6.3}{2(0.009 + 7.679 \times 0.000228)} = 64.1369 \quad \text{MW}$$
$$P_3^{(3)} = \frac{7.679 - 6.8}{2(0.007 + 7.679 \times 0.000179)} = 52.4834 \quad \text{MW}$$

The real power loss is

$$P_L^{(3)} = 0.000218(35.0965)^2 + 0.000228(64.1369)^2 + 0.000179(52.4834)^2 = 1.699$$

Since $P_D = 150$ MW, the error $\Delta P^{(3)}$ from (7.68) is

$$\Delta P^{(3)} = 150 + 1.6995 - (35.0965 + 64.1369 + 52.4834) = -0.01742$$

From (7.71)

$$\sum_{i=1}^{3}\left(\frac{\partial P_i}{\partial \lambda}\right)^{(3)} = \frac{0.008 + 0.000218 \times 7.0}{2(0.008 + 7.679 \times 0.000218)^2} + \frac{0.009 + 0.000228 \times 6.3}{2(0.009 + 7.679 \times 0.000228)^2}$$
$$+ \frac{0.007 + 0.000179 \times 6.8}{2(0.007 + 7.679 \times 0.000179)^2} = 154.624$$

From (7.65)

$$\Delta\lambda^{(3)} = \frac{-0.01742}{154.624} = -0.0001127$$

Therefore, the new value of λ is

$$\lambda^{(4)} = 7.679 - 0.0001127 = 7.6789$$

Since $\Delta\lambda^{(3)}$, is small the equality constraint is met in four iterations, and the optimal dispatch for $\lambda = 7.6789$ are

$$P_1^{(4)} = \frac{7.6789 - 7.0}{2(0.008 + 7.679 \times 0.000218)} = 35.0907 \quad \text{MW}$$
$$P_2^{(4)} = \frac{7.6789 - 6.3}{2(0.009 + 7.679 \times 0.000228)} = 64.1317 \quad \text{MW}$$
$$P_3^{(4)} = \frac{7.6789 - 6.8}{2(0.007 + 7.679 \times 0.000179)} = 52.4767 \quad \text{MW}$$

The real power loss is

$$P_L^{(4)} = 0.000218(35.0907)^2 + 0.000228(64.1317)^2 + 0.000179(52.4767)^2 = 1.699$$

and the total fuel cost is

$$C_t = 200 + 7.0(35.0907) + 0.008(35.0907)^2 + 180 + 6.3(64.1317) + 0.009(64.1317)^2 + 140 + 6.8(52.4767) + 0.007(52.4767)^2 = 1592.65 \text{ \$/h}$$

The **dispatch** program can be used to find the optimal dispatch of generation. The program is designed for the loss coefficients to be expressed in per unit. The loss coefficients are arranged in a matrix form with the variable name B. The base MVA must be specified by the variable name **basemva**. If base mva is not specified, it is set to 100 MVA.

We use the following commands

```
cost = [200  7.0    0.008
        180  6.3    0.009
        140  6.8    0.007];
mwlimits =[10  85
           10  80
           10  70];
Pdt = 150;
B = [0.0218        0        0
     0    0.0228        0
     0         0   0.0179];
basemva = 100;
dispatch
gencost
```


The result is

```
Incremental cost of delivered power(system lambda) =
7.678935$/MWh
Optimal Dispatch of Generation:

   35.0907
   64.1317
   52.4767

Total system loss = 1.6991 MW
Total generation cost = 1592.65 $/h
```

Example 7.8 (chp7ex8)

Figure 7.7 (page 295) shows the one-line diagram of a power system described in Example 7.9. The B matrices of the loss formula for this system are found in Example 7.9. They are given in per unit on a 100 MVA base as follows

$$B = \begin{bmatrix} 0.0218 & 0.0093 & 0.0028 \\ 0.0093 & 0.0228 & 0.0017 \\ 0.0028 & 0.0017 & 0.0179 \end{bmatrix}$$
$$B_0 = \begin{bmatrix} 0.0003 & 0.0031 & 0.0015 \end{bmatrix}$$
$$B_{00} = 0.00030523$$

Cost functions, generator limits, and total loads are given in Example 7.7. Use **dispatch** program to obtain the optimal dispatch of generation.

We use the following commands.

```
cost = [200  7.0    0.008
        180  6.3    0.009
        140  6.8    0.007];
mwlimits =[10  85
           10  80
           10  70];
Pdt = 150;
```

```
B =  [0.0218   0.0093   0.0028
      0.0093   0.0228   0.0017
      0.0028   0.0017   0.0179];
B0 = [0.0003   0.0031   0.0015];
B00 = 0.00030523;
basemva = 100;
dispatch
gencost
```

The result is

```
Incremental cost of delivered power (system lambda) =
7.767785 $/MWh
Optimal Dispatch of Generation:

   33.4701
   64.0974
   55.1011

Total generation cost =    1599.98 $/h
```

7.7 DERIVATION OF LOSS FORMULA

One of the major steps in the optimal dispatch of generation is to express the system losses in terms of the generator's real power outputs. There are several methods of obtaining the loss formula. One method developed by Kron and adopted by Kirchmayer is the *loss coefficient* or *B-coefficient* method.

The total injected complex power at bus i, denoted by S_i, is given by

$$S_i = P_i + jQ_i = V_i I_i^* \tag{7.72}$$

The summation of powers over all buses gives the total system losses

$$P_L + jQ_L = \sum_{i=1}^{n} V_i I_i^* = V_{bus}^T I_{bus}^* \tag{7.73}$$

where P_L and Q_L are the real and reactive power loss of the system. V_{bus} is the column vector of the nodal bus voltages and I_{bus} is the column vector of the injected bus currents. The expression for the bus currents in terms of bus voltage was derived in Chapter 6 and is given by (6.2) as

$$I_{bus} = Y_{bus} V_{bus} \tag{7.74}$$

where Y_{bus} is the bus admittance matrix with ground as reference. Solving for V_{bus}, we have

$$\begin{aligned} V_{bus} &= Y_{bus}^{-1} I_{bus} \\ &= Z_{bus} I_{bus} \end{aligned} \tag{7.75}$$

The inverse of the bus admittance matrix is known as the *bus impedance matrix*. The bus admittance matrix is nonsingular if there are shunt elements (such as shunt capacitive susceptance) connected to the ground (bus number 0). As discussed in Chapter 6, the bus admittance matrix is sparse and its inverse can be expressed as a product of sparse matrix factors. Actually Z_{bus}, which is also required for short-circuit analysis, can be obtained directly by the method of *building algorithm* without the need for matrix inversion. This technique is discussed in Chapter 9.

Substituting for V_{bus} from (7.75) into (7.73), results in

$$\begin{aligned} P_L + jQ_L &= [Z_{bus} I_{bus}]^T I_{bus}^* \\ &= I_{bus}^T Z_{bus}^T I_{bus}^* \end{aligned} \tag{7.76}$$

Z_{bus} is a symmetrical matrix; therefore, $Z_{bus}^T = Z_{bus}$, and the total system loss becomes

$$P_L + jQ_L = I_{bus}^T Z_{bus} I_{bus}^* \tag{7.77}$$

The expression in (7.77) can also be expressed with the use of index notation as

$$P_L + jQ_L = \sum_{i=1}^{n} \sum_{j=1}^{n} I_i Z_{ij} I_j^* \tag{7.78}$$

Since the bus impedance matrix is symmetrical, i.e., $Z_{ij} = Z_{ji}$, the above equation may be rewritten as

$$P_L + jQ_L = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} Z_{ij}(I_i I_j^* + I_j I_i^*) \tag{7.79}$$

The quantity inside the parentheses in (7.79) is real; thus the power loss can be broken into its real and imaginary components as

$$P_L = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} R_{ij}(I_i I_j^* + I_j I_i^*) \tag{7.80}$$

$$Q_L = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij}(I_i I_j^* + I_j I_i^*) \tag{7.81}$$

where R_{ij} and X_{ij} are the real and imaginary elements of the bus impedance matrix, respectively. Again, since $R_{ij} = R_{ji}$, the real power loss equation can be converted back into

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} I_i R_{ij} I_j^* \tag{7.82}$$

Or in matrix form, the equation for the system real power loss becomes

$$P_L = I_{bus}^T R_{bus} I_{bus}^* \tag{7.83}$$

where R_{bus} is the real part of the bus impedance matrix. In order to obtain the general formula for the system power loss in terms of generator powers, we define the total load current as the sum of all individual load currents, i.e.,

$$I_{L1} + I_{L2} + \cdots + I_{Ln_d} = I_D \tag{7.84}$$

where n_d is the number of load buses and I_D is the total load currents. Now the individual bus currents are assumed to vary as a constant complex fraction of the total load current, i.e.,

$$I_{Lk} = \ell_k I_D \qquad k = 1, 2, \ldots, n_d \tag{7.85}$$

or

$$\ell_k = \frac{I_{Lk}}{I_D} \tag{7.86}$$

Assuming bus 1 to be the reference bus (slack bus), expanding the first row in (7.75) results in

$$V_1 = Z_{11}I_1 + Z_{12}I_2 + \cdots + Z_{1n}I_n \tag{7.87}$$

If n_g is the number of generator buses and n_d is the number of load buses, the above equation can be written in terms of the load currents and generator currents as

$$V_1 = \sum_{i=1}^{n_g} Z_{1i}I_{gi} + \sum_{k=1}^{n_d} Z_{1k}I_{Lk} \tag{7.88}$$

Substituting for I_{Lk} from (7.85) into (7.88), we have

$$\begin{aligned} V_1 &= \sum_{i=1}^{n_g} Z_{1i}I_{gi} + I_D \sum_{k=1}^{n_d} \ell_k Z_{1k} \\ &= \sum_{i=1}^{n_g} Z_{1i}I_{gi} + I_D T \end{aligned} \tag{7.89}$$

where

$$T = \sum_{k=1}^{n_d} \ell_k Z_{1k} \tag{7.90}$$

If I_0 is defined as the current flowing away from bus 1, with all other load currents set to zero, we have

$$V_1 = -Z_{11}I_0 \tag{7.91}$$

Substituting for V_1 in (7.89) and solving for I_D, we have

$$I_D = -\frac{1}{T}\sum_{i=1}^{n_g} Z_{1i}I_{gi} - \frac{1}{T}Z_{11}I_0 \tag{7.92}$$

Substituting for I_D from (7.92) into (7.85), the load currents become

$$I_{Lk} = -\frac{\ell_k}{T}\sum_{i=1}^{n_g} Z_{1i}I_{gi} - \frac{\ell_k}{T}Z_{11}I_0 \tag{7.93}$$

Let

$$\rho = -\frac{\ell_k}{T} \tag{7.94}$$

Then

$$I_{Lk} = \rho_k \sum_{i=1}^{n_g} Z_{1i}I_{gi} + \rho_k Z_{11}I_0 \tag{7.95}$$

Augmenting the generator currents with the above relation in matrix form, we have

$$\begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gn_g} \\ I_{L1} \\ I_{L2} \\ \vdots \\ I_{Ln_d} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ \rho_1 Z_{11} & \rho_1 Z_{12} & \cdots & \rho_1 Z_{1n_g} & \rho_1 Z_{11} \\ \rho_2 Z_{11} & \rho_2 Z_{12} & \cdots & \rho_2 Z_{1n_g} & \rho_2 Z_{11} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_k Z_{11} & \rho_k Z_{12} & \cdots & \rho_k Z_{1n_g} & \rho_k Z_{11} \end{bmatrix} \begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gn_g} \\ I_0 \end{bmatrix} \tag{7.96}$$

Showing the above matrix by C, (7.96) becomes

$$I_{bus} = C I_{new} \tag{7.97}$$

Substituting for I_{bus} in (7.83), we have

$$P_L = [CI_{new}]^T R_{bus} C^* I^*_{new} \\ = I^T_{new} C^T R_{bus} C^* I^*_{new} \tag{7.98}$$

If S_{gi} is the complex power at bus i, the generator current is

$$I_{gi} = \frac{S^*_{gi}}{V^*_i} = \frac{P_{gi} - jQ_{gi}}{V^*_i} \\ = \frac{1 - j\frac{Q_{gi}}{P_{gi}}}{V^*_i} P_{gi} \tag{7.99}$$

or

$$I_{gi} = \psi_i P_{gi} \tag{7.100}$$

where

$$\psi_i = \frac{1 - j\frac{Q_{gi}}{P_{gi}}}{V^*_i} \tag{7.101}$$

Adding the current I_0 to the column vector current I_{gi} in (7.100) results in

$$\begin{bmatrix} I_{g1} \\ I_{g2} \\ \vdots \\ I_{gn_g} \\ I_0 \end{bmatrix} = \begin{bmatrix} \psi_1 & 0 & \cdots & 0 & 0 \\ 0 & \psi_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \psi_{n_g} & 0 \\ 0 & 0 & \cdots & 0 & I_0 \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn_g} \\ 1 \end{bmatrix} \tag{7.102}$$

or in short form

$$I_{new} = \Psi P_{G1} \tag{7.103}$$

where

$$P_{G1} = \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn_g} \\ 1 \end{bmatrix} \tag{7.104}$$

Substituting from (7.103) for I_{new} in (7.98), the loss equation becomes

$$P_L = [\Psi P_{G1}]^T C^T R_{bus} C^* \Psi^* P^*_{G1} \\ = P^T_{G1} \Psi^T C^T R_{bus} C^* \Psi^* P^*_{G1} \tag{7.105}$$

The resultant matrix in the above equation is complex and the real power loss is found from its real part, thus

$$P_L = P^T_{G1} \Re[H] P^*_{G1} \tag{7.106}$$

where

$$H = \Psi^T C^T R_{bus} C^* \Psi^* \tag{7.107}$$

Since elements of the matrix H are complex, its real part must be used for computing the real power loss. It is found that H is a *Hermitian matrix*. This means that H is symmetrical and $H = H^*$. Thus, real part of H is found from

$$\Re[H] = \frac{H + H^*}{2} \tag{7.108}$$

The above matrix is partitioned as follows

$$\Re[H] = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n_g} & B_{01}/2 \\ B_{21} & B_{22} & \cdots & B_{2n_g} & B_{02}/2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & B_{n_gn_g} & B_{0n_g}/2 \\ B_{01}/2 & B_{02}/2 & \cdots & B_{0n_g}/2 & B_{00} \end{bmatrix} \tag{7.109}$$

Substituting for $\Re[H]$ into (7.106), yields

$$P_L = [P_{g1}\ P_{g2} \cdots P_{gn_g}\ 1] \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n_g} & B_{01}/2 \\ B_{21} & B_{22} & \cdots & B_{2n_g} & B_{02}/2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & B_{n_gn_g} & B_{0n_g}/2 \\ B_{01}/2 & B_{02}/2 & \cdots & B_{0n_g}/2 & B_{00} \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \cdots \\ P_{gn_g} \\ 1 \end{bmatrix} \tag{7.110}$$

or

$$P_L = [\, P_{g1}\ \ P_{g2}\ \cdots\ P_{gn_g} \,] \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n_g} \\ B_{21} & B_{22} & \cdots & B_{2n_g} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n_g1} & B_{n_g2} & \cdots & B_{n_gn_g} \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \cdots \\ P_{gn_g} \end{bmatrix} \\ + [\, P_{g1}\ \ P_{g2}\ \cdots\ P_{gn_g} \,] \begin{bmatrix} B_{01} \\ B_{02} \\ \cdots \\ B_{0n_g} \end{bmatrix} + B_{00} \tag{7.111}$$

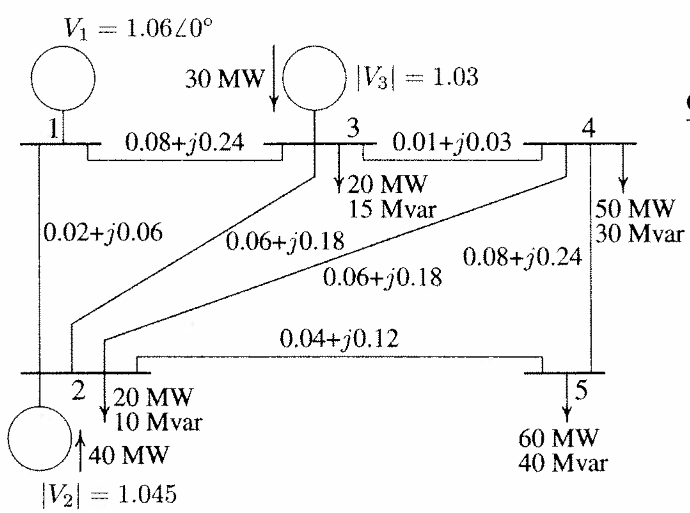
To find the loss coefficients, first a power flow solution is obtained for the initial operating state. This provides the voltage magnitude and phase angles at all buses. From these results, load currents I_{Lk}, the total load current I_D, and ℓ_k are obtained. Next the bus matrix Z_{bus} is found. This can be obtained by converting the bus admittance matrix found from **lfybus** or directly from the *building algorithm* described in Chapter 9. Next the transformation matrices C and Ψ and H are obtained. Finally the B-coefficients are evaluated from (7.109). It should be noted that the B-coefficients are functions of the system operating state. If a new scheduling of generation is not drastically different from the initial operating condition, the loss coefficients may be assumed constant. A program named **bloss** is developed for the computation of the B-coefficients. This program requires the power flow solution and can be used following any of the power flow programs such as **lfgauss**, **lfnewton**, or **decouple**. The B-coefficients obtained are based on the generation in per unit. When generation are expressed in MW, the loss coefficients are

$$B_{ij} = B_{ij\,pu}/S_B \qquad B_{0i} = B_{0i\,pu} \quad \text{and} \quad B_{00} = B_{00\,pu} \times S_B$$

where S_B is the base MVA.

Example 7.9 (chp7ex9)

Figure 7.7 shows the one-line diagram of a simple 5-bus power system with generator at buses 1, 2, and 3. Bus 1, with its voltage set at $1.06\angle 0°$ pu, is taken as the slack bus. Voltage magnitude and real power generation at buses 2 and 3 are 1.045 pu, 40 MW, and 1.030 pu, 30 MW, respectively.



Line	$\frac{1}{2}B$
1 – 2	0.030
1 – 3	0.025
2 – 3	0.020
2 – 4	0.020
2 – 5	0.015
3 – 4	0.010
4 – 5	0.025

(Capacitive susceptance)

FIGURE 7.7
One-line diagram of Example 7.9 (impedances in pu on 100-MVA base).

The load MW and Mvar values are shown on the diagram. Line impedances and one-half of the line capacitive susceptance are given in per unit on a 100-MVA base. Obtain the power flow solution and use the **bloss** program to obtain the loss coefficients in per unit.

We use the following commands

```
clear
basemva = 100;  accuracy = 0.0001;  maxiter = 10;

%        Bus Bus Voltage Angle -Load- ----Generator----Injected
%        No code Mag. Degree  MW  Mvar  MW Mvar  Qmin Qmax Mvar
busdata=[1   1  1.06   0.0    0    0     0   0    10   50   0
         2   2  1.045  0.0   20   10    40  30    10   50   0
         3   2  1.03   0.0   20   15    30  10    10   40   0
         4   0  1.00   0.0   50   30     0   0     0    0   0
         5   0  1.00   0.0   60   40     0   0     0    0   0];
%         Bus bus    R        X        1/2 B   1 for lines code or
%         nl  nr     pu       pu       pu     tap setting value
linedata=[1   2     0.02     0.06     0.030     1
          1   3     0.08     0.24     0.025     1
          2   3     0.06     0.18     0.020     1
          2   4     0.06     0.18     0.020     1
          2   5     0.04     0.12     0.015     1
          3   4     0.01     0.03     0.010     1
          4   5     0.08     0.24     0.025     1];
lfybus                 % form the bus admittance matrix
lfnewton     % Power flow solution by Newton-Raphson method
busout       % Prints the power flow solution on the screen
bloss               % Obtains the loss formula coefficients
```

The result is

```
          Power Flow Solution by Newton-Raphson Method
             Maximum Power mismatch = 1.43025e-05
                    No. of iterations = 3

Bus Voltage  Angle    -----Load-----    --Generation-- Injected
No.  Mag.    Degree     MW      Mvar      MW     Mvar    Mvar
1   1.060    0.000    0.000     0.00    83.051   7.271   0.00
2   1.045   -1.782   20.000    10.00    40.000  41.811   0.00
3   1.030   -2.664   20.000    15.00    30.000  24.148   0.00
4   1.019   -3.243   50.000    30.00     0.000   0.000   0.00
5   0.990   -4.405   60.000    40.00     0.000   0.000   0.00
Total               150.000    95.000  153.051  73.230   0.00
```

```
B =
      0.0218     0.0093     0.0028
      0.0093     0.0228     0.0017
      0.0028     0.0017     0.0179

B0 =
      0.0003     0.0031     0.0015
B00 =
      3.0523e-04
Total system loss = 3.05248 MW
```

As we have seen, any of the power flow programs, together with the **bloss** and **dispatch** programs can be used to obtain the optimal dispatch of generation. The **dispatch** program produces a variable named **dpslack**. This is the difference (absolute value) between the scheduled slack generation determined from the coordination equation, and the slack generation, obtained from the power flow solution. A power flow solution obtained with the new scheduling of generation results in a new loss coefficients, which can be used to solve the coordination equation again. This process can be continued until **dpslack** is within a specified tolerance. This procedure is demonstrated in the following example.

Example 7.10 (chp7ex10)

The generation cost and the real power limits of the generators of the power system in Example 7.9 is given in Example 7.4 and Example 7.6. Obtain the optimal dispatch of generation. Continue the optimization process until the difference (absolute value) between the scheduled slack generation, determined from the coordination equation, and the slack generation, obtained from the power flow solution, is within 0.001 MW.

We use the following commands

```
clear
basemva = 100;  accuracy = 0.0001;  maxiter = 10;

%       Bus Bus  Voltage Angle --Load-- --Generator-- Injected
%       No  code Mag.   Degree MW   Mvar MW   Mvar Qmin Qmax Mvar

busdata=[1   1   1.06   0.0    0     0   0    0    10   50    0
         2   2   1.045  0.0   20    10  40   30    10   50    0
         3   2   1.03   0.0   20    15  30   10    10   40    0
         4   0   1.00   0.0   50    30   0    0     0    0    0
         5   0   1.00   0.0   60    40   0    0     0    0    0];

%         Bus bus    R      X       1/2 B  1 for lines code or
%         nl  nr    pu     pu        pu    tap setting value

linedata=[1  2    0.02    0.06    0.030      1
          1  3    0.08    0.24    0.025      1
          2  3    0.06    0.18    0.020      1
          2  4    0.06    0.18    0.020      1
          2  5    0.04    0.12    0.015      1
          3  4    0.01    0.03    0.010      1
          4  5    0.08    0.24    0.025      1];

cost = [200  7.0    0.008
        180  6.3    0.009
        140  6.8    0.007];

mwlimits =[10  85
           10  80
           10  70];

lfybus                    % forms the bus admittance matrix
lfnewton    % Power flow solution by Newton-Raphson method
busout      % Prints the power flow solution on the screen
bloss              % Obtains the loss formula coefficients
gencost           % Computes the total generation cost $/h
dispatch          % Obtains optimum dispatch of generation
     % dpslack is the difference (absolute value) between
     % the scheduled slack generation determined from the
        % coordination equation, and the slack generation
                % obtained from the power flow solution.

while dpslack > 0.001                % Test for convergence
lfnewton                           % New power flow solution
bloss                        % Loss coefficients are updated
dispatch %Optimum dispatch of gen.with new B-coefficients
end
busout                % Prints the final power flow solution
gencost % Generation cost with optimum scheduling of gen.
```

The result is

```
     Power Flow Solution by Newton-Raphson Method
        Maximum Power mismatch = 1.43025e-05
               No. of iterations = 3
```

```
Bus Voltage  Angle     -----Load-----    --Generation-- Injected
No. Mag.     Degree    MW        Mvar      MW        Mvar     Mvar

1   1.060    0.000     0.000     0.00     83.051    7.271     0.00
2   1.045   -1.782    20.000    10.00     40.000   41.811     0.00
3   1.030   -2.664    20.000    15.00     30.000   24.148     0.00
4   1.019   -3.243    50.000    30.00      0.000    0.000     0.00
5   0.990   -4.405    60.000    40.00      0.000    0.000     0.00
Total                150.000    95.000   153.051   73.230     0.00

     B =
          0.0218    0.0093    0.0028
          0.0093    0.0228    0.0017
          0.0028    0.0017    0.0179

     B0 =
          0.0003    0.0031    0.0015
     B00 =
          3.0523e-04
     Total system loss = 3.05248 MW

     Total generation cost =    1633.24 $/h
     Incremental cost of delivered power (system lambda) =
     7.767608 $/MWh
     Optimal Dispatch of Generation:

        33.4558
        64.1101
        55.1005

     Absolute value of the slack bus real power mismatch,
     dpslack = 0.4960 pu
```

In this example the final optimal dispatch of generation was obtained in six iterations. The results for final loss coefficients and final optimal dispatch of generation is presented below

```
     B =
          0.0472    0.0130    0.0036
          0.0130    0.0130    0.0010
          0.0036    0.0010    0.0115

     B0 =
          0.0047    0.0012    0.0004

     B00 =
          3.0516e-04

     Total system loss = 2.15691 MW
     Incremental cost of delivered power (system lambda) =
     7.759051 $/MWh
     Optimal Dispatch of Generation:

        23.5581
        69.5593
        59.0368

     Absolute value of the slack bus real power mismatch,
     dpslack = 0.0009 pu

          Power Flow Solution by Newton-Raphson Method
             Maximum Power mismatch = 1.90285e-08
                    No. of iterations = 4

Bus Voltage  Angle     -----Load-----    --Generation-- Injected
No. Mag.     Degree    MW        Mvar      MW        Mvar     Mvar

1   1.060    0.000     0.000     0.000    23.649   25.727     0.00
2   1.045   -0.282    20.000    10.000    69.518   30.767     0.00
3   1.030   -0.495    20.000    15.000    58.990   14.052     0.00
4   1.019   -1.208    50.000    30.000     0.000    0.000     0.00
5   0.990   -2.729    60.000    40.000     0.000    0.000     0.00

Total                150.000    95.000   152.154   70.545     0.00

Total generation cost =    1596.96 $/h
```

The total generation cost for the initial operating condition is 1,633.24 \$/h and the total generation cost with optimal dispatch of generation is 1,596.96 \$/h. This results in a savings of 36.27 \$/h.

Example 7.11 (chp7ex11)

Figure 7.8 is the 26-bus power system network of Problem 6.14. Bus 1 is taken as the slack bus with its voltage adjusted to $1.025\angle 0^\circ$ pu. The data for the voltage-controlled buses is

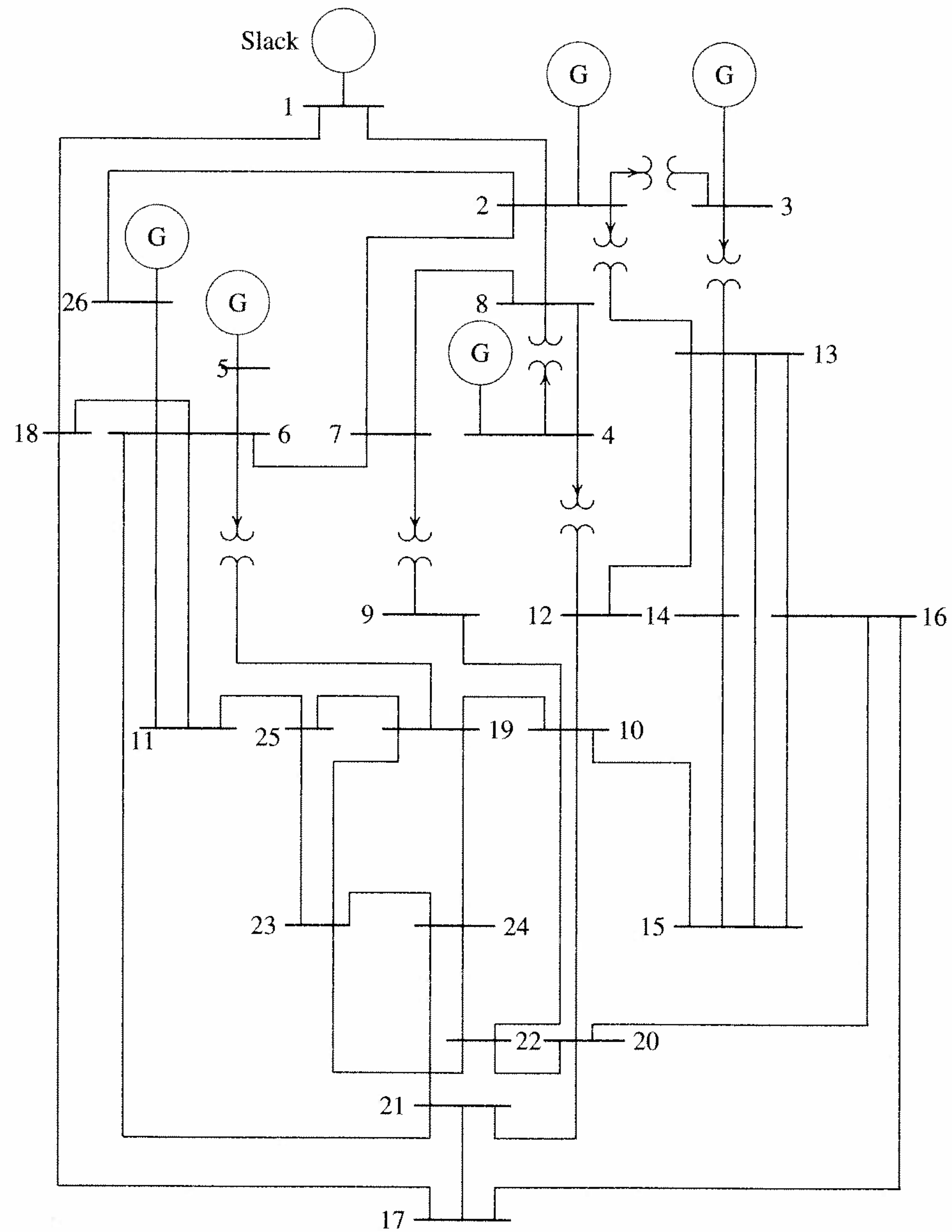


FIGURE 7.8
One-line diagram of Example 7.11.

REGULATED BUS DATA			
Bus No.	Voltage Magnitude	Min. Mvar Capacity	Max. Mvar Capacity
2	1.020	40	250
3	1.025	40	150
4	1.050	40	80
5	1.045	40	160
26	1.015	15	50

Transformer tap settings are given in the table below. The left bus number is assumed to be the tap side of the transformer.

TRANSFORMER DATA	
Transformer Designation	Tap Setting Per Unit
2 – 3	0.960
2 – 13	0.960
3 – 13	1.017
4 – 8	1.050
4 – 12	1.050
6 – 19	0.950
7 – 9	0.950

The shunt capacitive data is

SHUNT CAPACITOR DATA	
Bus No.	Mvar
1	4.0
4	2.0
5	5.0
6	2.0
9	3.0
11	1.5
12	2.0
15	0.5
19	5.0

Generation and loads are as given in the data prepared for use in the *MATLAB* environment in the matrix defined as **busdata**. Code 0, code 1, and code 2 are used for the load buses, the slack bus, and the voltage-controlled buses, respectively. Values for **basemva**, **accuracy**, and **maxiter** must be specified. Line data are as given in the matrix called **linedata**. The last column of this data must contain 1 for lines, or the tap setting values for transformers with off-nominal turn ratio. The generator's operating costs in \$/h, with P_i in MW are as follow:

$$C_1 = 240 + 7.0P_1 + 0.0070P_1^2$$
$$C_2 = 200 + 10.0P_2 + 0.0095P_2^2$$
$$C_3 = 220 + 8.5P_3 + 0.0090P_3^2$$
$$C_4 = 200 + 11.0P_4 + 0.0090P_4^2$$
$$C_5 = 220 + 10.5P_5 + 0.0080P_5^2$$
$$C_{26} = 190 + 12.0P_{26} + 0.0075P_{26}^2$$

The generator's real power limits are

GENERATOR REAL POWER LIMITS		
Gen.	Min. MW	Max. MW
1	100	500
2	50	200
3	80	300
4	50	150
5	50	200
5	50	120

Write the necessary commands to obtain the optimal dispatch of generation using **dispatch**. Continue the optimization process until the difference (absolute value) between the scheduled slack generation, determined from the coordination equation, and the slack generation, obtained from the power flow solution, is within 0.001 MW.

We use the following commands:

```
clear
basemva = 100;  accuracy = 0.0001;  maxiter = 10;

%       Bus Bus  Voltage Angle --Load-- --Generator---Injected
%       No  code Mag.  Degree MW Mvar  MW  Mvar Qmin Qmax Mvar
busdata=[1   1   1.025   0.0   51  41    0    0    0    0    4
         2   2   1.020   0.0   22  15   79    0   40  250    0
         3   2   1.025   0.0   64  50   20    0   40  150    0
         4   2   1.050   0.0   25  10  100    0   25   80    2
         5   2   1.045   0.0   50  30  300    0   40  160    5
         6   0   1.00    0.0   76  29    0    0    0    0    2
         7   0   1.00    0.0    0   0    0    0    0    0    0
         8   0   1.00    0.0    0   0    0    0    0    0    0
         9   0   1.00    0.0   89  50    0    0    0    0    3
        10   0   1.00    0.0    0   0    0    0    0    0    0
        11   0   1.00    0.0   25  15    0    0    0    0  1.5
        12   0   1.00    0.0   89  48    0    0    0    0    2
        13   0   1.00    0.0   31  15    0    0    0    0    0
        14   0   1.00    0.0   24  12    0    0    0    0    0
        15   0   1.00    0.0   70  31    0    0    0    0  0.5
        16   0   1.00    0.0   55  27    0    0    0    0    0
        17   0   1.00    0.0   78  38    0    0    0    0    0
        18   0   1.00    0.0  153  67    0    0    0    0    0
        19   0   1.00    0.0   75  15    0    0    0    0    5
        20   0   1.00    0.0   48  27    0    0    0    0    0
        21   0   1.00    0.0   46  23    0    0    0    0    0
        22   0   1.00    0.0   45  22    0    0    0    0    0
        23   0   1.00    0.0   25  12    0    0    0    0    0
        24   0   1.00    0.0   54  27    0    0    0    0    0
        25   0   1.00    0.0   28  13    0    0    0    0    0
        26   2   1.015   0.0   40  20   60    0   15   50    0];

%         Bus bus    R        X        1/2 B  1 for lines code or
%         nl  nr     pu       pu       pu     tap setting value
linedata=[1   2   0.00055  0.00480  0.03000   1
          1  18   0.00130  0.01150  0.06000   1
          2   3   0.00146  0.05130  0.05000   0.96
          2   7   0.01030  0.05860  0.01800   1
          2   8   0.00740  0.03210  0.03900   1
          2  13   0.00357  0.09670  0.02500   0.96
          2  26   0.03230  0.19670  0.00000   1
          3  13   0.00070  0.00548  0.00050   1.017
          4   8   0.00080  0.02400  0.00010   1.050
          4  12   0.00160  0.02070  0.01500   1.050
          5   6   0.00690  0.03000  0.09900   1
          6   7   0.00535  0.03060  0.00105   1
          6  11   0.00970  0.05700  0.00010   1
          6  18   0.00374  0.02220  0.00120   1
          6  19   0.00350  0.06600  0.04500   0.95
          6  21   0.00500  0.09000  0.02260   1
          7   8   0.00120  0.00693  0.00010   1
          7   9   0.00095  0.04290  0.02500   0.95
          8  12   0.00200  0.01800  0.02000   1
          9  10   0.00104  0.04930  0.00100   1
         10  12   0.00247  0.01320  0.01000   1
         10  19   0.05470  0.23600  0.00000   1
         10  20   0.00660  0.01600  0.00100   1
         10  22   0.00690  0.02980  0.00500   1
         11  25   0.09600  0.27000  0.01000   1
         11  26   0.01650  0.09700  0.00400   1
         12  14   0.03270  0.08020  0.00000   1
```

```
        12  15   0.01800  0.05980  0.00000   1
        13  14   0.00460  0.02710  0.00100   1
        13  15   0.01160  0.06100  0.00000   1
        13  16   0.01793  0.08880  0.00100   1
        14  15   0.00690  0.03820  0.00000   1
        15  16   0.02090  0.05120  0.00000   1
        16  17   0.09900  0.06000  0.00000   1
        16  20   0.02390  0.05850  0.00000   1
        17  18   0.00320  0.06000  0.03800   1
        17  21   0.22900  0.44500  0.00000   1
        19  23   0.03000  0.13100  0.00000   1
        19  24   0.03000  0.12500  0.00200   1
        19  25   0.11900  0.22490  0.00400   1
        20  21   0.06570  0.15700  0.00000   1
        20  22   0.01500  0.03660  0.00000   1
        21  24   0.04760  0.15100  0.00000   1
        22  23   0.02900  0.09900  0.00000   1
        22  24   0.03100  0.08800  0.00000   1
        23  25   0.09870  0.11680  0.00000   1];

cost = [240   7.0   0.0070
        200  10.0   0.0095
        220   8.5   0.0090
        200  11.0   0.0090
        220  10.5   0.0080
        190  12.0   0.0075];

mwlimits =[100  500
            50   200
            80   300
            50   150
            50   200
            50   120];

lfybus                   % Forms the bus admittance matrix
lfnewton   % Power flow solution by Newton-Raphson method
busout     % Prints the power flow solution on the screen
bloss              % Obtains the loss formula coefficients
gencost           % Computes the total generation cost $/h
dispatch          % Obtains optimum dispatch of generation
     % dpslack is the difference (absolute value) between
     % the scheduled slack generation determined from the
        % coordination equation, and the slack generation
                 % obtained from the power flow solution.

while dpslack>.001%Repeat till dpslack is within tolerance
lfnewton                            % New power flow solution
bloss                         % Loss coefficients are updated
dispatch %Optimum dispatch of gen. with new B-coefficients
end
busout                  % Prints the final power flow solution
gencost  % Generation cost with optimum scheduling of gen.
```

The result is

```
         Power Flow Solution by Newton-Raphson Method
           Maximum Power mismatch = 3.18289e-10
                  No. of iterations = 6

Bus Voltage  Angle    -----Load-----   --Generation-- Injected
No. Mag.     Degree    MW      Mvar      MW      Mvar     Mvar

1   1.025    0.000   51.000   41.000  719.534  224.011   4.00
2   1.020   -0.931   22.000   15.000   79.000  125.354   0.00
3   1.035   -4.213   64.000   50.000   20.000   63.030   0.00
4   1.050   -3.582   25.000   10.000  100.000   49.223   2.00
5   1.045    1.129   50.000   30.000  300.000  124.466   5.00
6   0.999   -2.573   76.000   29.000    0.000    0.000   2.00
7   0.994   -3.204    0.000    0.000    0.000    0.000   0.00
8   0.997   -3.299    0.000    0.000    0.000    0.000   0.00
9   1.009   -5.393   89.000   50.000    0.000    0.000   3.00
10  0.989   -5.561    0.000    0.000    0.000    0.000   0.00
11  0.997   -3.218   25.000   15.000    0.000    0.000   1.50
12  0.993   -4.692   89.000   48.000    0.000    0.000   2.00
13  1.014   -4.430   31.000   15.000    0.000    0.000   0.00
14  1.000   -5.040   24.000   12.000    0.000    0.000   0.00
15  0.991   -5.538   70.000   31.000    0.000    0.000   0.50
16  0.983   -5.882   55.000   27.000    0.000    0.000   0.00
17  0.987   -4.985   78.000   38.000    0.000    0.000   0.00
18  1.007   -1.866  153.000   67.000    0.000    0.000   0.00
19  1.004   -6.397   75.000   15.000    0.000    0.000   5.00
20  0.980   -6.025   48.000   27.000    0.000    0.000   0.00
21  0.977   -5.778   46.000   23.000    0.000    0.000   0.00
22  0.978   -6.437   45.000   22.000    0.000    0.000   0.00
23  0.976   -7.087   25.000   12.000    0.000    0.000   0.00
24  0.968   -7.347   54.000   27.000    0.000    0.000   0.00
25  0.974   -6.775   28.000   13.000    0.000    0.000   0.00
26  1.015   -1.803   40.000   20.000   60.000   32.706   0.00
Total               1263.000  637.000 1278.534  618.791  25.00
```

```
B =
   0.0014    0.0015    0.0009   -0.0001   -0.0004   -0.0002
   0.0015    0.0043    0.0050    0.0001   -0.0008   -0.0003
   0.0009    0.0050    0.0315   -0.0000   -0.0020   -0.0016
  -0.0001    0.0001   -0.0000    0.0029   -0.0006   -0.0009
  -0.0004   -0.0008   -0.0020   -0.0006    0.0085   -0.0001
  -0.0002   -0.0003   -0.0016   -0.0009   -0.0001    0.0176

B0 =
  -0.0002   -0.0008    0.0067    0.0001    0.0000   -0.0012

B00 =
   0.0056

  Total system loss = 15.53 MW

  Total generation cost =   16760.73 $/h
  Incremental cost of delivered power (system lambda) =
  13.911780 $/MWh
  Optimal Dispatch of Generation:

    474.1196
    173.7886
    190.9515
    150.0000
    196.7196
    103.5772

  Absolute value of the slack bus real power mismatch,
  dpslack = 2.4541 pu
```

In this example the final optimal dispatch of generation was obtained in three iterations. The results for final loss coefficients and final optimal dispatch of generation is presented below

```
B =
   0.0017    0.0012    0.0007   -0.0001   -0.0005   -0.0002
   0.0012    0.0014    0.0009    0.0001   -0.0006   -0.0001
   0.0007    0.0009    0.0031    0.0000   -0.0010   -0.0006
  -0.0001    0.0001    0.0000    0.0024   -0.0006   -0.0008
  -0.0005   -0.0006   -0.0010   -0.0006    0.0129   -0.0002
  -0.0002   -0.0001   -0.0006   -0.0008   -0.0002    0.0150

B0 =
   1.0e-03 *
  -0.3908   -0.1297    0.7047    0.0591    0.2161   -0.6635

B00 =
   0.0056

  Total system loss = 12.807 MW
  Incremental cost of delivered power (system lambda) =
  13.538113 $/MWh
  Optimal Dispatch of Generation:
    447.6919
    173.1938
    263.4859
    138.8142
    165.5884
     87.0260

  Absolute value of the slack bus real power mismatch,
  dpslack =  0.0008 pu

      Power Flow Solution by Newton-Raphson Method
         Maximum Power mismatch = 2.33783e-05
                No. of iterations = 3
```

Bus No.	Voltage Mag.	Angle Degree	Load MW	Load Mvar	Generation MW	Generation Mvar	Injected Mvar
1	1.025	0.000	51.000	41.000	447.611	250.582	4.00
2	1.020	-0.200	22.000	15.000	173.087	57.303	0.00
3	1.045	-0.639	64.000	50.000	263.363	78.280	0.00
4	1.050	-2.101	25.000	10.000	138.716	33.449	2.00
5	1.045	-1.453	50.000	30.000	166.099	142.890	5.00
6	1.001	-2.874	76.000	29.000	0.000	0.000	2.00
7	0.995	-2.406	0.000	0.000	0.000	0.000	0.00
8	0.998	-2.278	0.000	0.000	0.000	0.000	0.00
9	1.010	-4.387	89.000	50.000	0.000	0.000	3.00
10	0.991	-4.311	0.000	0.000	0.000	0.000	0.00
11	0.998	-2.824	25.000	15.000	0.000	0.000	1.50
12	0.994	-3.282	89.000	48.000	0.000	0.000	2.00
13	1.022	-1.261	31.000	15.000	0.000	0.000	0.00
14	1.008	-2.445	24.000	12.000	0.000	0.000	0.00
15	0.999	-3.229	70.000	31.000	0.000	0.000	0.50
16	0.990	-3.990	55.000	27.000	0.000	0.000	0.00
17	0.983	-4.366	78.000	38.000	0.000	0.000	0.00

```
18  1.007   -1.884  153.000   67.000    0.000     0.000     0.00
19  1.005   -6.074   75.000   15.000    0.000     0.000     5.00
20  0.983   -4.759   48.000   27.000    0.000     0.000     0.00
21  0.977   -5.411   46.000   23.000    0.000     0.000     0.00
22  0.980   -5.325   45.000   22.000    0.000     0.000     0.00
23  0.978   -6.388   25.000   12.000    0.000     0.000     0.00
24  0.969   -6.672   54.000   27.000    0.000     0.000     0.00
25  0.975   -6.256   28.000   13.000    0.000     0.000     0.00
26  1.015   -0.284   40.000   20.000   86.939    27.892     0.00

Total               1263.000  637.000 1275.800  590.396    25.00

Total generation cost =    15447.72 $/h
```

The total generation cost for the initial operating condition is 16,760.73 \$/h and the total generation cost with optimal dispatch of generation is 15,447.72 \$/h. This results in a savings of 1,313.01 \$/h. That is, with this loading, the total annual savings is over \$11 million.

PROBLEMS

7.1. Find a rectangle of maximum perimeter that can be inscribed in a circle of unit radius given by

$$g(x,y) = x^2 + y^2 - 1 = 0$$

Check the eigenvalues for sufficient conditions.

7.2. Find the minimum of the function

$$f(x,y) = x^2 + 2y^2$$

subject to the equality constraint

$$g(x,y) = x + 2y + 4 = 0$$

Check for the sufficient conditions.

7.3. Use the Lagrangian multiplier method for solving constrained parameter optimization problems to determine an isosceles triangle of maximum area that may be inscribed in a circle of radius 1.

7.4. For a second-order bandpass filter with transfer function

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

determine the values of the damping ratio and natural frequency, ζ and ω_n, corresponding to a Bode plot whose peak occurs at 7071.07 radians/sec and whose half-power bandwidth is 12,720.2 radians/sec.

7.5. Find the minimum value of the function

$$f(x,y) = x^2 + y^2$$

subject to the equality constraint

$$g(x,y) = x^2 - 6x - y^2 + 17 = 0$$

7.6. Find the minimum value of the function

$$f(x,y) = x^2 + y^2$$

subject to one equality constraint

$$g(x,y) = x^2 - 5x - y^2 + 20 = 0$$

and one inequality constraint

$$u(x,y) = 2x + y \geq 6$$

7.7. The fuel-cost functions in \$/h for two 800 MW thermal plants are given by

$$C_1 = 400 + 6.0P_1 + 0.004P_1^2$$
$$C_2 = 500 + \ \beta P_2 + \quad \gamma P_2^2$$

where P_1 and P_2 are in MW.

(a) The incremental cost of power λ is \$8/MWh when the total power demand is 550 MW. Neglecting losses, determine the optimal generation of each plant.

(b) The incremental cost of power λ is \$10/MWh when the total power demand is 1300 MW. Neglecting losses, determine the optimal generation of each plant.

(c) From the results of (a) and (b) find the fuel-cost coefficients β and γ of the second plant.

7.8. The fuel-cost functions in \$/h for three thermal plants are given by

$$C_1 = 350 + 7.20P_1 + 0.0040P_1^2$$
$$C_2 = 500 + 7.30P_2 + 0.0025P_2^2$$
$$C_3 = 600 + 6.74P_3 + 0.0030P_3^2$$

where P_1, P_2, and P_3 are in MW. The governors are set such that generators share the load equally. Neglecting line losses and generator limits, find the total cost in \$/h when the total load is

(i) P_D = 450 MW
(ii) P_D = 745 MW
(iii) P_D = 1335 MW

7.9. Neglecting line losses and generator limits, determine the optimal scheduling of generation for each loading condition in Problem 7.8
(a) by analytical technique, using (7.33) and (7.31).
(b) using Iterative method. Start with an initial estimate of $\lambda = 7.5$ \$/MWh.
(c) find the savings in \$/h for each case compared to the costs in Problem 7.8 when the generators shared load equally.
Use the **dispatch** program to check your results.

7.10. Repeat Problem 7.9 (a) and (b), but this time consider the following generator limits (in MW)

$$122 \leq P_1 \leq 400$$
$$260 \leq P_2 \leq 600$$
$$50 \leq P_3 \leq 445$$

Use the **dispatch** program to check your results.

7.11. The fuel-cost function in \$/h of two thermal plants are

$$C_1 = 320 + 6.2P_1 + 0.004P_1^2$$
$$C_2 = 200 + 6.0P_2 + 0.003P_2^2$$

where P_1 and P_2 are in MW. Plant outputs are subject to the following limits (in MW)

$$50 \leq P_1 \leq 250$$
$$50 \leq P_2 \leq 350$$

The per-unit system real power loss with generation expressed in per unit on a 100-MVA base is given by

$$P_{L(pu)} = 0.0125P_{1(pu)}^2 + 0.00625P_{2(pu)}^2$$

The total load is 412.35 MW. Determine the optimal dispatch of generation. Start with an initial estimate of $\lambda = 7$ \$/MWh. Use the **dispatch** program to check your results.

7.12. The 9-bus power system network of an Electric Utility Company is shown in Figure 7.9. The load data is tabulated below. Voltage magnitude, generation schedule and the reactive power limits for the regulated buses are also tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.03\angle 0°$, is taken as the slack bus.

LOAD DATA

Bus No.	Load MW	Load Mvar
1	0	0
2	20	10
3	25	15
4	10	5
5	40	20
6	60	40
7	10	5
8	80	60
9	100	80

GENERATION DATA

Bus No.	Voltage Mag.	Generation MW	Mvar Limits Min.	Mvar Limits Max.
1	1.03			
2	1.04	80	0	250
7	1.01	120	0	100

The Mvar of the shunt capacitors installed at substations are given below

SHUNT CAPACITORS

Bus No.	Mvar
3	1.0
4	3.0

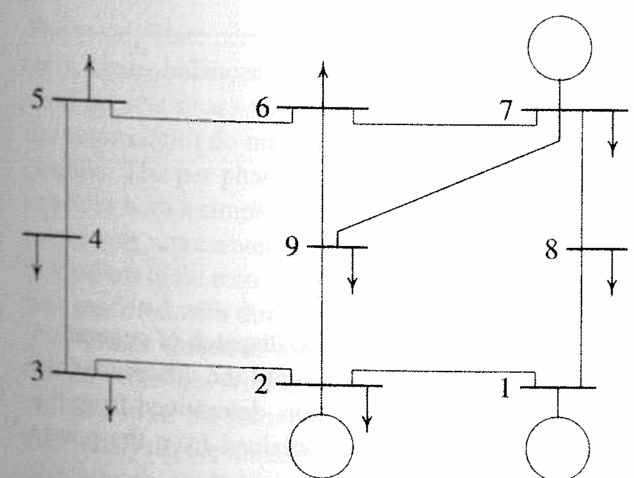


FIGURE 7.9
One-line diagram for Problem 7.12.

The line data containing the series resistance and reactance in per unit, and one-half of the total capacitance in per unit susceptance on a 100 MVA base is tabulated below.

LINE DATA				
Bus No.	Bus No.	R, PU	X, PU	$\frac{1}{2}$ B, PU
1	2	0.018	0.054	0.0045
1	8	0.014	0.036	0.0030
2	9	0.006	0.030	0.0028
2	3	0.013	0.036	0.0030
3	4	0.010	0.050	0.0000
4	5	0.018	0.056	0.0000
5	6	0.020	0.060	0.0000
6	7	0.015	0.045	0.0038
6	9	0.002	0.066	0.0000
7	8	0.032	0.076	0.0000
7	9	0.022	0.065	0.0000

The generator's operating costs in \$/h are as follows:

$$C_1 = 240 + 6.7P_1 + 0.009P_1^2$$
$$C_2 = 220 + 6.1P_2 + 0.005P_2^2$$
$$C_7 = 240 + 6.5P_7 + 0.008P_7^2$$

The generator's real power limits are

GENERATOR REAL POWER LIMITS		
Gen.	Min. MW	Max. MW
1	50	200
2	50	200
7	50	100

Write the necessary commands to obtain the optimal dispatch of generation using **dispatch**. Continue the optimization process until the difference (absolute value) between the scheduled slack generation, determined from the coordination equation, and the slack generation, obtained from the power flow solution, is within 0.001 MW.

CHAPTER

8

SYNCHRONOUS MACHINE TRANSIENT ANALYSIS

8.1 INTRODUCTION

The steady state performance of the synchronous machine was described in Chapter 3. Under balanced steady state operations, the rotor mmf and the resultant stator mmf are stationary with respect to each other. As a result, the flux linkages with the rotor circuit do not change with time, and no voltages are induced in the rotor circuits. The per phase equivalent circuit then becomes a constant generated emf in series with a simple impedance. In Chapter 3, for steady state operation the generator was represented with a constant emf behind the synchronous reactance X_s. For salient-pole rotor, because of the nonuniformity of the air gap, the generator was modeled with direct axis reactance X_d and the quadrature axis reactance X_q.

Under transient conditions, such as short circuits at the generator terminals, the flux linkages with the rotor circuits change with time. This result in transient currents in all the rotor circuits, which in turn reacts on the armature. For the transient analysis, the idealized synchronous machine is represented as a group of magnetically coupled circuits with inductances which depend on the angular position of the rotor. The resulting differential equations describing the machine have time-varying coefficients, and a closed form of solution in most cases is not feasible. A great simplification can be made by transformation of stator variables from phases a, b, and c into new variables the frame of reference of which moves with the rotor. The transformation is based on the so-called *two-axis theory*, which was pioneered by Blondel, Doherty, Nickle, and Park [20, 61]. The transformed equations are linear provided that the speed is assumed to be constant.

In this chapter, the voltage equation of a synchronous machine is first established. Reference frame theory is then used to establish the machine equations with the stator variables transformed to a reference frame fixed in the rotor (Park's equations). The Park's equations are solved numerically during balanced three-phase short circuit. If the speed deviation is taken into account, transformed equations become nonlinear and must be solved by numerical integration. In *MATLAB*, the nonlinear differential equations of the synchronous machine in matrix form can be simulated with ease. Also, there is the additional advantage that the original voltage equations can be used without the need for any transformations. In particular, the numerical solution is obtained for the line-to-line and the line-to-ground short circuits using direct-phase quantities.

Another objective of this chapter is to develop simple network models of the synchronous generator for the power system fault analysis and transient stability studies. For this purpose, the generator behavior is divided into three periods: the *subtransient period*, lasting only for the first few cycles; the *transient period* covering a relatively longer time; and, finally, the *steady state period*. Thus, the generator equivalent circuits during transient state are obtained.

8.2 TRANSIENT PHENOMENA

To better understand the synchronous machine transient phenomena, we first study the transient behavior of a simple RL circuit. Consider a sinusoidal voltage $v(t) = V_m \sin(\omega t + \alpha)$ applied to a simple RL circuit at time $t = 0$, as shown in Figure 8.1.

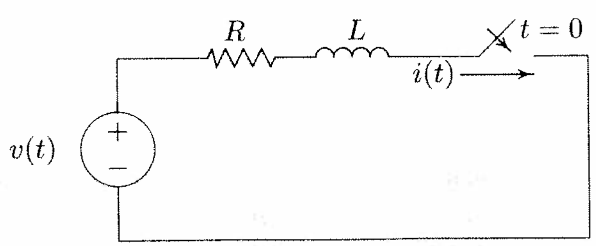


FIGURE 8.1
A simple series circuit with constant R and L.

The circuit consists of R in series with a constant L. The instantaneous voltage equation for the circuit is

$$Ri(t) + L\frac{di(t)}{dt} = V_m \sin(\omega t + \alpha) \tag{8.1}$$

The solution for the current may be shown to be

$$i(t) = I_m \sin(\omega t + \alpha - \gamma) - I_m e^{-t/\tau} \sin(\alpha - \gamma) \tag{8.2}$$

where $I_m = V_m/Z$, $\tau = L/R$, $\gamma = \tan^{-1}\omega L/R$, and $Z = \sqrt{R^2 + X^2}$. The first term is the steady state sinusoidal component. The second term is a dc transient component known as *dc offset* which decays exponentially. The dc and sinusoidal components are equal and opposite when $t = 0$, so that the condition for zero initial current is satisfied. The magnitude of the dc component depends on the instant of application of the voltage to the circuit, as defined by the angle α. The dc component is zero when $(\alpha = \gamma)$. This current waveform is shown in Figure 8.2(a). Similarly, the dc component will have a maximum initial value of V_m/Z which is the peak value of the alternating component, if the circuit is closed when $\alpha = \gamma - \pi/2$ radians. The current waveform with maximum dc offset is shown in Figure 8.2(b). If $\omega L \gg R$, then $\gamma \simeq \pi/2$, so that circuit closure at voltage maximum would give no dc component, and closure at voltage zero would cause the maximum dc transient current to flow.

Example 8.1 (chp8ex1)

In the circuit of Figure 8.1, let $R = 0.125\ \Omega$, $L = 10$ mH, and the source voltage be given by $v(t) = 151\sin(377t + \alpha)$. Determine the current response after closing the switch for the following cases.
(a) No dc offset.
(b) For maximum dc offset.

$$Z = 0.125 + j(377)(0.01) = 0.125 + j3.77 = 3.772\angle 88.1^\circ$$

$$I_m = \frac{151}{3.772} = 40 \text{ A}$$

and

$$\tau = \frac{L}{R} = 0.08 \text{ sec}$$

From (8.2) the response is

$$i(t) = 40\sin(\omega t + \alpha - 88.1^\circ) - 40e^{-t/0.08}\sin(\alpha - 88.1^\circ)$$

The response has no dc offset if switch is closed when $\alpha = 88.1^\circ$, and it has the maximum dc offset when $\alpha = 88.1^\circ - 90^\circ = -1.9^\circ$. The following commands produce the responses shown in Figures 8.2(a) and 8.2(b).

```
alf1 = 88.1*pi/180;
alf2 = -1.9*pi/180;
gamma = 88.1*pi/180;
t = 0:.001:.3;
i1 = 40*sin(377*t+alf1-gamma)-40*exp(-t/.08).*sin(alf1-gamma);
i2 = 40*sin(377*t+alf2-gamma)-40*exp(-t/.08).*sin(alf2-gamma);
subplot(2,1,1), plot(t, i1)
xlabel('t, sec'), ylabel('i(t)')
subplot(2,1,2), plot(t, i2)
xlabel('t, sec'), ylabel('i(t)')
subplot(111)
```

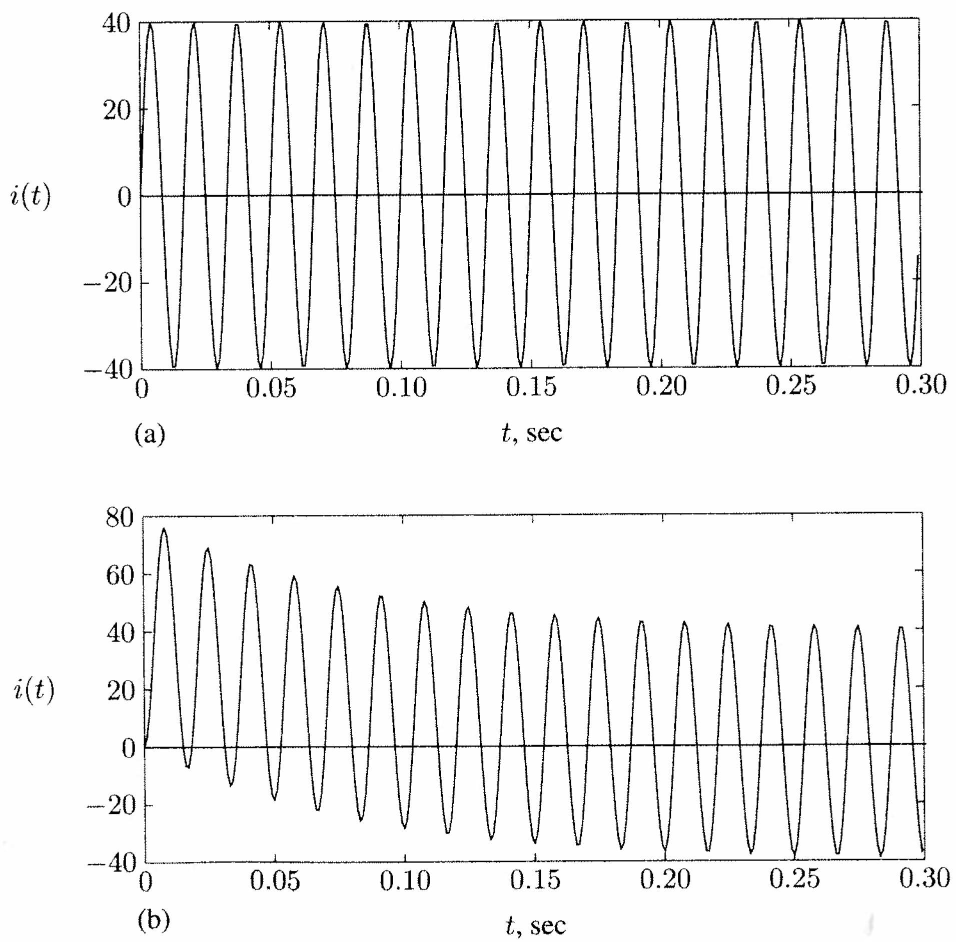


FIGURE 8.2
Current waveform, (a) with no dc offset, (b) with maximum dc offset.

8.3 SYNCHRONOUS MACHINE TRANSIENTS

The synchronous machine consists of three stator windings mounted on the stator, and one field winding mounted on the rotor. Two additional fictitious windings could be added to the rotor, one along the direct axis and one along the quadrature axis, which model the short-circuited paths of the damper windings. These windings are shown schematically in Figure 8.3.

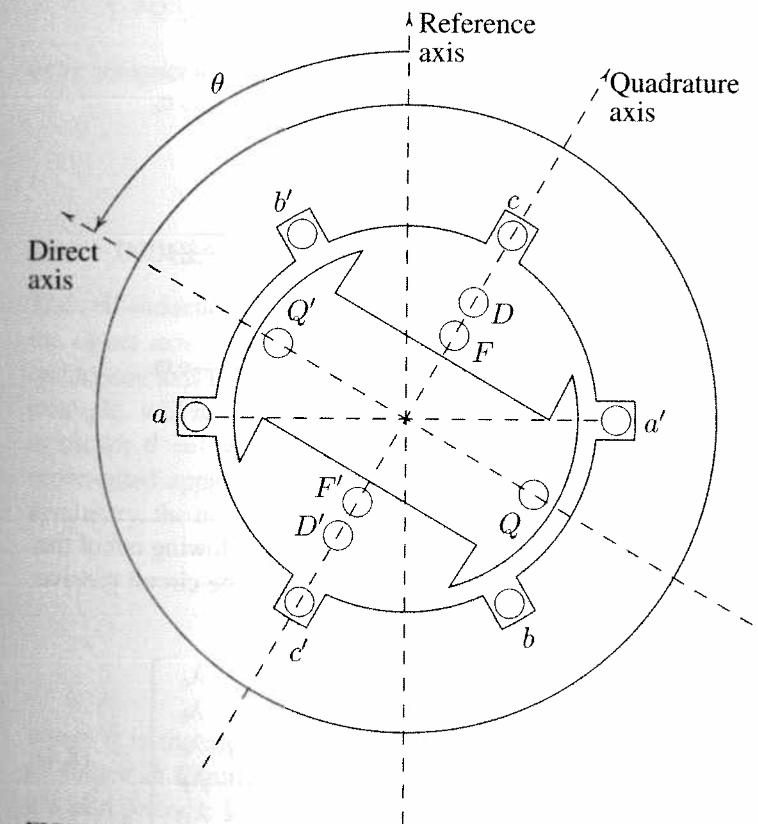


FIGURE 8.3
Schematic representation of a synchronous machine.

We shall assume a synchronously rotating reference frame (axis) rotating with the synchronous speed ω which will be along the axis of phase a at $t = 0$. If θ is the angle by which rotor direct axis is ahead of the magnetic axis of phase a, then

$$\theta = \omega t + \delta + \frac{\pi}{2} \tag{8.3}$$

where δ is the displacement of the quadrature axis from the synchronously rotating reference axis and $(\delta + \frac{\pi}{2})$ is the displacement of the direct axis.

In the classical method, the idealized synchronous machine is represented as a group of magnetically coupled circuits with inductances which depend on the angular position of the rotor. In addition, saturation is neglected and spatial distribution of armature mmf is assumed sinusoidal. The circuits are shown schematically in Figure 8.4.

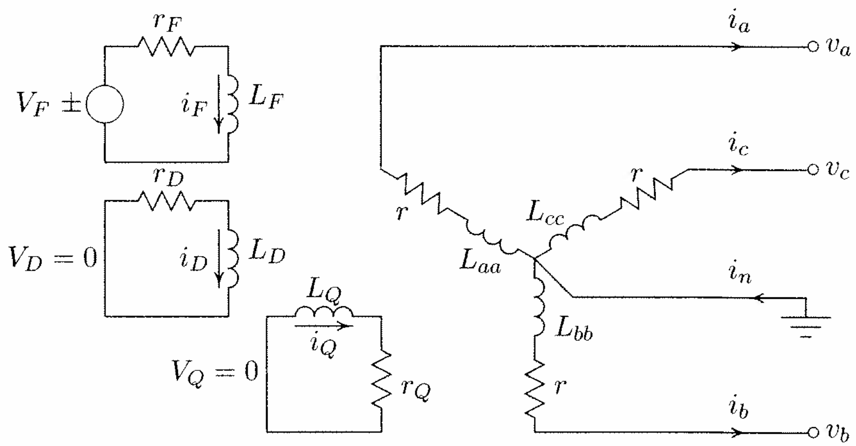


FIGURE 8.4
Schematic representation of mutually coupled circuits.

The stator currents are assumed to have a positive direction flowing out of the machine terminals. Since the machine is a generator, following the circuit passive sign convention, the voltage equation becomes

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ -v_F \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} \quad (8.4)$$

The above equation may be written in partitioned form as

$$\begin{bmatrix} \mathbf{v}_{abc} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} \quad (8.5)$$

where

$$\mathbf{v}_{FDQ} = \begin{bmatrix} -v_F \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{i}_{FDQ} = \begin{bmatrix} i_F \\ i_D \\ i_Q \end{bmatrix} \quad \lambda_{FDQ} = \begin{bmatrix} \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} \quad \text{etc.} \quad (8.6)$$

The flux linkages are functions of self- and mutual inductances given by

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} \quad (8.7)$$

or in compact form we have

$$\begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} \quad (8.8)$$

8.3.1 INDUCTANCES OF SALIENT-POLE MACHINES

The self-inductance of any stator coil varies periodically from a maximum (when the direct axis coincides with the coil magnetic axis) to a minimum (when the quadrature axis is in line with the coil magnetic axis). The self-inductance L_{aa}, for example, will be a maximum for $\theta = 0$, a minimum for $\theta = 90°$ and maximum, again for $\theta = 180°$, and so on. That is, L_{aa} has a period of 180° and can be represented approximately by cosines of second harmonics. Because of the rotor symmetry, the diagonal elements of the submatrix $\mathbf{L}_{SS}$ are represented as

$$\begin{aligned} L_{aa} &= L_s + L_m \cos 2\theta \\ L_{bb} &= L_s + L_m \cos 2(\theta - 2\pi/3) \\ L_{cc} &= L_s + L_m \cos 2(\theta + 2\pi/3) \end{aligned} \quad (8.9)$$

where θ is the angle between the direct axis and the magnetic axis of phase a, as shown in Figure 8.3. The mutual inductances between any two stator phases are also periodic functions of rotor angular position because of the rotor saliency. We can conclude from the symmetry considerations that the mutual inductance between phase a and b should have a negative maximum when the pole axis is lined up 30° behind phase a or 30° ahead of phase b, and a negative minimum when it is midway between the two phases. Thus, the variations of stator mutual inductances, i.e., the off-diagonal elements of the submatrix $\mathbf{L}_{SS}$ can be represented as follows.

$$\begin{aligned} L_{ab} = L_{ba} &= -M_s - L_m \cos 2(\theta + \pi/6) \\ L_{bc} = L_{cb} &= -M_s - L_m \cos 2(\theta - \pi/2) \\ L_{ca} = L_{ac} &= -M_s - L_m \cos 2(\theta + 5\pi/6) \end{aligned} \quad (8.10)$$

All the rotor self-inductances are constant since the effects of stator slots and saturation are neglected. They are represented with single subscript notation.

$$L_{FF} = L_F \quad L_{DD} = L_D \quad L_{QQ} = L_Q \tag{8.11}$$

The mutual inductance between any two circuits both in direct axis (or both in quadrature axis) is constant. The mutual inductance between any rotor direct axis circuit and quadrature axis circuit vanishes. Thus, we have

$$L_{FD} = L_{DF} = M_R \quad L_{FQ} = L_{QF} = 0 \quad L_{DQ} = L_{QD} = 0 \tag{8.12}$$

Finally, let us consider the mutual inductances between stator and rotor circuits, which are periodic functions of rotor angular position. Because only the space-fundamental component of the produced flux links the sinusoidally distributed stator, all stator-rotor mutual inductances vary sinusoidally, reaching a maximum when the two coils in question align. Thus, their variations can be written as follows.

$$\begin{aligned}
L_{aF} &= L_{Fa} = M_F \cos\theta \\
L_{bF} &= L_{Fb} = M_F \cos(\theta - 2\pi/3) \\
L_{cF} &= L_{Fc} = M_F \cos(\theta + 2\pi/3) \\
L_{aD} &= L_{Da} = M_D \cos\theta \\
L_{bD} &= L_{Db} = M_D \cos(\theta - 2\pi/3) \\
L_{cD} &= L_{Dc} = M_D \cos(\theta + 2\pi/3) \\
L_{aQ} &= L_{Qa} = M_Q \sin\theta \\
L_{bQ} &= L_{Qb} = M_Q \sin(\theta - 2\pi/3) \\
L_{cQ} &= L_{Qc} = M_Q \sin(\theta + 2\pi/3)
\end{aligned} \tag{8.13}$$

The resulting differential equations (8.4) describing the behavior of the machine have time-varying coefficients given by (8.9)–(8.13), and we are not able to use Laplace transforms directly to obtain a closed form of solution.

8.4 THE PARK TRANSFORMATION

A great simplification can be made by transformation of stator variables from phases a, b, and c into new variables the frame of reference of which moves with the rotor. The transformation is based on the so called *two-axis theory*, which was pioneered by Blondel, Doherty, Nickle, and Park [20, 61].

The transformed quantities are obtained from the projection of the actual variables on three axes; one along the direct axis of the rotor field winding, called the direct axis; a second along the neutral axis of the field winding, called the quadrature axis; and the third on a stationary axis. For example, the three armature currents i_a, i_b, and i_c are replaced by three fictitious currents with the symbols i_d, i_q, and i_0. They are found such that, in a balanced condition, when $i_a + i_b + i_c = 0$, they produce the same flux, at any instant, as the actual phase currents in the armature. The third fictitious current i_0 is needed to make the transformation possible when the sum of the three-phase current is not zero.

The Park transformation for currents is as follows

$$\begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} = \sqrt{2/3}\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin\theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{8.14}$$

or, in matrix notation

$$\mathbf{i}_{0dq} = \mathbf{P}\mathbf{i}_{abc} \tag{8.15}$$

Similarly for voltages and flux linkages, we have

$$\mathbf{v}_{0dq} = \mathbf{P}\mathbf{v}_{abc} \tag{8.16}$$

$$\lambda_{0dq} = \mathbf{P}\lambda_{abc} \tag{8.17}$$

The Park transformation matrix is orthogonal, i.e., $\mathbf{P}^{-1} = \mathbf{P}^T$ and thus, it is a power invariant transformation matrix. For the inverse Park transformation matrix we get

$$\mathbf{P}^{-1} = \sqrt{2/3}\begin{bmatrix} 1/\sqrt{2} & \cos\theta & \sin\theta \\ 1/\sqrt{2} & \cos(\theta - 2\pi/3) & \sin(\theta - 2\pi/3) \\ 1/\sqrt{2} & \cos(\theta + 2\pi/3) & \sin(\theta + 2\pi/3) \end{bmatrix} \tag{8.18}$$

We now wish to transform the time-varying inductances to a rotor frame of reference with the original rotor quantities unaffected. Thus, in (8.17) we augment the $\mathbf{P}$ matrix with a 3×3 identity matrix $\mathbf{U}$ to get

$$\begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix}\begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.19}$$

or

$$\begin{bmatrix} \lambda_{abc} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix}\begin{bmatrix} \lambda_{odq} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.20}$$

Substituting in (8.8), we get

$$\begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix}\begin{bmatrix} \lambda_{odq} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix}\begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix}\begin{bmatrix} \mathbf{i}_{odq} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.21}$$

or

$$\begin{bmatrix} \lambda_{odq} \\ \lambda_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{odq} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.22}$$

Substituting for $\mathbf{P}, \mathbf{P}^{-1}$ and the inductances given by (8.9)–(8.13), the above equation reduces to

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & KM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix} \tag{8.23}$$

where we have introduced the following new parameters

$$L_0 = L_s - 2M_s \tag{8.24}$$

$$L_d = L_s + M_s + \frac{3}{2}L_m \tag{8.25}$$

$$L_q = L_s + M_s - \frac{3}{2}L_m \tag{8.26}$$

and $k = \sqrt{3/2}$.

Transforming the stator-based currents ($\mathbf{i}_{abc}$) into rotor-based currents ($\mathbf{i}_{0dq}$), with rotor currents unaffected, we obtain

$$\begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.27}$$

or

$$\begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{odq} \\ \mathbf{i}_{FDQ} \end{bmatrix} \tag{8.28}$$

and similarly for voltages, we get

$$\begin{bmatrix} \mathbf{v}_{abc} \\ \mathbf{v}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{odq} \\ \mathbf{v}_{FDQ} \end{bmatrix} \tag{8.29}$$

Substituting (8.20), (8.28), and (8.29) into (8.5), we get

$$\begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.30}$$

or

$$\begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \begin{bmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.31}$$

Evaluating the first term, and obtaining the derivative of the second term in (8.31), yields

$$\begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \begin{bmatrix} \mathbf{P}\frac{d}{dt}\mathbf{P}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_{0dq} \\ \lambda_{FDQ} \end{bmatrix} \tag{8.32}$$

Next, the expression for $\mathbf{P}\frac{d}{dt}\mathbf{P}^{-1}$ can be written as

$$\mathbf{P}\frac{d}{dt}\mathbf{P}^{-1} = \mathbf{P}\frac{d\theta}{dt}\frac{d}{d\theta}\mathbf{P}^{-1} = \omega\mathbf{P}\frac{d}{d\theta}\mathbf{P}^{-1} \tag{8.33}$$

Substituting for $\mathbf{P}$ from (8.14), and for the derivative of $\mathbf{P}^{-1}$ from (8.18), we get

$$\begin{aligned} \mathbf{P}\frac{d}{dt}\mathbf{P}^{-1} &= 2/3\omega \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ \sin\theta & \sin(\theta - 2\pi/3) & \sin(\theta + 2\pi/3) \end{bmatrix} \begin{bmatrix} 0 & -\sin\theta & \cos\theta \\ 0 & -\sin(\theta - 2\pi/3) & \cos(\theta - 2\pi/3) \\ 0 & -\sin(\theta + 2\pi/3) & \cos(\theta + 2\pi/3) \end{bmatrix} \\ &= \omega \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \end{aligned} \tag{8.34}$$

Substituting (8.23) and (8.34) into (8.32), the machine equation in the rotor frame of reference becomes

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \\ -v_F \\ 0 \\ 0 \end{bmatrix} = - \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & \omega L_q & 0 & 0 & \omega k M_Q \\ 0 & -\omega L_d & r & -\omega k M_F & -\omega k M_D & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

$$-\begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & kM_D & 0 \\ 0 & 0 & L_q & 0 & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & M_R & 0 \\ 0 & kM_D & 0 & M_R & L_D & 0 \\ 0 & 0 & kM_Q & 0 & 0 & L_Q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix} \quad (8.35)$$

We now make some observations regarding the nature of the above equations. The most important one is that they have constant coefficients provided that speed is assumed constant. Also, the first equation

$$v_0 = -ri_0 - L_0 \frac{di_0}{dt}$$

is not coupled to the other equations. Therefore, it can be treated separately. The variables v_0, L_0, and i_0 are known as the *zero-sequence variables*. The name originally comes from the theory of symmetrical components, as discussed in Chapter 10. Finally, we note that while the transformation technique is a mathematical process, it provides valuable insight into internal phenomena and gives the effects of transients. Furthermore, it provides physical meaning to the new quantities.

8.5 BALANCED THREE-PHASE SHORT CIRCUIT

Consider a three-phase synchronous generator operating at synchronous speed with constant excitation. We will explore the nature of the three armature currents and the field current following a three-phase short circuit at the armature terminals. The machine is assumed to be initially unloaded, i.e.,

$$i_a(0^+) = i_b(0^+) = i_c(0^+) = 0$$

With reference to (8.15), this condition results in

$$i_0(0^+) = i_d(0^+) = i_q(0^+) = 0$$

The initial value of the field current is

$$i_F(0^+) = \frac{V_F}{r_F}$$

For balanced three-phase short circuit at the terminals of the machine

$$v_a = v_b = v_c = 0$$

With reference to (8.16), this condition results in

$$v_0 = v_d = v_q = 0$$

Since $i_0 = 0$, the machine equation in the rotor reference frame following a three-phase short circuit becomes

$$\begin{bmatrix} v_d \\ -v_F \\ 0 \\ v_q \\ 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & \omega L_q & \omega kM_Q \\ 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 \\ -\omega L_d & -\omega kM_F & -\omega kM_D & r & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix}$$
$$-\begin{bmatrix} L_d & kM_F & kM_D & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & L_q & kM_Q \\ 0 & 0 & 0 & kM_Q & L_Q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix} \quad (8.36)$$

This equation is in the state-space form and can be written in compact form as

$$\mathbf{v} = -\mathbf{R}\mathbf{i} - \mathbf{L}\frac{d}{dt}\mathbf{i} \quad (8.37)$$

or

$$\frac{d}{dt}\mathbf{i} = -\mathbf{L}^{-1}\mathbf{R}\mathbf{i} - \mathbf{L}^{-1}\mathbf{v} \quad (8.38)$$

If speed is assumed constant, the resulting state-space equation is linear and an analytical solution can be obtained by the Laplace transform technique. However, the availability of powerful simulation packages make it possible to simulate the nonlinear differential equations of the synchronous machine easily in matrix form. To consider the speed variation we need to include the dynamic equation of the machine. This is a second-order differential equation known as the *swing equation* which is described in Chapter 11. The swing equation can be expressed in the state-space form as two first-order differential equation and can easily be augmented with (8.36). Since the speed variation has very little effect in the momentary current immediately following the fault, speed variation may be neglected.

Once a solution is obtained for the direct axis and quadrature axis currents, the phase currents are obtained through the inverse Park transformation, i.e.,

$$\mathbf{i}_{abc} = \mathbf{P}^{-1}\mathbf{i}_{0dq} \quad (8.39)$$

Substituting for $\mathbf{P}^{-1}$ from (8.18), and noting $i_0 = 0$, the phase currents are

$$\begin{aligned} i_a &= i_d \cos\theta + i_q \sin\theta \\ i_b &= i_d \cos(\theta - 2\pi/3) + i_q \sin(\theta - 2\pi/3) \\ i_c &= i_d \cos(\theta + 2\pi/3) + i_q \sin(\theta + 2\pi/3) \end{aligned} \quad (8.40)$$

MATLAB provides two M-files named **ode23** and **ode45** for numerical solution of differential equations employing the Runge-Kutta-Fehlberg integration method. **ode23** uses a simple second and third order pair of formulas for medium accuracy and **ode45** uses a fourth and fifth order pair for higher accuracy. Synchronous machine simulation during balanced three-phase fault is demonstrated in the following example.

Example 8.2 (chp8ex2)

A 500-MVA, 30-kV, 60-Hz synchronous generator is operating at no-load with a constant excitation voltage of 400 V. A three-phase short circuit occurs at the armature terminals. Use **ode45** to simulate (8.36), and obtain the transient waveforms for the current in each phase and the field current. Assume the short circuit is applied at the instant when the rotor direct axis is along the magnetic axis of phase a, i.e., $\delta = 0$. Also, assume that the rotor speed remains constant at the synchronous value. The machine parameters are

Generator Parameters for Example 8.2		
$L_d = 0.0072$ H	$L_q = 0.0070$ H	$L_F = 2.500$ H
$L_D = 0.0068$ H	$L_Q = 0.0016$ H	$M_F = 0.100$ H
$M_D = 0.0054$ H	$M_Q = 0.0026$ H	$M_R = 0.125$ H
$r = 0.0020\ \Omega$	$r_F = 0.4000\ \Omega$	$r_D = 0.015\ \Omega$
$r_Q = 0.0150\ \Omega$	$L_0 = 0.0010$ H	

The dc field voltage is $V_F = 400$ V. The derivatives of the state equation given by (8.38), together with the coefficient matrices in (8.36), are defined in a function file named **symshort.m**, which returns the state derivatives. The initial value of the field current is

$$i_F(0^+) = \frac{V_F}{r_F} = \frac{400}{0.4} = 1000 \text{ A}$$

and since the machine is initially on no-load

$$i_0(0^+) = i_d(0^+) = i_q(0^0) = 0$$

The following file **chp8ex2.m** uses **ode45** to simulate the differential equations defined in **symshort** over the desired interval. The periodic nature of currents necessitates a very small step size for integration. The currents i_d and i_q are substituted in (8.40) and the phase currents are determined.

```
VF = 400; rF = 0.4; iF0 = VF/rF;
f = 60;   w=2.*pi*f;
d = 0;    d=d*pi/180;
t0 = 0 ; tfinal = 0.80;
tspan =[t0, tfinal];
i0 = [0; iF0; 0; 0; 0 ];          % Initial currents
[t,i] = ode45('symshort',tspan,i0);
theta = w*t + d + pi/2;
id = i(:,1), iq = i(:,4), iF = i(:,2);
ia = sqrt(2/3)*(id.*cos(theta) + iq.*sin(theta));
ib = sqrt(2/3)*(id.*cos(theta-2*pi/3)+ iq.*sin(theta-2*pi/3));
ic = sqrt(2/3)*(id.*cos(theta+2*pi/3)+ iq.*sin(theta+2*pi/3));
figure(1), plot(t,ia),  xlabel('Time - sec.'), ylabel('ia, A')
title(['Three-phase short circuit ia, ','delta =',num2str(d)])
figure(2), plot(t,ib),  xlabel('Time - sec.'), ylabel('ib, A')
title(['Three-phase short circuit ib, ','delta =',num2str(d)])
figure(3), plot(t,ic),  xlabel('Time - sec.'), ylabel('ic, A')
title(['Three-phase short circuit ic, ','delta =',num2str(d)])
figure(4), plot(t,iF),  xlabel('Time - sec.'), ylabel('iF, A')
title(['Three-phase short circuit iF,','delta = ',num2str(d)])
```

Results of the simulations are shown in Figure 8.5.

Armature currents in the various phases vary with time in a rather complicated way. Analysis of the waveforms show that they consist of

- A fundamental-frequency component.
- A dc component.
- A double-frequency component.

The fundamental-frequency component is symmetrical with respect to the time axis. Its superposition on the dc component will give an unsymmetrical waveform. The degree of asymmetry depends upon the point of the voltage cycle at which the short circuit takes place. The field current shown in Figure 8.5, like the stator current, consists of dc and ac components. The ac component is decaying and is comprised of a fundamental and a second harmonic. The second harmonic components in the field current as well as the armature currents are relatively small and are usually neglected. Furthermore, in Section 8.7 we see that during short circuit, the effective reactance of the machine may be assumed only along the direct axis and very simple models are obtained for power system fault studies and transient stability analysis. Before we obtain these simplified models, we consider the unbalanced short circuit of synchronous machine.

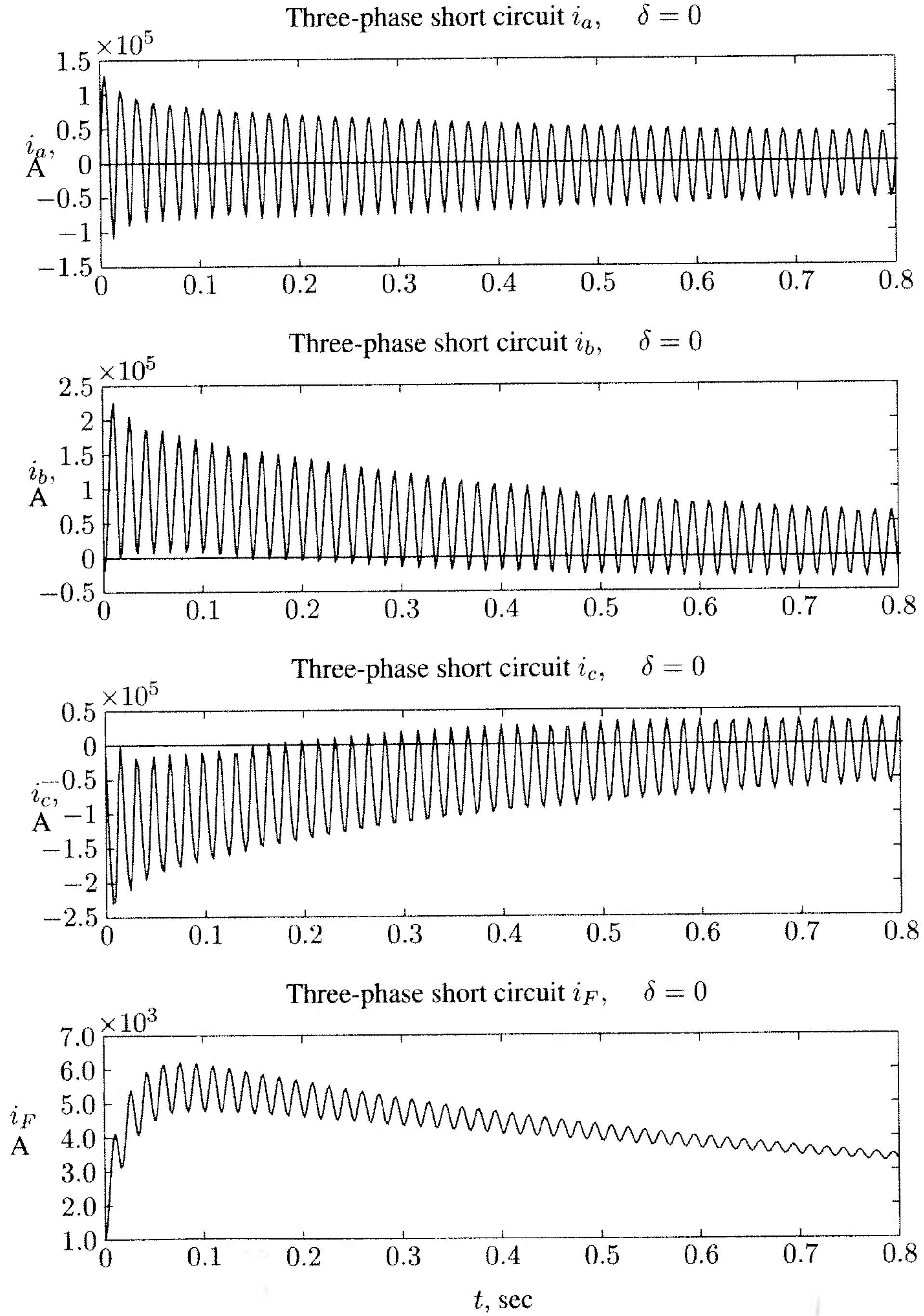


FIGURE 8.5
Balanced three-phase short-circuit current waveforms.

8.6 UNBALANCED SHORT CIRCUITS

Most frequent faults on synchronous machines are phase-to-phase and phase-to-neutral short circuits. These unbalanced faults are most difficult to analyze. The d-q-0 model is not well suited for the study of unbalanced fault and requires further transformation. The analytical solution becomes exceptionally complicated and at the end of it all the solutions are still only approximate. In the numerical solution the original voltage equations can be used without the need for any transformations. In the following section the machine equations are developed in direct-phase quantities for simulation of the synchronous machine for the line-to-line and the line-to-ground short circuits.

8.6.1 LINE-TO-LINE SHORT CIRCUIT

For a solid short circuit between phases a and b,

$$v_b = v_c = 0$$

and

$$i_b = -i_c$$

Since phase a is not involved in the short circuit and the generator is initially on no-load, $i_a = 0$, thus

$$i_0 = i_a + i_b + i_c = 0$$

and from (8.35), $v_0 = 0$. Substituting the above conditions into (8.15) and (8.16) yields

$$v_d \sin\theta - v_q \cos\theta = 0 \tag{8.41}$$

and

$$i_d = \sqrt{2}\, i_b \sin\theta \tag{8.42}$$

$$i_q = \sqrt{2}\, i_b \cos\theta \tag{8.43}$$

Derivatives of the direct axis and the quadrature axis currents are

$$\frac{di_d}{dt} = \sqrt{2}\,\frac{di_b}{dt}\sin\theta + \sqrt{2}\,\omega i_b \cos\theta \tag{8.44}$$

$$\frac{di_q}{dt} = \sqrt{2}\,\frac{di_b}{dt}\cos\theta - \sqrt{2}\,\omega i_b \sin\theta \tag{8.45}$$

Substituting (8.42)–(8.45) into (8.36) and applying (8.41) to the first and fourth equations in (8.36), the voltage equation for a line-to-line fault in direct-phase quantities is obtained.

$$\begin{bmatrix} -v_F \\ 0 \\ 0 \\ 0 \end{bmatrix} = - \begin{bmatrix} \sqrt{2}k\omega M_F \cos\theta & r_F & 0 & 0 \\ \sqrt{2}k\omega M_D \cos\theta & r_D & 0 & 0 \\ \sqrt{2}k\omega M_Q \sin\theta & 0 & 0 & r_Q \\ \sqrt{2}[r + \omega(L_d - L_q)]\sin 2\theta & k\omega M_F \cos\theta & k\omega M_D \cos\theta & k\omega M_Q \sin\theta \end{bmatrix} \begin{bmatrix} i_b \\ i_F \\ i_D \\ i_Q \end{bmatrix} - \begin{bmatrix} \sqrt{2}kM_F \sin\theta & L_F & M_R & 0 \\ \sqrt{2}kM_D \sin\theta & M_R & L_D & 0 \\ -\sqrt{2}kM_Q \cos\theta & 0 & 0 & L_Q \\ \sqrt{2}(L_d \sin^2\theta + L_q \cos^2\theta) & kM_F \sin\theta & kM_D \sin\theta & -kM_Q \cos\theta \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_b \\ i_F \\ i_D \\ i_Q \end{bmatrix} \quad (8.46)$$

This equation is in the state-space form and is written in compact form as (8.37). The state derivatives is given by (8.38), which is suitable for numerical integration.

Example 8.3 (chp8ex3)

The synchronous generator in Problem 8.2 is operating at no-load with a constant excitation voltage of 400 V. A line-to-line short circuit occurs between phases b and c at the armature terminals. Use **ode45** to simulate (8.46), and obtain the waveforms for current in phase b and the field current. Assume the short circuit is applied at the instant when the rotor direct axis is along the magnetic axis of phase a, i.e., $\delta = 0$. Also, assume that the rotor speed remains constant at the synchronous value.

The dc field voltage is $V_F = 400$ V. The derivatives of the state equation given by (8.38), together with the coefficient matrices in (8.46) are defined in a function file named **llshort.m** which returns the state derivatives. The following file **chp8ex3.m** uses **ode45** to simulate the differential equation defined in **llshort** over the desired interval. The current in phase b and the field current are determined and their plots are shown in Figure 8.6.

```
VF = 400; rF = 0.4; iF0 = VF/rF;
f = 60;    w = 2.*pi*f;
d = 0;     d = d*pi/180;
t0 = 0 ; tfinal = 0.80;
tspan = [t0, tfinal];
i0 = [0; iF0; 0; 0;];            % Initial currents
[t,i] = ode45('llshort', tspan, i0);
ib=i(:,1);  iF=i(:,2);
figure(1), plot(t,ib), xlabel('t, sec'), ylabel('ib, A')
title(['Line-line short circuit ib, ','delta = ', num2str(d)])
figure(2), plot(t,iF), xlabel('t, sec.'),ylabel('iF, A')
title(['Line-line short circuit iF,  ','delta = ',num2str(d)])
```

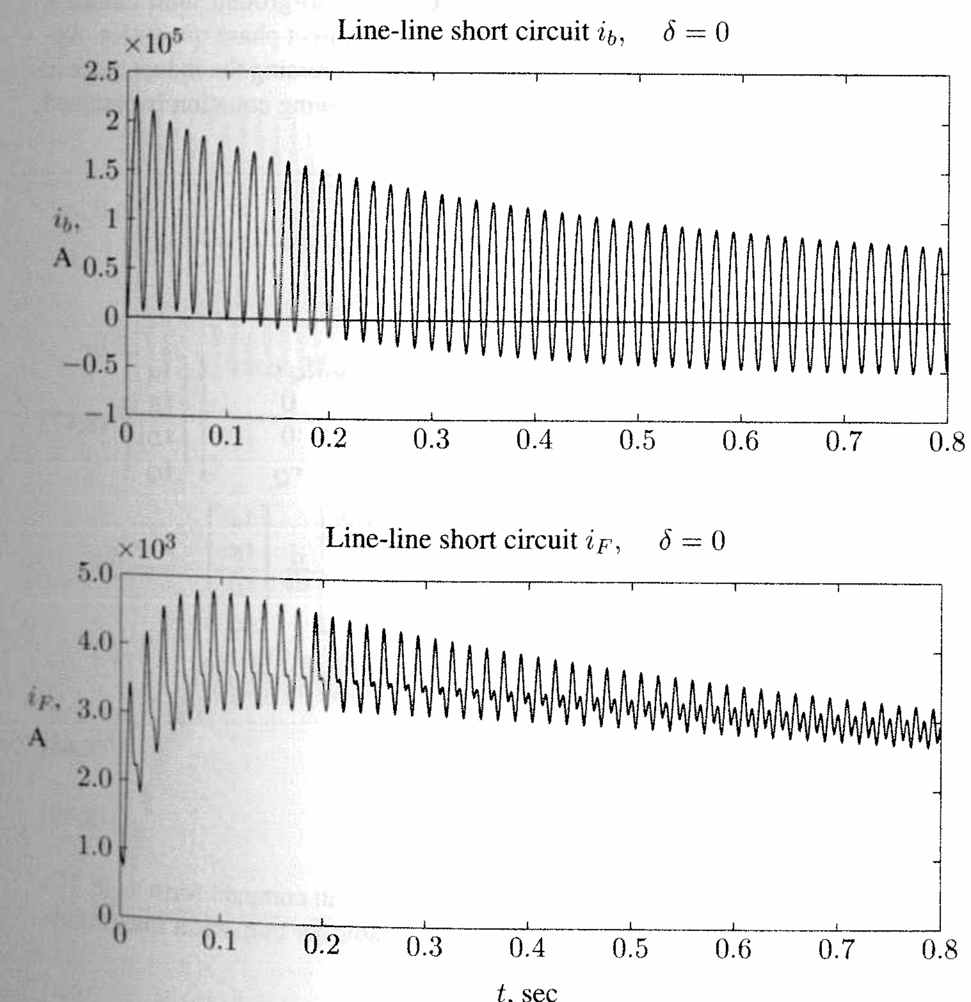


FIGURE 8.6
Line-to-line short-circuit current waveforms.

8.6.2 LINE-TO-GROUND SHORT CIRCUIT

For a solid short circuit between phases a and ground

$$v_a = 0$$

and with the machine initially on no-load

$$i_b = i_c = 0$$

A convenient way to obtain the voltage equation for line-to-ground short circuit is to start with (8.4), i.e., the machine voltage equation in direct phase quantities. Applying the short circuit condition to this equation and expressing the inductances in terms of the more commonly d-q-0 reactances, the following equation is obtained for the line-to-ground fault on phase a.

$$\begin{bmatrix} 0 \\ -v_F \\ 0 \\ 0 \end{bmatrix} = - \begin{bmatrix} r - 2\omega L_m \sin 2\theta & -\omega M_F \sin\theta & -\omega M_D \sin\theta & \omega M_Q \cos\theta \\ -\omega M_F \sin\theta & r_F & 0 & 0 \\ -\omega M_D \sin\theta & 0 & r_D & 0 \\ \omega M_Q \cos\theta & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_F \\ i_D \\ i_Q \end{bmatrix} - \begin{bmatrix} L_s + L_m \cos 2\theta & M_F \cos\theta & M_D \cos\theta & M_Q \sin\theta \\ M_F \cos\theta & L_F & M_R & 0 \\ M_D \cos\theta & M_R & L_D & 0 \\ M_Q \sin\theta & 0 & 0 & L_Q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_F \\ i_D \\ i_Q \end{bmatrix} \quad (8.47)$$

where

$$L_s = \frac{1}{3}(L_0 + L_d + L_q) \quad (8.48)$$

$$L_m = \frac{1}{3}(L_d - L_q) \quad (8.49)$$

Equation (8.47) is in the state-space form and is written in compact form as (8.37). The state derivatives is given by (8.38) which is suitable for numerical integration.

Example 8.4 (chp8ex4)

The synchronous generator in Problem 8.2 is operating at no-load with a constant excitation voltage of 400 V. A line-to-ground short circuit occurs on phase a of the armature terminals. Use **ode45** to simulate (8.47), and obtain the waveforms for the current in phase a and the field current. Assume the short circuit is applied at the instant when the rotor direct axis is along the magnetic axis of phase a, i.e., $\delta = 0$. Also, assume that the rotor speed remains constant at the synchronous value.

The dc field voltage is $V_F = 400$ V. The derivative of the state equation given by (8.38), together with the coefficient matrices in (8.47), are defined in a function file named **lgshort.m** which returns the state derivatives. The following file **chp8ex4.m** uses **ode45** to simulate the differential equation defined in **lgshort** over the desired interval. The phase and the field currents are determined and their plots are shown in Figure 8.7.

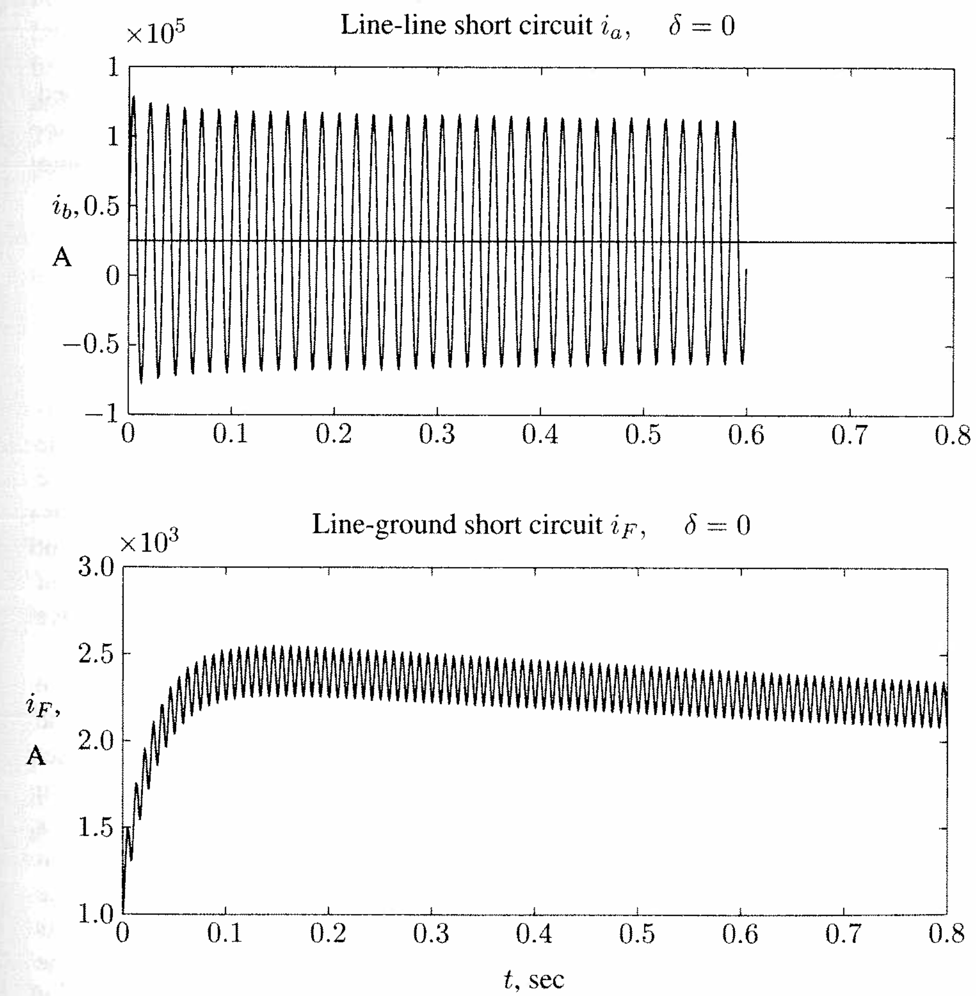


FIGURE 8.7
Line-to-ground short-circuit current waveforms.

```
VF = 400; rF = 0.4; iF0 = VF/rF;
f = 60;     w = 2.*pi*f;
d = 0;      d = d*pi/180;
t0 = 0 ; tfinal = 0.80; tspan = [t0, tfinal];
i0 = [0; iF0; 0; 0;];             % Initial currents
tol =  0.0001;                    % accuracy
[t,i] = ode45('lgshort', tspan, i0, tol);
ia=i(:,1);  iF=i(:,2);
figure(1), plot(t,ia), xlabel('t, sec'), ylabel('ia, A')
title(['Line-ground short circuit ia,','delta =', num2str(d)])
figure(2), plot(t,iF), xlabel('t, sec'), ylabel('iF, A')
title(['Line-ground short circuit iF,','delta = ',num2str(d)])
```

A three-phase model, which uses direct physical parameters, is well suited for simulation on a computer, and it is not necessary to go through complex transformations. The analysis can easily be extended to take the speed variation into account by including the dynamic equations of the machine.

8.7 SIMPLIFIED MODELS OF SYNCHRONOUS MACHINES FOR TRANSIENT ANALYSES

In Chapter 3, for steady-state operation, the generator was represented with a constant emf behind a synchronous reactance X_s. For salient-pole rotor, because of the nonuniformity of the air gap, the generator was modeled with direct axis reactance X_d and the quadrature axis reactance X_q. However, under short circuit conditions, the circuit reactance is much greater than the resistance. Thus, the stator current lags nearly $\pi/2$ radians behind the driving voltage, and the armature reaction mmf is centered almost on the direct axis. Therefore, during short circuit, the effective reactance of the machine may be assumed only along the direct axis.

The three-phase short circuit waveform shown in Figure 8.5 shows that the ac component of the armature current decays from a very high initial value to the steady state value. This is because the machine reactance changes due to the effect of the armature reaction. At the instant prior to short circuit, there is some flux on the direct axis linking both stator and rotor, due only to rotor mmf if the machine is on open circuit, or due to the resultant of rotor and stator mmf if some stator current is flowing. When there is a sudden increase of stator current on short circuit, the flux linking stator and rotor cannot change instantaneously due to eddy currents flowing in the rotor and damper circuits, which oppose this change. Since, the stator mmf is unable at first to establish any armature reaction, the reactance of armature reaction is negligible, and the initial reactance is very small and similar in value to the leakage reactance. As the eddy current in the damper circuit and eventually in the field circuit decays, the armature reaction is fully established. The armature reaction which is produced by a nearly zero power factor current provides mostly demagnetizing effect and the machine reactance increases to the direct axis synchronous reactance.

For purely qualitative purposes, a useful picture can be obtained by thinking of the field and damper windings as the secondaries of a transformer whose primary is the armature winding. During normal steady state conditions, there is no transformer action between stator and rotor windings of the synchronous machine as the resultant field produced by the stator and rotor both revolve with the same synchronous speed. This is similar to a transformer with open-circuited secondaries. For this condition, its primary may be described by the synchronous reactance X_d. During disturbance, the rotor speed is no longer the same as that of the revolving field produced by stator windings resulting in the transformer action. Thus, field and damper circuits resemble much more nearly as short-circuited secondaries. The equivalent circuit for this condition, referred to the stator side, is shown in Figure 8.8. Ignoring winding resistances, the equivalent reactance of Figure 8.8,

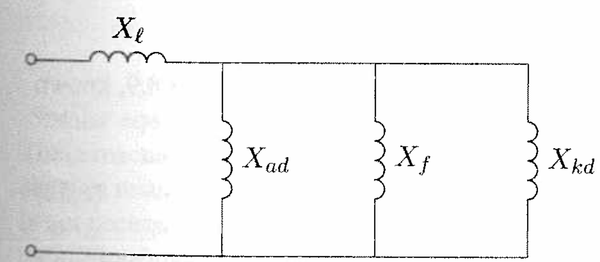


FIGURE 8.8
Equivalent circuit for the subtransient period.

known as the *direct axis subtransient reactance*, is

$$X''_d = X_\ell + (\frac{1}{X_{ad}} + \frac{1}{X_f} + \frac{1}{X_{kd}})^{-1} \tag{8.50}$$

If the damper winding resistance R_k is inserted in Figure 8.8 and the Thévenin's inductance seen at the terminals of R_k is obtained, the circuit time constant, known as the *direct axis short circuit subtransient time constant*, becomes

$$\tau''_d = \frac{X_{kd} + (\frac{1}{X_\ell} + \frac{1}{X_f} + \frac{1}{X_{ad}})^{-1}}{R_k} \tag{8.51}$$

In (8.51) reactances are assumed in per unit and, therefore, they have the same numerical values as inductances in per unit. For a 2-pole, turbo-alternators X''_d may be between 0.07 and 0.12 per unit, while for water-wheel alternators the range may be 0.1 to 0.35 per unit. The direct axis subtransient reactance X''_d is only used in calculations if the effect of the initial current is important, as for example, when determining the circuit breaker short-circuit rating.

Typically, the damper circuit has relatively high resistance and the direct axis short circuit subtransient time constant is very small, around 0.035 second. Thus, this component of current decays quickly. It is then permissible to ignore the branch of the equivalent circuit which takes account of the damper windings, and the equivalent circuit reduces to that of Figure 8.9.

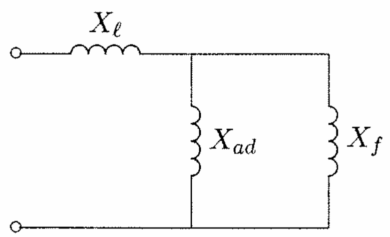


FIGURE 8.9
Equivalent circuit for the transient period.

Ignoring winding resistances, the equivalent reactance of Figure 8.9, known as the *direct axis short circuit transient reactance*, is

$$X'_d = X_\ell + \left(\frac{1}{X_{ad}} + \frac{1}{X_f}\right)^{-1} \tag{8.52}$$

If the field winding resistance R_f is inserted in Figure 8.9, and the Thévenin's inductance seen at the terminals of R_f is obtained, the circuit time constant, known as the *direct axis short circuit transient time constant*, becomes

$$\tau'_d = \frac{X_f + (\frac{1}{X_\ell} + \frac{1}{X_{ad}})^{-1}}{R_f} \tag{8.53}$$

The direct axis transient short circuit reactance X'_d may lie between 0.10 to 0.25 per unit. The short circuit transient time constant τ'_d is usually in order of 1 to 2 seconds.

The field time constant which characterizes the decay of transients with the armature open-circuited is called the *direct axis open circuit transient time constant*. This is given by

$$\tau'_{d0} = \frac{X_f}{R_f} \tag{8.54}$$

Typical values of the direct axis open circuit transient time constant are about 5 seconds. τ'_d is related to τ'_{d0} by

$$\tau'_d = \frac{X'_d}{X_d}\tau'_{d0} \tag{8.55}$$

Finally, when the disturbance is altogether over, there will be no hunting of the rotor, and, hence there will not be any transformer action between the stator and the rotor, and the circuit reduces to that of Figure 8.10.

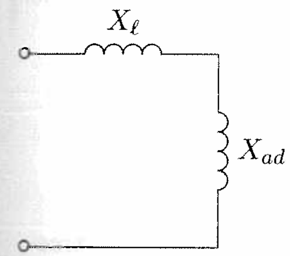


FIGURE 8.10
Equivalent circuit for the steady state.

The equivalent reactance becomes the direct axis synchronous reactance, given by

$$X_d = X_\ell + X_{ad} \tag{8.56}$$

Similar equivalent circuits are obtained for reactances along the quadrature axis. These reactances X''_q, X'_q, and X_q may be considered for cases when the circuit resistance results in a power factor appreciably above zero and the armature reaction is not necessarily totally on the direct axis.

The fundamental-frequency component of armature current following the sudden application of a short circuit to the armature of an initially unloaded machine can be expressed as

$$i_{ac}(t) = \sqrt{2}E_0\left[\left(\frac{1}{X''_d} - \frac{1}{X'_d}\right)e^{-t/\tau''_d} + \left(\frac{1}{X'_d} - \frac{1}{X_d}\right)e^{-t/\tau'_d} + \frac{1}{X_d}\right]\sin(\omega t + \delta) \tag{8.57}$$

A typical symmetric trace of the short circuit waveform obtained for the data of Example 8.5 is shown in Figure 8.11 (page 340).

It should be recalled that in the derivation of the above results, resistance was neglected except in consideration of the time constant. In addition, in the above treatment the dc and the second harmonic components corresponding to the decay of the trapped armature flux has been neglected. It should also be emphasized that the representation of the short-circuited paths of the damper windings and the solid iron rotor by a single equivalent damper circuit is an approximation to the actual situation. However, this approximation has been found to be quite valid in many cases. The synchronous machine reactances and time constants are provided by the manufacturers. These values can be obtained by a short circuit test described in the next section.

Example 8.5 (chp8ex5)

A three-phase, 60-Hz synchronous machine is driven at constant synchronous speed by a prime mover. The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 per unit). The machine has the following per unit reactances and time constants.

$$X_d'' = 0.15 \text{ pu} \qquad \tau_d'' = 0.035 \text{ sec}$$
$$X_d' = 0.40 \text{ pu} \qquad \tau_d' = 1.0 \text{ sec}$$
$$X_d = 1.20 \text{ pu}$$

a) Determine the steady state, transient and subtransient short circuit currents.
b) Obtain the fundamental-frequency waveform of the armature current for a three-phase short circuit at the terminals of the generator. Assume the short circuit is applied at the instant when the rotor direct axis is along the magnetic axis of phase a, i.e., $\delta = 0$.

$$I_d = \frac{E_0}{X_d} = \frac{1.0}{1.2} = 0.8333 \text{ pu}$$
$$I_d' = \frac{E_0}{X_d'} = \frac{1.0}{0.4} = 2.5 \text{ pu}$$
$$I_d'' = \frac{E_0}{X_d''} = \frac{1.0}{0.15} = 6.666 \text{ pu}$$

To obtain the short circuit waveform, we write the following commands.

```
w0 = 2*pi*60;
E0 = 1.0;        delta = 0;
Xd2dash = 0.15;
Xddash = 0.4;
Xd = 1.2;
tau2dash = 0.035; taudash = 1.0;
t=0:1/(4*240):1.0;
iac = sqrt(2)*E0*((1/Xd2dash-1/Xddash)*exp(-t/tau2dash)+...
(1/Xddash-1/Xd)*exp(-t/taudash) + 1/Xd).*sin(w0*t + delta);
plot(t, iac), xlabel('t, sec'), ylabel('iac, A')
end
```

The result is shown in Figure 8.11.

The trace is obtained up to 1 second. If the simulation period is extended to about $5\tau_d' = 5.0$ seconds, the short circuit will reach to its steady state with a peak value of $I_{d(max)} = \sqrt{2}\,E_0/X_d = \sqrt{2}\,(1.0)/1.2 = 1.1785$ per unit.

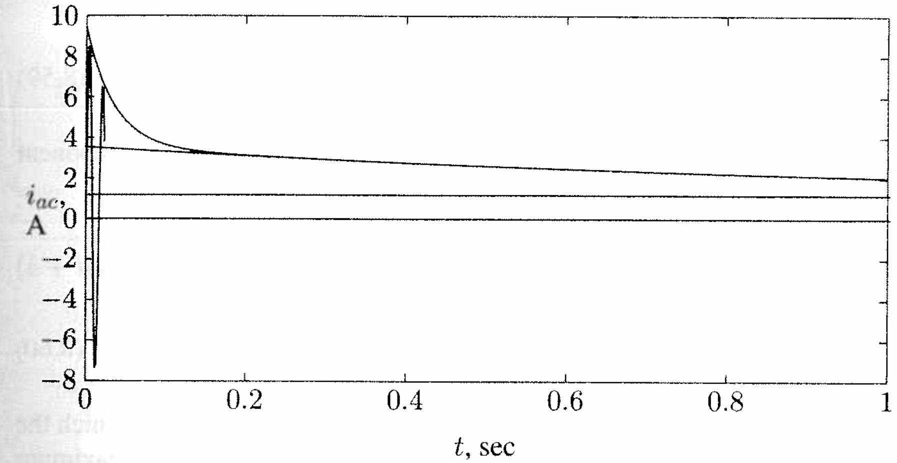


FIGURE 8.11
The 60-Hz component of the short-circuit current of a synchronous generator.

8.8 DC COMPONENTS OF STATOR CURRENTS

In the expression for the armature current as given by (8.57), the unidirectional transient component has not been taken into account. As seen from consideration of the simple RL circuit of Figure 8.1, there will in general be a dc offset depending on when the voltage is applied. Similarly, in the synchronous machine, the dc offset component depend on the instantaneous value of the stator voltage at the time of the short circuit. The rotor position is given by $\theta = \omega t + \delta + \pi/2$. The dc component depends on the rotor position δ when the short circuit occurs at time $t = 0$. The time constant associated with the decay of the dc component of the stator current is known as the *armature short circuit time constant*, τ_a. Most of the decay of the dc component occurs during the subtransient period. For this reason the average value of the direct axis and quadrature axis subtransient reactances is used for finding τ_a. It is approximately given by

$$\tau_a = \frac{X_d'' + X_q''}{2R_a} \tag{8.58}$$

Typical values of the armature short circuit time constant is around 0.05 to 0.17 second.

Since the three-phase voltages are each separated by $2\pi/3$ radians, the amount of the dc component of the armature current is different in each phase and depends upon the point of the voltage cycle at which the short circuit occurs. The dc component for phase a is given by

$$I_{dc} = \sqrt{2}\frac{E_0}{X_d''}\sin\delta\, e^{-t/\tau_a} \tag{8.59}$$

The superposition of the dc component on the fundamental frequency component will give an asymmetrical waveform.

$$i_{asy}(t) = \sqrt{2}E_0\left[\left(\frac{1}{X_d''} - \frac{1}{X_d'}\right)e^{-t/\tau_d''} + \left(\frac{1}{X_d'} - \frac{1}{X_d}\right)e^{-t/\tau_d'} + \frac{1}{X_d}\right]\sin(\omega t + \delta)$$
$$+\sqrt{2}\frac{E_0}{X_d''}\sin\delta e^{-t/\tau_a} \tag{8.60}$$

The degree of asymmetry depends upon the point of the voltage cycle at which the fault takes place. The worst possible transient condition is $\delta = \pi/2$. The maximum possible initial magnitude of the dc component is

$$I_{dc_{(max)}} = \sqrt{2}\,\frac{E_0}{X_d''} \tag{8.61}$$

Therefore, the maximum rms current (ac plus dc) at the beginning of the short circuit is

$$I_{asy} = \sqrt{I_d''^2 + I_{dc}^2} = \sqrt{\left(\frac{E_0}{X_d''}\right)^2 + \left(\sqrt{2}\frac{E_0}{X_d''}\right)^2} \tag{8.62}$$

from which

$$I_{asy} = \sqrt{3}\,\frac{E_0}{X_d''} = \sqrt{3}I_d'' \tag{8.63}$$

In practice, the *momentary duty* of a circuit breaker is given in terms of the asymmetrical short circuit current.

Example 8.6 (chp8ex6)

For the machine in Example 8.5, assume that a three-phase short circuit is applied at the instant when the rotor quadrature axis is along the magnetic axis of phase a, i.e., $\delta = \pi/2$ radians. Obtain the asymmetrical waveform of the armature current for phase a. The armature short circuit time constant is $\tau_a = 0.15$ sec.

In the *MATLAB* program of Example 8.5, we make $\delta = \pi/2$ and use (8.60) in place of the i_{ac} statement. Running this example results in the waveform shown in Figure 8.12.

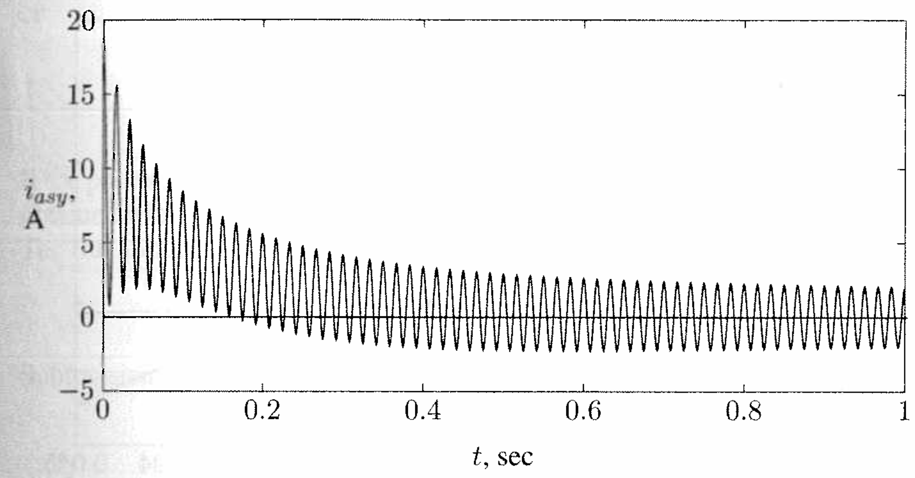


FIGURE 8.12
Synchronous generator asymmetrical short-circuit current $\delta = \pi/2$.

8.9 DETERMINATION OF TRANSIENT CONSTANTS

A sudden three-phase short circuit is applied to the terminals of an unloaded generator and the oscillogram of the current in one phase is obtained. The test is repeated until a symmetric waveform is obtained which does not contain the dc offset. This occurs when the voltage is near maximum at the instant of fault. The machine reactances X_d'', X_d', and X_d and the time constants τ_d'' and τ_d' are determined by analyzing the oscillogram waveform as follow.

The waveform is divided into three periods: the subtransient period, lasting only for the first two cycles, during which the current decrement is very rapid; the transient period, covering a relatively longer time, during which the current decrement is more moderate; and finally, the steady state period.

The no-load generated voltage E_0 is obtained by measuring the phase voltage and expressing it in per unit. The direct axis synchronous reactance X_d is determined from the part of the oscillogram where the envelope of the current has become constant. Denoting this amplitude with $I_{d_{(max)}}$, the rms value of the steady short circuit is $I_d = I_{d_{(max)}}/\sqrt{2}$. From this the direct axis synchronous reactance is found

$$X_d = \frac{E_0}{I_d} \tag{8.64}$$

The peak steady short circuit current is subtracted from two points after approximately the 10th cycle where the subtransient component has decayed. Dividing these values by $\sqrt{2}$ results in the following term

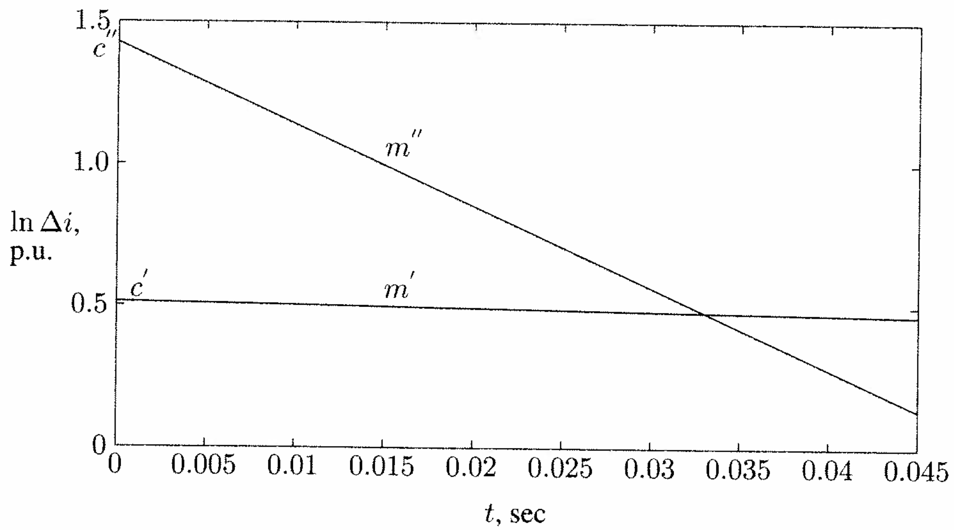


FIGURE 8.13
Current difference logarithm, $\ln \Delta i'$ and $\ln \Delta i''$.

$$\Delta i' = (I'_d - I_d)e^{-t/\tau'_d}$$

or

$$\ln \Delta i' = \ln(I'_d - I_d) - t/\tau'_d$$
$$= c' - m't$$

If the points given by $\ln \Delta i'$ are plotted against a linear time scale, a straight line is obtained with y-intercept $c' = \ln(I'_d - I_d)$ and slope $-m'$, as shown in Figure 8.13. The rms transient component of current is obtained from

$$I'_d = e^{c'} + I_d \quad (8.65)$$

Transient reactance and time constant are then obtained by

$$X'_d = \frac{E_0}{I'_d} \quad (8.66)$$

and

$$\tau'_d = \frac{1}{m'} \quad (8.67)$$

To find the subtransient components, the peak current of the first two cycles are divided by $\sqrt{2}$. Subtracting the steady short circuit current and the rms transient currents found earlier from these points results in

$$\Delta i'' = (I''_d - I'_d)e^{-t/\tau''_d}$$

or

$$\ln \Delta i'' = \ln(I''_d - I'_d) - t/\tau''_d$$
$$= c'' - m''t$$

If the points given by $\ln \Delta i''$ are plotted against a linear time scale, a straight line is obtained with y-intercept $c'' = \ln(I''_d - I'_d)$ and slope $-m''$, shown in Figure 8.13. The rms subtransient component of current is given by

$$I''_d = e^{c''} + I'_d \quad (8.68)$$

Subtransient reactance and time constant are

$$X''_d = \frac{E_0}{I''_d} \quad (8.69)$$

and

$$\tau''_d = \frac{1}{m''} \quad (8.70)$$

The above procedure is demonstrated in the following example.

Example 8.7 (chp8ex7)

A three-phase, 60-Hz synchronous machine is driven at constant synchronous speed by a prime mover. The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 per unit). The generator is suddenly subjected to a symmetrical three-phase short circuit at the instant when direct axis is along the magnetic axis of phase a, i.e., $\delta = 0$. An oscillogram of the short-circuited current is obtained. The peak values at the first two cycles, at the 20th and 21st cycles, and the steady value after a long time were recorded as tabulated in the following table.

I_{max}, pu	8.7569	6.7363	···	2.8893	2.8608	···	1.1785
Time, sec	0.0042	0.0208	···	0.3208	0.3375	···	5.0000

Determine the transient and the subtransient reactances and time constants.

The following statements are written with reference to the above procedure.

```
E0 = 1.0;
Im=[8.7569   6.7363  2.8893  2.8608   1.1785];
 t=[0.0042   0.0208  0.3208  0.3375   5.0000];
I = Im/sqrt(2);            % The rms value of the above envelope
id=I(5);                   % rms value of the steady short circuit
```

```
Dt2=[t(3) t(4)];                % Time for 20th and 21st cycles
Di2=[I(3)-id I(4)-id];%Diff. between transient envelope and id
LDi2= log(Di2);             %Natural log of the above two points
c2=polyfit(Dt2, LDi2, 1);
                        %Finds coefficients of a 1st-order polynomial
                    % i.e. the slope and intercept of a straight line
iddash=(exp(c2(2))+id)    % rms value of the transient current
Xddash=E0/iddash             % Direct axis transient reactance
taudash=abs(1/c2(1))   %Direct axis sc transient time constant
Di=(iddash-id)*[exp(-t(1)/taudash)  exp(-t(2)/taudash)];
Di1=[I(1)-Di(1)-id   I(2)-Di(2)-id];   % Subtransient envelope
LDi1=log(Di1);
Dt1 =[t(1)  t(2)];        % Natural log of the first two points
c1=polyfit(Dt1, LDi1, 1);
                        %Finds coefficients of a 1st-order polynomial
                    % i.e. the slope and intercept of a straight line

id2dash=exp(c1(2))+iddash  %rms value of  subtransient current
Xd2dash= E0/id2dash         % Direct axis subtransient reactance
tau2dash=abs(1/c1(1))% direct axis sc subtransient time const.

t=0:.005:.045;
fit2 = polyval(c2, t); % line C2 evaluated for all values of t
fit1 = polyval(c1, t); % line C1 evaluated for all values of t
plot(t, fit1, t, fit2),grid % Logarithmic plot of id'' and id'
ylabel('ln(I)  pu' )     % intercepts are ln(Id'') and ln(Id')
xlabel('t, sec')      %slopes are reciprocal of time constants
```

The result is

$$I'_d = 2.5038 \text{ pu} \qquad X'_d = 0.3994; \text{ pu} \qquad \tau'_d = 0.9941 \text{ sec}$$
$$I''_d = 6.6728 \text{ pu} \qquad X''_d = 0.1499; \text{ pu} \qquad \tau''_d = 0.0348 \text{ sec}$$

Example 8.8 (chp8ex8)

A 100-MVA, 13.8-kV, 60-Hz, Y-connected, three-phase synchronous generator is connected to a 13.8-kV/220-kV, 100-MVA, Δ-Y connected transformer. The reactances in per unit to the machine's own base are

$$X_d = 1.0 \text{ pu} \quad X'_d = 0.25 \text{ pu} \quad X''_d = 0.12 \text{ pu}$$

and its time constants are

$$\tau_a = 0.25 \text{ sec} \quad \tau''_d = 0.4 \text{ sec} \quad \tau'_d = 1.1 \text{ pu}$$

The transformer reactance is 0.20 per unit on the same base. The generator is operating at the rated voltage and no-load when a three-phase fault occurs at the secondary terminals of the transformer as shown in Figure 8.14.

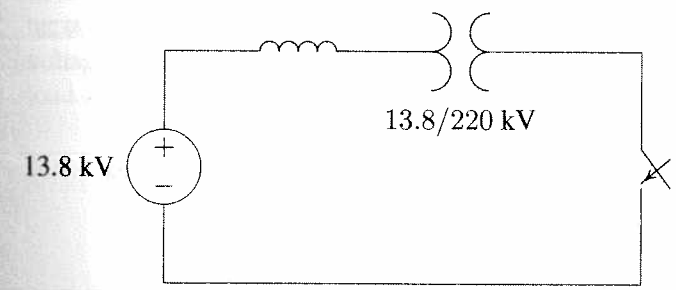


FIGURE 8.14
One-line diagram for Example 8.8.

(a) Find the subtransient, transient, and the steady state short circuit currents in per unit and actual amperes on both sides of the transformer.
(b) What is the maximum rms current (ac plus dc) at the beginning of the fault?
(c) Obtain the instantaneous expression for the short circuit current including the dc component. Assume $\delta = \pi/2$ radians.

(a) The base current on the generator side is

$$I_{B1} = \frac{S_B}{\sqrt{3}\,V_{B1}} = \frac{100 \times 10^3}{\sqrt{3}\,13.8} = 4184 \text{ A}$$

The base current on the secondary side of the transformer is

$$I_{B2} = \frac{13.8}{220}(4184) = 262.4 \text{ A}$$

the subtransient, transient and the steady state short circuit currents are

$$I''_d = \frac{1.0}{0.12 + 0.2} = 3.125 \text{ pu} = 13,075 \text{ A} \quad \text{on the generator side}$$
$$= 820 \text{ A} \quad \text{on the 220-kV side}$$
$$I'_d = \frac{1.0}{0.25 + 0.2} = 2.22 \text{ pu} = 9,288.5 \text{ A} \quad \text{on the generator side}$$
$$= 582.5 \text{ A} \quad \text{on the 220-kV side}$$
$$I_d = \frac{1.0}{1.0 + 0.2} = 0.833 \text{ pu} = 3,486.6 \text{ A} \quad \text{on the generator side}$$
$$= 218.6 \text{ A} \quad \text{on the 220-kV side}$$

(b) The maximum rms current (ac plus dc) at the beginning of the fault is

$$I_{asy} = \sqrt{3}\, I''_d = \sqrt{3}\,(3.125) = 5.4 \text{ pu} = 22,646 \text{ A} \quad \text{on the generator side}$$

(c) The instantaneous short circuit current including the dc component from (8.60), for $\delta = \pi/2$ is

$$i(t) = \sqrt{2}[(I''_d - I'_d)e^{-t/0.4} + (I'_d - I_d)e^{-t/1.1} + I_d]\sin(377t + \pi/2) + \sqrt{2}I''_d e^{-t/0.25}$$

or

$$i(t) = [1.28e^{-2.5t} + 1.96e^{-0.91t} + 1.18]\sin(377t + \pi/2) + 4.42e^{-4t} \quad \text{pu}$$

Use *MATLAB* to obtain a plot of $i(t)$.

8.10 EFFECT OF LOAD CURRENT

If the fault occurs when the generator is delivering a prefault load current, two methods might be used in the solution of three-phase symmetrical fault currents.

(a) Use of internal voltages behind reactances

When there is a prefault load current, three fictitious internal voltages E'', E', and E_0 may be considered to be effective during the subtransient, transient, and the steady state periods, respectively. These voltages are known as the *voltage behind subtransient reactance*, *voltage behind transient reactance*, and *voltage behind synchronous reactance*. Consider the one-line diagram of a loaded generator shown in Figure 8.15(a). The internal voltages shown by the phasor diagram in Figure 8.15(b) are given by

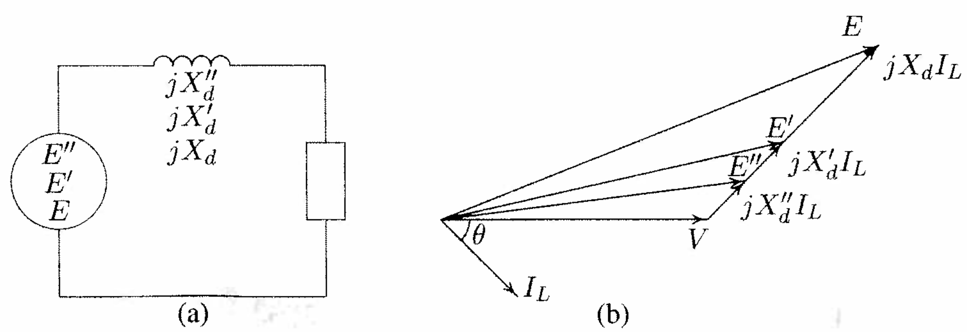


FIGURE 8.15
(a) One-line diagram of a loaded generator, (b) phasor diagram.

$$\begin{aligned} E'' &= V + jX''_d I_L \\ E' &= V + jX'_d I_L \\ E &= V + jX_d I_L \end{aligned} \tag{8.71}$$

Example 8.9 (chp8ex9)

In Example 8.8, a three-phase load of 100 MVA, 0.8 power factor lagging is connected to the transformer secondary side as shown in Figure 8.16. The line-to-line voltage at the load terminals is 220 kV. A three-phase short circuit occurs at the load terminals. Find the generator transient current including the load current.

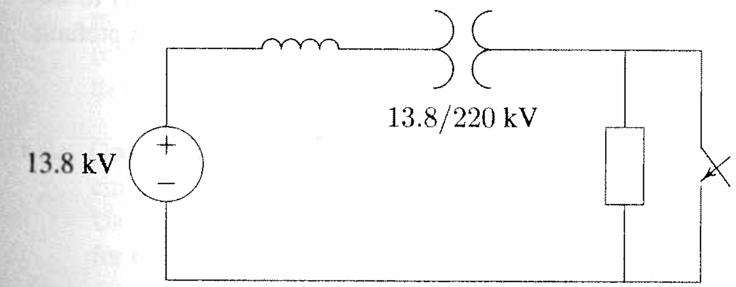


FIGURE 8.16
One-line diagram for Example 8.9.

The load may be represented by a per unit impedance as shown in Figure 8.16.

$$S_L = \frac{100\angle 36.87^\circ}{100} = 1\angle 36.87^\circ \text{ pu}$$

$$V = \frac{220}{220} = 1\angle 0^\circ \text{ pu}$$

$$Z_L = \frac{|V^2|}{S^*} = \frac{1}{1\angle -36.87^\circ} = 0.8 + j0.6 \text{ pu}$$

Before fault, the load current is

$$I_L = \frac{V}{Z_L} = \frac{1.0\angle 0^\circ}{0.8 + j0.6} = 0.8 - j0.6 = 1\angle -36.87^\circ \text{ pu}$$

The emf behind transient reactance is

$$\begin{aligned} E' &= V + j(X'_d + X_t)I_L \\ &= 1.0\angle 0^\circ + j(0.25 + 0.2)(0.8 - j0.6) = 1.27 + j0.36 = 1.32\angle 15.83^\circ \text{ pu} \end{aligned}$$

When the fault is applied by closing switch S, the generator short circuit transient current is

$$I'_g = \frac{E'}{j(X'_d + X_t)} = \frac{1.32\angle 15.83^\circ}{j(0.25 + 0.2)} = 0.8 - j2.822 = 2.93\angle -74.17^\circ \text{ pu}$$

(b) Using Thévenin's theorem and superposition with load current

The fault current is found in the absence of the load by obtaining the Thévenin's equivalent circuit to the point of fault. The total short circuit current is then given by superimposing the fault current with the load current.

Example 8.10 (chp8ex10)

Find the generator transient current in Problem 8.9 using Thévenin's method.

The one-line diagram of Example 8.10 without the load is shown in Figure 8.17(a). The circuit for the Thévenin's equivalent impedance with respect to the point of fault is shown in Figure 8.17(b). The Thévenin's voltage is the prefault

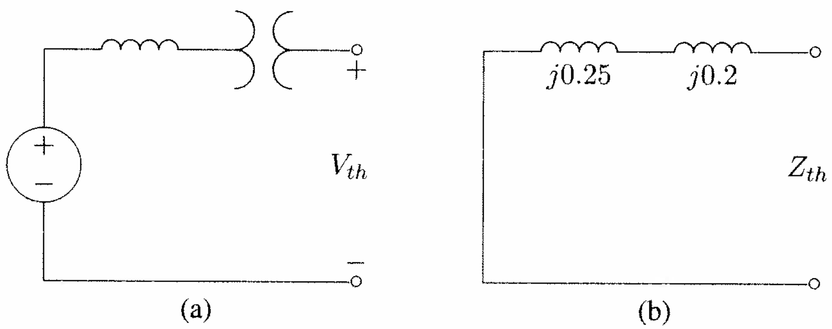


FIGURE 8.17
(a) One-line diagram for Example 8.10 without the load and (b) Thévenin's equivalent impedance to the point of fault.

terminal voltage, i.e.,

$$V_{th} = \frac{220}{220} = 1\angle 0^\circ \text{ pu}$$

and the Thévenin's impedance is

$$Z_{th} = j(0.25 + 0.2) = j0.45$$

The fault contribution is

$$I_f' = \frac{V_{th}}{Z_{th}} = \frac{1.0\angle 0^\circ}{j0.45} = -j2.222 \text{ pu}$$

Now superimposing the load current with the fault current results in

$$I_g' = I_f' + I_L = -j2.222 + (0.8 - j0.6) = 0.8 - j2.822 = 2.93\angle{-74.17^\circ} \text{ pu}$$

which checks with the result in Example 8.9.

PROBLEMS

8.1. A sinusoidal voltage given by $v(t) = 390\sin(315t + \alpha)$ is suddenly applied to a series RL circuit. $R = 32\ \Omega$ and $L = 0.4$ H.
(a) The switch is closed at such a time as to permit no transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.
(b) The switch is closed at such a time as to permit maximum transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.
(c) What is the maximum value of current in part (b) and at what time does this occur after the switch is closed?

8.2. Consider the synchronous generator in Example 8.2. A three-phase short circuit is applied at the instant when the rotor direct axis position is at $\delta = 30^\circ$. Use **ode45** to simulate (8.36), and obtain and plot the transient waveforms for the current in each phase and the field current.

8.3. Consider the synchronous generator in Example 8.2. A line-to-line short circuit occurs between phases b and c at the instant when the rotor direct axis position is at $\delta = 30^\circ$. Use **ode45** to simulate (8.46), and obtain the transient waveforms for the current in phase b and the field current.

8.4. Consider a line-to-ground short circuit between phase a and ground in a synchronous generator. Apply the short circuit conditions

$$v_a = 0$$
$$i_b = i_c = 0$$

to the voltage equation of the synchronous machine given by (8.4). Substitute for all the flux linkages in terms of the inductances given by (8.9)–(8.13) and verify Equation (8.47).

8.5. Consider the synchronous generator in Example 8.2. A line-to-ground short circuit occurs between phase a and ground at the instant when the rotor direct axis position is at $\delta = 30^\circ$. Use **ode45** to simulate (8.47), and obtain the transient waveforms for the current in phase a and the field current.

8.6. A three-phase, 60-Hz synchronous machine is driven at constant synchronous speed by a prime mover. The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 per unit). The machine has the following per unit reactances and time constants.

$X_d'' = 0.25$ pu $\quad \tau_d'' = 0.04$ sec
$X_d' = 0.45$ pu $\quad \tau_d' = 1.4$ sec
$X_d = 1.50$ pu

(a) Determine the steady state, transient, and subtransient short circuit currents.
(b) Obtain and plot the fundamental-frequency waveform of the armature current for a three-phase short circuit at the terminals of the generator. Assume the short circuit is applied at the instant when the rotor direct axis is along the magnetic axis of phase a, i.e., $\delta = 0$.

8.7. For the machine in Problem 8.6, assume that a three-phase short circuit is applied at the instant when the rotor quadrature axis is along the magnetic axis of phase a, i.e., $\delta = \pi/2$ radians. Obtain and plot the asymmetrical waveform of the armature current for phase a. The armature short circuit time constant is $\tau_a = 0.3$ sec.

8.8. A three-phase, 60-Hz synchronous machine is driven at constant synchronous speed by a prime mover. The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 per unit). The generator is suddenly subjected to a symmetrical three-phase short circuit at the instant when direct axis is along the magnetic axis of phase a, i.e., $\delta = 0$. An oscillogram of the short-circuited current is obtained. The peak values at the first two cycles, at the 20th and 21st cycles, and the steady value after a long time were recorded as tabulated in the following table.

I_{max}, pu	5.4016	4.6037	⋯	2.6930	2.6720	⋯	0.9445
Time, sec	0.0042	0.0208	⋯	0.3208	0.3375	⋯	10.004

Determine the transient and the subtransient reactances and time constants.

8.9. A 100-MVA, three-phase, 60-Hz generator driven at constant speed has the following per unit reactances and time constants

$X_d'' = 0.20$ pu $\quad \tau_d'' = 0.04$ sec
$X_d' = 0.30$ pu $\quad \tau_d' = 1.0$ sec
$X_d = 1.20$ pu $\quad \tau_a = 0.25$ sec

The armature windings are initially open-circuited and field voltage is adjusted so that the armature terminal voltage is at the rated value (i.e., 1.0 per unit). The generator is suddenly subjected to a symmetrical three-phase short circuit at the instant when $\delta = \pi/2$. Obtain and plot the asymmetrical waveform of the armature current for phase a. Determine

(a) The rms value of the ac component in phase a at $t = 0$.
(b) The dc component of the current in phase a at $t = 0$.
(c) The rms value of the asymmetrical current in phase a at $t = 0$.

8.10. A 100-MVA, 20-kV, 60-Hz three-phase synchronous generator is connected to a 100-MVA, 20/400 kV three-phase transformer. The machine has the following per unit reactances and time constants.

$X_d'' = 0.15$ pu $\quad \tau_d'' = 0.035$ sec
$X_d' = 0.25$ pu $\quad \tau_d' = 0.50$ sec
$X_d = 1.25$ pu $\quad \tau_a = 0.3$ sec

The transformer reactance is 0.25 per unit. The generator is operating at the rated voltage and no-load when a three-phase short circuit occurs at the secondary terminals of the transformer.
(a) Find the subtransient, transient, and the steady state short circuit currents in per unit and actual amperes on both sides of the transformer.
(b) What is the maximum asymmetrical rms current (ac plus dc) at the beginning of the short circuit?
(c) Obtain and plot the instantaneous expression for the short circuit current including the dc component. Assume $\delta = \pi/2$ radians.

8.11. In Problem 8.10, a three-phase load of 80-MVA, 0.8 power factor lagging is connected to the transformer secondary side. The line-to-line voltage at the load terminals is 400 kV. A three-phase short circuit occurs at the load terminals. Find the generator transient current including the load current.

8.12. A 100-MVA, 20-kV synchronous generator is connected through a transmission line to a 100-MVA, 20-kV synchronous motor. The per unit transient reactances of the generator and motor are 0.25 and 0.20, respectively. The line reactance on the base of 100 MVA is 0.1 per unit. The motor is taking 50 MW at 0.8 power factor leading at a terminal voltage of 20 kV. A three-phase short circuit occurs at the generator terminals. Determine the transient currents in each of the two machines and in the short circuit.