Chapter 4: Project Scheduling

4.1 Introduction

In chapter 3, the AOA and AON networks were presented, also the time and cost of individual activities based were calculated. Yet, however, we do not know how long is the total project duration. Also, we need to evaluate the early and late times at which activities start and finish. In addition, since real-life projects involve hundreds of activities, it is important to identify the group of critical activities so that special care is taken to make sure they are not delayed. All these statements are the basic objectives of the scheduling process, which adds a time dimension to the planning process. In other words, we can briefly state that: Scheduling = Planning + Time.

Scheduling is the determination of the timing of the activities comprising the project to enable managers to execute the project in a timely manner. The project scheduling id sued for:

- Knowing the activities timing and the project completion time.
- Having resources available on site in the correct time.
- Making correction actions if schedule shows that the plan will result in late completion.
- Assessing the value of penalties on project late completion.
- Determining the project cash flow.
- Evaluating the effect of change orders on the project completion time.
- Determining the value pf project delay and the responsible parties.

4.2 The Critical Path Method

The most widely used scheduling technique is the critical path method (CPM) for scheduling. This method calculates the minimum completion time for a project along with the possible start and finish times for the project activities. Many texts and managers regard critical path scheduling as the only usable and practical scheduling procedure. Computer programs and algorithms for critical path scheduling are widely available and can efficiently handle projects with thousands of activities.

The critical path itself represents the set or sequence of activities which will take the longest time to complete. The duration of the critical path is the sum of the activities' durations along the path. Thus, the critical path can be defined as the longest possible path through the "network" of project activities. The duration of the critical path represents the minimum time required to complete a project. Any delays along the critical path would imply that additional time would be required to complete the project.

There may be more than one critical path among all the project activities, so completion of the entire project could be delayed by delaying activities along any one of the critical paths. For example, a project consisting of two activities performed in parallel that each requires three days would have each activity critical for a completion in three days. Formally, critical path scheduling assumes that a project has been divided into activities of fixed duration and well defined predecessor relationships. A predecessor relationship implies that one activity must come before another in the schedule.

The CPM is a systematic scheduling method for a project network and involves four main steps:

- A forward path to determine activities early-start times;
- A backward path to determine activities late-finish times;
- Float calculations; and
- Identifying critical activities.

4.3 Calculations for the Critical Path Method

The inputs to network scheduling of any project are simply the AOA or the AON networks with the individual activity duration defined. The network scheduling process for AOA and AON networks, however, is different. To demonstrate these two techniques, let's consider a simple 5-activity project, with activity A at the start, followed by three parallel activities B, C, and D; which are then succeeded by activity E. The AOA or the AON networks of this example are presented in Figure 4.1. Detailed analysis of theses AOA or the AON networks are presented in the following subsections. It is noted that the example at hand involves only simple finish-to-start relationships among activities.

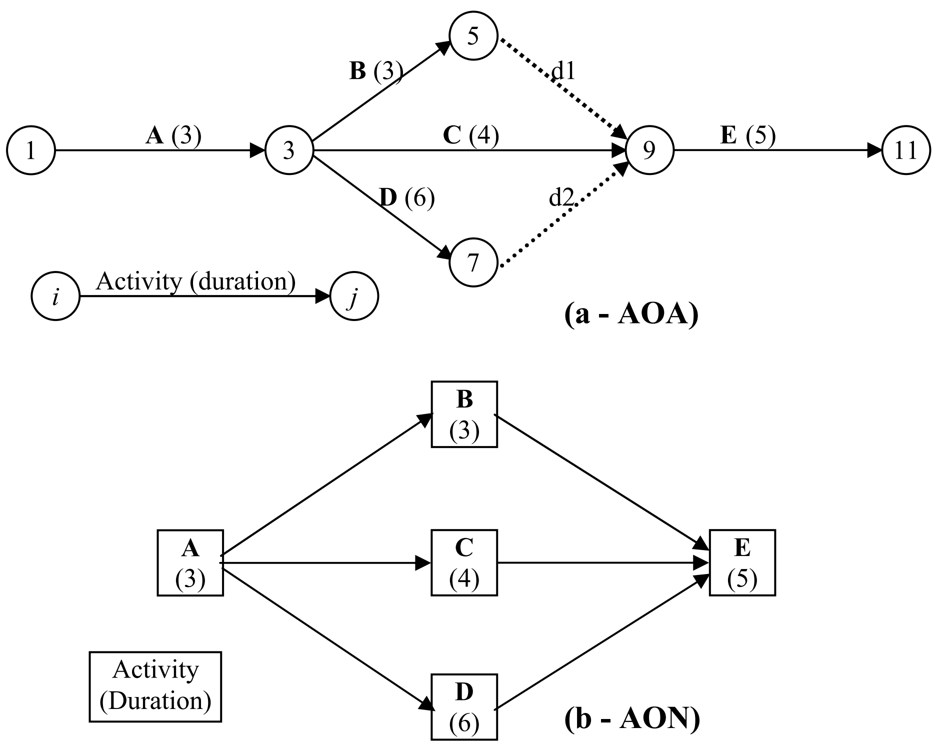


Figure 4.1: Network example

4.3.1 Activity-on-node networks calculations

The objective of arrow network analysis is to compute foe each event in the network its early and late timings. These times are defined as:

Early event time (ET) is the earliest time at which an event can occur, considering the duration of preceding activities.
Late event time (LT) is the latest time at which an event can occur if the project is to be completed on schedule.

Forward Path

The forward path determines the early-start times of activities. The forward path precedes from the most left node in the network (node 1 – Figure 4.2) and moves to the right, putting the calculations inside the shaded boxes to the left.

Each node in the network, in fact, is a point at which some activities end (head arrows coming into the node), as shown in Figure 4.3. That node is also a point at which some activities start (tail arrows of successor activities). Certainly, all successor activities can start only after the latest predecessor is finished. Therefore, for the forward path to determine the early-start (ES) time of an activity, we have to look at the head arrows coming into the start node of the activity. We then have to set the activity ES time as the latest finish time of all predecessors.

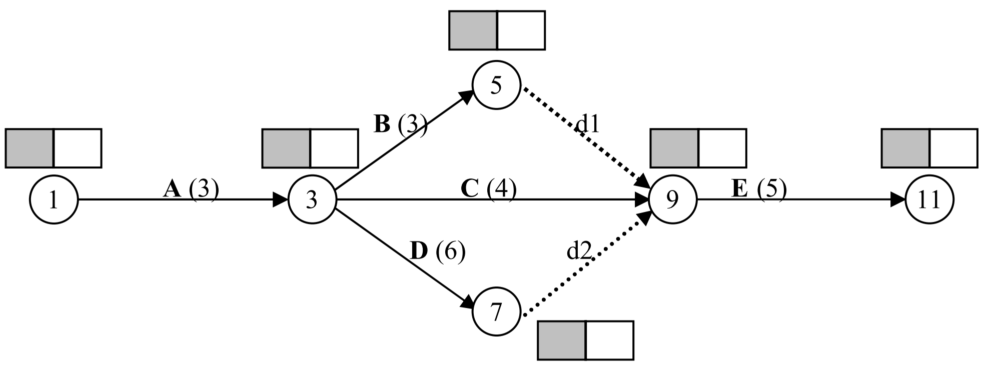


Figure 4.2: Preparation for the forward path

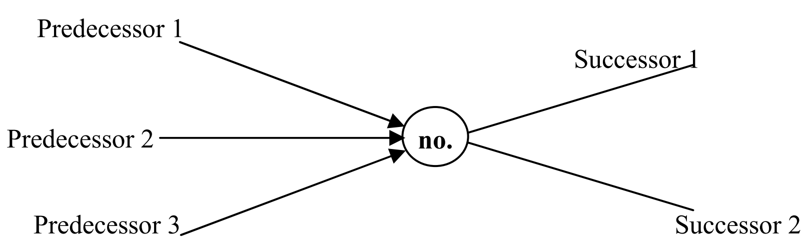


Figure 4.3: A node in an AOA network

In our example, the forward path calculations are as follows:

- We begin at node 1, the start node of the project, and assign it an early-start time of zero. Here, all activity times use an end-of-day notation. Therefore, the ES of activity A is zero means that activity starts at end of day zero, or the beginning of day 1 in the project.

- We now move to node 3. This node receives one head arrow, and as such, it has one predecessor, activity A. Since the predecessor started on time zero and has 3 days duration, then, it ends early at time 3 (Early-Finish (EF) = Early-Start (ES) + d). Accordingly, the ES time of all successor activities to node 3 (activities B, C, and D) is time 3. This value is therefore, put in the shaded box on top of node 3, as shown in Figure 4.4.

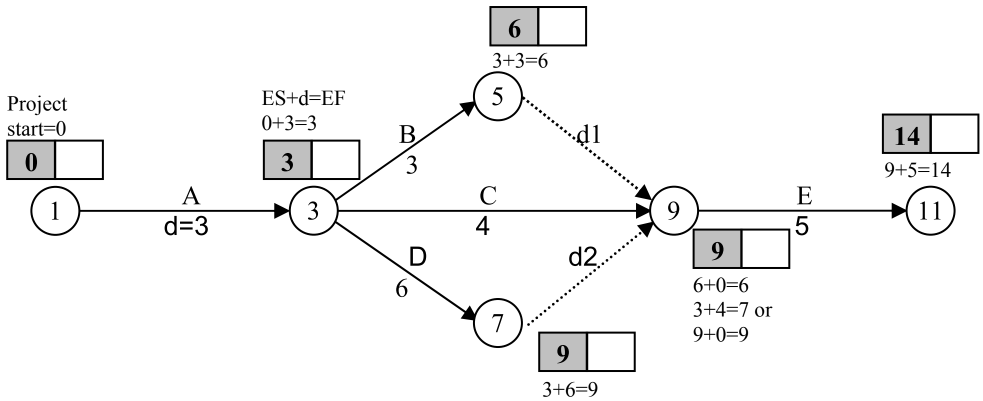


Figure 4.4: Forward path calculations in AOA networks

- We now move forward to successor nodes 5, 7, and 9. However, since node 9 is linked to nodes 5 and 9 by dummy activities, we begin with nodes 5 and 7. Node 5 receives one head arrow from its predecessor activity B, we evaluate the EF time of B as 6 (ES (3) + d (3)). Successor activities to node 5, therefore, can have an ES time of 6. Similarly, the ES time at node 7 is calculated as time 9.

- Moving to node 9, we evaluate the EF times of its 3 predecessors (d1, C, and d2) as time 6, 7, and 9, respectively. Accordingly, the ES time of successor activities is the largest value 9. Notice that only the largest EF value of predecessor activities is used to calculate the ES of successor activities and all other values not used. As such, only ES values can be directly read from the calculations in Figure 4.4. EF values, on the other hand can be calculated as EF = ES + d.

- We now move to the last node 11. It receives one head arrow, activity E which has an ES value of 9. The EF time of activity E, therefore =9 + 5 = time 14. Since node 11 is the last node, the EF of this node becomes the end of the project, reaching a total project duration of 14 days.

Generally, for any activity *x* connecting between nodes *i* and *j* as shown in Figure 4.5, the calculations as follows:

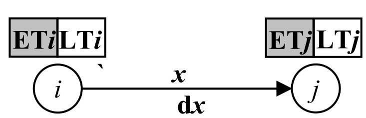


Figure 4.5: Activity times

$$ET_j = ET_i + d_x \qquad (4.1)$$

In case of more than one arrow terminating at node *j*, then consider the largest value.

Accordingly,

$$ES_x = ET_i \qquad (4.2)$$

$$EF_x = ES_x + d_x \qquad (4.3)$$

Backward Path

The backward path determines the late-finish (LF) times of activities by proceeding backward from the end node to the starting node of the AOA network. We put the LF values in the right side boxes adjacent to the nodes, as shown in Figure 4.6. For the example at hand, we do the following:

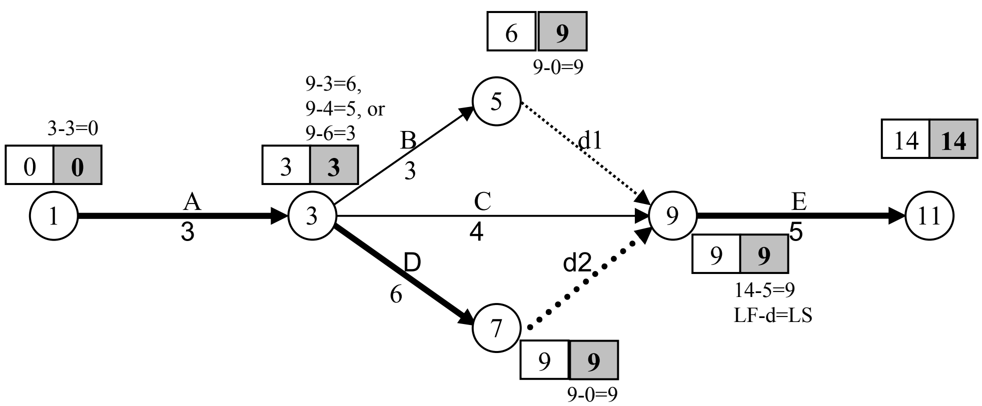


Figure 4.6: Backward path calculations in AOA networks

- We begin at the last node of the network (node 11) and we transfer the early-finish value from the left box to be the late-finish (LF) value at the right side box.

- We then move backward to node 9 which has only one tail arrow of activity E. With the LF time of E being time 14, its LS time becomes LS = LF - d = 14 – 5 = time 9. At node 9, therefore, time 9 becomes the LF time of the predecessor activities of this node.

- Moving backward to predecessor nodes 5, and 7. Node 5 has one tail arrow of the dummy activity d1, and as such, the LF time value to be used at node 5 becomes 9. Similarly, the LF time value of node 7 becomes 9.

- Moving to node 3, we evaluate the LS time of its 3 successor activities B, C, and D as 6, 5, and 3, respectively. The LF time at node 3, therefore, becomes the smallest value 3. With other LS values not used, the values in the calculation boxes, as such, directly show the LF times of activities. LS times can be calculated as LS = LF – d.

- We now proceed to the first node in the network (node 1). It connects to one tail arrow of activity A. The LS time of A, therefore, is LS = LF – d = 3 – 3 = 0, a necessary check to ensure the correctness of the calculation.

Having Figure 4.5 again in mind and to generalize the calculations, for any activity x connecting between nodes i and j, the calculations as follows:

$$LT_i = LT_j - d_x \quad (4.4)$$

In case of more than one arrow leaving node i, then consider the smallest value.

Accordingly,

$$LF_x = LT_j \quad (4.5)$$

$$LS_x = LF_x - d_x \quad (4.6)$$

Float Calculations

Once forward path and backward path calculations are complete, it is possible to analyze the activity times. First, let's tabulate the information we have as shown in Table 4.1. One important aspect is Total-Float (TF) calculations, which determine the flexibility of an activity to be delayed. Notice in Table 4.1 that some activities such as activity A has ES time = LS time, and its EF time = LF time, indicating no slack time for the activity. Other activities such as B can start early at time 3 and late at time 6, indicating a 3-day of total float. Float calculations can be illustrated as shown in Figure 4.7 for any activity.

Table 4.1: CPM results

Activity	Duration	early Start (ES)	Late Finish (LF)	Late Start (LS)	Early Finish (EF)	Total Float (TF)	Critical Activity
A	3	0	3	0	3	0	Yes
B	3	3	9	6	6	3	No
C	4	3	9	5	7	2	No
D	6	3	9	3	9	0	Yes
E	5	9	14	9	14	0	Yes

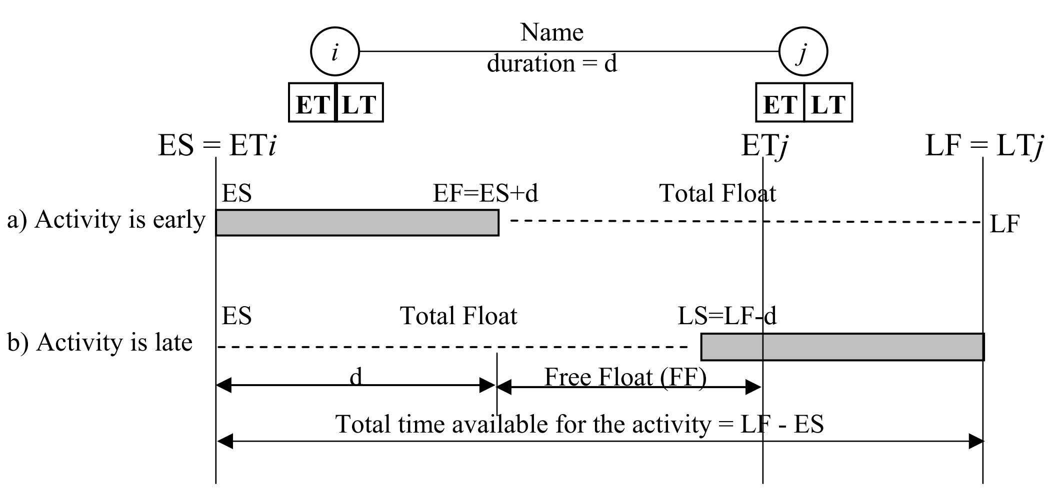


Figure 4.7: Float calculations

Figure 4.7 shows two ways of scheduling each activity using its activity times. One way is to schedule it as early as possible (using its ES time). The other way is as late as possible (using its LS time). The activity float can, therefore, be represented by the following relationships:

$$\text{Total Float (TF)} = LF - EF \qquad (4.7)$$

$$= LS - ES \qquad (4.8)$$

Also, with the ES and LF times directly read from the boxes used in forward and backward path calculations, the total float can also be calculated as; TF = LF – ES – d. Using these relationships, activities total floats are calculated as shown in Table 4.1.

Another type of float often used in network analysis is the Free Float, which can be calculated as:

$$\text{Free Float (FF)} = ET_j - ET_i - d \qquad (4.9)$$

$$\text{or FF} = \text{smallest ES (of succeeding activities)} - \text{EF (of current activity)} \qquad (4.10)$$

The free float defines the amount of time that an activity can be delayed without affecting any succeeding activity. With free float available for an activity, a project manager knows that the float can be used without changes the status of any non-critical activity to become critical.

Identifying Critical Activities

Activities with zero total floats mean that they have to be constructed right at their schedule times, without delays. These activities are considered to be critical. They deserve the special attention of the project manager because any delay in critical activities causes a delay in the project duration.

One interesting observation in the results of CPM analysis is that critical activities form a continuous path of the critical activities that spans from the beginning to the end of the network. In our example, activities A, D, and E (excluding dummy activities) are critical and the critical path is indicated by bold lines on Figure 4.6. Notice that among the 3 paths in this example (A-B-E; A-C-E; and A-D-e), the critical path is the longest one, an important characteristic of the critical path. In real-life projects with many activities, it is possible that more than one critical path are formed. By definition, the length of these critical paths is the same.

4.3.2 Precedence Diagram Method (PDM)

Precedence Diagram Method (PDM) is the CPM scheduling method used for AON networks and it follows the same four steps of the CPM for AOA method.

Forward Path

Forward path can proceed from one activity to the other; the process is as follow (Figure 4.8):

- At activity A. It is the first activity in the network. We give it an early-start (ES) of 0 in the left top box. Adding the activity duration, we determine the EF time of the activity and we put it in the top right box.

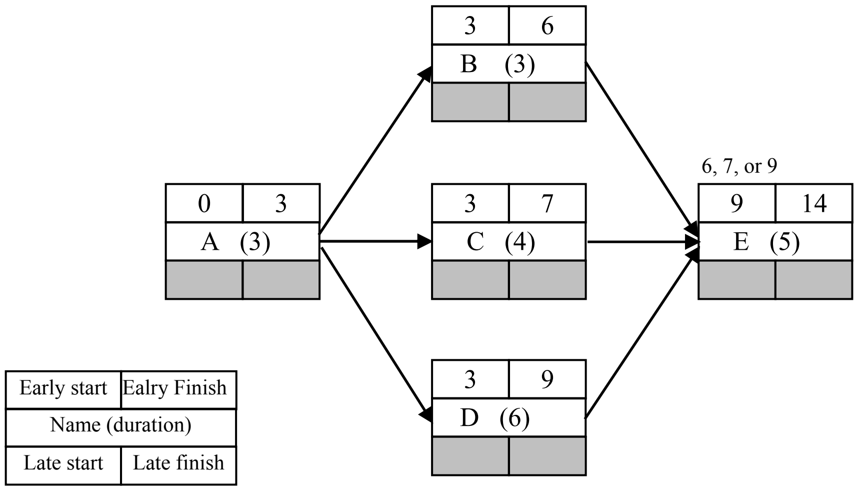


Figure 4.8: Forward Path in PDM Analysis

- We move forward to the succeeding activities B, C, and D. These three activities have only A as a predecessor with time 3 as its EF. As such, all the three activities can start as early as time 3 (ES = 3). Each activity, accordingly, has its own EF time based on its duration.

- Moving forward to activity E. This activity has 3 predecessors (3 head arrows) of activities B, C, and D with their largest EF time being 9. The ES of activity E, thus, with becomes time 9. Adding its duration, the EF becomes time 14.

To generalize the calculations consider Figure 4.9, of two activities i and j with relationship finish to start and overlap between them. Overlaps will have a positive sign, while lags will have a negative sign. The forward path calculations are as follows:

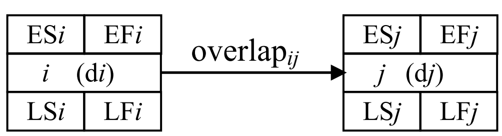


Figure 4.9: Activities times in PDM Analysis

$$ES_j = EF_i - overlap_{ij} \quad (4.11)$$

In case of more than one activity precedes activity *j* then consider the maximum. Then, apply Equation 4.3 to calculate the early finish times.

Backward Path

Once the forward path is finished, the backward path can start, moving from the last activity to the first, putting the calculations in the bottom two boxes of each activity, as shown in Figure 4.10. The process is as follows:

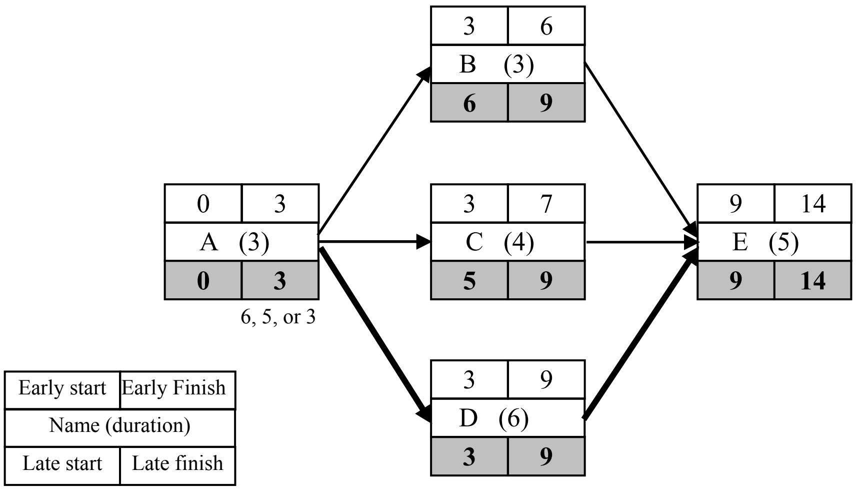


Figure 4.10: Backward path in PDM analysis

- We start at the last activity E and we transfer the early-finish value to become the activity's late-finish (LF) time. Then, subtracting the activity's own duration, the late-start (LS) time is calculated as time 9 and put in the bottom left box of the activity.

- Moving backward to activities B, C, and D all have one successor (activity E) with LS time of 9. The LF of all these activities becomes time 9. Each activity then has its own LS time, as shown in Figure 4.10.

- Moving to activity A. The activity is linked to 3 tail arrows (i.e., has 3 successors) of activities B, C, and D. The LF of activity A, thus, is the smallest of its successors' LS times, or time 3. Activity A then has LS equals zero.

Considering Figure 4.9 again, the backward path calculations are as follows:

$$LF_i = LS_j + overlap_{ij} \qquad (4.12)$$

In case of more than one activity succeeds activity *j* then consider the minimum. Then, apply Equation 4.6 to calculate the late start times.

Notice that by the end of the backward path, all activity times can be read directly from the boxes of information on the activity, without additional calculations. This also, makes it simple to calculate the total float of each activity using the same relationships used in the AOA analysis, basically, Identifying Critical Activities

Critical activities can also be easily determined as the ones having zero float times, activities A, D, and E. The critical path is then shown in bold as Figure 4.10. The PDM analysis, as explained, is a straight forward process in which each activity is considered as an entity that stores its own information.

4.4 Time-Scaled Diagrams

Time-scaled diagrams are used extensively in the construction industry. Such diagrams enable one to determine immediately which activities are scheduled to proceed at any point in time and to monitor field progress. Also, it can be used to determine resources need. The time scale used in time-scaled diagrams can be either the calendar dates or the working periods (ordinary dates), or using both at the same time.

The activities are represented as arrows that drawn to scale to reflect the activity duration it represents. The horizontal dashed lines represent total float for groups of activities and free float for the immediate activity to the left of the dashed line. The precedence of an activity is the immediate activities before it or that linked to it through vertical dashed lines. The name and the duration of an activity are written above and below the arrow representing it respectively (Figure 4.11). The ES, EF, and FF times of the activities can be easily read directly from the diagram. The TF for an activity is the smallest sum of succeeding FF on all paths. Accordingly, the LS and LF times can be easily calculated as follows:

$$LSi = ESi + TFi \tag{4.13}$$

$$LFi = LSi + Di \tag{4.14}$$

The critical path can be easily determined as the continuous lines from the beginning to the end of the network with any dashed lines. The main advantage of this diagram I its simple representation and it can be sued directly for determining resources need. However, its disadvantage is that it needs a great effort to be modified or updated. Also, it can not be used to represent overlapping activities.

Figure 4.11 shows the time-scaled diagram for the same 5-activities project solved previously using AOA and AON networks.

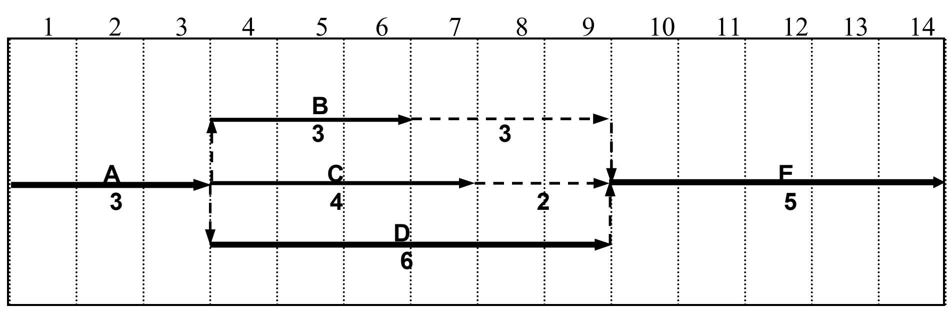


Figure 4.11: Time-scaled diagram

The TF for activity A equals the smallest of the sum of the floats along all paths from the end of activity A to the end of the project. The float on path ABE = 3, path ACE = 2 and path ADE = 0, then the TF of activity A = 0. The calculations are shown in Table 4.2.

Table 4.2 Time-scaled diagram calculations

Activity	ES	EF	FF	TF	LF=EF+TF	LS=LF-d
A	0	3	0	0	3	0
B	3	6	3	3	9	6
C	3	7	2	2	9	5
D	3	9	0	0	9	3
E	9	14	0	0	14	9

4.5 Schedule Presentation

After the AOA and AON calculations are made, it is important to present their results in a format that is clear and understandable to all the parties involved in the project. The simplest form is the Bar chart or Gantt chart, named after the person who first used it. A bar chart is a time versus activity chart in which activities are plotted using their early or late times, as shown in Figures 4.12 a and b. Early bar chart is drawn using the ES times of activities, while the late bar chart is drawn using the LS times.

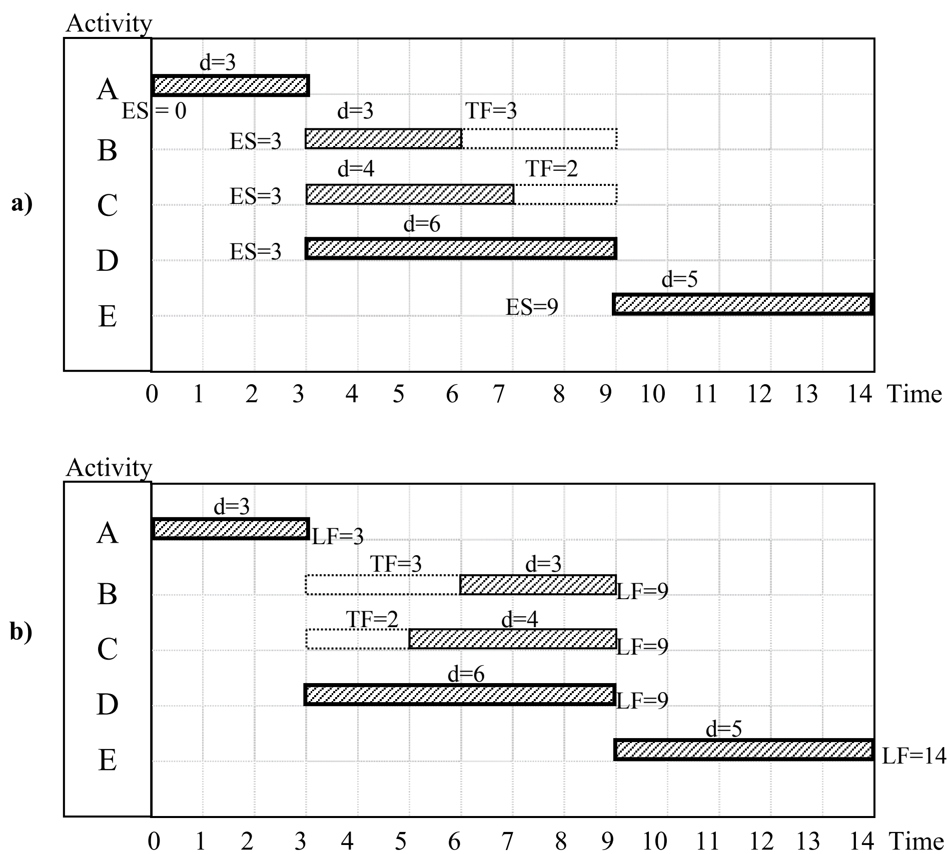


Figure 4.12: a) Early bar chat b) Late bar chart

The bar chart representation, in fact, shows various details. Float times of activities, critical activities can be shown in a different color, or bold borders, as shown in Figure 4.12. The bar chart can also be used for accumulating total daily resources and / or costs, as shown at the bottom part of Figure 4.13. In this figure, the numbers on each activity represent the number of labors needed.

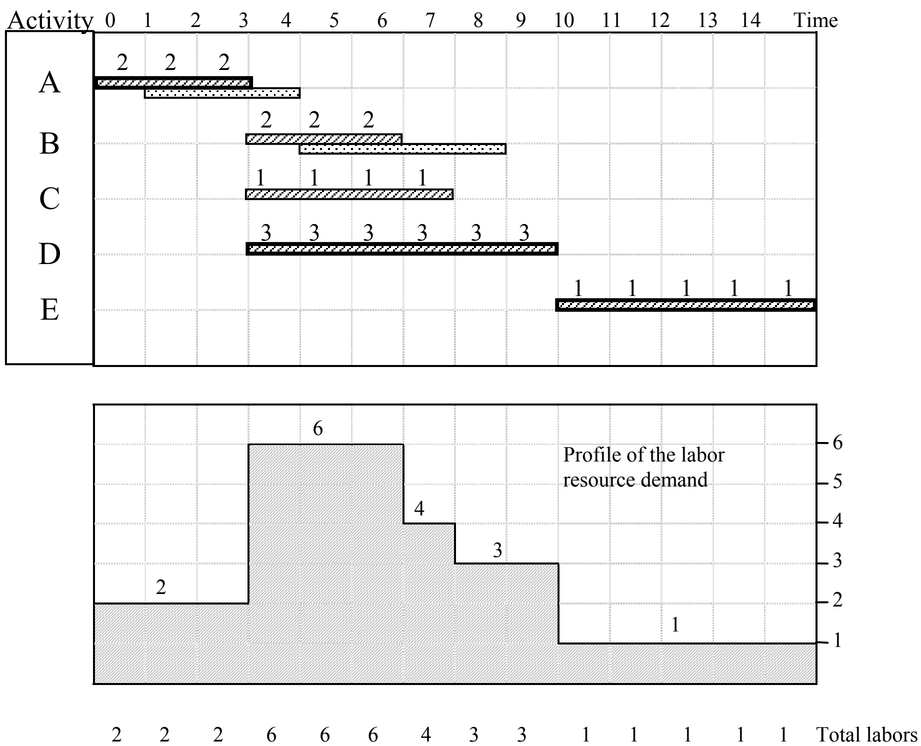


Figure 4.13: Using bar chart to accumulate resources

One additional benefit of the bar chart is its use on site to plot and compare the actual progress in the various activities to their scheduled times. An example is shown on Figure 4.13, showing actual bars plotted at the bottom of the original bars of the schedule.

4.6 Criticisms to Network Techniques

The CPM and PDM analyses for network scheduling provide very important information that can be used to bring the project to success. Both methods, however, share some drawbacks that require special attention from the project manager. These drawbacks are:

- Assume all required resources are available: The CPM calculations do not incorporate resources into their formulation. Also, as they deal with activity durations only, it can result in large resource fluctuations. Dealing with limited resources and resource leveling, therefore, has to be done separately after the analysis;

- Ignore project deadline: The formulations of CPM and PDM methods do not incorporate a deadline duration to constrain project duration;

- Ignore project costs: Since CPM and PDM methods deal mainly with activities durations, they do not deal with any aspects related to minimize project cost;

- Use deterministic durations: The basic assumption in CPM and PDM formulations is that activity durations are deterministic. In reality, however, activity durations take certain probability distribution that reflect the effect of project conditions on resource productivity and the level of uncertainty involved in the project; and

4.7 Solved Examples

Example 3.1

For the project data in Table 4.3, answer the following questions:

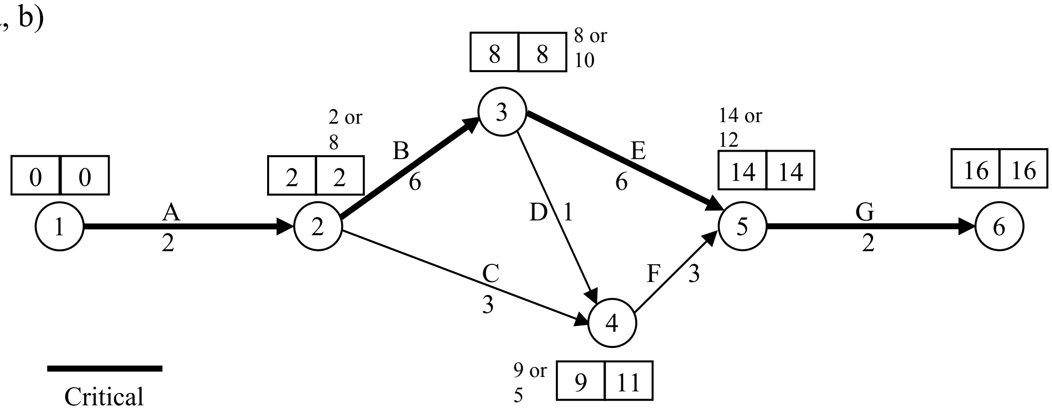
a) Draw an AOA network of the project?
b) Perform forward path and backward path calculations?
c) What is the effect of delaying activity D by 3 days?

Table 4.3: Data for Example 3.1

Activity	Duration	Immediate predecessor
A	2	-
B	6	A
C	3	A
D	1	B
E	6	B
F	3	C, D
G	2	E, F

Solution

a, b)

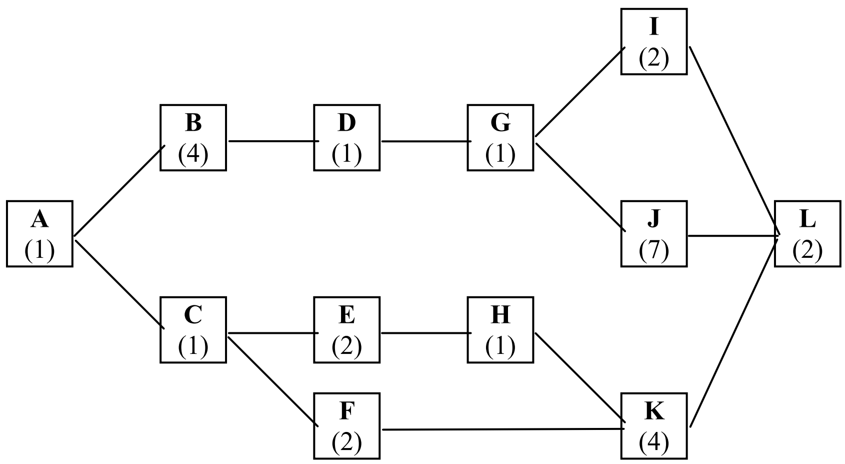


c) Total float of activity D = LF – ES – d = 11 – 8 – 1 = 2.

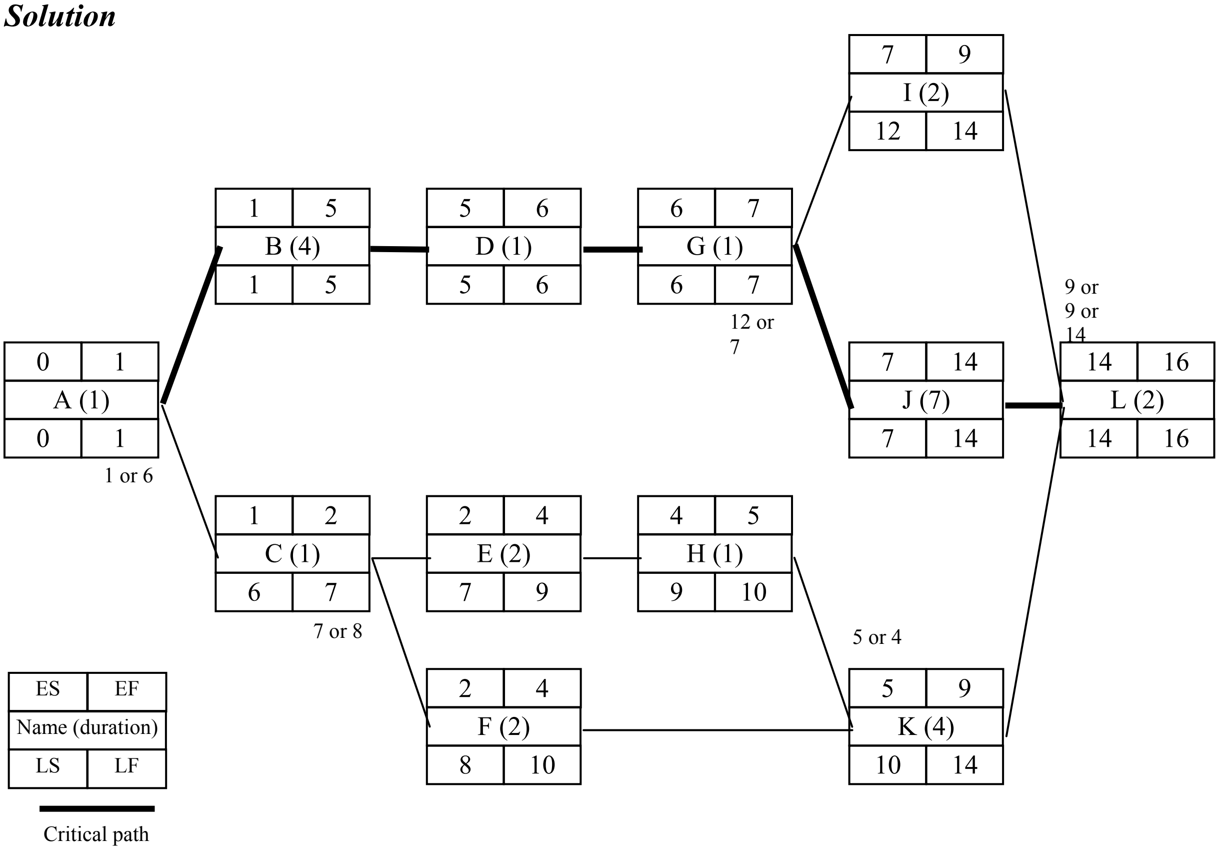
Then delaying activity D by 1 day more than its total float will cause a net delay in the whole project by 1 day to become 17 days.

Example 3.2

Perform PDM calculations for the small project below and determine activity times. Durations are shown on the activities.



Solution

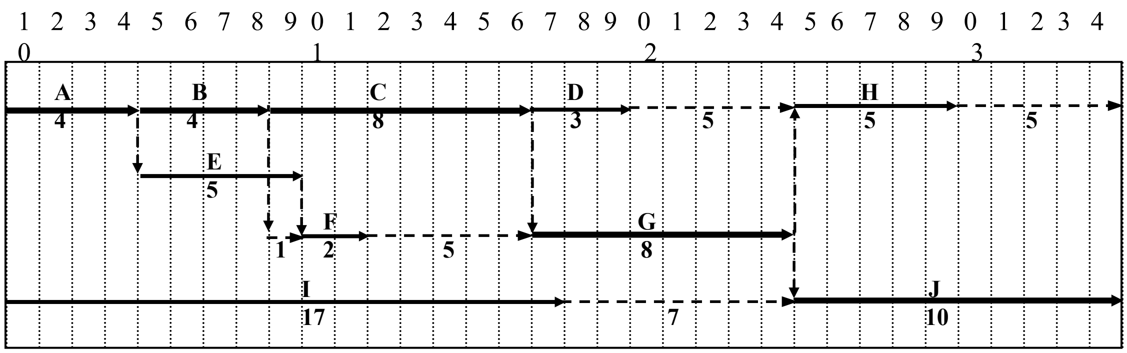


Example 3.3

For the activities listed in the table below, draw the time-scaled diagram and mark the critical path. Determine the completion time for the project. Tabulate activities times and floats.

Activity	Duration	Predecessor
A	4	-
B	4	A
C	8	B
D	3	C
E	5	A
F	2	B, E
G	8	C, F
H	5	D, G
I	17	-
J	10	G, I

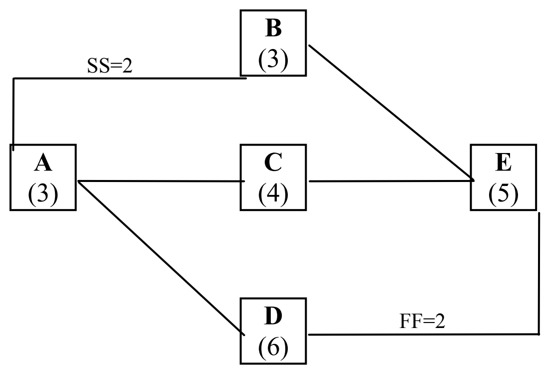
Solution



Activity	ES	EF	TF	FF	LS	LF
A	0	4	0	0	0	4
B	4	8	0	0	4	8
C	8	16	0	0	8	16
D	16	19	10	5	26	29
E	4	9	5	0	9	14
F	9	11	5	5	14	16
G	16	24	0	0	16	24
H	24	29	5	5	29	34
I	0	17	7	7	7	24
J	24	34	0	0	24	34

Example 3.4

Perform PDM calculations for the small AoN network shown here. Pay special attention to the different relationships and the lag times shown on them.



Solution