

## **Chapter 5: Scheduling of Repetitive Projects**

### **5.1 Introduction**

This chapter introduces new techniques for scheduling of multiple and linear projects that involve a number of repetitive activities. These techniques include: the summary diagrams and the line of balance (LOB). Examples of these projects are highways, pipelines, and high-rise buildings. The objective of the LOB technique is to determine a balanced mix of resources and synchronize their work so that they are fully employed and non-interrupted. As such, it is possible to benefit from repetition, and the crews will likely be able to spend less time and money on later units once they develop a learning momentum. Another benefit of the LOB technique is its interesting representation of the schedule, given the large amount of data for the repetitive units. This chapter introduces the summary diagrams calculations presented on AON networks and integrated CPM-LOB calculations that combine the benefits of CPM network analysis of a single unit and the LOB analysis and representation.

### **5.2 Linear Projects**

Linear projects are projects involving repetitive activities. They take their name from either: (a) involving several uniform units of work such as multiple houses or typical floors in a building; or (b) being geometrically linear such as highway, pipeline, and utility projects. In both categories, however, some non-typical units could be involved such as a non-typical floor in a high-rise building or a non-standard station in a highway project. The activities in these non-typical units may certainly involve higher or lower quantity of work than their counterparts in the typical units. To simplify the scheduling task in these situations, we can assume that the project is comprised of (n) typical units, with the activities in each unit having average quantity of the work in all units. As the number of units in a project increases, eventually the project becomes more complex and more challenging.

### **5.3 Resource-Driven Scheduling**

As we have seen in network scheduling, the basic inputs to critical-path analysis are the individual project activities, their durations, and their dependency relationships. Accordingly, the forward-path and backward-path calculations determine the start and finish times of the activities. The CPM algorithm, therefore, is duration-driven. Activities' durations here are function of the resources that are required (rather than available) to complete each activity. The CPM formulation, therefore, assumes that resources are in abundance and cannot be used to determine what resources are needed in order to meet a known project deadline duration.

Resource-driven scheduling, on the other hand, is different and is more focused on resources. Its objective is to schedule the activities (determine their start and finish times) so that a project deadline is met using predefined resource availability limits. The line of balance technique dealt with in this chapter is a resource-driven schedule.

## 5.4 Line of Balance (LOB)

### 5.4.1 Basic representation

Let's consider a medium-sized high-rise building of 40 typical floors. The construction of each typical floor involves various inter-related activities. If a CPM network is to be developed for the whole project, certainly it will be so complex and will be composed of copies of the activities in a single floor. A Bar Chart of the project will still be so complex and will not serve the purpose of a good communication tool between planners and execution personnel.

A schedule representation that suits projects with repetitive activities is shown in [Figure 5.1](#) between time on the horizontal axis and units on the vertical axis. This representation shows the following information:

- Each sloping bar represents one activity (A, B, C, or D) in the project and the width of the bar is the activity duration of one unit, which is uniform along all units;
- A horizontal line at any unit intersects with the activity bars at the planned start and finish times of the work in that unit;
- A vertical line at any date (time) shows the planned work that should be completed/started before and on that date;
- The slope of each activity represents its planned rate of progress and this is direct function of the number of crews involved in the activity. The slope of the last activity is the rate of delivery of the various units; and

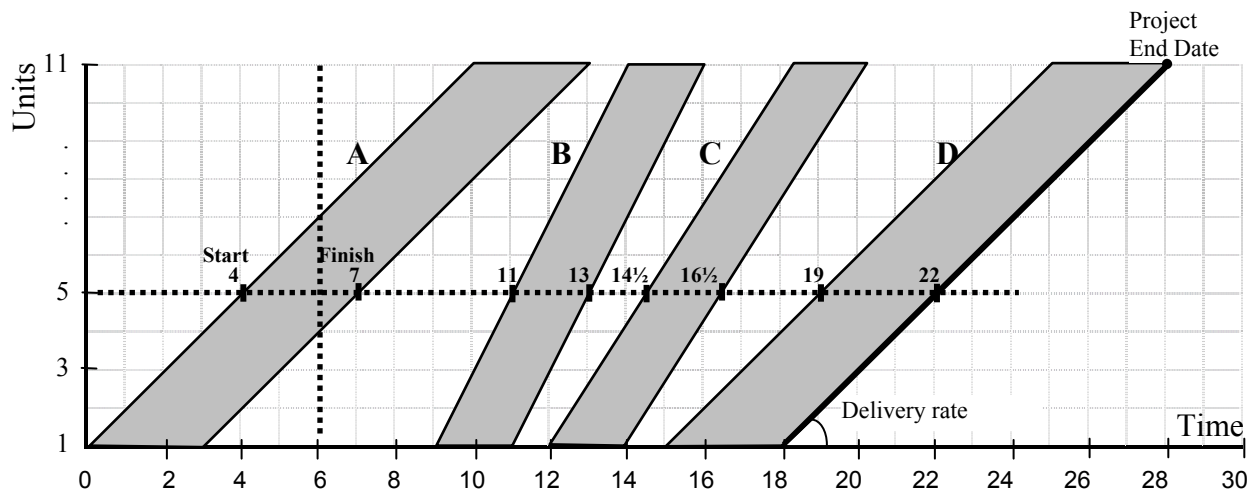


Figure 5.1: LOB representation

- The finish time of the last unit in the last activity represents the end date of the project.

It is possible also to add more details to the basic LOB schedule as shown in [Figure 5.2](#). The modified figure shows interesting information, as follows:

- The number of crews employed in each task is graphically represented with each crew indicated by a different pattern. As such, the movement of the crews from one unit to the other is shown;

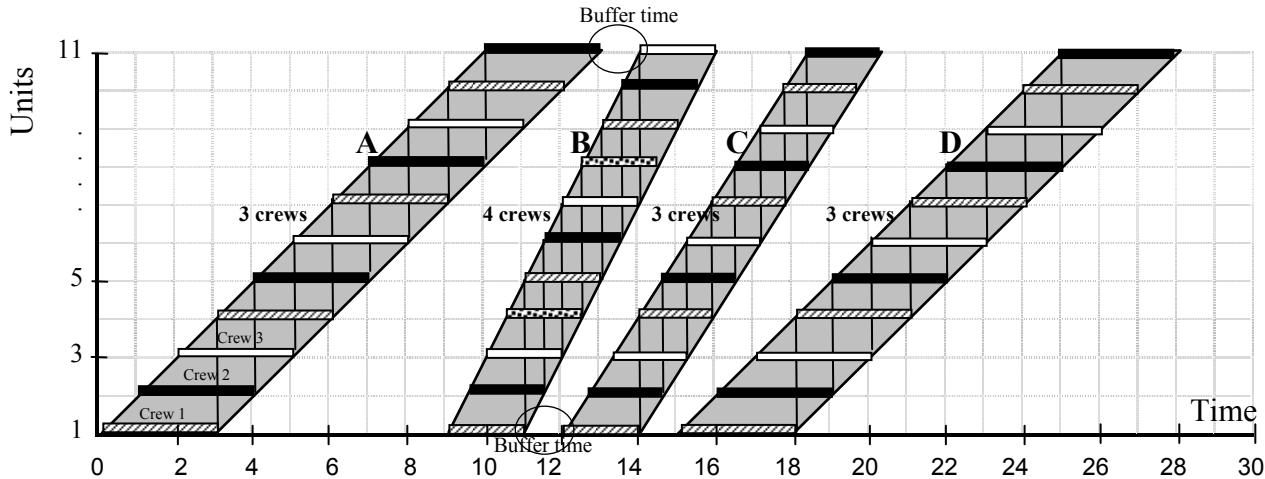


Figure 5.2: LOB Schedule with crew details

- The three crews employed in activity (A) have different work assignments. Crew 1 works in four units (numbers 1, 4, 7, and 10) and leaves site on day 12. Similarly, Crew 2 works on four units (numbers 2, 5, 8, and 11) then leaves site on day 13. Crew 3, on the other hand, works on three units only (numbers 3, 6, and 9) and leaves site on day 11;
- Each crew moves to a new unit as soon as it finishes with the previous one, without interruption. As such, work continuity is maintained and the learning phenomenon can lead to some savings in cost and time;
- To prevent interference among the sequential tasks of the LOB schedule in case an activity is slightly delayed, a buffer time may be introduced as shown, to act as a float time;
- When a slower activity is to follow a faster activity (e.g., C follows B), the activity C can be scheduled starting from unit 1, immediately following the predecessor B. Since interference can happen at unit 1, buffer time may be added to the start of unit 1;
- When a faster activity is to follow a slower activity (e.g., B follows A), the activity B needs to be scheduled starting at the top unit. If buffer time is to be added, it will be added at top. Notice that the start of unit 1 in activity B has been delayed to allow the task the proceed without interruption;
- Changing the production rate (slope) of any activity changes the project duration. Even speeding one task may prove to be harmful to the project when the conflict point changes from bottom to top; and

- If speeding an activity or relaxing it may result in a delay in the project, a good scheduling strategy is to schedule the activities as parallel as possible to each other and also parallel to a desired project delivery.

### 5.4.2 LOB Calculations

The objective of using LOB is to achieve a resource-balanced schedule by determining the suitable crew size and number of crews to employ in each repetitive activity. This is done such that: (1) the units are delivered with a rate that meets a pre-specified deadline; (2) the logical CPM network of each unit is respected; and (3) crews' work continuity is maintained. The analysis also involves determining the start and finish times of all activities in all units and the crews' assignments.

The CPM-LOB formulation that achieves the above objective involves four main issues, which are discussed in the next sections:

- Crew synchronization and work continuity equation;
- Computation of a project delivery rate that meets a given deadline duration;
- Calculating resource needs for critical and non-critical activities; and
- Drawing the LOB schedule.

#### Crew synchronization

A simple relationship between the duration taken by a crew in one unit ( $D$ ) and the number of crews ( $C$ ) to employ in a repetitive activity can be derived from the illustration in Figure 5.3. In this figure, we have a 5-unit activity and 3 crews to use. Only one crew is assumed to work in a single unit and the crew spends time ( $D$ ) on the unit before moving to another unit.

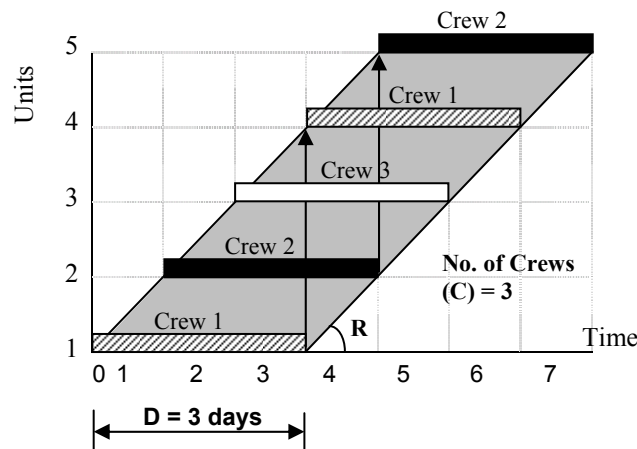


Figure 5.3: Crew synchronization

Having 3 crews available for this activity, it is possible to schedule their movements in and out of each unit, as shown in the figure, so that they are not interrupted and the work progresses at a rate ( $R$ ). For that work synchronization to happen, the following simple relationship applies:

$$\text{Number of Crews } (C) = D \times R \quad (5.1)$$

In the example shown,  $C = 3$ ;  $D = 3$  days; then,  $R$  becomes 1 unit/day, according to Equation 5.1. Therefore, it is possible to achieve work continuity given any change in the number of crews ( $C$ ) or crew formation (affects  $D$ ) by adjusting the rate of progress ( $R$ ). For example, if 4 crews become available, we can apply the same Equation 5.1 to determine a faster progress rate of 1.25 units/day.

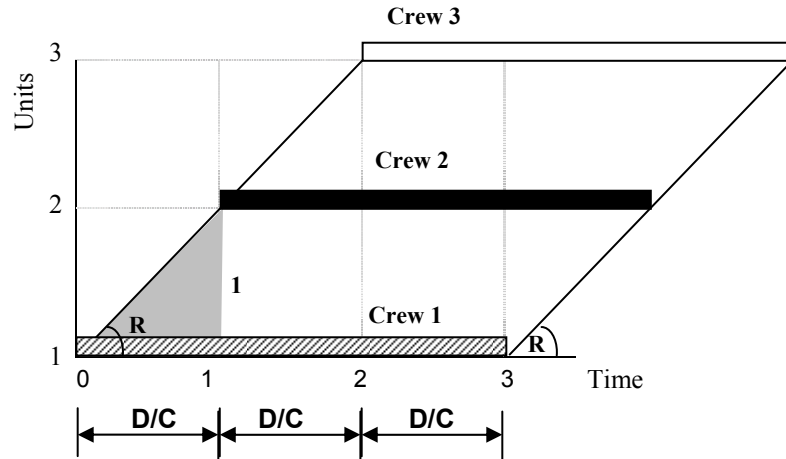


Figure 5.4: Deriving Equation 5.1

Deriving the relationship of Equation 5.1 is simple. By enlarging part of Figure 5.3 and dividing the duration ( $D$ ) among the ( $C$ ) crews, the slope of the shaded triangle in Figure 5.4 becomes:

$$R = 1 / (D / C) \quad (5.2)$$

and the time  $D/C$  becomes:

$$D / C = 1 / R \quad (5.3)$$

Both equations lead to our formulation of  $C = D \times R$ . Equation 5.2 also means that work continuity is achieved by shifting the start of each unit from its previous one by a time  $D/C$  or  $1/R$ . This shift also has another practical meaning. Since each crew has part of its duration non-shared with other crews, the chance of work delay is reduced when two crews need the same equipment, or other resource, such as a crane on site.

#### Meeting a deadline duration

A basic objective in CPM-LOB calculation is to meet a given deadline for finishing a number of ( $n$ ) repetitive units; each has its own CPM network of component activities. Using the illustration in Figure 5.5, it is possible to formulate a strategy for meeting the deadline by calculating a desired rate of delivery ( $R_d$ ) for the units, as follows:

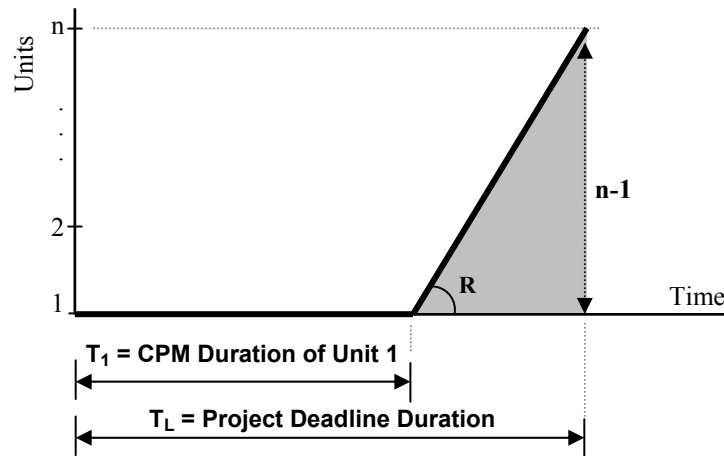


Figure 5.5: Calculating a desired rate of delivery

$$R_d = (n - 1) / (T_L - T_1) \quad (5.4)$$

where,  $T_L$  is the deadline duration of the project and  $T_1$  is the CPM duration of the first unit. The delivery rate determined from Equation 5.4 is, in fact, the minimum rate required to meet the desired deadline. Any higher rate can expectedly produce shorter project duration, however, more crews may need to be used and the schedule can be more costly.

#### Calculating resource needs

Once a minimum delivery rate ( $R_d$ ) is calculated, it is desirable to enforce this rate on the schedule of the repetitive activities to determine the resources needed to complete the project on time. Equation 5.4, therefore, needs to be applied particularly to the critical activities, which are the sequential tasks that take the longest path in the CPM network of each unit. Non-critical activities, on the other hand, have float (TF) times and as such, we can afford to relax them according to their float times to reduce cost. It is, therefore, possible to modify Equation 5.4 and generalize it to determine a desired rate ( $R_i$ ) for any repetitive task (i), as follows:

$$R_i = (n - 1) / (T_L - T_1) + TF_i \quad (5.5)$$

The physical meaning of Equation 5.5 is illustrated in Figure 5.6. In this figure, a 5-unit project is shown with each unit consisting of a simple four-activity network. Three of the four activities A, B, and C are sequential and each has 5-days duration. The fourth activity D runs parallel to B and has a duration of 2 days only. Accordingly, A, B, and C are critical activities while activity D is non-critical with Total Float (TF) of 3 days. As shown in Figure 5.6, the slopes of activities A, B, and C are the same and are steep up. The slope of activity (D), on the other hand, has been relaxed by simply starting unit 1 of task D as early as possible while starting the last unit as late as possible (notice the difference in the CPM networks of the first and the last units). In this manner, simple analysis of the slope of activity D in the figure leads us to the formulation of Equation 5.5. Using this approach, the relaxation of non-critical activities can be performed without violating any logical relationships or crew work continuity requirements.

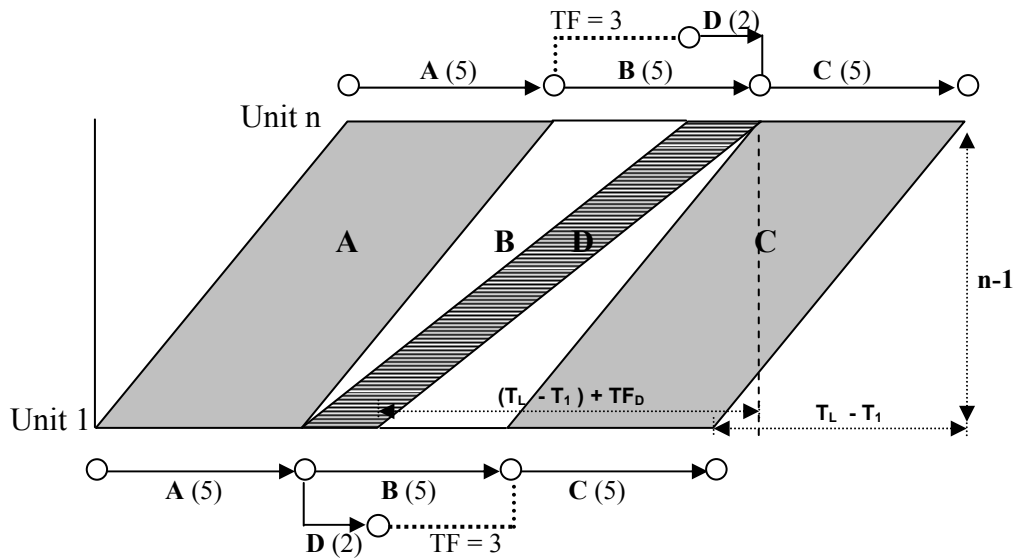


Figure 5.6: Utilization of float in LOB calculations

With the desired rates calculated for the individual activities, a generalized form of Equation 5.1 can be used to determine the necessary number of crews ( $C_i$ ) to use in each activity (i), as follows:

$$C_i = D_i \times R_i \quad (5.6)$$

Another important consideration is that, in most cases, the number of crews calculated using Equation 5.6 is not an integer value. Since a fraction of a crew is not possible, the number of crews ( $C_i$ )'s has to be rounded up to determine the actual number of crews ( $C_{ai}$ )'s. As a consequence to that, the actual rates of progress in the activities ( $R_{ai}$ )'s need to be adjusted, as follows:

$$C_{ai} = \text{Round Up } (C_i) \quad (5.7)$$

$$R_{ai} = C_{ai} / D_i \quad (5.8)$$

Equations 5.5 to 5.8, therefore, become the basis of integrated CPM-LOB calculations.

### Drawing the LOB Schedule

A LOB schedule becomes simple to draw when all activities run with an exactly similar rate (i.e., activities run parallel to each other). However, due to the rounding of number of crews in Equation 5.4, the activities' actual rates ( $R_{ai}$ )s calculated using Equation 5.5 will not be parallel. Drawing the LOB schedule as such requires extra care as conflict points, either at the top unit or at the first unit, will be introduced due to the difference in progress rates from one activity to the other. As explained earlier, sometimes speeding an activity will cause a net delay in the whole project, if work continuity is to be maintained. Therefore, some non-critical activities may end up being delayed even in some situations violating the logical relationships or becomes critical

themselves. Also, in some situations, the end schedule may slightly extend beyond the deadline. In this case, a simple approach to use is to re-schedule the project with a deadline duration that is slightly (one or two days) shorter than originally desired.

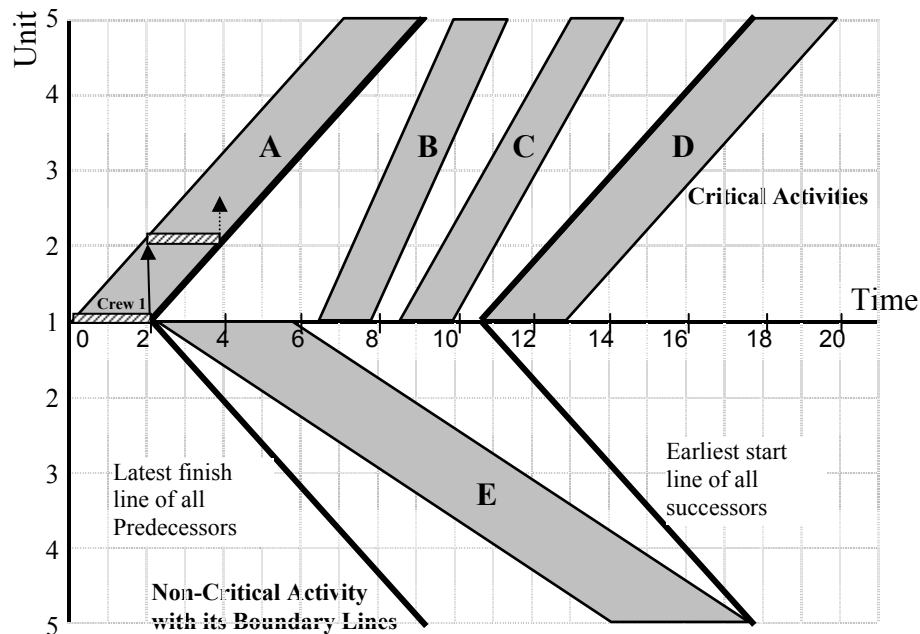


Figure 5.7: Alternative LOB representation

To draw the LOB schedule using the activities actual rates ( $R_{ai}$ )s, we need to proceed in a forward path, following the logical relationships in the CPM network. When an activity is considered, its predecessors are first examined to identify their largest finish times, which are then considered as a boundary on the start of the current activity. Drawing the schedule by hand is simple when the network is small and can be done with varying levels of detail as shown in [Figures 5.1 and 5.2](#).

In terms of presentation, showing all the activities on the same grid results in a crowded schedule and can be confusing even for a small network. Two interesting approaches can be used to circumvent this problem. One approach is to draw the critical path on one grid and draw the other paths, each on a different grid. The benefit of drawing these paths is to help visualize the successor/predecessor relations for any given task, and accordingly facilitate any desired changes to rates or crews. The second approach is to extend the LOB representation to show the non-critical activities on a mirrored grid as shown in [Figure 5.7](#).

### 5.4.3 Example Application

Now, let's describe the systematic CPM-LOB procedure and apply it to a complete example. Consider a subcontractor involved in a 5-unit housing project. The CPM network of the activities in a single unit is shown in [Figure 5.8](#).



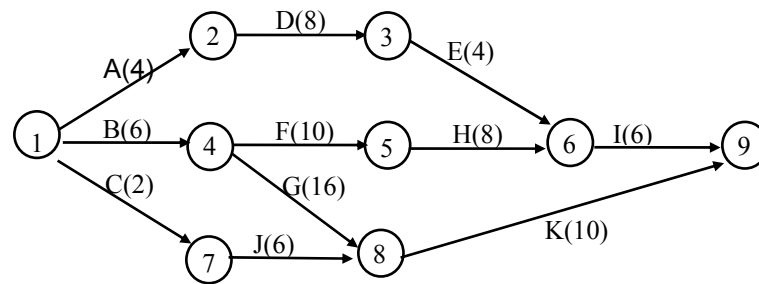


Figure 5.8: AOA network of a single unit

Activity durations, in days, are shown inside the brackets. The contractor signed a contract with the developer to finish all the work in 50 days. Calculate and draw a LOB schedule. Also, determine how many crews are needed and show on the LOB schedule how the crews move from one unit to the other.

### Solution

Step1: CPM calculations for a single unit: In this step, we determine the duration of a single unit and identify the critical path. As shown in Figure 5.12, the CPM duration (T1) of a single unit is 32 days and the critical path is B - G - K.

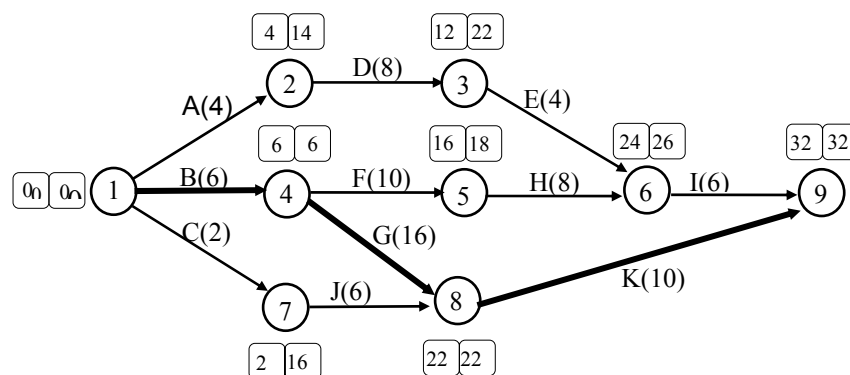


Figure 5.9: CPM Calculations for a single unit

Step 2: LOB calculations: In this step, apply the continuity equation (Equation 5.1) to determine the number of crews, and then determine the actual rate of progress in each activity. The calculations are shown in Table 5.1. Given that a project duration (TL) of 50 days is desirable, the 32 days taken by the first unit (T1) will leave only 18 days (TL - T1) to deliver the remaining 4 (i.e.,  $n - 1$ ) units. This gives a rate of delivery ( $4/18 = 0.222$  units per day) or simply one unit every 4.5 days. As discussed before, this rate will be applied to the critical activities as a desired rate required to meet the deadline. Non-critical activities will have smaller rates depending on their float times (column 4 of Table 5.1).

Table 5.1: LOB calculations

Activity	Activity Duration (D)	Total Float (TF)	Desired Rate (R) = $(n-1) / (T_L - T_1 + TF)$	Required Crews (C) = $D \times R$	Actual Crews ( $C_a$ )	Actual Rate ( $R_a$ ) = $C_a / D$
A	4	10	0.143	0.572	1	0.25
*B	6	0	0.222	1.332	2	0.333
C	2	14	0.125	0.250	1	0.5
D	8	10	0.143	1.144	2	0.25
E	4	10	0.143	0.572	1	0.25
F	10	2	0.200	2	2	0.2
*G	16	0	0.222	3.552	4	0.25
H	8	2	0.200	1.6	2	0.25
I	6	2	0.200	1.2	2	0.333
J	6	14	0.125	0.75	1	0.167
*K	10	0	0.222	2.22	3	0.3

$T_L$  = deadline duration = 50 days;  $T_1$  = CPM duration of a single unit; \* = Critical activity

Step 3: Draw the LOB Schedule: In this step, we draw the LOB schedule, starting with the critical path. First, we construct an empty grid and then plot the activities one by one as a four-point parallelogram (Figure 5.10). The two points on the left side represent the line connecting the start times of all units while the right side line connects the finish times. We start with activity B, the first critical activity. The first unit starts at time 0 (lower-left point) since this is the first activity in the path. The finish time of the first unit (bottom-right point), therefore, is at time 6 since the activity has a duration of 6 days. Knowing the two bottom points of B, the top two points are then determined, considering the actual progress rate of this activity ( $R = 0.333$ , see Table 5.1). As discussed earlier, each unit starts after  $(1/R)$  days from the start of its previous unit. Therefore, the last unit (unit 5) starts after  $4 \times (1/0.333)$  days from the start of unit 1 ( $12 + 0$ ) = day 12 (Figure 5.10). The finish time of unit 5 (top-right point) then becomes  $12 + 6$  (duration) = day 18.

Once an activity is plotted, we proceed with its successor, activity G in our case. Since G depends only on B, its start is bounded only by the finish line of activity B, which is the line between day 6 on unit 1 and day 18 on unit 5. Now, since G has a slower progress rate (0.25) than the boundary line (0.333), we can start the first unit of G (lower-left point) right after the work in activity B has finished, which is day 6 (notice that conflict point is at bottom). The finish of unit 1 (lower-right point of G), then, becomes day 22 (starts at day 6 + a duration of 16 days). Following that, similar to what we did for activity B, we can plot the top two points, considering the progress rate of activity G.

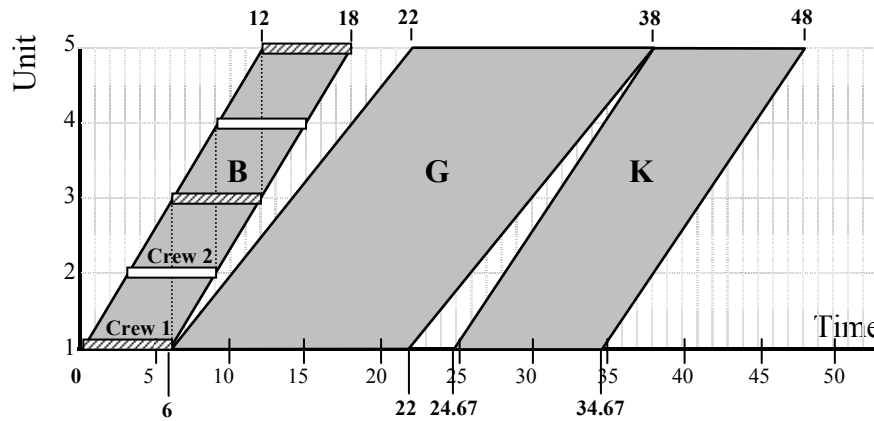


Figure 5.10: LOB schedule for the critical path

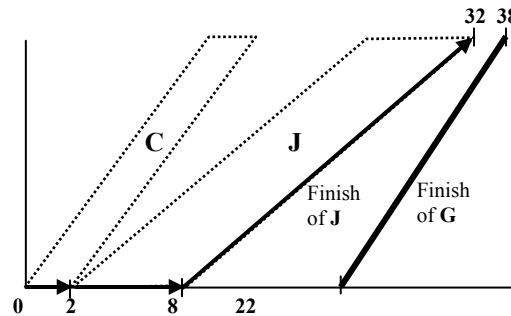


Figure 5.11: Determining the boundary line on activity K

After plotting activity G, we continue with activity K. Since K depends on both G and J, the start of K has to be bounded by the largest finish times of G and J. For G, the finish times are connected by the line between day 22 on unit 1 and day 38 on unit 5. For J, on the other hand, simple calculations have to be made to determine its finish times. As illustrated in Figure 5.11, J follows C and has a slower rate than C. Then, without doing any calculations for C, we sketch the duration of C as 2 days, then proceed with J at unit 1 with 6 days duration, then we draw the sloped line of J's finish times, from day 8 to day 32, which are smaller than those for activity G. As such, the start of activity K is bounded by day 22 on unit 1 and day 38 on unit 5.

Activity K has a higher progress rate than that of its boundary line, and as such, is expected to have a conflict point at top unit. Therefore, we start plotting that activity starting from the top unit and then subtract the slope of this activity to determine the start of unit 1 (lower point), as shown on Figure 5.10. Following that process, we can see that the project is planned to end at day 48, thus meeting the 50-day deadline. Also, after drawing the lines representing the activities on the LOB schedule, it is possible to show the activities' crew assignments and their movement along the different units. Activity B in Figure 5.10, for example, shows the work assignments for its two crews. Each crew can be given a different pattern or color to be easily identified.

Now consider the path connecting activities A-D-E-I and will use the same procedure described for drawing the LOB schedule of the critical path. Activities A, D and E are sequential, have no more than one predecessor, and have identical progress rates. As such, they can be easily plotted as shown in Figure 5.12. Activity I on the other hand has two predecessors, E and H. The finish line of activity E is shown in Figure 5.12 and spans from day 16 till day 32. The finish line of activity H, however, can be calculated considering the B-F-H path, as illustrated in Figure 5.13. Accordingly, the start of activity I is bounded by activity H (not activity E). Now, since the activity being considered (I) has a higher progress rate (0.333) than its boundary line (0.25), we need to draw the activity starting from the top point at day 44. The resulting LOB schedule of this path (Figure 5.12) extends the project duration till day 50, which still meets the scheduling goal. The significance of this change in project duration is that the critical path has also changed for unit 5 (all other units end before day 48). This change in the critical path is due to the relaxation of the slopes of non-critical activities and the rounding of the crew numbers. As demonstrated by this example, extra care has to be taken when drawing the LOB schedule to determine the planned project duration.

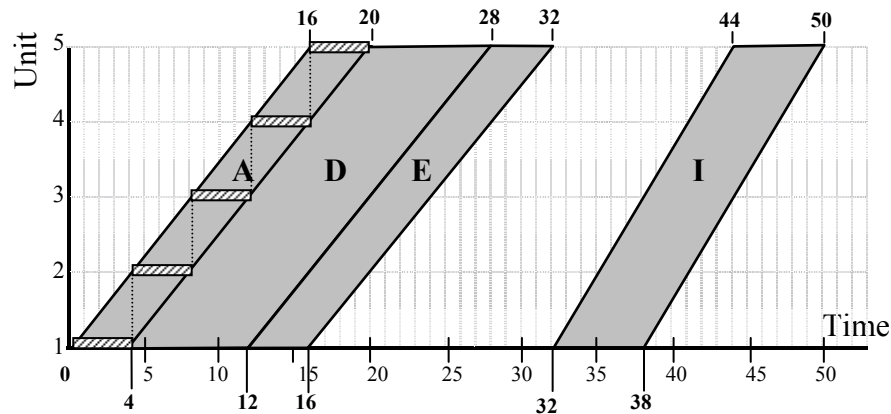


Figure 5.12: LOB schedule for path A-D-E-I

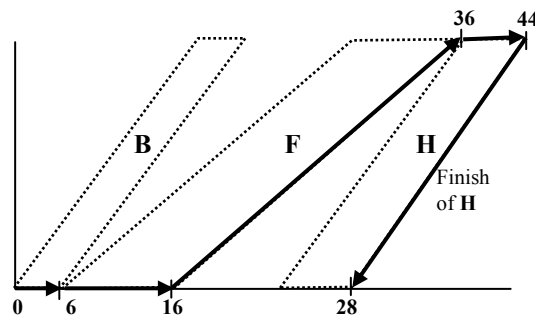


Figure 5.13: Determining the boundary line on activity I

### Example 5.1

The activities involved in the construction of one kilometer of a pipeline are given together with their estimated durations in the table below. The project consists of 10 similar kilometers.

Calculate the number of crews needed for each activity if the deadline for completing the project is 40 days and draw the LOB schedule. Assume one day buffer time between activities.

Table 5.2: Data for Example 5.1

Activity no.	Activity name	Duration (days)	Preceding activities
1	Locate and clear	1	-
2	Excavate	3	1
3	String pipe	1	1
4	Lay pipe	4	2,3
5	Pressure test	1	4
6	Backfill	2	5

### Solution

Figure 5.14 shows the CPM calculations for a single unit of the project. In this step, we determine the duration of a single unit and identify the critical path.

Note that the one day buffer time is set as a lag between activities.

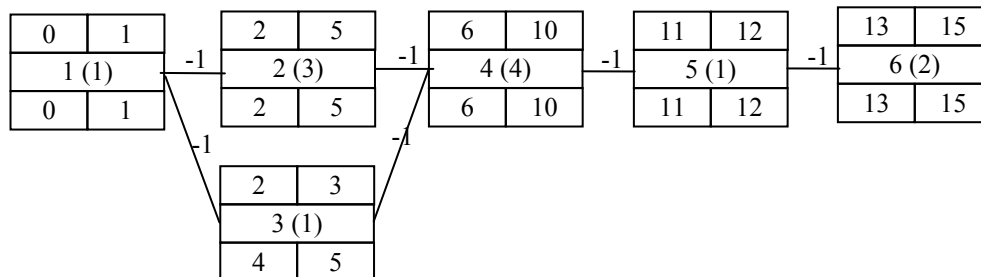


Figure 5.14: CPM calculations for Example 5.1

$$T_1 = 15 \text{ day} \quad T_L = 40 \text{ day} \quad N = 10 \text{ units}$$

$$R_i = (n-1) / T_L - T_1 + TF_i = 9 / (25 + TF_i)$$

Table 5.3: LOB calculations for Example 5.1

Activity	Duration $D_i$	Total Float	$R_i = 4 / (25+TF_i)$	$C_i = D_i \times R_i$	$C_{ai} =$ Round up $C_i$	$R_{ai} = C_{ai} / D_i$
1	1	0	0.36	0.36	1	1
2	3	0	0.36	1.08	2	0.667
3	1	2	0.333	0.333	1	1
4	4	0	0.36	1.44	2	0.5
5	1	0	0.36	0.36	1	1
6	2	0	0.36	0.72	1	0.5

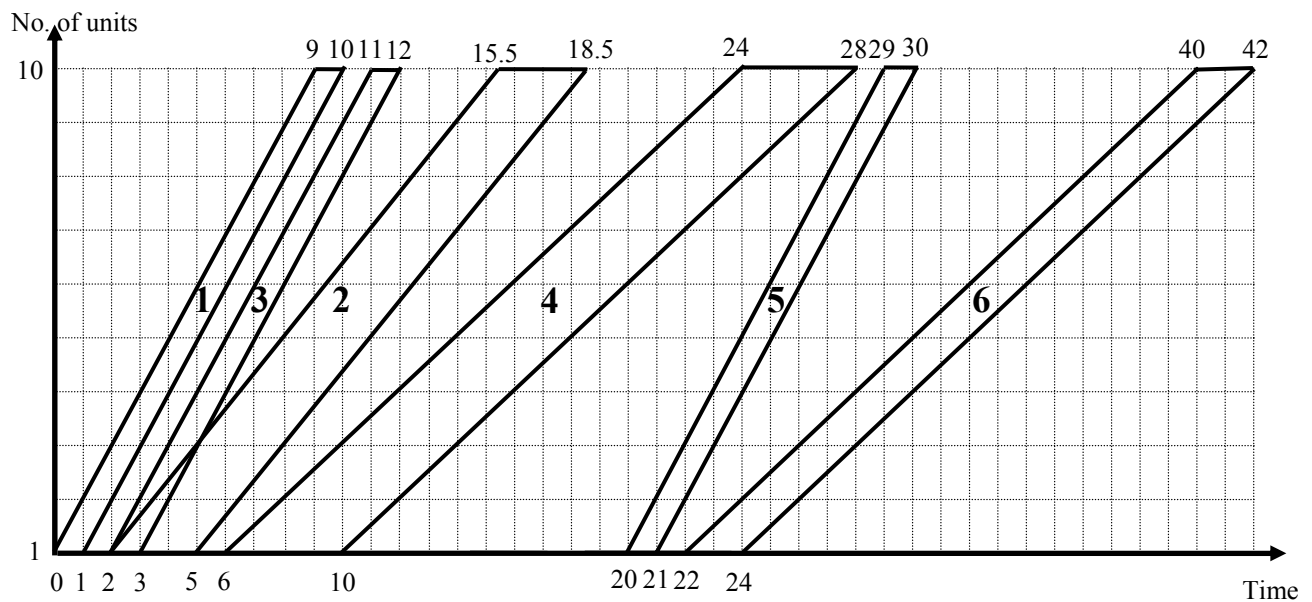


Figure 5.15: LOB for Example 5.1