HISTORICAL BACKGROUND ON MECHANICS OF MATERIALS

Dr. Osama Mohammed Elmardi Suleiman Khayal
Department of Mechanical Engineering, Faculty of Engineering and Technology, Nile Valley University, Atbara – Sudan
E – mail address: osamakhayal66@nilevalley.edu.sd

Abstract

Strength of materials, also called mechanics of materials, deals with the behavior of solid objects subject to stresses and strains. The complete theory began with the consideration of the behavior of one and two dimensional members of structures, whose states of stress can be approximated as two dimensional, and was then generalized to three dimensions to develop a more complete theory of the elastic and plastic behavior of materials. An important founding pioneer in mechanics of materials was Stephen Timoshenko. The study of strength of materials often refers to various methods of calculating the stresses and strains in structural members, such as beams, columns, and shafts. The methods employed to predict the response of a structure under loading and its susceptibility to various failure modes takes into account the properties of the materials such as its yield strength, ultimate strength, Young's modulus, and Poisson's ratio; in addition the mechanical element's macroscopic properties (geometric properties), such as its length, width, thickness, boundary constraints and abrupt changes in geometry such as holes are considered.

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1. Introduction

In mechanics of materials, the strength of a material is its ability to withstand an applied load without failure or plastic deformation. The field of strength of materials deals with forces and deformations that result from their acting on a material. A load applied to a mechanical member will induce internal forces within the member called stresses when those forces are expressed on a unit basis. The stresses acting on the material cause deformation of the material in various manners including breaking them completely. Deformation of the material is called strain when those deformations too are placed on a unit basis. The applied loads may be axial (tensile or compressive), or rotational (strength shear). The stresses and strains that develop within a mechanical member must be calculated in order to assess the load capacity of that member. This requires a complete description of the geometry of the member, its constraints, the loads applied to the member and the properties of the material of which the member is composed. With a complete description of the loading and the geometry of the member, the state of stress and state of strain at any point within the member can be calculated. Once the state of stress and strain within the member is known, the strength (load carrying capacity) of that member, its deformations (stiffness qualities), and its stability (ability to maintain its original configuration) can be calculated. The calculated stresses may then be compared to some measure of the strength of the member such as its material yield or ultimate strength. The calculated deflection of the member may be compared to a deflection criteria that is based on the member's use. The calculated buckling load of the member may be compared to the applied load. The calculated stiffness and mass distribution of the member may be used to calculate the member's dynamic response and then compared to the acoustic environment in which it will be used. Material strength refers to the point on the engineering stress–strain curve (yield stress) beyond which the material experiences deformations that will not be completely reversed upon removal of the loading and as a result the member will have a permanent deflection. The ultimate strength of the material refers to the maximum value of stress reached. The fracture strength is the stress value at fracture (the last stress value recorded).

Mechanics of materials or solid mechanics was developed in the outpouring of mathematical and physical studies following the great achievement of Isaac Newton (1642-1727) in stating the laws of motion, although it has earlier roots. The need to understand and control the fracture of solids seems to have been a first motivation. Leonardo da Vinci (1452-1519) sketched in his notebooks a possible test of the tensile strength of a wire. The Italian experimental scientist Galileo Galilei

(1564-1642), who died in the year of Newton's birth, had investigated the breaking loads of rods in tension and concluded that the load was independent of length and proportional to the cross section area, this being a first step towards a concept of stress. He also investigated how the breaking of heavy stone columns, laid horizontally in storage as beams, depended on the number and condition of their supports.

2. Concepts of Stress, Strain and Elasticity

The English scientist Robert Hooke discovered in 1660, but published only in 1678, the observation that for many materials that displacement under a load was proportional to force, thus establishing the notion of (linear) elasticity but not yet in a way that was expressible in terms of stress and strain. E. Mariotte in France published similar discoveries in 1680 and, also, reached an understanding of how beams like those studied by Galileo resisted transverse loadings or, more precisely, resist the torques caused by those transverse loadings, by developing extensional and compressional deformations, respectively, in material fibers along their upper and lower portions. It was for Swiss mathematician and mechanical engineer James Bernoulli (1654-1705) to observe, in the final paper of his life, in 1705, that the proper way of describing deformation was to give force per unit area, or stress, as a function of the elongation per unit length, or strain, of a material fiber under tension. Swiss mathematician and mechanical engineer Leonhard Euler (1707-1783), who was taught mathematics by James' brother John Bernoulli (1667-1748), proposed, among many contributions, a linear relation between stress ε and strain σ in 1727, of form $\sigma = E\ \varepsilon$ where the coefficient E is now generally called Young's modulus after English naturalist Thomas Young who developed a related idea in 1807.

The notion that there is an internal tension acting across surfaces in a deformed solid was expressed by German mathematician and physicist Gottfried Wilhelm Leibniz in 1684 and James Bernoulli in 1691. Also, Bernoulli and Euler (see below) introduced the idea that at a given section along the length of a beam there were internal tensions amounting to a net force and a net torque. Euler introduced the idea of compressive normal stress as the pressure in a fluid in 1752. The French engineer and physicist Charles-Augustine Coulomb (1736-1806) was apparently the first to relate the theory of a beam as a bent elastic line to stress and strain in an actual beam, in a way never quite achieved by Bernoulli and, although possibly recognized, never published by Euler. He developed the famous expression $\aleph = M\ y\ /\ I$ for the stress due to the pure bending of a homogeneous linear elastic beam; here M is the torque, or bending moment, y is the distance of a point from an axis that passes through the section centroid, parallel to the torque axis, and I is the integral of $y2$ over the section area. The French mathematician Parent introduced the concept of shear stress in 1713, but Coulomb was the one who extensively developed the idea in connection with beams and with the stressing and failure of soil in 1773, and studies of frictional slip in 1779. It was the great French mathematician Augustin Louis Cauchy (1789-1857), originally educated as an engineer, who in 1822 formalized the stress concept in the context of a general three-dimensional theory, showed its properties as consisting of a 3 by 3 symmetric array of numbers that transform as a tensor, derived the equations of motion for a continuum in terms of the components of stress, and gave the specific development of the theory of linear elastic response for isotropic solids. As part of this work, Cauchy also introduced the equations which express the six components of strain, three extensional and three shear, in terms of derivatives of displacements for the case when all those derivatives are much smaller than unity; similar expressions had been given earlier by Euler in expressing rates of straining in terms of the derivatives of the velocity field in a fluid.

3. Beams, Columns, Plates and Shells

The 1700's and early 1800's were a productive period in which the mechanics of simple elastic structural elements were developed well before the beginnings in the 1820's of the general three-dimensional theory. The development of beam theory by Euler, who generally modeled beams as elastic lines which resist bending, and by several members of the Bernoulli family and by Coulomb, remains among the most immediately useful aspects of solid mechanics, in part for its simplicity and in part because of the pervasiveness of beams and columns in structural technology. James Bernoulli proposed in his final paper of 1705 that the curvature of a beam was proportional to bending moment. Euler in 1744 and John's son, Daniel Bernoulli (1700-1782) in 1751 used the theory to address the transverse vibrations of beams, and Euler gave in 1757 his famous analysis of the buckling of an initially straight beam subjected to a compressive loading; the beam is then commonly called a column. Following a suggestion of Daniel Bernoulli in 1742, Euler in 1744 introduced the strain energy per unit length for a beam, proportional to the square of its curvature, and regarded the total strain energy as the quantity analogous to the potential energy of a discrete mechanical system. By adopting procedures that were becoming familiar in analytical mechanics, and following from the principle of virtual work as introduced by John Bernoulli for discrete systems such as pin-connected rigid bodies in 1717, Euler rendered the energy stationary and in this way developed the calculus of variations as an approach to the equations of equilibrium and motion of elastic structures.

That same variation approach played a major role in the development by French mathematicians in the early 1800's of a theory of small transverse displacements and vibrations of elastic plates. This theory was developed in preliminary form by Sophie Germain and partly improved upon by Simeon Denis Poisson in the early 1810's; they considered a flat plate as an elastic plane which resists curvature. Navier gave a definitive development of the correct energy expression and governing differential equation a few years later. An uncertainty of some duration in the theory arose from the fact that the final partial differential equation for the transverse displacement is such that it is impossible to prescribe, simultaneously, along an unsupported edge of the plate, both the twisting moment per unit length of middle surface and the transverse shear force per unit length. This was finally resolved in 1850 by German physicist Gustav Robert Kirchhoff in an application of virtual work and variation calculus procedures, in the framework of simplifying kinematic assumptions that fibers initially perpendicular to the plate middle surface remain so after deformation of that surface. As first steps in the theory of thin shells, in the 1770's Euler addressed the deformation of an initially curved beam, as an elastic line, and provided a simplified analysis of vibration of an elastic bell as an array of annular beams. John's grandson, through a son and mathematician also named John, James Bernoulli "the younger" (1759-1789) further developed this model in the last year of his life as a two dimensional network of elastic lines, but could not develop an acceptable treatment. Shell theory was not to attract attention for a century after Euler's work, as the outcome of many researches following the first consideration of shells from a three-dimensional elastic viewpoint by H. Aron in 1873. Acceptable thin-shell theories for general situations, appropriate for cases of small deformation, were developed by English mathematician, mechanical engineer and geophysicist A. E. H. Love [1] in 1888 and mathematician and physicist Horace Lamb in 1890 (there is no uniquely correct theory as the Dutch applied mechanical engineer and engineer W. T. Koiter and Russian mechanical engineer V. V. Novozhilov were to clarify in the 1950's; the difference between predictions of acceptable theories is small when the ratio of shell thickness to a typical length scale is small). Shell theory remained of immense interest well beyond the mid-1900's in part because so many problems lay beyond the linear theory (rather small transverse displacements often dramatically alter the way that a shell supports load by a combination of bending and membrane action), and in part because of the interest in such light-weight structural forms for aeronautical technology.

4. General Theory of Elasticity

Linear elasticity as a general three-dimensional theory was in hand in the early 1820's based on Cauchy's work. Simultaneously, Navier had developed an elasticity theory based on a simple corpuscular, or particle, model of matter in which particles interacted with their neighbors by a central-force attractions between particle pairs. As was gradually realized following works by Navier, Cauchy and Poisson in the 1820's and 1830's, the particle model is too simple and makes predictions concerning relations among elastic moduli which are not met by experiment. In the isotropic case it predicts that there is only one elastic constant and that the Poisson ratio has the universal value of 1/4. Most subsequent development of the subject was in terms of the continuum theory. Controversies concerning the maximum possible number of independent elastic moduli in the most general anisotropic solid were settled by English mathematician George Green [2] in 1837, through pointing out that the existence of an elastic strain energy required that of the 36 elastic constants, relating the six stress components to the six strains, at most 21 could be independent. Scottish physicist Lord Kelvin (William Thomson) put this consideration on sounder ground in 1855 as part of his development of macroscopic thermodynamics, in much the form as it is known today, showing that a strain energy function must exist for reversible isothermal or adiabatic (isentropic) response, and working out such results as the (very modest) temperature changes associated with isentropic elastic deformation.

The middle and late 1800's were a period in which many basic elastic solutions were derived and applied to technology and to the explanation of natural phenomena. French mathematician Barre de Saint-Venant derived in the 1850's solutions for the torsion of non-circular cylinders, which explained the necessity of warping displacement of the cross section in the direction parallel to the axis of twisting, and for flexure of beams due to transverse loadings; the latter allowed understanding of approximations inherent in the simple beam theory of Bernoulli, Euler and Coulomb. The German physicist Heinrich Rudolph Hertz developed solutions for the deformation of elastic solids as they are brought into contact, and applied these to model details of impact collisions. Solutions for stress and displacement due to concentrated forces acting at an interior point of a full space were derived by Kelvin, and on the surface of a half space by mathematicians J. V. Bousinesq (French) and V. Cerruti (Italian). The Prussian mathematician L. Pochhammer analyzed the vibrations of an elastic cylinder and Lamb and the Prussian physicist P. Jaerisch derived the equations of general vibration of an elastic sphere in the 1880's, an effort that was continued by many seismologists in the 1900's to describe the vibrations of the Earth. Kelvin derived in 1863 the basic form of the solution of the static elasticity equations for a spherical solid, and these were applied in following years to such problems as deformation of the Earth due to rotation and to tidal forcing, and to effects of elastic deformability on the motions of the Earth's rotation axis.

The classical development of elasticity never fully confronted the problem of finite elastic straining, in which material fibers change their lengths by other than very small amounts. Possibly this was because the common materials of construction would remain elastic only for very small strains before exhibiting either plastic straining or brittle failure. However, natural polymeric materials show elasticity over a far wider range (usually also with enough time or rate effects that they would more accurately be characterized as viscoelastic), and the widespread use of natural rubber and like materials motivated the development of finite elasticity. While many roots of the subject were laid in the classical theory, especially in the work of Green [2], G. Piola and Kirchhoff in the mid-1800's, the development of a viable theory with forms of stress-strain relations for specific rubbery elastic materials, and an understanding of the physical effects of the nonlinearity in simple problems like torsion and bending, is mainly the achievement of British - American engineer and applied mathematician Ronald S. Rivlin in the 1940's and 1950's.

5. Waves

Poisson, Cauchy and George G. Stokes showed that the equations of the theory predicted the existence of two types of elastic deformation waves which could propagate through isotropic elastic solids. These are called body waves. In the faster type, called longitudinal waves, the particle motion is in the same direction as that of wave propagation; in the slower, called transverse, or shear, or rotational waves, it is perpendicular to the propagation direction. No analog of the shear wave exists for propagation through a fluid medium, and that fact led seismologists in the early 1900's to understand that the Earth has a liquid core (at the center of which there is a solid inner core).

Lord Rayleigh (John Strutt) showed in 1887 that there is a wave type that could propagate along surfaces, such that the motion associated with the wave decayed exponentially with distance into the material from the surface. This type of surface wave, now called a Rayleigh wave, propagates typically at slightly more than 90% of the shear wave speed, and involves an elliptical path of particle motion that lies in planes parallel to that defined by the normal to the surface and the propagation direction. Another type of surface wave, with motion transverse to the propagation direction and parallel to the surface, was found by Love [1] for solids in which a surface layer of material sits atop an elastically stiffer bulk solid; this defines the situation for the Earth's crust. The shaking in an earthquake is communicated first to distant places by body waves, but these spread out in three-dimensions and must diminish in their displacement amplitude as r^{-1}, where r is distance from the source, to conserve the energy propagated by the wave field. The surface waves spread out in only two dimensions and must, for the same reason, diminish only as fast as $r^{-1/2}$. Thus shaking in surface waves is normally the more sensed, and potentially damaging, effect at moderate to large distances from a crustal earthquake. Indeed, well before the theory of waves in solids was in hand, Thomas Young had suggested in his 1807 Lectures on Natural Philosophy that the shaking of an earthquake "is probably propagated through the earth in the same manner as noise is conveyed through air". (It had been suggested by American mathematician and astronomer Jonathan Winthrop, following his experience of the "Boston" earthquake of 1755, that the ground shaking was due to a disturbance propagated like sound through the air).

With the development of ultrasonic transducers operated on piezoelectric principles, the measurement of the reflection and scattering of elastic waves has developed into an effective engineering technique for the non-destructive evaluation of materials for detection of potentially dangerous defects such as cracks. Also, very strong impacts, whether from meteorite collision, weaponry, or blasting and the like in technological endeavors, induce waves in which material response can be well outside the range of linear elasticity, involving any or all of finite elastic strain, plastic or viscoplastic response, and phase transformation. These are called shock waves; they can propagate much beyond the speed of linear elastic waves and are accompanied with significant heating.

6. Stress concentrations and fracture

In 1898 G. Kirsch derived the solution for the stress distribution around a circular hole in a much larger plate under remotely uniform tensile stress. The same solution can be adapted to the tunnel-like cylindrical cavity of circular section in a bulk solid. His solution showed a significant concentration of stress at the boundary, by a factor of three when the remote stress was uniaxial tension. Then in 1907 the Russian mathematician G. Kolosov, and independently in 1914 the British engineer Charles Inglis, derived the analogous solution for stresses around an elliptical hole. Their solution showed that the concentration of stress could become far greater as the radius of curvature at an end of the hole becomes small compared to the overall length of the hole. These results provided the insight to sensitize engineers to the possibility of dangerous stress concentrations at, for example, sharp re-entrant corners, notches, cut-outs, keyways, screw threads, and the like in structures for which the nominal stresses were at otherwise safe levels. Such stress concentration sites are places from which a crack can nucleate.

The elliptical hole of Kolosov and Inglis defines a crack in the limit when one semi-axis goes to zero, and the Inglis solution was adopted in that way by British aeronautical engineer A. A. Griffith in 1921 to describe a crack in a brittle solid. In that work Griffith made his famous proposition that spontaneous crack growth would occur when the energy released from the elastic field just balanced the work required to separate surfaces in the solid. Following a hesitant beginning, in which Griffith's work was initially regarded as important only for very brittle solids such as glass, there developed, largely under the impetus of American engineer and physicist George R. Irwin, a major body of work on the mechanics of crack growth and fracture, including fracture by fatigue and stress corrosion cracking, starting in the late 1940's and continuing into the 1990's. This was driven initially by the cracking of American fleet of Liberty ships during the Second World War, by the failures of the British Comet airplane, and by a host of reliability and safety issues arising in aerospace and nuclear reactor technology. The new complexion of the subject extended beyond the Griffith energy theory and, in its simplest and most widely employed version in engineering practice, used Irwin's stress intensity factor as the basis for predicting crack growth response under service loadings in terms of laboratory data that is correlated in terms of that factor. That stress intensity factor is the coefficient of a characteristic singularity in the linear elastic solution for the stress field near a crack tip, and is recognized as providing a proper characterization of crack tip stressing in many cases, even though the linear elastic solution must be wrong in detail near the crack tip due to non-elastic material response, large strain, and discreteness of material microstructure [3] – [7].

7. Dislocations

The Italian elasticity scientist and mathematician V. Volterra introduced in 1905 the theory of the elastostatic stress and displacement fields created by dislocating solids. This involves making a cut in a solid, displacing its surfaces relative to one another by some fixed amount, and joining the sides of the cut back together, filling in with material as necessary. The initial status of this work was simply as an interesting way of generating elastic fields but, in the early 1930's, Geoffrey Ingram Taylor, Egon Orowan and Michael Polanyi realized that just such a process could be going on in ductile crystals and could provide an explanation of the low plastic shear strength of typical ductile solids, much like Griffith's cracks explained low fracture strength under tension. In this case the displacement on the dislocated surface corresponds to one atomic lattice spacing in the crystal. It quickly became clear that this concept provided the correct microscopic description of metal plasticity and, starting with Taylor in the 1930's and continuing into the 1990's, the use of solid mechanics to explore dislocation interactions and the microscopic basis of plastic flow in crystalline materials has been a major topic, with many distinguished contributors.

The mathematical techniques advanced by Volterra are now in common use by Earth scientists in explaining ground displacement and deformation induced by tectonic faulting. Also, the first elastodynamic solutions for the rapid motion of a crystal dislocations by South African materials scientist F. R. N. Nabarro, in the early 1950's, were quickly adapted by seismologists to explain the radiation from propagating slip distributions on faults. Japanese seismologist H. Nakano had already shown in 1923 how to represent the distant waves radiated by an earthquake as the elastodynamic response to a pair of force dipoles amounting to zero net torque. (All of his manuscripts were destroyed in the fire in Tokyo associated with the great Kwanto earthquake in that same year, but some of his manuscripts had been sent to Western colleagues and the work survived).

8. Continuum Plasticity Theory

The macroscopic theory of plastic flow has a history nearly as old as that of elasticity. While in the microscopic theory of materials, the word "plasticity" is usually interpreted as denoting deformation by dislocation processes, in macroscopic continuum mechanics it is taken to denote any type of permanent deformation of materials, especially those of a type for which time or rate of deformation effects are not the most dominant feature of the phenomenon (the terms viscoplasticity or creep or viscoelasticity are usually used in such cases). Coulomb's work of 1773 on the frictional yielding of soils under shear and normal stress has been mentioned; yielding denotes the occurrence of large shear deformations without significant increase in applied stress. This work found applications to explaining the pressure of soils against retaining walls and footings in work of the French mathematician and engineer J. V. Poncelot in 1840 and Scottish engineer and physicist W. J. M. Rankine in 1853. The inelastic deformation of soils and rocks often takes place in situations for which the deforming mass is infiltrated by groundwater, and Austrian-American civil engineer Karl Terzaghi in the 1920's developed the concept of effective stress, whereby the stresses which enter a criterion of yielding or failure are not the total stresses applied to the saturated soil or rock mass, but rather the effective stresses, which are the difference between the total stresses and those of a purely hydrostatic stress state with pressure equal to that in the pore fluid. Terzaghi also introduced the concept of consolidation, in which the compression of a fluid-saturated soil can take place only as the fluid slowly flows through the pore space under pressure gradients, according to the law of Darcy; this effect accounts for the time-dependent settlement of constructions over clay soils.

Apart from the earlier observation of plastic flow at large stresses in the tensile testing of bars, the continuum plasticity of metallic materials begins with Henri Edouard Tresca in 1864. His experiments on the compression and indentation of metals led him to propose that this type of plasticity, in contrast to that in soils, was essentially independent of the average normal stress in the material and dependent only on shear stresses, a feature later rationalized by the dislocation mechanism. Tresca proposed a yield criterion for macroscopically isotropic metal polycrystals based on the maximum shear stress in the material, and that was used by Saint- Venant to solve a first elastic-plastic problem, that of the partly plastic cylinder in torsion, and also to solve for the stresses in a completely plastic tube under pressure. German applied mechanical engineer Ludwig Prandtl developed the rudiments of the theory of plane plastic flow in 1920 and 1921, with an analysis of indentation of a ductile solid by a flat-ended rigid indenter, and the resulting theory of plastic slip lines was completed by H. Hencky in 1923 and Hilda Geiringer in 1930. Additional developments include the methods of plastic limit analysis, which allowed engineers to directly calculate upper and lower bounds to the plastic collapse loads of structures or to forces required in metal forming. Those methods developed gradually over the early 1900’s on a largely intuitive basis, first for simple beam structures and later for plates, and were put on a rigorous basis within the rapidly developing mathematical theory of plasticity around 1950 by Daniel C. Drucker and William Prager [8] in the United States and Rodney Hill [9] in England.

German applied mathematician Richard von Mises proposed in 1913 that a mathematically simpler theory of plasticity than that based on the Tresca yield criterion could be based on the second tensor invariant of the deviator stresses (that is, of the total stresses minus those of a hydrostatic state with pressure equal to the average normal stress over all planes). An equivalent yield condition had been proposed independently by Polish engineer M.-T. Huber. The Mises theory incorporates a proposal by M. Levy in 1871 that components of the plastic strain increment tensor are in proportion to one another just as are the components of deviator stress. This criterion was found to generally provide slightly better agreement with experiment than did that of Tresca, and most work on the application of plasticity theory uses this form. Following a suggestion of Prandtl, E. Reuss completed the theory in 1930 by adding an elastic component of strain increments, related to stress increments in the same way as for linear elastic response. This formulation was soon generalized to include strain hardening, whereby the value of the second invariant for continued yielding increases with ongoing plastic deformation, and was extended to high-temperature creep response in metals or other hot solids by assuming that the second invariant of the plastic (now generally called “creep”) strain rate is a function of that same invariant of the deviator stress, typically a power law type with Arrhenius temperature dependence. This formulation of plastic and viscoplastic or creep response has been applied to all manner of problems in materials and structural technology and in flow of geological masses. Representative problems addressed include the large growth to coalescence of microscopic voids in the ductile fracture of metals, the theory of the indentation hardness test, the extrusion of metal rods and rolling of metal sheets, the auto-frettage of gun tubes, design against collapse of ductile steel structures, estimation the thickness of the Greenland ice sheet, and modeling the geologic evolution of the Tibetan plateau. Other types of elastic-plastic theories intended for analysis of ductile single crystals originate from the work of G. I. Taylor and Hill [9], and bases the criterion for yielding on E. Schmid’s concept from the 1920’s of a critical resolved shear stress along a crystal slip plane, in the direction of an allowed slip on that plane; this sort of yield condition has approximate support from the dislocation theory of plasticity.

9. Viscoelasticity

The German physicist Wilhelm Weber noticed in 1835 that a load applied to a silk thread produced not only an immediate extension but also a continuing elongation of the thread with time. This type of viscoelastic response is especially notable in polymeric solids but is present to some extent in all types of solids and often does not have a clear separation from what could be called viscoplastic or creep response. In general, if all the strain is ultimately recovered when a load is removed from a body, the response is termed viscoelastic, but the term is also used in cases for which sustained loading leads to strains which are not fully recovered. The Austrian physicist Ludwig Boltzmann developed in 1874 the theory of linear viscoelastic stress-strain relations. In their most general form these involve the notion that a step loading (suddenly imposed stress, subsequently maintained constant) causes an immediate strain followed by a time-dependent strain which, for different materials, may either have a finite long time limit or may increase indefinitely with time. Within the assumption of linearity, the strain at time t in response to a general time dependent stress history

$\sigma(t)$ can then be written as the sum (or integral) of terms that involve the step-loading strain response at time t-t' due to a step loading dt' $d\sigma(t')/dt'$ at time t'. The theory of viscoelasticity is important for consideration of the attenuation of stress waves and the damping of vibrations.

A new class of problems arose with the mechanics of very long molecule polymers, without significant cross-linking, existing either in solution or as a melt. These are fluids in the sense that they cannot long support shear stress but have, at the same time, remarkable properties like those of finitely deformed elastic solids. A famous demonstration is to pour one of these fluids slowly from a bottle and to suddenly cut the flowing stream with scissors; if the cut is not too far below the place of exit from the bottle, the stream of falling fluid immediately contracts elastically and returns to the bottle. The molecules are elongated during flow but tend to return to their thermodynamically preferred coiled configuration when forces are removed. The theory of such materials came under intense development in the 1950’s after British applied mathematician James Gardner Oldroyd showed in 1950 how viscoelastic stress-strain relations of a memory type could be generalized to a flowing fluid. This involves subtle issues on assuring that the constitutive relation, or rheological relation, between the stress history and the deformation history at a material “point” is properly invariant to a superposed history of rigid rotation , which should not affect the local physics determining that relation (the resulting Coriolis and centrifugal effects are quite negligible at the scale of molecular interactions). Important contributions on this issue were made by S. Zaremba and G. Jaumann in the first decade of the 1900’s; they showed how to make tensor definitions of stress rate that were invariant to superposed spin and thus were suitable for use in constitutive relations. But it was only during the 1950’s that these concepts found their way into the theory of constitutive relations for general viscoelastic materials and, independently and a few years later, properly invariant stress rates were adopted in continuum formulations of elastic-plastic response.

10. Computational mechanics

The digital computer revolutionized the practice of many areas of engineering and science, and solid mechanics was among the first fields to benefit from its impact. Many computational techniques have been used in that field, but the one which emerged by the end of the 1970’s as, by far, the most widely adopted is the finite element method. This method was outlined by the mathematician Richard Courant in 1943 and was developed independently, and put to practical use on computers, in the mid-1950’s by aeronautical structures engineers M. J. Turner, R. W. Clough, H. C. Martin and L. J. Topp in the United States and by J. H. Argyris and S. Kelsey in Britain. Their work grew out of earlier attempts at systematic structural analysis for complex frameworks of beam elements. The method was soon recast in a variation framework and related to earlier efforts at approximate solution of problems described by variation principles, by substituting trial functions of unknown amplitude into the variation functional which is then rendered stationary as an algebraic function of the amplitude coefficients. In the most common version of the finite element method, the domain to be analyzed is divided into cells, or elements, and the displacement field within each element is interpolated in terms of displacements at a few points around the element boundary, and sometimes within it, called nodes. The interpolation is done so that the displacement field is continuous across element boundaries for any choice of the nodal displacements. The strain at every point can thus be expressed in terms of nodal displacements, and it is then required that the stresses associated with these strains, through the stress-strain relations of the material, satisfy the principle of virtual work for arbitrary variation of the nodal displacements. This generates as many simultaneous equations as there are degrees of freedom in the finite element model, and numerical techniques for solving such systems of equations are programmed for computer solution. The finite element method and other computational techniques (finite differences, spectral expansions, boundary integral equations) have made a major change in the practice of, and education for, engineering in the various areas that draw on solid mechanics. Previously, many educators saw little point in teaching engineers much of the subject beyond the techniques of elementary beam theory developed in the 1700’s by Bernoulli, Euler and Coulomb. More advanced analyses involved sufficiently difficult mathematics as to be beyond the reach of the typical practitioner, and were regarded as the domain of advanced specialists who would, themselves, find all but the simpler cases intractable. The availability of software incorporating the finite element method, and other procedures of computational mechanics and design analysis, has placed the advanced concepts of solid mechanics into the hands of a far broader community of engineers. At the same time, it has created a necessity for them and other users to have a much deeper education in the subject, so that the computational tools are used properly and at full effectiveness.

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Author

Osama Mohammed Elmardi Suleiman Khayal was born in Atbara, Sudan in 1966. He received his diploma degree in mechanical engineering from Mechanical Engineering College, Atbara, Sudan in 1990. He also received a bachelor degree in mechanical engineering from Sudan University of science and technology – Faculty of engineering in 1998, and a master degree in solid mechanics from Nile valley university (Atbara, Sudan) in 2003, and a PhD in structural engineering in 2017. He contributed in teaching some subjects in other universities such as Red Sea University (Port Sudan, Sudan), Kordofan University (Obayed, Sudan), Sudan University of Science and Technology (Khartoum, Sudan), Blue Nile University (Damazin, Sudan) and Kassala University (Kassala, Sudan). In addition, he supervised more than hundred and fifty under graduate studies in diploma and B.Sc. levels and about fifteen master theses. The author wrote about forty engineering books written in Arabic language, and fifteen books written in English language and more than hundred research papers in fluid mechanics, thermodynamics, internal combustion engines and analysis of composite structures. He authored more than thousands of lectures notes in the fields of mechanical, production and civil engineering He is currently an associated professor in Department of Mechanical Engineering, Faculty of Engineering and Technology, Nile Valley University Atbara, Sudan. His research interest and favorite subjects include structural mechanics, applied mechanics, control engineering and instrumentation, computer aided design, design of mechanical elements, fluid mechanics and dynamics, heat and mass transfer and hydraulic machinery. The author also works as a technical manager and superintendent of Al – Kamali mechanical and production workshops group which specializes in small, medium and large automotive overhaul maintenance and which situated in Atbara town in the north part of Sudan, River Nile State.